The coordination of centralized and distributed electricity generation

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FiME seminar

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Introduction

1. Three optimization problems

1.1. The consumer

1.2 The energy company

1.3 The social planner

2. Looking for an equilibrium

3. Generalizing the model

Conclusions

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Electricité : EDF lance une offre d'autoconsommation

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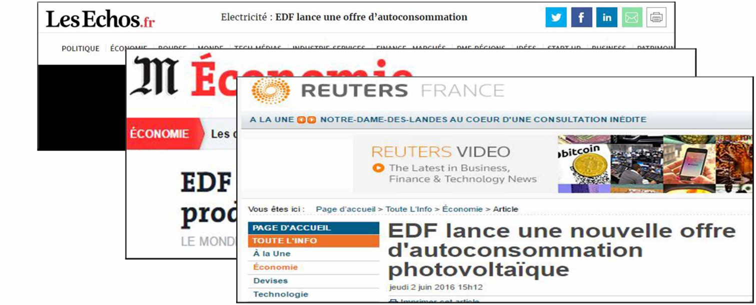

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Our goals. Solution to the three problems above? Do the planner's suggestions coincide with the consumer/company's choices?
Framework: McKean-Vlasov stochastic control problems.

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- If D is the consumer's electricity demand (constant), $D - X_t^\alpha$ is the amount of electricity still needed and bought in the market, at price P_t ($\mathcal{F}^{W^0}$-adapted process, $W^0 \perp\!\!\!\perp W$). Important: no model on P.

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- As the consumer wants a stable production of energy from solar panels, the variance of the production $\mathrm{Var}[X_t^\alpha]$ is penalized.

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$$\inf_{\alpha} \mathbb{E}\left[\int_0^{\infty} e^{-\rho t}\Big(c\alpha_t + \gamma\alpha_t^2 + P_t\big(D - X_t^{\alpha}\big) + \eta \mathrm{Var}[X_t^{\alpha}]\Big)\, dt\right]$$

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So, the consumer has to solve

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$$dX_t^{\alpha} = b\alpha_t dt + \sigma X_t^{\alpha} dW_t, \qquad P \text{ stochastic.}$$

Consumer: cost function. Recall that α_t is the installation rate in t (how many panels the consumer buys/sells in t). Let $c(\alpha_t)$ be the corresponding installation/dismiss costs. Which model for $c(\cdot)$?

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- For $\alpha < 0$ big, the consumer is trying to suddenly sell a large amount of panels, which is practically impossible, so that he actually loses money; hence, we ask $c(\alpha) > 0$ in $]-\infty, -\bar{\alpha}[$.

To sum up, we want the cost function c to be:

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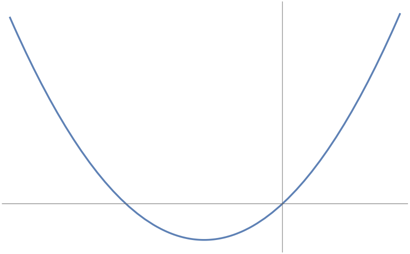
The simplest function with all these properties is $c(\alpha) = c\alpha + \gamma\alpha^2$.



Consumer: SDE. The SDE for X_t^α is $dX_t^\alpha = b\alpha_t + \sigma X_t^\alpha dW_t$. The noise term is $\sigma X_t^\alpha dW_t$ and not $\sigma \alpha_t dW_t$: why?

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Consumer: Brownian motions. The production depends on W, the market price depends on W^0. We assume $W \perp\!\!\!\perp W^0$: why?

The production basically depends on the weather. Conversely, as we consider a big international company, the price is not influenced by local issues (like today's weather) but only by wider elements (fuels, status of power plants,...). So, the noises are independent.

Consumer: how to solve the problem. Recall the problem:

$$V_0 = \inf_{\alpha} \mathbb{E}\left[\int_0^{\infty} e^{-\rho t}\Big(c\alpha_t + \gamma\alpha_t^2 + P_t(D - X_t^{\alpha}) + \eta \mathrm{Var}[X_t^{\alpha}]\Big)dt\right].$$

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How to solve the problem? To characterize the optimal control, we use the following formulation of the verification theorem.

Statement. Let $\{w_t^\alpha\}_{\alpha,t}$ be a family of processes in the form $w_t^\alpha = w_t(X_t^\alpha, \mathbb{E}[X_t^\alpha])$ and such that:

- $\mathbb{E}\left[e^{-\rho T} w_T^\alpha\right] \to 0$ as $T \to \infty$, for each α;
- $t \mapsto \mathbb{E}\left[e^{-\rho t} w_t^\alpha + \int_0^t e^{-\rho s}\left(c\alpha_s + \gamma\alpha_s^2 - P_s(D - X_s^\alpha) + \eta \mathrm{Var}[X_s^\alpha]\right) ds\right]$ is increasing for each α and constant for some $\alpha = \hat{\alpha}$.

