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Gas storage hedging

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1 Introduction

Gaz storage valuation has been an intense subject of research during the recent years. This problem is related to optimal control problems [17], [15] and more precisely to the class of optimal switching problem. On the energy market, the gaz storage management can be seen as a so called swing option [12] with some operational contraints : each day the manager of the gas storage has to decide either to inject gaz in the storage, buying it on the gas market, either to withdraw gas from the storage selling it on the market, either do nothing. Moreover it has to deal with some operational contraints :

- compressor used to inject or withdraw gas may break down,
- in order to preserve the storage cavity used due to thermomecanics contraints for example in a salt cavity, complete withdrawal can not be achieved at full speed,
- the formation of hydrates has to be limited for safety reasons.

Respecting these constraints, the manager will try to maximize its earnings on average having to deal with the stochasticity of the gas prices. Historically, gas storage valuation was coarsely valuated as a strip of call spread options [9] totally ignoring operational contraints and some properties of the storage. Because of the fact that injection capacity decreases as the storage is fulled and the withdrawal capacity decreases as the the pressure in the cavity decreases, the problem is fully non linear and the classical dynamic programming method [3] is generally used as a solver. In fact these physical contraints assure convexity of the problem and permits also to use the stochastic dual programming method [16] , [19] : some commercial sofware as QEM use this approach [20]. During the late nineties, a lot of research has been devoted to the search of efficient numerical procedures to solve the swing option

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problem. First trinomial tree were used [12], then Lonstaff Schwarz [13] and PDE [21] were used. As for the more complex case of gaz storage valuation, during the last year Monte Carlo methods [2], [14] and PDE methods [6] have been used to accuretaly valuate the assets. Practionners now classically use Monte Carlo to price swing option [11] or gas storage but in order to calculate the hedge at the valuation date they use classical finite different methods. Besides practionners are interested in evaluating the expected effectiveness of their hedge. Most of the time classical mean reverting one or two factor models are used to describe gas price models [18], [7]. But even within the Black Scholes framework some source of incompleness occurs :

- Hedging is permitted only once a day. As explained for example in [4], future and spot prices are set only once a day. The hedging periodicity can not be shorter than one day and it is well know that the hedging error in Black Scholes framework converges to zero at a rate proportional to the square root of hedging frequency [22] [10],
- Daily futures contracts are not available for the following days. Day ahead product can be seen as spot and depending on market,
  - either only monthly future products are available for the 72 following months for example at Henry Hub,
  - either monthly future products are available for the 10-12 following months, 11-12 quarter products and 6 seasons for example at Intercontinental Exchange. A season corresponds either to summer ranging from april to september, either to winter ranging from october to march.
  - either the three following months, the two following quarters and three following quarters for example at Powernext.

  Notice that even if week ahead products are not quoted on the market for hedging till the end of the month, it is possible to find such products over the counter.

For this two reasons even in the perfect gaussian world hedge can not be perfect and practionners are very interested in studying its effectiveness. In order to simulate the hedging strategy, practionners generate some price scenarios. They try to use classical finite different method to compute the hedging strategy on each scenario each day of the studied period for all available future products available. Even with powerful cluster, this task cannot be achieved in reasonnable time. In this article we first recall how to use tangent processes to calculate delta of American options to avoid the use of finite difference and show that the hedge calculated is efficient. In a second part we explain how to use this tangent process to accurately calculate conditionnal delta for gas storage. We give algorithms to calculate efficiently this conditionnal delta. We explain how to use this conditionnal delta in simulation and give some numerical results on its efficiency for salt cavities and depleted gas field. To our knowledge it is the first time that numerical results are given for the hedge of american style options with conditional delta. Some comparisons with classical finite difference methods are given on some scenarios.
In this paper most of theoretical background can be found in the articles listed in

[5].
In the sequel we suppose that conditional expectation are calculated by the Longstaff and Schwartz method [13] adapted as explained in [5]. The algorithm given here are associated to this method and so, are based on a Monte Carlo estimation of the option valuation and its hedging. Using trees methods for example would lead to another representation of the conditional delta (the one presented here is heavily using the estimation of optimal stopping time associated to the american type option). For some other representations of the delta see [5] and references inside.

2 Recall on american and bermudean options and delta hedging

2.1 Formulas

All over this section, we shall consider a one-dimensional Brownian motion W on a probability space $(\Omega, \mathscr{F}, \mathbb{P})$ endowed with the natural (completed and right-continuous) filtration $\mathbb{F} = (\mathscr{F}_t)_{t\leq T}$ generated by W up to some fixed time horizon $T > 0$. The dynamic of the stock is given by

$$S_t = S_0 \int_0^t \sigma(s) S_s dW_s \quad t \leq T, \tag{1}$$

In the case of american options with pay off g, maturity T, the risk free rate being taken equal to 0, it is well known that the price of an american option P_t is given by

$$P_t = \text{esssup}_{\tau\in\mathscr{T}_{[t,T]}} \mathbb{E}\left[g(S_\tau) \mid \mathscr{F}_t\right] \quad \text{for } t \leq T \;\; \mathbb{P}-\text{a.s.}\,, \tag{2}$$

where $\mathscr{T}_{[t,T]}$ denotes the set of stopping times with values in $[t, T]$ and $\mathscr{F}_t$. As recalled in [5], using tangent process Y_t solution of this equation

$$\begin{aligned} dY_t &= \sigma(s) Y_t dW_t, \\ Y_0 &= 1, \end{aligned}$$

the delta can estimated using this representation if g is C^1_b :

$$\Delta_t = \mathbb{E}\left[g'(S_{\tau_t}) Y_{\tau_t} \mid \mathscr{F}_t\right] (Y_t)^{-1} \;\;, t \leq T\,. \tag{3}$$

where τ_t is the first optimal stopping time after t such that

$$\mathbb{E}\left[g(S_{\tau_t}) \mid \mathscr{F}_t\right] = \text{esssup}_{\tau\in\mathscr{T}_{[t,T]}} \mathbb{E}\left[g(S_\tau) \mid \mathscr{F}_t\right].$$

This results remains true if g can be approximated uniformly by a sequence of C^1_b functions.
The finite difference approach consists in estimating the price process for different initial conditions. More precisely, let P^δ be defined as in equation 1 with S_0 replaced

by $S_0 + \varepsilon$, $\varepsilon \in \mathbb{R}$. Then, following the standard approach for European options, one can approximate Δ_0 by $(P_0^{\varepsilon} - P_0)/\varepsilon$ or $(P_0^{\varepsilon} - P_0^{-\varepsilon})/2\varepsilon$ where ε is a small scalar of $\mathbb{R}$. A large literature is available on this approach for European type options, see e.g. [8] and the references therein.

