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A Principal–Agent approach to study Capacity Remuneration Mechanisms

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Abstract

We propose to study electricity capacity remuneration mechanism design through a Principal-Agent approach. The Principal represents the aggregation of electricity consumers (or a representative entity), subject to the physical risk of shortage, and the Agent represents the electricity capacity owners, who invest in capacity and produce electricity to satisfy consumers' demand, and are subject to financial risks. Following the methodology of Cvitanić et al. (2017), we propose an optimal contract, from consumers' perspective, which complements the revenue capacity owners achieved from the spot energy market, and incentivizes both parties to perform an optimal level of investments while sharing the physical and financial risks. Numerical results provide insights on the necessity of a capacity remuneration mechanism and also show how this is especially true when the level of uncertainties on demand or production side increases.

Key words. Capacities Market, Capacity Remuneration Mechanism, Principal-Agent problem, Contract Theory.

1 Introduction

Electricity market is characterized by the constraint that production must be equal to the consumption at any time. In case of non respect of this constraint, the system can incur a power outage whose consequences might be highly problematic. For example, the total economic cost of the August 2003 blackout in the US was estimated to be between $7 and $10 billion (see [Cou04]). This blackout resulted in the loss of around 62 GW of electric load that served more than 50 million people at the US-Canada border. Besides, it took 2 days for major affected areas to have the power restored, while some regions had to wait up to a full week.

As electricity can hardly be stored; hydro storage is limited in size, and developing a large fleet of batteries is still highly costly, the power production capacity must be high enough to cope with major peak load events which can reach extreme levels compared to the average load. In France for example the average load was around 55GW in 2017[1], whereas the peak of electricity consumption record was above 100 GW in February 2012. Indeed electrical load is characterized by a high variability implied by meteorological variations and economic conditions on different time scales. Again, in France for example, the difference

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[1]See the French TSO website http://bilan-electrique-2017.rte-france.com.

between peak load in 2012 and 2014 is around 20 GW, which corresponds to an equivalent capacity of around 40 Combined Cycle Gas Turbines (CCGT) of 500 MW[2]. To ensure security of supply, most electricity systems specify the *Loss of Load Expectation* (LoLE) which is a reliability target for the electricity system, and has been fixed in some countries at three hours at most per year (see for instance [New16]).

The consequence of such constraints on the production system is that some power plants are used rarely (only during extreme peak load) but remain necessary for the system security, and insuring their economical viability with energy markets only is not guaranteed. This question has already motivated a great amount of economic literature under the name of "missing money" (see for example [Jos06]). This lack of revenue can occur because of energy market imperfections such as price caps or out-of-market actions made by the transmission system operator as well as reliability targets going beyond reliability outcome provided by the market.

The "missing money" issue might be further increased when the share of renewable energies increases in the system (see [New16]). Indeed, renewable energies have low variable costs so their introduction has made electricity prices lower (see [Bro18] or [LB15] for a proof that subsidized renewable capacity pushes downward energy prices). This could result in the withdrawal of most expensive power plants, jeopardizing the security of electricity system and the lack of incentives to invest in new capacities.

In addition, even without the "missing money" problem, electricity markets are highly volatile for the reasons already stated (i.e. low level of storage, high uncertainties on load and production levels induced by outages and meteorological conditions impacting the production capacities) and suffer from *missing markets* issues (see [New16]) such as the horizon shortness of contracts proposed by electricity markets compared to the lifetime of power-plants. For all these reasons, the financial risk is particularly high for investors in electricity capacities and may lead to high hurdle rates (see [Hob+07] for a model which studies how capacity markets variations can lower capital costs for generators by reducing risks).

For all the reasons cited above, several regions of the world have decided to put in place a Capacity Remuneration Mechanism (CRM in the sequel), in addition to energy markets. This kind of markets aims at insuring a payment for electricity generating assets for the capacities they provide, regardless of their actual production. This market can be thought of as a payment for an insurance provided by the power plant against shortage and blackout risk. However, no consensus on the design of such CRM arose so far, see for example [Bub+19] for a review of theoretical studies and implementations of CRM or [Bha+17] for a survey of different capacity markets implemented in the US.

In [Sco18], the author argues that CRM do improve security of supply, in exchange of a significant impact on consumer's bill in the US, as opposed (surprisingly) to the EU where the impact on the end users price is not significant. An analysis of the impact of capacity on welfare under a price-capped electricity market is made on the Texas market (ERCOT) in [BB17] showing that capacity markets have several effect: an increase of the wholesale electricity price and reliability and a reduction of price volatility. Several mechanisms and their corresponding conditions for achieving efficiency are studied in [Léa16]. In [BK13], distortion of capacity markets implied by subsidies of base load capacities are pointed out and correction mechanisms such as the MOPR (Minimal Offer Price Rule) tested in PJM markets are studied. Currently in Europe, several designs of CRM have been adopted such as a market capacity for example in the UK, France,

[2]See `https://www.ecologique-solidaire.gouv.fr/securite-dapprovisionnement-en-electricite`.