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Idea behind. As the expectation above is increasing, we have

$$w_0 \le \mathbb{E}\left[e^{-\rho t} w_t^\alpha + \int_0^t e^{-\rho s}\big(c\alpha_s + \gamma\alpha_s^2 - P_s(D - X_s^\alpha) + \eta \mathrm{Var}[X_s^\alpha]\big)\,ds\right],$$

which leads ($t \to \infty$) to $w_0 \le J(\alpha)$, and then $w_0 \le V_0$. Similarly, for $\hat{\alpha}$ we get $w_0 = J(\hat{\alpha})$ and then $w_0 \ge V_0$. Finally, $w_0 = V_0 = J(\hat{\alpha})$.

Strategy. The key-point of this approach is to prove that $t \mapsto \mathbb{E}\left[e^{-\rho t} w_t^\alpha + \int_0^t e^{-\rho s}\big(c\alpha_s+\gamma\alpha_s^2 - P_s(D-X_s^\alpha)+\eta \mathrm{Var}[X_s^\alpha]\big)ds\right]$ is increasing/constant. Our strategy is as follows.

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- Step 3. We impose that $\mathbb{E}[\mathcal{D}_t^\alpha]$ is positive/zero, since we have
$$\mathbb{E}[S_t^\alpha] \text{ is increasing/constant} \iff \mathbb{E}[\mathcal{D}_t^\alpha] \text{ is positive/zero.}$$

Step 1. As we deal with a LQ problem, we guess a quadratic form for $w_t^\alpha = w_t(X_t^\alpha, \mathbb{E}[X_t^\alpha])$, with stochastic coefficients:

$$w_t^\alpha = K_t(X_t^\alpha - \mathbb{E}[X_t^\alpha])^2 + \Lambda_t \mathbb{E}[X_t^\alpha]^2 + Y_t(X_t^\alpha - \mathbb{E}[X_t^\alpha]) + \Gamma_t \mathbb{E}[X_t^\alpha] + R_t,$$

where we assume $d\xi_t = \dot{\xi}_t dt + \hat{\xi}_t dW_t^0$, for $\xi \in \{K, \Lambda, Y, \Gamma, R\}$.
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Recall the goal: we want $\mathbb{E}[\mathcal{D}_t^\alpha]$ to be positive for each α; in this form, it is complicated... Idea: completing the square.

Step 3. By completing the square we get, for explicit functions ξ_i,

$$\mathbb{E}[\mathcal{D}_t^\alpha] = \mathbb{E}\Big[\gamma\Big(\alpha_t + \xi_0(X_t^\alpha, K_t, \Lambda_t, Y_t, \Gamma_t)\Big)^2 + \xi_1(K_t)(X_t^\alpha - \mathbb{E}[X_t^\alpha])^2 + \xi_2(K_t, \Lambda_t)\mathbb{E}[X_t^\alpha]^2$$
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As we want $\mathbb{E}[\mathcal{D}_t^\alpha] \geq 0$ for each α, we set the coefficients $\xi_1, \ldots, \xi_5$ to be identically zero. This corresponds to a system of conditions which completely characterizes the coefficients $K_t, \Lambda_t, Y_t, \Gamma_t, R_t$.

Step 3. By completing the square we get, for explicit functions ξ_i,

$$\mathbb{E}[\mathcal{D}_t^\alpha] = \mathbb{E}\Big[\gamma\Big(\alpha_t + \xi_0(X_t^\alpha, K_t, \Lambda_t, Y_t, \Gamma_t)\Big)^2 + \xi_1(K_t)(X_t^\alpha - \mathbb{E}[X_t^\alpha])^2 + \xi_2(K_t, \Lambda_t)\mathbb{E}[X_t^\alpha]^2 + \xi_3(K_t, Y_t, \Gamma_t)(X_t^\alpha - \mathbb{E}[X_t^\alpha]) + \xi_4(\Lambda_t, \Gamma_t)\mathbb{E}[X_t^\alpha] + \xi_5(Y_t, \Gamma_t, R_t)\Big].$$

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We can now apply the theorem, since we have

$$\mathbb{E}[\mathcal{D}_t^\alpha] = \mathbb{E}\Big[\gamma\Big(\alpha_t + \xi_0(X_t^\alpha, K_t, \Lambda_t, Y_t, \Gamma_t)\Big)^2\Big],$$

which is always positive and equals zero for the (optimal) control

$$\hat{\alpha}_t = -\xi_0\big(X_t^{\hat{\alpha}}, K_t, \Lambda_t, Y_t, \Gamma_t\big).$$