2.2 Classical Longstaff Schwarz and conditional delta

In what follows, we approximate the value of the american option by estimating the optimal stopping time associated to the corresponding bermudean option with grid π with κ meshes. We denote by $\hat{\mathbb{E}}[\cdot \mid \mathscr{F}_{t_i}]$ an approximation of the true conditional expectation operator $\mathbb{E}[\cdot \mid \mathscr{F}_{t_i}]$. $\hat{\tau}$ will represent the estimation of the optimal stopping time associated to the bermudean option. We note M the number of Monte Carlo simulations. $S_i^j = S_{t_i}^j, i = 0..\kappa, j = 1$ to M is the j th simulation of the asset at time step i corresponding to date $t_i = i\pi$. The algorithm 1 estimates the option value, the delta at initial date 0 and calculates conditional delta during optimization. This is a classical dynamic programming method [3] : at each time step, we compare the gain obtained by immediate exercise with the expected gain if the exercise is postponed. With exercising, cash flows on trajectories are updated. This methodology is equivalent to keep in memory the optimal stopping time on each trajectory. At time step zero, cash flows generated on each trajectory at each optimal stopping times are averaged to give the option value. Besides, during this backward recursion, conditional delta $C\Delta$ are stored using a regression approach to approximate (3). In the sequel we call conditional delta at date $i\pi$ a function of the prices approximated at date 0 giving the hedge at date $i\pi$ depending on the asset price if the option has not been exercised at this date. The hedge at initial date is simply obtained by average.

The second algorithm 2 simulates the optimal exercise of the american option and its hedging for some Monte Carlo scenarios S_i^j, for $i = 0, ..., \kappa$, $j = 1... M$.
In order to validate this approach, we use the classical Black Scholes model for an asset S_t following classical SDE

$$dS_t/S_t = \mu dt + \sigma dW_t, \tag{4}$$

with trend $\mu = 0.1$, volatility $\sigma = 0.2$, risk free rate $r = 0.05$. We study the hedging effectiveness of at the money put option with strike 1 with maturity one year. The value of the option is 0.06104. We can check on numerical results in table 1 that as the frequency of the hedge inscreases, the expected gain obtained with optimal continous exercise converges to the value of the option and that the standard deviation of the gain goes to zero. The number of simulations used for results in table 1 in optimization and simulation is taken equal to one million and the number of time step used to valuate the american option is taken equal to 720 so that results are converged.

Algorithm 1 Algorithm to price american option, calculate delta and conditional deltas

Require: Option and asset parameters
Ensure: Calculate the option and conditional delta at each time step

$CF^j = g(S_\kappa^j)$ for $j = 1$ to M // final cash flow
$\Delta^j = Y_\kappa^j g'(S_\kappa^j)$ for $j = 1$ to M // final delta
for $i = \kappa - 1 ... 0$ **do**
  Calculate and store cash flow conditional expectation $Esp_i = \hat{\mathbb{E}}[CF|\mathscr{F}_{i\pi}]$
  Calculate and store conditional deltas $C\Delta_i = \hat{\mathbb{E}}[\Delta|\mathscr{F}_{i\pi}]/Y_i$
  for $j = 1...M$ **do**
    if $Esp_i^j < g(S_i^j)$ **then**
      $CF^j = g(S_i^j)$
      $\Delta^j = Y_i^j g'(S_i^j)$
    end if
  end for
end for
Final value $\frac{\sum_j^M CF^j}{M}$, delta $\frac{\sum_j^M \Delta^j}{M}$ at $t = 0$.

Number of hedges	1	5	10	20	40	80	160
Average cash flow with hedge	0.04822	0.06176	0.06138	0.06117	0.06111	0.06108	0.06106
Standard deviation	0.06605	0.02997	0.02369	0.01933	0.01656	0.01490	0.01397

Table 1 Efficiency of delta hedging for the Black Scholes model

Remark 1. Of course if $\mu = r$, we would always have an average hedging portfolio equal to the option value.

3 Gas storage valuation and hedging methodology

We first give in this section the price model used for valuation and hedging. Similarly to the previous section we recall the mathematical formulation of gas storage problematic valuation in a continuous time framework. In a second part we give the algorithm used to valuate a gas storage asset by dynamic programming. We explain how to calculate Bellman value and conditional delta associated to the problem. At last we give the algorithm used to hedge the asset dealing with the availability of the futures products on the market.

Algorithm 2 Algorithm to simulate and hedge an american option

Require: Option and asset parameters, Bellman values and conditional delta calculated in optimization part
Ensure: Simulate the option exercise, portfolio with and without hedging for one scenario k

```
PF = 0. // Portfolio without hedge initialisation
PFH = 0. // Portfolio with hedge initialisation
bcash(j) = true for j = 1...M // Continue time step exploration till exercise
for i = 0 to κ−1 do
   Get back continuation function depending on asset value at time i stored in Algorithm 1 Esp_i(.)
   Get back conditional delta depending on asset value at time i stored in Algorithm 1 CΔ_i(.)
   for j = 1...M do
      if bcash(j) = true then
         if g(S_i^j) > Esp_i(S_i^j) then
            PF = g(S_i^j)
            PFH+ = g(S_i^j)
            bcash(j) = false
         else
            PFH− = CΔ_i(S_i^j)(S_{i+1}^j − S_i^j)
         end if
      end if
   end for
end for
for j = 1...M do
   if bcash(j) = true then
      PFH+ = g(S_κ^j)
      PF = g(S_κ^j)
   end if
end for
```

3.1 Price model

3.1.1 Stochastic Differential Equation

We suppose that the daily price of gas S_t follows under the risk neutral measure a n dimensional OU process [7]. The following SDE describes our uncertainty model for the forward curve $F(t,T)$ giving the prices of a MWh at day t for delivery at date T:

$$\frac{dF(t,T)}{F(t,T)} = \sum_{i=1}^{n} \sigma_i(t) e^{-a_i(T-t)} dz_t^i, \tag{5}$$

with $z_t^i, i = 1...n$ brownian motions with correlation matrix ρ, σ_i some volatility parameters and a_i mean reverting parameters.

Remark 2. Most of the time a two factors model is used. In this model, the first brownian motion describes swift changes in the future curve, the second one describes structural changes in the gas market and deals with long term changes in the curve. The mean reverting parameter is generally taken equal to zero in the second term.