Italy, Belgium and Ireland[3] (under construction); a capacity payment as in Spain or Portugal; strategic reserves in Sweden or in Germany.

In the literature, several papers study different CRM designs with distinct modeling approaches. For example in [HRS16], the authors compare the benefits of capacity markets or strategic reserves versus energy-only design in terms of security of supply, investment and generation costs in a dynamic model of investing. This is the same approach developed by [HCS16] and applied to the UK market. [Hös+17] analyse the impact of capacity mechanism on energy markets and on the remuneration of flexibility and emission-neutral renewable capacities. In [Bha+17], the authors implement the UK capacity markets in an Agent-based model where Agents have a limited vision of the future. In [HPP07], the authors model two CRMs–in particular under information asymmetry–using agency theory. They model capacity payments as a menu of contracts and strategic reserves as a retention rule of a bilateral contract between the TSO and a producer and then compare these CRMs. The information asymmetry is mainly on the "type" of the generator, namely its access to the capital market which impacts its efficiency.

In this work, we propose a Principal-Agent framework to shed light on the design of capacity remuneration mechanisms in a context of information asymmetry and external uncertainties (in production and demand). Using the recent developments of contract theory (see for instance [CPT18]), we model and solve the problem in a continuous time setting, which allows us to dive deeper into the incentive mechanism, and provide a recommended policy for investment in electric power plants, with an optimal dynamic capacity payment allowing for an efficient (financial and physical) risk sharing between consumers and producers.

In the scope of the paper, producers and retailers are fully separated and exchange electricity through spot markets and the electricity demand is considered to be inelastic. The relationship between consumers and producers is modeled by a Principal-Agent problem, with the Principal being the aggregation of power consumers (or an equivalent entity representing them), and the Agent the collection of producers. Note that the Transmission System Operator (TSO) which operates in real-time in many electricity markets could be considered as a representative entity of the aggregation of consumers.

Our model accounts for the information asymmetry on the actions of the Agent –moral hazard–, i.e., consumers do not observe producers' actions but only the results of their actions. In fact, consumers want to incentivize producers in an optimal way to provide electricity when needed, but at the same time they have no information on the commitment of the latter (producers) to build or maintain power plants, as they only observe the volume of electricity produced, not the effort of capacity owners to install new power plants or to keep the existing ones in good operation conditions. The model developed enables us to specify an optimal contract which incentivizes the producers to make the right level of investment to achieve a certain level of security for the system.

This proposed contract remunerates the capacity owners depending on realized uncertainties on the demand and available capacities while sharing the financial risks between consumers and producers. It is also shown that the more uncertainties on the system, such as the increase share of variable renewable capacities, the more a capacity remuneration is needed to ensure correct levels of investment and maintenance.

Finally, we provide a numerical illustration of the optimal capacity payment obtained with our proposed

[3] See for instance https://www.sem-o.com/markets/capacity-market-overview and https://ec.europa.eu/commission/presscorner/detail/en/IP_17_4944.

optimal contract, compared with the payment supplied by the spot market. This numerical illustration is inspired by the French electricity system.

The second section of this paper is devoted to the presentation of the model and the objective functions of both the Agent and the Principal, with a brief summary of the resolution methodology (the details of which are left to the Appendix). In the third section, we present our case study; the French electricity system and provide some numerical interpretations of the optimal capacity payment. Mathematical proofs and details are included in the Appendix for the sake of clarity.

2 The model

In order to study capacity remuneration mechanisms in a context of information asymmetry, we propose a non-zero sum Stackelberg game with a Principal-Agent formulation, i.e., the gain of one party does not come necessarily from the loss of the other. In this setting, the aggregation of consumers or an entity representing them such as the TSO, proposes to producers a capacity payment which optimally complements the revenue they (producers) obtain on the spot energy market. This payment incentivizes them to invest optimally in power plants management (construction, maintenance, etc..) to ensure an acceptable level (for consumers) of shortage occurrences. The proposed payment is a way to correct the information asymmetry faced by consumers (as they cannot observe directly producers decisions concerning the capacities of the production mix, thus the need for incentive), and to share the risks coming from demand and available capacities uncertainties between the two parties. Moreover, the proposed payment limits producers' potential abuse of market power. Indeed, without capacity payment, producers may decide to under-invest in order to obtain high remunerations from a spot market with more shortages and price spikes.