Consumer: optimal control. After precise computations, the optimal control $\hat{\alpha}$ is ($K, \Lambda > 0$ explicit, $\bar{P}_s := \mathbb{E}[P_s]$, $\hat{X} := X^{\hat{\alpha}}$)

$$
\begin{aligned}
\hat{\alpha}_t = &- \frac{bK}{\gamma}(\hat{X}_t - \mathbb{E}[\hat{X}_t]) \\
&+ \frac{b}{2\gamma} \int_t^\infty e^{-(\rho + b^2 K/\gamma)(s-t)} \mathbb{E}[P_s | \mathcal{F}_t^0] ds \\
&+ \frac{b}{2\gamma} \int_t^\infty \left(e^{-(\rho + b^2 \Lambda/\gamma)(s-t)} - e^{-(\rho + b^2 K/\gamma)(s-t)} \right) \bar{P}_s ds \\
&- \frac{b\Lambda}{\gamma} \mathbb{E}[\hat{X}_t] - \frac{\rho c \Lambda}{2\gamma\sigma^2 K}.
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\hat{\alpha}_t = & -\frac{bK}{\gamma}(\hat{X}_t - \mathbb{E}[\hat{X}_t]) \qquad \textit{(mean-reverting term)} \\
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Notice that we can compute $\mathbb{E}[\hat{X}_t]$. Also, the mean-reverting coefficient $\frac{bK}{\gamma}$ is increasing w.r.t. η. Reasonable: big η means big penalty on the variance, so need to reduce the oscillations.

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The formulas are quite complicated, but we can deduce some interesting limit results...

Consumer: limits. If there exists $\bar{P} := \lim_t \mathbb{E}[P_s]$, then

$$\lim_{t\to\infty} \mathbb{E}[\hat{\alpha}_t] = 0, \qquad \lim_{t\to\infty} \mathbb{E}[\hat{X}_t] = \frac{b\bar{P} - \rho c}{2b\sigma^2 K} =: \overline{\hat{X}}.$$

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- To have a meaningful model, we need $\bar{\hat{X}} \in]0, D[$; indeed, beside the obvious positivity condition, producing more than D is not admissible in a limit situation (but may happen locally) .
- The limit production belongs to $]0, D[$ under weak assumptions on the coefficients, namely $\bar{P} \in]\frac{\rho c}{b}, \frac{\rho c}{b} + 2\sigma^2 KD[$.

Consumer: simulations. We run some numerical simulations, in the case where P_t is a scaled Brownian motion:

$$dP_s = \xi dW_s, \qquad P_t = p_0 + \xi W_s.$$

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$$dP_s = \xi dW_s, \qquad P_t = p_0 + \xi W_s.$$

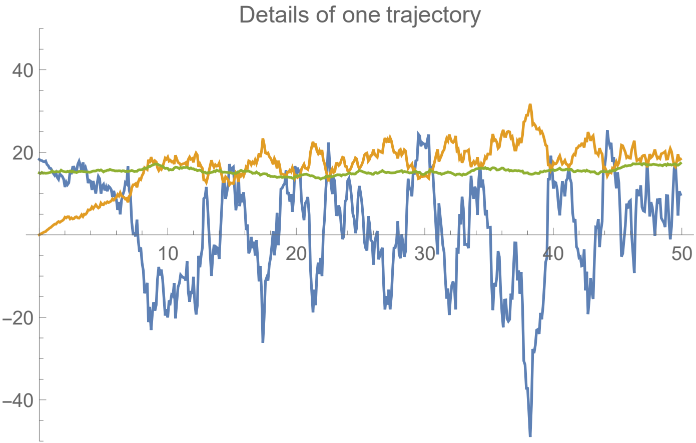
The optimal control here writes

$$\hat{\alpha}_t = \tilde{A}\hat{X}_t + \tilde{B}P_t + \tilde{C}e^{-\frac{b^2\Lambda}{\gamma}t} + \tilde{D},$$

$$\tilde{A} = -\frac{bK}{\gamma}, \qquad \tilde{B} = \frac{b}{2(\rho\gamma + b^2K)},$$

$$\tilde{C} = \frac{b(K-\Lambda)}{\gamma}\left(x_0 - \frac{bp_0 - \rho c}{2b\sigma^2 K}\right), \qquad \tilde{D} = \frac{bp_0 - \rho c}{2\gamma\sigma^2} - \frac{bp_0}{2(\rho\gamma + b^2K)}.$$

We here plot a sample trajectory (blue: $\hat{\alpha}_t$, orange: $\hat{X}_t$, green: P_t).