With the following notations:

$$V(t1,t2) = \int_0^{t_1} \sum_{i=1}^{n} \sigma_i(u)^2 e^{-2a_i(t_2-u)} + 2\sum_{i=1}^{n} \sum_{j=i+1}^{n} \rho_{i,j} \sigma_i(u) e^{-a_i(t_2-u)} \sigma_j(u) e^{-a_j(t_2-u)} du,$$

$$W_t^i = \int_0^{t} \sigma_i(u) e^{-a_i(t-u)} dz_u^i, \text{ for } i = 1,\dots,n \tag{6}$$

the integration of the previous equation gives :

$$F(t,T) = F(t_0,T) e^{-\frac{1}{2}V(t,T) + \sum_{i=1}^{n} e^{-a_i(T-t)} W_t^i}. \tag{7}$$

With this modelization, the spot price is defined as the limit of the future price :

$$S_t = \lim_{T \downarrow t} F(t,T) \tag{8}$$

In the sequel we note $X_t = (W_t^1,\dots,W_t^n)$ the stochastic state vector.

3.1.2 Tangent process

Similarly to the Black Scholes model we introduce the “ forward tangent” process noted Y_t^T satisfying :

$$Y_t^T = e^{-\frac{1}{2}V(t,T) + \sum_{i=1}^{n} e^{-a_i(T-t)} W_t^i}. \tag{9}$$

With this new notation, the conditional delta at date t dealing with delivery at date T for a classical European option with pay off g can be written :

$$C\Delta_t^T = \mathbb{E}\left[g'(S_T) Y_T^T \mid \mathscr{F}_t\right] / Y_t^T \ , \ t \le T \ . \tag{10}$$

3.2 Gas storage modelization

We note C_t the gas level in the gas storage at date t. We suppose that the asset management is such that C_t satisfies $C_{min} \le C_t \le C_{max}$. At each stock level C_t a set of possible commands is associated. Noting a_{in} the injection rate per time unit (a day typically), and a_{out} the withdrawal rate, this command to be executed during a time step π is choosen inside an interval $[\max(-a_{out}\pi, C_{min} - C_t), \min(a_{in}\pi, C_{max} - C_t)]$. The bang bang strategy is a good approximation of the optimal strategy [2] [1] .

This strategy when not dealing with global constraints C_{min} and C_{max} supposes that at each time step, the optimal command per time unit is to be choosen so that gains per time unit are between the following ones :

$$\begin{cases} \text{Injection} & a_{in} & \text{for a cost of } \phi_{-1}(S_t) = -S_t a_{in} - K_{in} & (\text{regime } -1) \\ \text{Do nothing} & & \text{for a cost of } \phi_0(S_t) = -K_s & (\text{regime } 0) \\ \text{Withdraw} & a_{out} & \text{for a cost of } \phi_1(S_t) = S_t a_{out} - K_{out} & (\text{regime } 1) \end{cases} \tag{11}$$

where K_{in}, K_s and $-K_{out}$ are some costs that we will suppose null for simplification.

Remark 3. Notice that injection rate a_{in} and withdrawal rate a_{out} depends on the stock level C_t. To simplify the notations we drop the C_t dependence.

In the sequel we suppose that this bang bang strategy is used. In [6] some numerical examples are given to estimate the error associated with this supposition on real assets.
At date t the regime u_t can take three values

- 1 in withdrawal mode,
- 0 in storage mode,
- -1 in injection mode.

We note u a given strategy to manage the asset: this strategy consists in a set of regimes and some associated stopping time where regimes change. We note u_t the regime number at date t belonging to $(-1,0,1)$. For this given strategy u, knowing that at date t the factors of the price model are given by $x = (w^1, \dots, w^n)$, that the stock level is c , neglecting switching cost, the expected profit associated to the asset management between dates t and T is :

$$J(t,x,c,u) = \mathbb{E}\left[\int_t^T \phi_{u_r}(S_r)dr + J(T,X_T,C_T,u_T)|X_t = x, C_t = c\right] \tag{12}$$

In the sequel we will suppose that the final value of the asset is 0. The asset operator will try to find a strategy in the set $\mathscr{U}_t$ of the admissible non anticipative strategy in order to maximize its gains and solve the problem

$$J^*(t,x,c) = \sup_{u \in \mathscr{U}_t} J(t,x,c,u) \tag{13}$$

3.2.1 Dynamic programming and daily hedging for gas storage

We suppose in the sequel that dates where the regime switches are allowed are discrete t_i , $i = 0,...\kappa - 1 = T/\pi - 1$. According to Bellman principe, at a date t_i, for some given random factors $x = (w^1, \dots w^n)$, the value of the asset at a given time step $t_i = i\pi$ with a stock level c follows :

$$J^*(t_i,x,c) = \sup_{k \in \{-1,0,1\}} \left\{\phi_k(S_{t_i})\pi + \mathbb{E}\left[J^*(t_{i+1}, X_{t_{i+1}}, \tilde{c}_k, k) \mid X_{t_i} = x, C_{t_i} = c\right]\right\} \tag{14}$$

where

$$\tilde{c}_{-1} = \min(c + a_{in}\pi, C_{max}),$$
$$\tilde{c}_0 = c,$$
$$\tilde{c}_1 = \max(c - a_{out}\pi, C_{min}), \quad (15)$$
$$(16)$$

In the sequel $k_i^* \in \{-1,0,1\}$ will denote the optimal regime number at time step i.
We define $V^*(t,x,c)$ the optimal volume exercised at date t, with $X_t = x$ and $C_t = c$, and taking three different possible values $-a_{in}\pi$ in injection regime, $a_{out}\pi$ in withdrawal regime, 0 when doing nothing when not dealing with global stock constraints. We note $C_m^{*,i}(x,c)$ the optimal stock level at date t_m starting at level c at date t_i following X_t trajectory with $X_{t_i} = x$. The optimal stock level $C_m^{*,i}(x,c)$ is $\mathscr{F}_{t_m}$ adapted and follows

$$C_i^{*,i}(x,c) = c,$$
$$C_m^{*,i}(x,c) = c - \sum_{k=i}^{m-1} V^*(t_k, X_{t_k}, C_k^{*,i}(x,c)) \text{ for } 0 \le i \le m \le \kappa, \quad (17)$$
$$(18)$$

Thus $\sum_{k=0}^{\kappa-1} V^*(t_k, X_{t_k}, C_k^{*,0}(x,c))$ corresponds to the sum of the optimal volumes exercised following the optimal strategy starting from a volume c at time step 0 where $X_{t_0} = x$.
Noticing that the optimal volume exercised corresponds to the derivative of the gain fonction $\pi\phi$ in the optimal regime in equation (14), similarly to equation (3), we introduce the $\mathscr{F}_{t_m}$ adapted random variable.