2.1 Principal-Agent Problem: a brief review

Contract theory, or Principal–Agent problem, is a classical moral hazard problem in microeconomics. The simplest formulation involves a controlled process X and two parties; the Principal (she) and the Agent (he). The controlled process is called output process and represents the value of the firm for example. Principal owns the firm, and delegates its management to Agent, i.e., the control of the output process. So she hires him at time $t = 0$ for the period $[0, T]$, in exchange for a terminal payment (a contract) ξ paid at time T, based upon the evolution of the output process during the contracting time period. In other words, ξ is an $\mathcal{F}_T$–measurable random variable[4], and thus can be a function of the firm value (a percentage of the final gain for example, or a function of the whole trajectory, etc..). However, Agent's effort is not observable and/or not contractible for Principal, which means that ξ cannot depend on the effort (work) of Agent, hence the moral hazard.

Each of the parties aims at maximizing his/her utility function. The Agent acts on the output process X via some control α (his management decision). He has to pay a cost $c^A(\alpha)$ as a function of his efforts, and expects a payment ξ from Principal at time T. He also has a reservation utility $U_A(\mathcal{R})$, to be thought of as a participation constraint, with $\mathcal{R}$ the cash equivalent of this constraint: Agent accepts the contract only if it satisfies $V^A(\xi) \geq U_A(\mathcal{R})$, otherwise he will refuse it. In the case where Agent accepts the contract, we can formulate his problem as follow:

[4] ξ is a function of the realized uncertainties on X up to time T.

$$V^A(\xi) \quad = \quad \sup_{\alpha} \mathbb{E}\left[U_A\left(\xi - \int_0^T c^A(\alpha_t)\, dt\right)\right]. \tag{2.1}$$

The Principal benefits from the output process X, so she incites Agent to put the effort α (to work hard) with the contract ξ. She tries to find the optimal incentive, while respecting his participation constraint. Principal's problem is written therefore as:

$$V^P = \sup_{\substack{\xi \\ V^A(\xi) \geq U_A(\mathcal{R})}} \sup_{\alpha^\star(\xi)} \mathbb{E}\left[U_P\left(-\xi + X_T^{\alpha^\star(\xi)}\right)\right], \tag{2.2}$$

where the contract ξ satisfies the participation constraint $V^A(\xi) \geq U_A(\mathcal{R})$ and $\alpha^\star(\xi)$ denotes Agent's optimal effort (response) given the contract ξ, i.e., the solution to (2.1). The first supremum (over $\alpha^\star(\xi)$) expresses the fact that given a contract ξ, Agent solves his problem (we will see later that the existence of at least one solution is guaranteed). Then in case of existence of multiple solutions to (2.1), we assume that Agent would choose the one which maximizes Principal's value function (once his utility maximized, Agent is cooperative with Principal). We refer the interested reader to [CZ13], and [CPT18] for a more detailed exposition of contract theory and Principal-Agent problem.

As mentioned before, in our model, Principal is the aggregation of consumers or an equivalent entity representing them, and Agent is the collection of producers. In the sequel we will ease the presentation by referring to these two parties by simply saying "the consumer" and "the producer".

So Agent ("he") is the producer who exerts an effort (a process which we will denote α), to build or invest in the maintenance of peak power plants, to increase the total capacity of the fleet. He is compensated an amount ξ by Principal ("she", the consumer) for the utility she receives; the satisfaction of consumption and the insurance against shortage risk. We also account for the moral-hazard (second best in Principal-Agent terminology), in the sense that effort performed by the Agent is not observable by Principal. Therefore, she doesn't observe α, and is not able to know if the available capacity is the result of decisions of maintenance and investments made by Agent, or if it is due to market conditions not controlled by the producer, such as unanticipated failures, good or bad weather conditions for renewable energy sources of production. In mathematical words, the capacity compensation ξ given to the producer, cannot be a function of the effort α.

2.2 Model, state variables and control

We fix a maturity $T \in (0, +\infty)$, and describe the system with two continuous processes X^C and X^D, denoting respectively the electricity generation capacity available at each time t and the instantaneous electricity demand, both in GigaWatt (GW from now on). X^C represents the aggregation of all production capacities, regardless of the corresponding production technology. The uncertainty of X^C represents the power outages of conventional power plants and the variability of the availability factor of renewable productions. We denote the state variable $X := \left(X^C, X^D\right)^T$, a stochastic process valued in $\mathbb{R}^2$.[5]

[5]We will work on the space of processes valued in $\mathbb{R}^2$, but our model will ensure that X takes only positive values, i.e., in $\mathbb{R}^2_+$.