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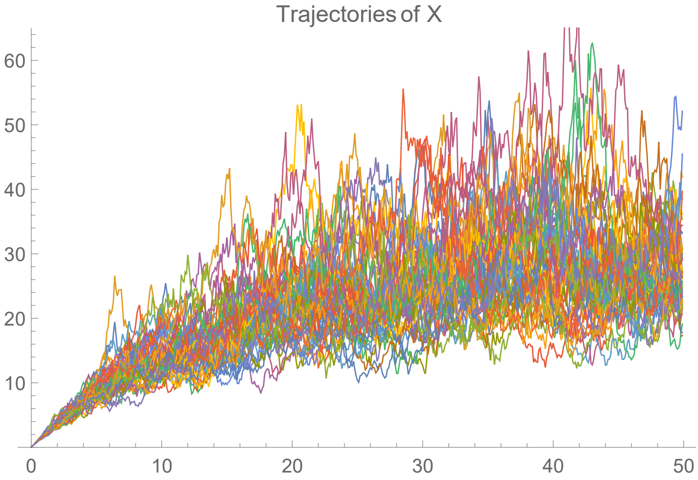
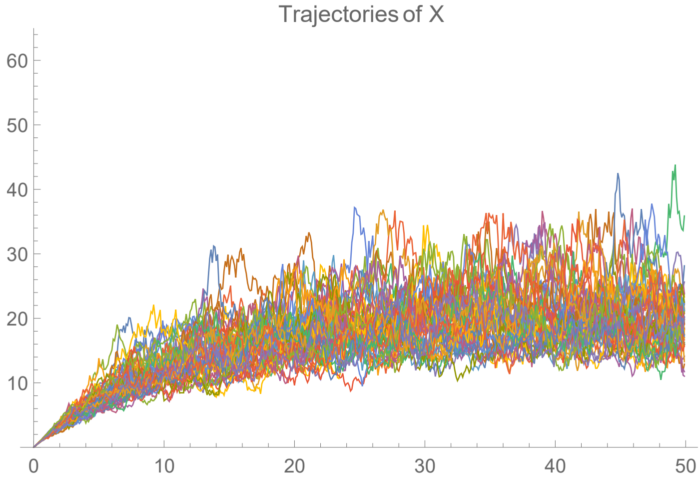
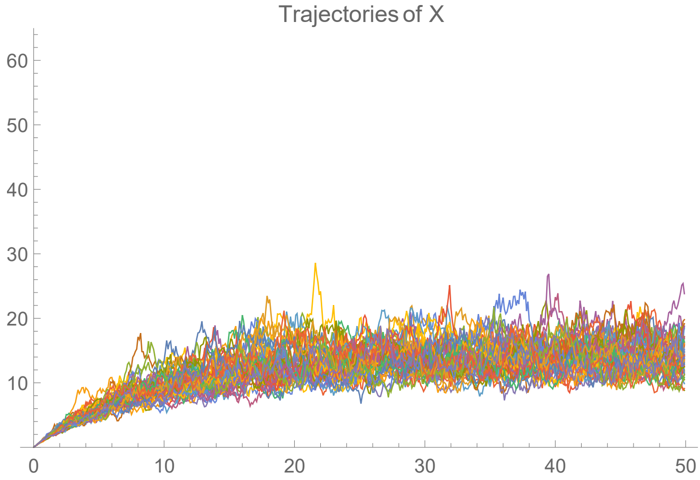


We now see the effect of penalizing the variance: $\eta = 2, \eta = 4$,



We now see the effect of penalizing the variance: $\eta = 2, \eta = 4, \eta = 8$.



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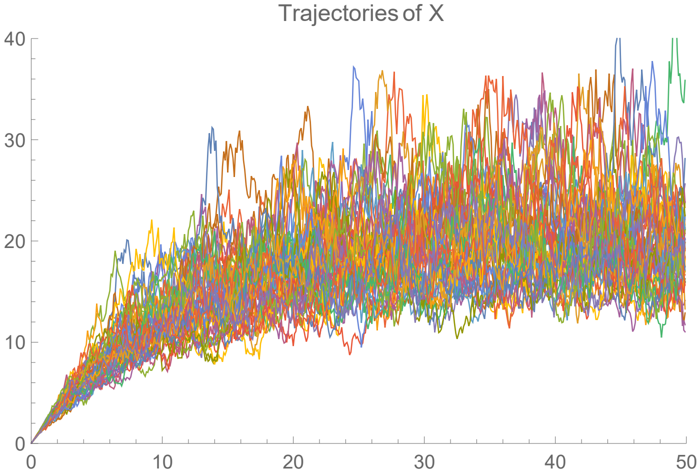