$$D(t_i, t_m, x, c_l) = V^*(t_m, X_{t_m}, C_m^{*,i}(x, c_l)) Y_{t_m}^{t_m}. \quad (19)$$

Conditional delta at date t_i for delivery at date t_m , $m = i+1, \dots \kappa - 1$ is easily calculated by equation (10)

$$C\Delta(t_i, t_m, x, c_l) = \mathbb{E}[D(t_i, t_m, X_{t_i}, c_l) \mid X_{t_i} = x] / Y_{t_i}^{t_m}. \quad (20)$$

In order to solve equation (14), the classical dynamic backward programming method using Longstaff Schwarz methodology [13] can be used as in the case of american options. The main difficulty comes from the fact that we don't know what the stock is at a future date $\tilde{t}$. This stock level depends on the strategy applied between dates t and $\tilde{t}$. So we have to store for each stock level the Bellman values and the hedging strategies associated as shown by equation (14).
The stock is discretized on a grid

$$c_l = C_{min} + l\delta, \quad l = 0, \dots, lc = (C_{max} - C_{min})/\delta$$

where δ is the mesh size. Similarly to the case of american option, a Monte Carlo method is used to get some gas prices simulations S_i^j for $j = 1, \dots M$ at dates t_i. Conditional expectation is estimated using regression as in [5] and cash flows are

estimated as in subsection (2.2).
We note $\hat{V}^*$ and $\hat{C}^*$ the estimation of the optimal volume V^* and optimal stock levels C^* obtained by the Longstaff Schwarz method and note $\hat{D}$ the function storing the optimal volume $\hat{V}^*$ multiplied by tangent process along trajectories :

$$\hat{D}(t_i, t_m, X_{t_i}^j, c_l) = \hat{V}^*(t_m, X_{t_m}^j, \hat{C}_m^{*,i}(X_{t_i}^j, c_l)) Y_{t_m}^{t_m, j} \tag{21}$$

The $\hat{D}$ values at date t_i can be calculated by the following backward recursion knowing $\hat{D}(t_{i+1}, t_m, X_{t_{i+1}}^j, c_l)$ for $m = i+1, \ldots \kappa - 1$, $l = 1, \ldots lc$, $j = 1, \ldots M$ and the optimal volume to exercise $\hat{V}^*(t_i, X_{t_i}^j, c_l)$:

$$\begin{aligned} \hat{D}(t_i, t_i, X_{t_i}^j, c_l) &= \hat{V}^*(t_i, X_{t_i}^j, c_l) Y_{t_i}^{t_i} \\ \hat{D}(t_i, t_m, X_{t_i}^j, c_l) &= \hat{D}(t_{i+1}, t_m, X_{t_{i+1}}^j, c_l + \hat{V}^*(t_i, X_{t_i}^j, c_l)), m = i+1, \ldots \kappa - 1 \end{aligned} \tag{22}$$

Remark 4. Equation (22) is simply obtained by using equation (21) and the fact that $\hat{C}_m^{*,i}(X_{t_i}, c_l) = \hat{C}_m^{*,i+1}(X_{t_{i+1}}, c_l + \hat{V}^*(t_i, X_{t_i}^j, c_l))$ for $\kappa > m > i$.

Remark 5. The $\hat{D}$ value is an approximation in equation (22) :

- $\hat{D}$ is the result of the approximated Longstaff Schwarz procedure,
- $\hat{D}$ is only available at some discretized stocks points and an interpolation in the delta values is needed.

The Longstaff Schwarz estimator of the conditional delta is then evaluated for $m > i$ by

$$C\hat{\Delta}(t_i, t_m, x, c_l) = \hat{\mathbb{E}}[\hat{D}(t_i, t_m, X_{t_i}, c_l) \mid X_{t_i} = x] / Y_{t_i}^{t_m}. \tag{23}$$

The algorithm 3 gives the entire procedure to calculate the gas storage value and the deltas. It is a simplified one not dealing with special cases and interpolation needed between stock level.

3.2.2 Cash flow simulation and delta hedging

As for american options, an algorithm can be derived to calculate the cash flow generated by the gas storage management and by the delta hedging. Because the model is a daily model, hedging could be theoretically done with daily product if daily product were available on future market for all maturities. As we will see later on numerical results the algorithm can be used with the real product available on the gas market.
Two options are possible :

- Calculate as proposed in algorithm 3 the daily deltas. Then do as in the algorithm 4 and approximate the Δ at date t for the delivery period $\mathscr{P}_p$ as the sum of all the daily delta ponderated by the future value at the date t :

Algorithm 3 Algorithm to valuate gas storage , calcule delta and conditional deltas

Require: Asset parameters
Ensure: Calculate the option and conditional delta at each time step
// initialisation of option
for $l = 0$ to lc **do**
  $J^j_\kappa(l) = 0, \Delta^j_\kappa(\kappa, l) = 0$, $j = 1$ to M
end for
for $i = \kappa - 1$ to 0 **do**
  for $l = 0$ to c **do**
    Calculate A^l_{out}, A^l_{in}, the maximum quantity of gas available for withdrawal, injection
    $PIn^j = -A^l_{in} S^j_i + \hat{\mathbb{E}}[J_{i+1}(l + A^c_{in}/\delta) | X_{t_i} = X^j_{t_i}]$ for $j = 1$ to M (injection)
    $PId^j = \hat{\mathbb{E}}[J_{i+1}(l) | X_{t_i} = X^j_{t_i}]$ for $j = 1$ to M (idle)
    $PW^j = A^l_{out} S^j_i + \hat{\mathbb{E}}[J_{i+1}(l - A^c_{out}/\delta) | X_{t_i} = X^j_{t_i}]$ for $j = 1$ to M (withdrawal)
    for $j = 1, M$ **do**
      if $PIn^j \geq PW^j$ and $PIn^j \geq PId^j$ **then**
        $J^j_i(l) = -A^l_{in} S^j_i + J^j_{i+1}(l + A^l_{in}/\delta)$
        $\Delta^j_i(i, l) = -A^l_{in} Y^{t_i, j}_{t_i}$,
        $\tilde{l} = l + A^l_{in}/\delta$
        for $m = i + 1$ to κ **do**
          $\Delta^j_i(m, l) = \Delta^j_{i+1}(m, \tilde{l})$
        end for
      else if $PId^j \geq PIn^j$ and $PId^j \geq PW^j$ **then**
        $J^j_i(l) = J^j_{i+1}(j, l)$
        $\Delta^j_i(i, l) = 0$
        for $m = i + 1$ to κ **do**
          $\Delta^j_i(m, l) = \Delta^j_{i+1}(m, l)$
        end for
      else
        $J^j_i(l) = A^l_{out} S^j_i + J^j_{i+1}(l - A_{out}/\delta)$
        $\Delta^j_i(i, l) = A^l_{out} Y^{t_i, j}_{t_i}$,
        $\tilde{l} = l - A^l_{out}/\delta$
        for $m = i + 1$ to $\kappa - 1$ **do**
          $\Delta^j_i(m, l) = \Delta^j_{i+1}(m, \tilde{l})$
        end for
      end if
    end for
  end for
  Store continuation values function $J(x, c_l) = \hat{\mathbb{E}}[J_{i+1}(l) | X_{t_i} = x]$ for all l
  Calculate and store conditional delta $C\hat{\Delta}(t_i, t_m, x, c_l) = \hat{\mathbb{E}}[\Delta_i(m, l) | X_{t_i} = x]/Y^{t_m}_{t_i} \quad \forall m > i, \quad \forall l,$
end for
Option value $\sum_{j=1}^{M} \frac{J^j_0(\tilde{l})}{M}$, $\tilde{l}$ initial index stock number
Delta for each day $\sum_{j=1}^{M} \frac{\Delta^j_0(m, \tilde{l})}{M} \quad \forall m > 0$