Agent (electricity producers) controls the generation capacity X^C via an $\mathbb{F}$–predictable process[6] The control α is expressed as a yield in $[\text{Year}]^{-1}$ and represents the decision at each time t to change the generation capacity by building or dismantling peak power plants; gas turbines for instance. This restriction in the choice of only one technology for the control (maintaining/building or destroying) simplifies parameters calibration and allows for the use of a continuous time setup. Numerically, this is approximated by a small-step discretization as peak power plants are quite rapidly adjustable. The control is only on the average value of the available capacity and not the volatility, which spares us a lot of technicalities. Indeed, one could expect that investing in wind power or solar panels would increase the uncertainty of available generation capacity (volatility), as opposed to thermal plants which have a more controllable production.

The instantaneous available capacity process X^C is driven by a controlled geometric Brownian motion, which has the property of staying positive consistent with available capacity. X^C starts from x_0^C and has the infinitesimal increments over dt:

$$dX_t^C = \alpha_t X_t^C dt + \sigma^C X_t^C dW_t^{C,\alpha}, \text{ for } t \text{ in } [0,T],$$

where $\alpha_t X_t^C dt$ is the variation on average capacity implied by the effort α and $\sigma^C X_t^C dW_t^{C,\alpha}$ is the stochastic part in the available capacities due to uncertainties, with $\sigma^C > 0$ the volatility parameter. Remark that we overlook ageing and deterioration in our model, which is justified by taking a short maturity T compared to the average lifetime of power plants.

The demand X^D is modeled as the exponential of a mean reverting process to ensure that $X_t^D > 0$, for t in $[0,T]$, and that the demand oscillates around some average level. The initial condition is fixed as $X_0^D = x_0^D$, and the infinitesimal variation over dt is modeled by:

$$d\log\left(X_t^D\right) = \mu^D\left(m^D - \log\left(X_t^D\right)\right)dt + \sigma^D dW_t^D, \text{ for } t \text{ in } [0,T].$$

The term $\sigma^D dW_t^D$ is the random part in the variation with $\sigma^D > 0$, and $m^D \in \mathbb{R}$ the long term average (of $\log\left(X_t^D\right)$) and $\mu^D > 0$ the speed of mean reversion.

We denote by $\mathcal{U}$ the set of admissible control processes α, i.e., the set of $\mathbb{F}$–predictable processes valued in $\mathbb{R}$ satisfying some integrability conditions[7], and we write in a more compact form the dynamic of the state variables

$$X_t = x_0 + \int_0^t \mu\left(X_s, \alpha_s\right) ds + \int_0^t \sigma\left(X_s\right) dW_s^\alpha, \ \forall t \in [0,T], \tag{2.3}$$

with α the control process, $x_0 \in \mathbb{R}_+^2$ a fixed initial condition and

$$\mu(x,\alpha) := \tilde{\mu}(x) + \begin{pmatrix} \alpha x^C \\ 0 \end{pmatrix}, \text{ and } \sigma(x) := \begin{pmatrix} \sigma^C x^C & 0 \\ 0 & \sigma^D x^D \end{pmatrix},$$

with

$$\tilde{\mu}(x) := \begin{pmatrix} 0 \\ \left(\mu^D\left(m^D - \log\left(x^D\right)\right) + \frac{\sigma_D^2}{2}\right) x^D \end{pmatrix},$$

[6]Roughly speaking, at each time $t \in [0,T]$ the control is only based on information prior to t without knowledge of the future. α valued in $\mathbb{R}$, where $\mathbb{F}$ is the completed filtration generated by X.

[7]A rigorous definition of $\mathcal{U}$ is provided in Appendix 5.1, along with the weak formulation.

or equivalently

$$\begin{pmatrix} X_t^C \\ \log\left(X_t^D\right) \end{pmatrix} = \begin{pmatrix} x_0^C \\ \log\left(x_0^D\right) \end{pmatrix} + \int_0^t \begin{pmatrix} \alpha_s X_s^C \\ \mu^D\left(m^D - \log\left(X_s^D\right)\right) \end{pmatrix} ds + \int_0^t \begin{pmatrix} \sigma^C X_s^C & 0 \\ 0 & \sigma^D \end{pmatrix} dW_t^\alpha.$$

Remark 2.1. *In reality, the processes X^C and X^D exhibit a strong seasonal behavior (annual, weekly and daily patterns). These seasonalities are explained by patterns of electricity consumption in day to day life and the weather conditions (heating in winter, solar production in the day ...). For expository purposes, we consider a deseasonalized version of state variables. The aim of this simplification is to focus the analysis on random consumption peaks, and how they should be dealt with, as opposed to seasonal variations which can be anticipated.*

Remark 2.2. *Power demand is considered to be inelastic with respect to electricity prices.*

2.3 Spot energy payment

Without capacity payment, the only transaction between consumers and producers would have been the reward for energy production. We assume that this reward corresponds to the spot price of electricity. Therefore, for a time interval $[0, T]$, the consumer pays for his consumption the amount

$$S_T := \int_0^T P\left(X_t\right) X_t^C \wedge X_t^D dt,$$

and the producer receives S_T, where $P : \mathbb{R}^2 \to \mathbb{R}_+$ is the spot price function which we define as[8]

$$P\left(X\right) := \beta_0 e^{-\beta_1\left(X^C - X^D\right)} \mathbf{1}_{\{\|X\| < x_\infty\}}, \text{ with } \beta_0, \beta_1, x_\infty > 0. \tag{2.4}$$

Note that the spot energy payment only accounts for delivered energy, i.e., the minimum between the demand and available capacity; the requested power X^D in standard situations, and just the available capacity X^C in the case of a shortage.