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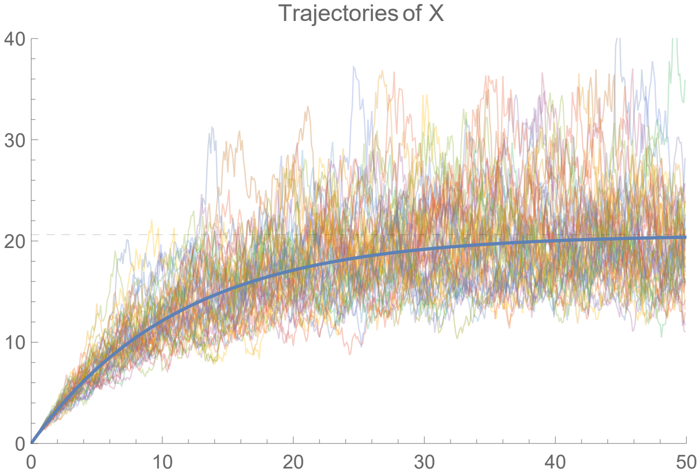

Introduction

1. Three optimization problems

1.1. The consumer

1.2 The energy company

1.3 The social planner

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Conclusions

Company: the model. Recall: the company has to adjust its production strategy according to the consumer's behaviour, so as to minimize the costs. Notice that the company knowns X_t^α, as the consumer buys an amount $D - X_t^\alpha$ of energy. Our model is as follows.

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- For each unity of energy produced at time t, the company has to pay the amount $\pi_t^{\alpha,u}$ (carbon tax). We assume (details later) $\pi_t^{\alpha,u} = \frac{X_t^\alpha}{D}\pi_0 + \frac{Q_t^u}{D}\pi_1$, where $[\pi_0, \pi_1]$ is a given interval.

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- The consumer buys an amount $D - X_t^\alpha$ at price P_t: we have a corresp. gain for the company, at price $\tilde{P}_t$, with $\tilde{P} = (1 - \varepsilon)P_t$.
- As the quantity Q_t^u should correspond to $D - X_t^\alpha$, there is a penalty in case of overproduction or underproduction.

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- quadratic installation costs;
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$$hu_t^2 - \tilde{P}_t\left(D - X_t^\alpha\right) + \pi_t^{\alpha,u}\left(D - X_t^\alpha\right) + \lambda\left(D - X_t^\alpha - Q_t^u\right)^2$$

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$$\mathbb{E}\left[\int_0^{\infty} e^{-\rho t}\left(hu_t^2 - \tilde{P}_t\left(D - X_t^{\alpha}\right) + \pi_t^{\alpha,u}\left(D - X_t^{\alpha}\right) + \lambda\left(D - X_t^{\alpha} - Q_t^u\right)^2\right) dt\right]$$

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So, the company has to solve

$$\inf_u \mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(hu_t^2 - \tilde{P}_t(D - X_t^\alpha) + \pi_t^{\alpha,u}(D - X_t^\alpha) + \lambda(D - X_t^\alpha - Q_t^u)^2\right)dt\right],$$

$$dQ_t^u = u_t dt, \qquad \tilde{P}_t, X_t^\alpha \ \textit{stochastic.}$$

Company: carbon tax. Recall: π_t is the amount the company pays for each unity of energy produced in t. Practically, $\pi_t \in [\pi_0, \pi_1]$ (fixed interval) and is increasing w.r.t. the production Q_t^u.

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A simple but reasonable model is: $\pi_t = \pi_t^{\alpha,u} = \frac{X_t^\alpha}{D}\pi_0 + \frac{Q_t^u}{D}\pi_1$.
Notice: $\pi_t^{\alpha,u} = \pi_0$ if the company does not work ($Q_t^u = 0, X_t^\alpha = D$).
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Company: optimal control. We solve the problem as above. Let $\hat{u}$ be the optimal control; we have ($\tilde{K} > 0$ explicit, $\hat{Q} = Q^{\hat{u}}$):

$$\hat{u}_t = -\frac{\tilde{K}}{h}\hat{Q}_t + \frac{2\lambda D - \pi_1}{2hD}\int_t^\infty e^{-\left(\rho+\frac{\tilde{K}}{h}\right)(s-t)}\mathbb{E}\left[D - X_s^\alpha \middle| \mathcal{F}_t\right] ds.$$

Second term: (over)discounted energy expected to be sold in $[t, \infty[$.