$$\Delta_p = \frac{\sum_{t_i \in \mathscr{P}_p} \Delta_i F(t, t_i)}{\sum_{t_i \in \mathscr{P}_p} F(t, t_i)}. \tag{24}$$

- Instead of aggregating the daily deltas in simulation it is possible to modify the algorithm 3 directly to store the hedge for the products available during optimization. Because the conditional delta in the algorithm 3 at date t are calculated for each date $\tilde{t} > t$, the data storage can become cumbersome if maturity of the asset is longer than a few months. It is possible to reduce the amount of conditional expectation to calculate and data storage if the future products satisfy some rules described below.
We note $\mathscr{Q}_t$ the set of future products available at date t, and for all $p \in \mathscr{Q}_t$, $\mathscr{P}_p$ the delivery period associated to product p, η_p the beginning of the delivery period. Supposing that $\forall t > 0, \forall p \in \mathscr{Q}_t, \forall \tilde{t} > t$ there exist $\mathscr{Q}^p \subset \mathscr{Q}_{\tilde{t}}$ such that $\mathscr{P}_p = \cup_{\tilde{p} \in \mathscr{Q}^p} \mathscr{P}_{\tilde{p}}$ then it is possible to aggregate an approximated conditional delta at date t per product with an addhoc rule so that a dynamic programming approach is still usable :

$$C\tilde{\Delta}(t_i, p, x, c) = \hat{\mathbb{E}}[\sum_{t_m \in \mathscr{P}_p}^{\kappa} \hat{V}^*(t_m, X_{t_m}, \hat{C}_m^{*,i}(X_{t_i}, c)) Y_{t_m}^{t_m} F(0, t_m) \mid X_{t_i} = x]$$
$$C\hat{\Delta}(t_i, p, x, c) = C\tilde{\Delta}(t_i, p, x, c) / (Y_{t_i}^{\eta_p} \sum_{t_m \in \mathscr{P}_p} F(0, t_m)) \tag{25}$$

for $p \in \mathscr{Q}_t$. where $C\hat{\Delta}(t_i, p, x, c)$ represents the power to invest at date t_i for product p for a gas stock level c and a stochastic state vector x. Noticing that

$$C\tilde{\Delta}(t_i, p, x, c) = \hat{\mathbb{E}}[1_{t_{i+1} \in \mathscr{P}_p} \hat{V}^*(t_{i+1}, X_{t_{i+1}}, \hat{C}_{i+1}^{*,i}(X_{t_i}, c)) Y_{t_{i+1}}^{t_{i+1}} F(0, t_{i+1}) \mid X_{t_i} = x]$$
$$+ \sum_{\tilde{p} \in \mathscr{Q}^p} \hat{\mathbb{E}}[\sum_{t_m \in \mathscr{P}_{\tilde{p}}, m > i+1} \hat{\mathbb{E}}[\hat{V}^*(t_m, X_{t_m}, \hat{C}_m^{*,i}(X_{t_i}, c)) Y_{t_m}^{t_m} F(0, t_m) \mid X_{t_i+1}] \mid X_{t_i} = x]$$

we get

$$C\tilde{\Delta}(t_i, p, x, c) = \hat{\mathbb{E}}[1_{t_{i+1} \in \mathscr{P}_p} \hat{V}^*(t_{i+1}, X_{t_{i+1}}, \hat{C}_{i+1}^{*,i}(X_{t_i}, c)) Y_{t_{i+1}}^{t_{i+1}} F(0, t_{i+1}) \mid X_{t_i} = x]$$
$$+ \sum_{\tilde{p} \in \mathscr{Q}^p} \hat{\mathbb{E}}[C\tilde{\Delta}(t_{i+1}, \tilde{p}, X_{t_{i+1}}, c + \hat{V}(t_i, X_{t_i}, c)) \mid X_{t_i} = x]$$

the function $C\tilde{\Delta}$ can be evaluated by backward recursion.

Remark 6. The direct use of the equation (24) is not possible within a dynamic programming framework.

We suppose in the algorithm 4 that simulates an hedged portfolio that M new simulations prices S_i^j $j = 1, \ldots M$ have been generated by Monte Carlo under the real world probability for this simulation part. $F^j(t, T)$ is the forward curve seen at date t for delivery at date $T \geq t$ and simulation $j \in [1, M]$. The algorithm given doesn't suppose the previous aggregation property.