Different choices of electricity spot price functions can be found in [Aïd15] or [Aïd+09] and our model is directly inspired by them. In particular, the function P in (2.4) captures a key feature in our problem: the relationship between the spot price and the residual capacity ($X^C - X^D$). In the sequel, we will call S_T the spot payment. Remark that S_T represents a cost for the Principal and a reward for the Agent.

2.4 Producer's problem

Electricity producer is the Agent, and provides consumer with electricity, for a terminal payment $\xi + S_T$. He is in charge of choosing the investment policy in power plants via the process α, and is subject to its costs.

We model his costs as a quadratic function of state variables. More specifically, we define Agent's instantaneous cost as

$$c^A(x, \alpha) := \tilde{c}^A(x) + \kappa_1(\alpha x^C) + \kappa_2 \frac{(\alpha x^C)^2}{2}, \text{ for } (x, \alpha) \in \mathbb{R}_+^2 \times \mathbb{R}, \text{ and } \kappa_1, \kappa_2 > 0.$$

[8] The parameter x_∞ is a positive large constant and the term $\mathbf{1}_{\{\|x\| < x_\infty\}}$ is used only to insure integrability conditions in our problem; x_∞ can be set arbitrarily large without impacting the results and so does not imply (in practice) the boundedness of capacity or demand.

The term $\kappa_1(\alpha x^C) + \kappa_2 \frac{(\alpha x^C)^2}{2}$ is the cost of building or dismantling peak power plants, where κ_1 is the cost per unit, and κ_2 is a penalization adjustment term[9]. We define then

$$\tilde{c}^A(x) := \left(ax^C + b(x^C \wedge x^D)\right) \mathbf{1}_{\{\|x\| < x_\infty\}}, \text{ with } a, b > 0,$$

where the first term is the cost of maintenance and the second models the variable cost of production. These variable costs of production are proportional to the available generation capacity x^C and the minimum between this capacity and demand $x^C \wedge x^D$.

During the time period $[0, T]$, producer provides electricity to consumer, and receives the payment $\xi + S_T$ at time T. The amount ξ represents the payment producer receives for the availability of his capacity in addition to what he receives as a spot payment S_T.

To include the moral hazard in our problem, we will need to use the *weak formulation* and so we introduce $\mathbb{P}^\alpha$, the law of the process X, weak solution of the stochastic differential equation (2.3) corresponding to a control process α, and $\mathcal{P}$ the set of probability measures $\mathbb{P}^\alpha$.
The producer's objective function or his average perceived utility is defined as

$$J_0^A(\xi, \mathbb{P}^\alpha) := \mathbb{E}^{\mathbb{P}^\alpha}\left[U_A\left(\xi + \int_0^T P(X_t)\, X_t^C \wedge X_t^D dt - \int_0^T c^A(X_t, \alpha_t)\, dt\right)\right], \text{ for } (\xi, \mathbb{P}^\alpha) \in \Xi \times \mathcal{P}, \tag{2.5}$$

for a given contract $\xi \in \Xi$[10] and a choice of $\alpha \in \mathcal{U}$ to which we associate the probability measure $\mathbb{P}^\alpha \in \mathcal{P}$, and where U_A is a utility function expressing the risk aversion; increasing and concave. For tractability, we choose an exponential utility function, $U_A(x) := -\exp(-\eta_A x)$ with $\eta_A > 0$, the Agent's risk aversion.

The producer is encouraged to provide enough capacity, otherwise the consumer would reduce his payment ξ. The moral hazard is modeled by adding the restriction that the payment ξ is a function only of X, not α or $\mathbb{P}^\alpha$. In technical words, ξ is $\mathcal{F}_T$–measurable[11]. That is to say the consumer only observes the state variables X as stochastic processes, and has no access to the control α or $\mathbb{P}^\alpha$ and so she cannot see if the randomness of X is coming from external uncertainties or from producer's actions. Formally, the producer control the *law* $\mathbb{P}^\alpha$ of the process X, i.e., the probability of having some trajectories rather than others, and the consumer observes the realized trajectory and fixes the payment ξ as a function of X.[12]

In addition, the producer has a participation constraint $U_A(\mathcal{R}) \in \mathbb{R}_-$, with $\mathcal{R} \geq 0$ its cash equivalent. Thus he will accept the contract only if he can expect to retrieve from it a utility above the level $U_A(\mathcal{R})$. Indeed, the producer has no obligation to accept the contract and is free to refuse it before the start of the time period $[0, T]$.