Consumer: limits. If there exists $\bar{X} := \lim_t \mathbb{E}[X_t^\alpha]$, then

$$\lim_{t\to\infty} \mathbb{E}[\hat{u}_t] = 0, \qquad \lim_{t\to\infty} \mathbb{E}[\hat{Q}_t] = \left(1 - \frac{\pi_1}{2\lambda D}\right)(D - \bar{X}).$$

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- Interpretation of the second limit: the limit production is the $1 - \frac{\pi_1}{2\lambda D}$ ratio of the quantity actually bought by the consumer. Notice: ratio increasing w.r.t. λ, decreasing w.r.t. π_1.

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- To have a meaningful model, the limit prod. must be positive.
- The limit production is positive under weak assumptions. Also notice it is always smaller than $D - \bar{X}$ (reasonable: no interest in producing more than the quantity bought by the consumer).

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Social planner: model and problem. Recall: the social planner wants to minimize the sum of the two payoffs.

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Hence, we have the following problem:

$$\inf_{(\alpha,u)} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(c\alpha_t + \gamma\alpha_t^2 + hu_t^2 + (\pi_t^{\alpha,u} + P_t - \tilde{P}_t)(D - X_t^\alpha)\right.\right.$$
$$\left.\left. + \eta \mathrm{Var}[X_t^\alpha] + \lambda\left(D - X_t^\alpha - Q_t^u\right)^2\right) dt\right],$$

$$dX_t^\alpha = b\alpha_t dt + \sigma X_t^\alpha dW_t, \qquad dQ_t^u = u_t dt, \qquad P \text{ stochastic.}$$

Notice: optimization with respect to (α, u), two-dimensional problem. Also recall that P is a generic stochastic process.

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Let $\beta^* = (\alpha^*, u^*)$ be the optimal control and $Z^* = (X^*, Q^*)$ be the corresponding optimal process. After some computations, we find

$$\beta_t^* = -\Xi_1(Z_t^* - \mathbb{E}[Z_t^*]) - \Xi_2\mathbb{E}[Z_t^*] - \frac{1}{2}N^{-1}\int_t^\infty e^{-\Xi_3(s-t)}\mathbb{E}[M_s|\mathcal{F}_t^0]ds - \frac{1}{2}N^{-1}\int_t^\infty \left(e^{-\Xi_4(s-t)} - e^{-\Xi_3(s-t)}\right)\bar{M}_s ds - \xi_1.$$

Here, N, M are known matrices, whereas Ξ_i are solution to an algebraic matrix equation (easy numerical computations). Also, notice the mean-reverting term.

Social planner: limits. If there exists $\bar{P} := \lim_t \mathbb{E}[P_t]$, then

$$\lim_{t\to\infty} \mathbb{E}[\alpha_t^*] = 0, \qquad \lim_{t\to\infty} \mathbb{E}[X_t^*] = \frac{2\lambda D^2\left(2\pi_1 - \pi_0 + \varepsilon\bar{P} - \frac{\rho c}{b}\right) - D\pi_1^2}{4\lambda D\left(\pi_1 - \pi_0 + \sigma^2 K^{11} D\right) - \pi_1^2} =: \bar{X}^*;$$

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- To have a meaningful model, we need $\bar{X}^* \in \,]0, D[$ and $\bar{Q}^* > 0$, for the reasons seen in the consumer's case.
- These admissibility conditions hold under weak assumptions; namely, we just need D or π_1 big enough.

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1. Three optimization problems

1.1. The consumer

1.2 The energy company

1.3 The social planner

2. Looking for an equilibrium

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A suitable definition. Recall the results of the three problems.

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We are interested in a price process P such that the social planner's suggestions for the consumer (the company) coincide with the optimal control of the consumer himself (the company himself).

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We are interested in a price process P such that the social planner's suggestions for the consumer (the company) coincide with the optimal control of the consumer himself (the company himself).

Definition (first attempt). A *Pareto equilibrium* is a price process P such that $\hat{X}_t(P) = X_t^*(P)$ and $\hat{Q}_t(\alpha^*(P)) = Q_t^*(P)$, for $t \geq 0$.

Conditions: $\hat{X}_t(P) = X_t^*(P)$ and $\hat{Q}_t\big(\alpha^*(P)\big) = Q_t^*(P)$. By def., the second one is satisfied for any P. We focus on the first one: very hard to solve. Idea: weaker definition, in term of limits.