Algorithm 4 Cash flow simulation with and without hedging of the gas storage

Require: Asset parameters, continuation values and delta calculated in optimization,
Ensure: Calculate cash flow generated with or without hedging.
  ST^j initialize stock at initial stock level for all $j = 1$ to M
  $PF^j = 0$ initialize portfolio at zero for all $j = 1$ to M
  $PFH^j = 0$ initialize portfolio with hedge at zero for all $j = 1$ to M
  for $i = 0$ to $\kappa - 1$ **do**
    Get back the continuation value function $J(.,c_l)$ at date t_i for all l
    Get back conditional delta function at date $t_i : C\Delta(t_i,t_m,.,c_l)$ for all l, $m > i$
    for $j = 1$ to M **do**
      for $p \in \mathscr{Q}_{t_i}$ // nest on future delivery period **do**
        Hedge for period $\mathscr{P}_p$
        $\Delta = \frac{\sum_{t_k \in \mathscr{P}_p} C\Delta(t_i,t_k,X^j_{t_i},ST^j)F^j(t_i,t_k)}{\sum_{t_k \in \mathscr{P}_p} F^j(t_i,t_k)}$
        Integrated future product on delivery periode date $i : F_i = \sum_{t_k \in \mathscr{P}_p} F^j(t_i,t_k)$
        Integrated future product on delivery periode date $i+1 : F_{i+1} = \sum_{t_k \in \mathscr{P}_p} F^j(t_{i+1},t_k)$
        Add hedge $PFH^j = PFH^j - (F_{i+1} - F_i)\Delta$
      end for
      Calculate A_{out}, A_{in}, the maximum quantity of gas available for withdrawal, injection
      $PIn^j = -A_{in}S^j_{t_i} + J(X^j_{t_i}, ST^j + A_{in})$ for $j = 1$ to M (injection)
      $PId^j = J(X^j_{t_i}, ST^j)$ for $j = 1$ to M (idle)
      $PW^j = A^c_{out}S^j_{t_i} + J(X^j_{t_i}, ST^j - A_{out})$ for $j = 1$ to M (withdrawal)
      if $PIn^j \geq PW^j$ and $PIn^j \geq PId^j$ **then**
        $PF^j = -A_{in}S^j_{t_i} + PF^j$ // update portfolio without hedge if injection,
        $PFH^j = -A_{in}S^j_{t_i} + PFH^j$ // update portfolio with hedge if injection,
        $ST^j = ST^j + A_{in}$ // update stock level
      else if $PW^j \geq PIn^j$ and $PW^j \geq PId^j$ **then**
        $PF^j = A_{out}S^j_{t_i} + PF^j$ // update portfolio without hedge if withdrawal,
        $PFH^j = A_{out}S^j_{t_i} + PFH^j$ // update portfolio with hedge if withdrawal
        $ST^j = ST^j - A_{out}$ // update stock level
      end if
    end for
  end for

4 Numerical results

In a first part we give the parameters used for gas modelling, the parameters used to describe two kinds of gas storage and the hedging products used. In a second part, we compare our method here called conditional tangent process to the finite difference method and the tangent process method.

4.1 Market representation

All numerical results are given with a two factor model. Parameters for this model are given in figure 1 and the initial forward curve is given in figure 2 starting the first of july 2010. The annual interest rate is taken equal to 0.06%.

long term volatility	29 % / $\sqrt{year}$
long term mean reverting	0 / *year*
short term volatility	94 % / $\sqrt{year}$
short term mean reverting	7.4 / *year*
correlation	-0.13

Fig. 1 Gas model parameters

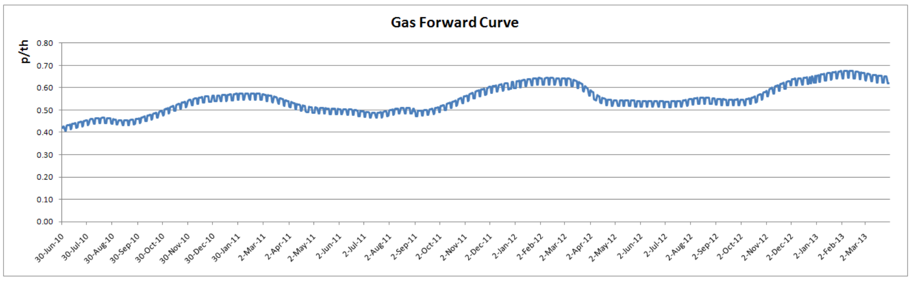


Fig. 2 Initial forward curve

We assumed daily hedging. We assumed the following:

- Day products are available for delivery from tomorrow to the end of the running week.
- Week products are available for delivery from next week to the end of the month. For simplification purpose, we avoid overlapping products by truncating the last available week product at the last day of the month,

- Month products are available from next month and up to the next quarter.
- Quarter products are available for delivery starting next quarter and until next year.
- Finally, next year, Y+2, and Y+3 are available.

The inclusion property given in remark **??** is respected, so all calculations with conditional delta can be carried out on a small laptop.

4.2 Gas storage description

Two sets of parameters are used to describe two typical kinds of gas storage: Salt cavities and gas storages tanks will be represented by a "fast storage" set of parameters, whereas depleted gas field will be described with the "seasonal storage" set of parameters (See Fig.3). Prices are given in pence per therm and the different capacities in therm.

	Fast storage	Seasonal Storage
Working gas capacity (th)	36,600,000 th	32,637,363 th
withdrawal rate (th/day)	4,500,000 th/day	400,000 th/day
injection rate (th/day)	6,000,000 th/day	140,659 th/day
withdrawal cost (p/th)	0.35 p/day	0.31 p/day
injection cost (p/th)	0.35 p/day	0.72 p/day

Fig. 3 Parameters for the fast and seasonal gas storage asset.

Fast storages and seasonal storages differ greatly in their injection/withdrawal rate. It requires around two weeks to totally fill and empty a fast storage, against 10 months for the seasonal storage. Hence, seasonal storages's asset manager will basically arbitrage the seasonal price differences (e.g. store in summer, withdraw in winter) (Fig.4), whereas fast storages's will rather take benefit of the volatility and the weekly seasonality (Fig.5). Thus the fast gas storage will be optimized and simulated during one year whereas the seasonal storage will be optimized and simulated during three years.

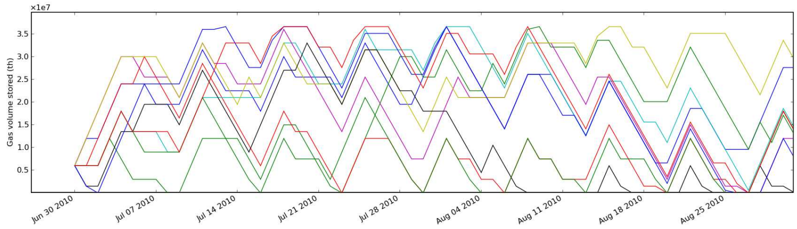


Fig. 4 10 simulations of the optimal fast gas storage levels (two months)

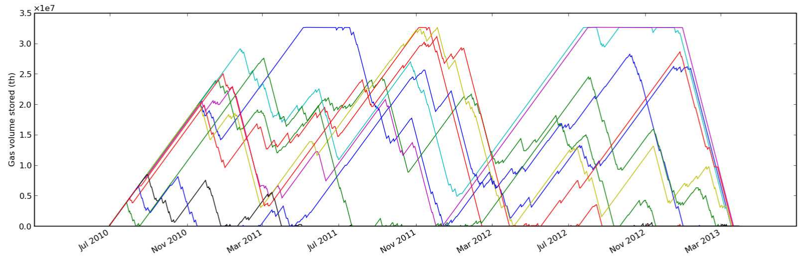


Fig. 5 10 simulations of the optimal seasonal gas storage levels (3 years)

4.3 Comparison with finite difference and tangent process

All calculations except when specified are achieved with 70000 simulations during optimization parts with 70 (10×7) local basis functions for the uncertainty $x = (w^1, w^2)$ discretization. As for the gas levels used for valorization, the fast gas storage is discretized with 24 steps, and the seasonal one with 80 steps. We compare the three methods for a simulation of the forward curve deformation. During this forward curve deformation the prices of the future products are obtained by averaging the curve on the delivery period.