[9] The parameter κ_2 is useful for computation since it adds convexity to the cost function and brings us to the familiar linear-quadratic framework. We can also provide an economical interpretation to this term as the quadratic cost of construction, since the marginal cost of building at a given time step is increasing.

[10] A rigorous definition of the set of admissible contracts Ξ will be provided in the next subsection.

[11] $\mathcal{F}_T$ models the information gathered from the observation of the process X up to time T (see Appendix 5.1). So ξ is a function of this information.

[12] An important feature of our model is that the structure of the contract is defined *ex-ante* while its exact value is provided only *ex-post* depending on the realized uncertainties.

Whenever he accepts a given contract ξ, the producer wants to make the optimal investment by choosing an appropriate control $\mathbb{P}^\alpha$. He solves the problem

$$V_0^A(\xi) := \sup_{\mathbb{P}^\alpha \in \mathcal{P}} J_0^A(\xi, \mathbb{P}^\alpha). \tag{2.6}$$

An Agent's control $\mathbb{P}^{\alpha^\star} \in \mathcal{P}(\xi)$ (or equivalently $\alpha^\star(\xi)$) is said to be optimal if it satisfies

$$V_0^A(\xi) = J_0^A\left(\xi,\ \mathbb{P}^{\alpha^\star}\right).$$

We denote by $\mathcal{P}^\star(\xi)$ the set of Agent optimal controls for some admissible contract ξ.

2.5 Consumer's problem

The consumer ("she") buys electricity from the producer during the time period $[0, T]$. She pays for the energy she consumes at the spot price $\int_0^T P(X_t)\, X_t^C \wedge X_t^D dt$, and a capacity remuneration given by the contract ξ. She gets an instantaneous utility from her electricity consumption, and a disutility in case of shortage. We therefore model her overall instantaneous utility as

$$c^P(x) := \left(\theta\left(x^C \wedge x^D\right) - k\left(x^D - x^C\right)^+\right) \mathbf{1}_{\{\|x\| < x_\infty\}} \text{ with } \theta, k > 0.$$

The first term represents consumers' reservation value or their willingness to pay; the larger θ, the more valuable consumption to the consumers. In the literature, θ is often set using the Value of Lost Load (VOLL) for the case of perfect elastic demand (see [Fab18]).

The second term can be thought of as the "cost of shortage/blackout" which is negative if the available capacity is less than the demand level, i.e., the disutility which is triggered when the residual capacity $(x^C - x^D)$ is negative. Depending on how the coefficient k is defined, this term represents the disutility caused only by a shortage ($k =$ VOLL for the case of inelastic demand) or the disutility integrating also a higher risk of total blackout ($k >$ VOLL). This term plays a role of "punishment"–via the contract– for producer whenever he fails to provide sufficient generation capacity to cover the instantaneous demand.

Altogether, consumer's objective function or expected utility is defined as

$$J_0^P(\xi, \mathbb{P}^\alpha) := \mathbb{E}^{\mathbb{P}^\alpha}\left[U_P\left(-\xi - \int_0^T P(X_t)\, X_t^C \wedge X_t^D dt + \int_0^T c^P(X_t)\, dt\right)\right], \text{ for } (\xi, \mathbb{P}^\alpha) \in \Xi \times \mathcal{P}, \tag{2.7}$$

with U_P denoting Principal's utility function, similar to Agent's utility function with a risk aversion η_P;

$$U_P(x) := -\exp(-\eta_P x) \text{ with } \eta_P > 0.$$

Principal's goal is to choose the optimal incentive (payment) for the Agent to make an optimal effort. Her problem is written

$$V_0^P := \sup_{\xi \in \Xi} \sup_{\mathbb{P}^\alpha \in \mathcal{P}^\star(\xi)} J_0^P(\xi, \mathbb{P}^\alpha), \tag{2.8}$$

i.e., given the optimal response of the Agent to the compensation scheme, the Principal chooses the best contract which serves in her own interest. Furthermore, (as stated earlier) we have assumed that when given different optimal controls, Agent will choose the one that maximizes Principal objective function, which is a standard assumption in contract theory, (see for instance [HM87], [San08] or [CZ13]).