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Definition (second attempt). An *asymptotic Pareto equilibrium* is a real number $\bar{P}$ such that $\lim_{t\to\infty} \mathbb{E}[\hat{X}_t](\bar{P}) = \lim_{t\to\infty} \mathbb{E}[X_t^*](\bar{P})$.

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Definition. An *admissible asymp. Pareto equilibrium* is a real $\bar{P}$ s.t.

- $\lim_t \mathbb{E}[\hat{X}_t](\bar{P}) = \lim_t \mathbb{E}[X_t^*](\bar{P})$;
- $\lim_t \mathbb{E}[X_t^*](\bar{P}) \in]0, D[$;
- $\lim_t \mathbb{E}[Q_t^*](\bar{P}) \in]0, +\infty[$;
- $\bar{P} \in]0, +\infty[$.

Conditions and formulas. We now look for admissible asymptotic Pareto equilibria for our problem. The equation

$$\lim_{t\to\infty} \mathbb{E}[\hat{X}_t](\bar{P}) = \lim_{t\to\infty} \mathbb{E}[X_t^*](\bar{P})$$

corresponds, by the formulas above, to

$$\frac{b\bar{P} - \rho c}{2b\sigma^2 K} = \frac{2\lambda D^2\big(2\pi_1 - \pi_0 + \varepsilon\bar{P} - \rho c/b\big) - D\pi_1^2}{4\lambda D\big(\pi_1 - \pi_0 + \sigma^2 K^{11} D\big) - \pi_1^2}.$$

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This finally leads to

$$\bar{P} = \frac{2\sigma^2 KD\Big(2\lambda D\big(2\pi_1 - \pi_0 - \frac{\rho c}{b}\big) - \pi_1^2\Big) + \frac{\rho c}{b}\Big(4\lambda D\big(\pi_1 - \pi_0 + \sigma^2 K^{11} D\big) - \pi_1^2\Big)}{4\lambda D\big(\pi_1 - \pi_0 + \sigma^2 K^{11} D\big) - \pi_1^2 - 4\varepsilon\lambda\sigma^2 KD^2}.$$

We just have to check the (three) admissibility conditions...

Proposition

A necessary and sufficient condition for the existence of an admissible asymptotic Pareto equilibrium is that:

$$\begin{cases} 2\sigma^2 KD\Big(2\lambda D\big(2\pi_1-\pi_0-\frac{\rho c}{b}\big)-\pi_1^2\Big)+\frac{\rho c}{b}\Big(4\lambda D\big(\pi_1-\pi_0+\sigma^2 K^{11} D\big)-\pi_1^2\Big) > 0, \\ -\pi_0 + 2\sigma^2 D(K^{11} - \varepsilon K) + (1-\varepsilon)\frac{\rho c}{b} > 0, \\ 2\lambda D\big(2\pi_1 - \pi_0 - \frac{\rho c}{b}\big) - \pi_1^2 + 2\varepsilon\lambda D(\rho c/b) > 0, \\ \pi_1 < 2\lambda D, \end{cases}$$

or

$$\begin{cases} 2\sigma^2 KD\Big(2\lambda D\big(2\pi_1-\pi_0-\frac{\rho c}{b}\big)-\pi_1^2\Big)+\frac{\rho c}{b}\Big(4\lambda D\big(\pi_1-\pi_0+\sigma^2 K^{11} D\big)-\pi_1^2\Big) < 0, \\ -\pi_0 + 2\sigma^2 D(K^{11} - \varepsilon K) + (1-\varepsilon)\frac{\rho c}{b} < 0, \\ 2\lambda D\big(2\pi_1 - \pi_0 - \frac{\rho c}{b}\big) - \pi_1^2 + 2\varepsilon\lambda D\frac{\rho c}{b} < 0, \\ \pi_1 < 2\lambda D. \end{cases}$$

In this case, the equilibrium is unique and defined as above:

$$\bar{P} = \frac{2\sigma^2 KD\Big(2\lambda D\big(2\pi_1-\pi_0-\frac{\rho c}{b}\big)-\pi_1^2\Big) + \frac{\rho c}{b}\Big(4\lambda D\big(\pi_1-\pi_0+\sigma^2 K^{11} D\big)-\pi_1^2\Big)}{4\lambda D\big(\pi_1 - \pi_0 + \sigma^2 K^{11} D\big) - \pi_1^2 - 4\varepsilon\lambda\sigma^2 KD^2}.$$

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The proposition above provides a complete and explicit solution to our questions. However, the conditions are a bit complicated. We then rewrite the statement in a stronger but simpler version.