- In conditional tangent process, all commands and hedges are calculated during the optimization part,
- As for the tangent process, at each time step, a valorization is achieved calculating the hedge applied at this date using the algorithm 4 not storing conditional hedges.
- As for finite difference, at each time step t a first calculation is achieved with the current forward curve $F(t,.)$ at this date giving a first valorization V. Then for each available future product with delivery period p, the future curve is shifted by

$$F_\varepsilon(t,T) = F(t,T)(1+\varepsilon) \text{ for all } T \in p, \tag{26}$$

$$= F(t,T) \text{ if } T \text{ not in } p, \tag{27}$$

leading to a second valorization V_ε. The p's sensibility is thus given by $\frac{V_\varepsilon - V}{\varepsilon}$. The ε parameter is not easily fit. A small parameter will lead to a less biased value estimation but the variance of the valorization increases the variance of this estimator. We take $\varepsilon = 0.005$ except when specified as our finite difference resolution parameter.

4.3.1 Fast storage results

We take a week product, a month product and a quarter product to achieve the comparisons. All figures give the hedging volume HV per day for the product used so that the volume associated to a given product with delivery length n days is nHV. The figure 6 compares the three methods for the fourth week of july 2010. We observe that conditional tangent process and tangent process give very similar results. Finite difference gives results quite different from other methods but these results

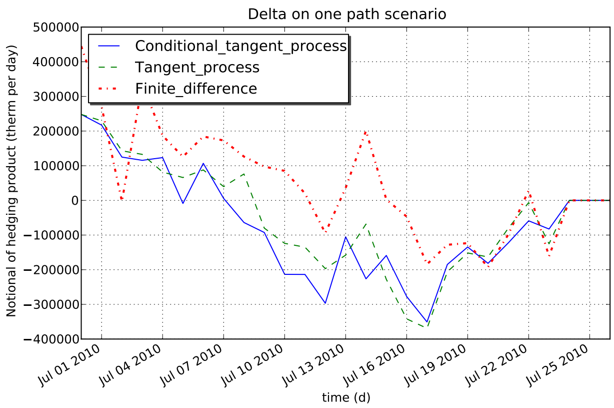
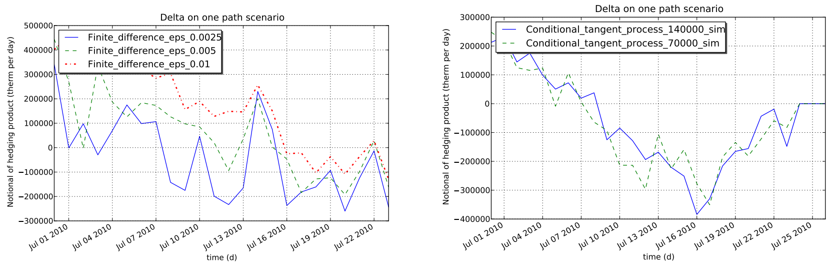


Fig. 6 Fast storage: Hedging strategy for week product with delivery the last week of july 2010

are unstable when changing ε parameter as shown on figure 7. Accuracy of each valuation in the finite difference procedure should be increase as ε decreases. Results obtained by conditional tangent process appears to be quite stable when changing the number of trajectories used in optimization as shown on figure 7. The figure 8 compares the three methods for december 2010. Notice that this product only appears on the first of october. The figure 9 compares the three methods for the last quarter of 2010. Results for the month and quarter product results are very similar for the three methods, finite difference still being little further from the two other methods. As for the time needed for each method calculating the hedge for all the products, the conditional delta took 10 minutes for the fast storage valorization and less than 20 secondes for one simulation. As for tangent process, the simulation took 15 hours and the finite difference simulation took more than 4 days.



(a) Influence of ε parameter in finite difference (b) Influence of the number of simulations for the conditional tangent process

Fig. 7 Fast storage: Example of delta evolution for the weekly product (july 26th 2010- july 31th 2010) on one scenario.

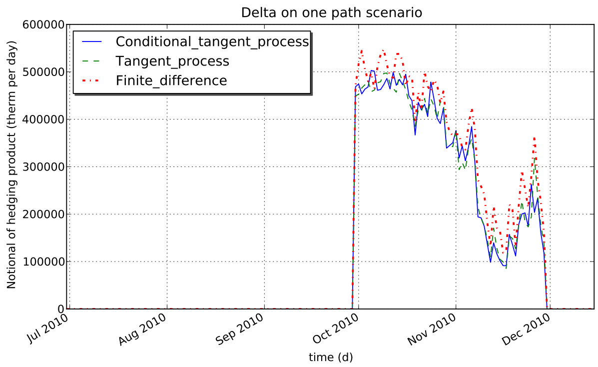


Fig. 8 Fast storage: Example of delta evolution for the monthly product december 2010 on one scenario

Besides this method comparison a simulation phase with 10000 simulations was achieved with the conditional delta method in less than 600 seconds. The standard deviation of the cash flow was reduced by a factor 5 from 8.21e+08 to 1.64e+08. Distribution of the cash flow with and without hedge is given in figure 10.