We can finally provide a natural definition of Ξ, the set of admissible contracts as the $\mathcal{F}_T$–measurable random variables ξ satisfying

$$\begin{cases} V_0^A(\xi) \geq U_A(\mathcal{R}), \\ \mathcal{P}^\star(\xi) \neq \emptyset, \\ \sup_{\mathbb{P}^\alpha \in \mathcal{P}} \mathbb{E}^{\mathbb{P}^\alpha}\left[e^{-\eta_A \xi}\right] + \sup_{\mathbb{P}^\alpha \in \mathcal{P}} \mathbb{E}^{\mathbb{P}^\alpha}\left[e^{\eta_P \xi}\right] < \infty. \end{cases} \tag{2.9}$$

This definition of admissible contract imposes that participation constraint is satisfied, the existence of an optimal control for Agent, and non-degeneracy conditions for both Principal and Agent problems[13].

2.6 Optimal contract and capacity payment

To find producer's and consumer's optimal policy, we follow the approach presented in [CPT18] for Principal-Agent problems. We start by considering a special class of contracts; the "revealing contracts" as capacity payments. These contracts satisfy the incentive compatibility property, which means that consumer provide them with a recommended policy (or effort) for producer, and producer's optimal response to these contracts corresponds to the recommended effort. Therefore, the consumer can identify producers' response and are able to maximize their utility by choosing the response which fits them best.

We next use a representation result to prove that any contract can be represented as "revealing", and thus there is no loss of generality or utility for consumers in optimizing only over such contracts.

In mathematical words we solve the problem for contracts which can be written as a terminal value of a (special) controlled forward stochastic differential equation (SDE) designed to make the Agent's response "predictable". Then we prove that we can associate to any admissible contract such a controlled SDE, obtained by solving an appropriate backward SDE.

The revealing contracts are introduced via a change of control variables. Instead of controlling the terminal payment, the Principal considers the contract as an additional state variable, and controls its initial level and the increments linear in the state variable. The class of *revealing contracts* $\mathcal{Z}$ is then defined as

$$\mathcal{Z} := \left\{ Y_T^{Y_0,Z} \text{ for some } (Y_0, Z) \in \mathbb{R} \times \mathcal{V} \text{ with } Y_T^{Y_0,Z} \in \Xi \right\}, \tag{2.10}$$

where $\mathcal{V}$ is the set of $\mathbb{F}$-predictable processes Z valued in $\mathbb{R}^2$ satisfying some integrability conditions[14] and

$$Y_t^{Y_0,Z} := Y_0 + \int_0^t Z_s \cdot dX_s - \int_0^t H(X_s, Z_s)\, ds, \text{ for all } t \in [0,T], \tag{2.11}$$

with H the Agent's *Hamiltonian*, a classical function in control theory defined by

$$H(x,z) := \sup_{\alpha \in \mathbb{R}} h(x,z,\alpha), \quad \text{for } (x,z) \in \mathbb{R}_+^2 \times \mathbb{R}^2, \tag{2.12}$$

[13]See Appendix 5.2.

[14] Detailed in the Appendix 5.3.1.

and $h : \mathbb{R}_+^2 \times \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}$ defined as

$$h(x, z, \alpha) := z \cdot \mu(x, \alpha) + P(x) x^C \wedge x^D - c^A(x, \alpha) - \frac{\eta_A}{2} |\sigma(x) z|^2, \text{ for } (x, z, \alpha) \in \mathbb{R}_+^2 \times \mathbb{R}^2 \times \mathbb{R}. \tag{2.13}$$

We will sometimes abuse notations and say that $(Y_0, Z) \in \mathcal{Z}$, meaning that $Y_T^{Y_0,Z} \in \mathcal{Z}$. For completeness, we provide a formal derivation of the class of revealing contracts in Appendix 5.3.2.

We denote by $\hat{\alpha} : \mathbb{R}_+^2 \times \mathbb{R}^2 \to \mathbb{R}$ the maximizer of h which can be easily computed

$$\hat{\alpha}(x, z) := \frac{z^C - \kappa_1}{x^C \kappa_2}, \text{ for } (x, z) \in \mathbb{R}_+^2 \times \mathbb{R}^2 \text{ and } x^C > 0. \tag{2.14}$$

where x^C (respectively z^C) denotes the first component of x (respectively z). The function $\hat{\alpha}$ will be called the *recommended effort* in the sequel (Sannikov's words, see [San08]).

Remark that the process $(Y_t^{Y_0,Z})_{t \in [0,T]}$ depends only on observations on X (consumption and available capacities) which are observable by Principal, as opposed to the effort α and the Brownian motion W^α. This is consistent with the moral-hazard of this problem.

The revealing contracts class $\mathcal{Z}$ plays a central role in Principal-Agent problems. Not only does it allow Principal to predict Agent's optimal control, but also to overcome the main difficulty; the *non-Markovianity* of ξ, i.e., the dependence of the payment on the whole paths of the demand X^D and the capacity X^C.