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Proposition

A sufficient condition for the existence of an admissible asymptotic Pareto equilibrium is that:

$$\begin{cases} 2\lambda D\big(2\pi_1 - \pi_0 - \rho c/b\big) - \pi_1^2 > 0, \\ -\pi_0 + 2\sigma^2 D(K^{11} - \varepsilon K) + (1-\varepsilon)\rho c/b > 0, \\ \pi_1 < 2\lambda D. \end{cases}$$

In this case, the equilibrium is unique and defined as above.

Notice: conditions easily satisfied, we just need D big enough!

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New costs. Recall the installation costs for the consumer:

$$c\alpha_t + \gamma\alpha_t^2.$$

They only depend on the present choice α_t, not on the past. It is reasonable to add a path-dependence: there should be a discount linked to the total number of panel bought in the past, i.e. $\int_0^t \alpha_s ds$ (in the long run: lot of sales, better technologies, cheaper prices).

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This suggests the following new definition for the installation costs:

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Problem: a new state variable. But $dX_t^\alpha = b\alpha_t dt + \sigma X_t^\alpha dW_t$, so that $\mathbb{E}[\int_0^t \alpha_s ds] = (\mathbb{E}[X_t^\alpha] - x_0)/b$ and we can rewrite as $(\mu = \tilde{\mu}/b)$

$$c\alpha_t + \gamma\alpha_t^2 - \mu\alpha_t\mathbb{E}[X_t^\alpha].$$

Further generalization. We could also consider a similar change for the company's costs: from hu_t^2 to $hu_t^2 - \nu u_t \mathbb{E}[Q_t^u]$. Formulas are similar, but more complicated. So we here focus on the case where only the consumer's costs change.

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Consumer. We have a new problem for the consumer.

- The payoff is

$$\inf_{\alpha} \mathbb{E}\left[\int_0^{\infty} e^{-\rho t}\Big(c\alpha_t + \gamma\alpha_t^2 - \mu\alpha_t\mathbb{E}[X_t^{\alpha}] + P_t\big(D_t - X_t^{\alpha}\big) + \eta \mathrm{Var}[X_t^{\alpha}]\Big)\,dt\right].$$

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- The limit for the optimal control is

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$$+(\pi_t^{\alpha,u}+P_t-\tilde{P}_t)\big(D_t-X_t^\alpha\big)+\eta\mathrm{Var}[X_t^\alpha]+\lambda\big(D_t-X_t^\alpha-Q_t^u\big)^2\Big)dt\Bigg].$$

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Pareto equilibria. We have new formulas for the equilibria.

- The sufficient conditions for the existence/uniqueness of an admissible asymptotic Pareto equilibrium are

$$\begin{cases} 2\lambda D\left(2\pi_1 - \pi_0 - \frac{\rho c}{b}\right) - \pi_1^2 > 0, \\ 2\sigma^2 K - \frac{\rho\mu}{b} > 0, \\ -\pi_0 + 2\sigma^2 D(K^{11} - \varepsilon K) + (1-\varepsilon)\frac{\rho c}{b} + \varepsilon\frac{\rho\mu D}{b} > 0, \\ \pi_1 < 2\lambda D. \end{cases}$$

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$$\bar{P} = \frac{D\left(2\sigma^2 K - \frac{\rho\mu}{b}\right)\left(2\lambda D\left(2\pi_1 - \pi_0 - \frac{\rho c}{b}\right) - \pi_1^2\right)}{4\lambda D\left(\pi_1 - \pi_0 + \sigma^2 K^{11} D - \frac{\rho\mu D}{2b}\right) - \pi_1^2 - 2\varepsilon\lambda D^2\left(2\sigma^2 K - \frac{\rho\mu}{b}\right)} + \frac{\frac{\rho c}{b}\left(4\lambda D\left(\pi_1 - \pi_0 + \sigma^2 K^{11} D - \frac{\rho\mu D}{2b}\right) - \pi_1^2\right)}{4\lambda D\left(\pi_1 - \pi_0 + \sigma^2 K^{11} D - \frac{\rho\mu D}{2b}\right) - \pi_1^2 - 2\varepsilon\lambda D^2\left(2\sigma^2 K - \frac{\rho\mu}{b}\right)}.$$

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Conclusion. All the results still hold!

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1. Three optimization problems

- Consumer's demand satisfied by self-production and market
- Point of view of a consumer, a company, a social planner
- Framework: McKean-Vlasov stochastic optimal control
- Explicit formula for the optimal controls

2. Looking for an equilibrium

- Definition of admissible asymptotic Pareto equilibrium
- Necessary and sufficient conditions for existence and uniqueness
- Explicit formulas for the equilibrium

3. Generalizing the model

- A more general model with path-dependence in the installation costs
- All the results still hold

Thank you!