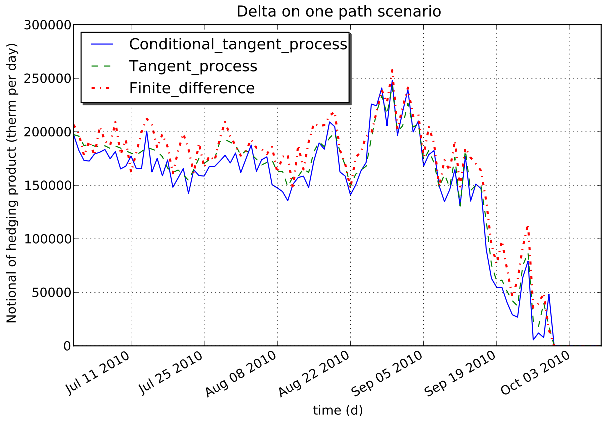


Fig. 9 Fast storage: Example of delta evolution for the quarter product Q4 2010 on one scenario

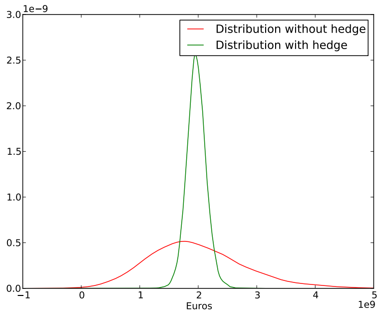


Fig. 10 Fast storage: cash flow distribution with and without hedge computed with the conditional tangent method

4.3.2 Seasonal storage results

We took the same products as for the fast storage plus the year 2011. Results are given on figure 11, 12, 13, 14. The figure 12 compares the three methods for decem-

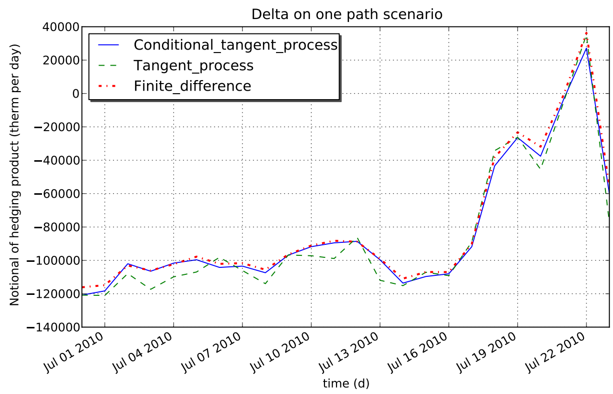


Fig. 11 Seasonal storage: Example of delta evolution for the weekly product (july 26th 2010- july 31th 2010) on one scenario.

ber 2010. The figure 13 compares tangent process and conditional tangent process

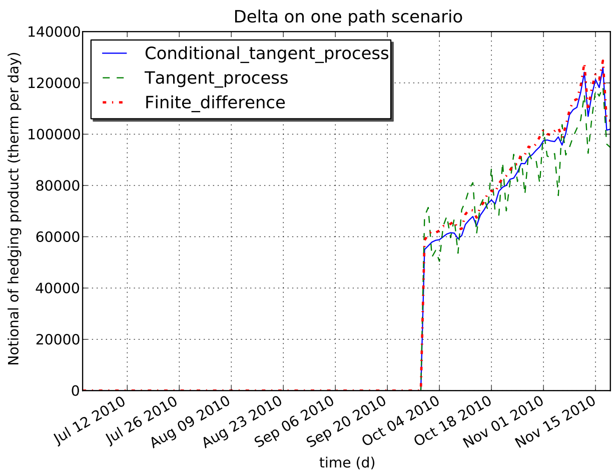


Fig. 12 Seasonal storage: Example of delta evolution for the monthly product december 2010 on one scenario

for the 2010 last quarter product, figure 14 for the year 2011 product. Due to time consideration, results with finite difference were only available for the week and the month product. The convergence for the week product of the three methods to

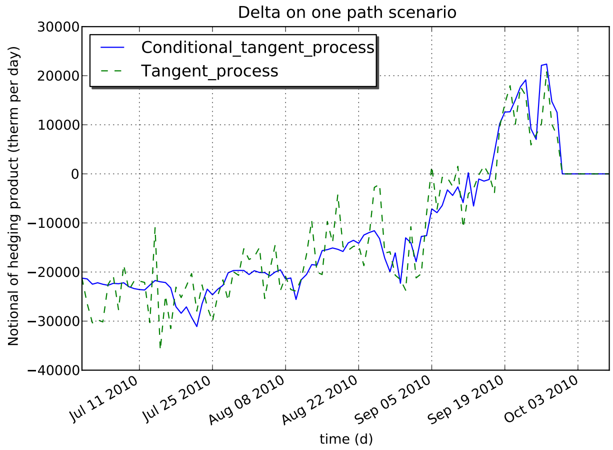


Fig. 13 Seasonal storage: Example of delta evolution for the quarter product Q4 2010 on one scenario

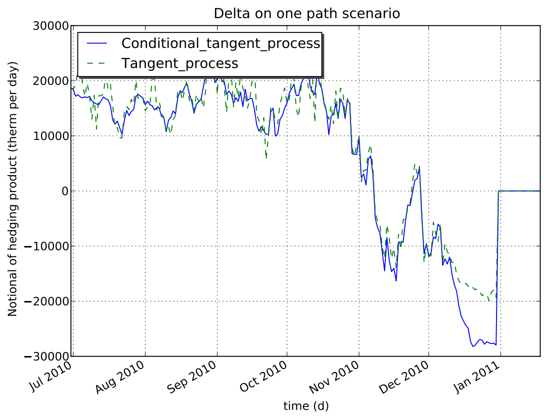


Fig. 14 Seasonal storage: Example of delta evolution for the year 2011 product on one scenario

the same solution is more easily obtained for the seasonal storage than for the fast storage. On the different figures, we see a kind of smoothing effect due to the con-

ditional tangent process method on the hedging strategy not so obvious on the fast storage results. As for the time needed for each method, the conditional delta took nearly two hours for valorization and 60 seconds for the simulation. As for tangent process, the simulation took 7 days for the seasonal storage and the hedge for all the products would have taken longer than a month for the finite difference method.
Once again a simulation phase with 10000 simulations taking 1400 seconds was achieved with the conditional delta method. It led to a reduction of the standard deviation of the results by more than a factor 8 from 9.60e+08 to 1.11e+08. Distribution of the cash flow with and without hedge is given in figure 15.

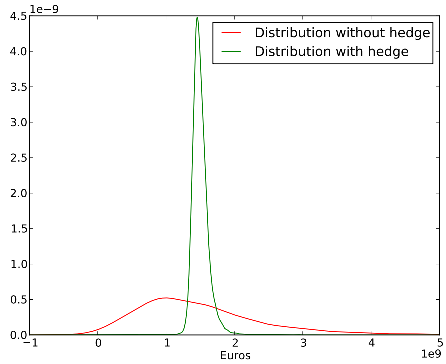


Fig. 15 Seasonal storage: cash flow distribution with and without hedge computed with the conditional tangent method

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