We can interpret the revealing contract as a performance index, closely related to *Agent's continuation value*, which comes with a *recommended effort* $\hat{\alpha}$ defined in (2.14); recall that the actual effort provided by Agent is neither observable, nor contractible, and therefore Principal can only propose $\hat{\alpha}$ as a recommendation and not an obligation.

Nevertheless, proposition 2.3 proves that whenever Agent (producer) is rational -which is a reasonable assumption- he will follow the recommended effort since it allows him to maximize his expected utility, and so Principal (consumer) can predict his effort. Furthermore, Theorem 2.4, identifies $\mathcal{Z}$ to Ξ, meaning that any admissible contract can be represented as a revealing one. We present these two results and provide their proofs in Appendix 5.5 and 5.7.

Proposition 2.3. *Under contracts in the class $\mathcal{Z}$ (any admissible contract as we will see in Theorem 2.4), producer's optimal effort is the same as the recommended effort by consumer, and his value function depends only on the initial value of the process $Y_T^{Y_0,Z}$, i.e.,*

$$V_0^A\left(Y_T^{Y_0,Z}\right) = U_A(Y_0), \text{ and } \alpha_t^\star = \hat{\alpha}(X_t, Z_t) = \frac{Z_t^C - \kappa_1}{\kappa_2 X_t^C}, \text{ for all } t \in [0,T]. \tag{2.15}$$

Theorem 2.4. *Let $\xi \in \Xi$. Then there exists a pair $(Y_0, Z) \in \mathcal{Z}$ such that*

$$\begin{cases} Y_T^{Y_0,Z} = \xi, \\ dY_t^{Y_0,Z} = Z_t \cdot dX_t - H(X_t, Z_t)\, dt. \end{cases} \tag{2.16}$$

In particular, we have $\mathcal{Z} = \Xi$, and therefore

$$\begin{aligned} V_0^P = & \sup_{\xi \in \Xi} \sup_{\mathbb{P}^\alpha \in \mathcal{P}^\star(\xi)} J_0^P\left(\xi, \mathbb{P}^\alpha\right), \\ = & \sup_{Y_0 \geq \mathcal{R}} \sup_{Z \in \mathcal{V}} \sup_{\mathbb{P}^\alpha \in \mathcal{P}^\star\left(Y_T^{Y_0,Z}\right)} J_0^P\left(Y_T^{Y_0,Z}, \mathbb{P}^\alpha\right). \end{aligned}$$

The main conclusion here is that there is no loss of generality in restricting consumer's problem to contracts in $\mathcal{Z}$. So she only needs to maximize her objective function over the set $\mathcal{Z}$ (with the two new control variables Y_0 and Z). This allows us to use standard tools of markovian stochastic control to tackle this problem. Once this is done, Principal's optimal control can be decomposed in different parts as in Proposition 2.5.

Proposition 2.5. *Principal's optimal contract $\xi^\star$ can be written as $\xi^\star := Y_T^{\mathcal{R},Z^\star}$ with the following decomposition:*

$$Y_T^{\mathcal{R},Z^\star} = \mathcal{R} + \int_0^T Z_t^\star \cdot dX_t - \int_0^T H\left(X_t, Z_t^\star\right) dt,$$

or equivalently,

$$\begin{aligned} Y_T^{\mathcal{R},Z^\star} + \int_0^T P\left(X_t\right) X_t^C \wedge X_t^D dt = \mathcal{R} + \int_0^T c^A\left(X_t, \hat{\alpha}\left(X_t, Z_t^\star\right)\right) dt \\ + \int_0^T Z_t^\star \cdot \sigma\left(X_t\right) dW_t^{\mathbb{P}^{\hat{\alpha}}} + \frac{\eta_A}{2} \int_0^T \left|\sigma\left(X_t\right) Z_t^\star\right|^2 dt. \end{aligned} \tag{2.17}$$

with $Z_t^\star$ computed using standard tools of stochastic control, and given by (5.17) below, and $\hat{\alpha}$ the recommended effort function defined by (2.14).

This optimal contract consists in a terminal payment to the producer of the random amount $Y_T^{\mathcal{R},Z^\star}$, which incites him to follow the recommended effort. Note that any different effort would be sub-optimal in terms of utility by Proposition 2.3.

Remark 2.6. *We can make some observations on the remuneration of the producers:*
(i) $Y_T^{\mathcal{R},Z^\star}$ *covers all the costs the producer has to pay to follow the recommended capacity policy (2.14) and to produce electricity to match the demand. Therefore, the optimal contract compensates those costs taking into account what the producer is earning on the spot market. We recall below producer's costs*[15]

$$\begin{aligned} \underbrace{\int_0^T c^A\left(X_t, \hat{\alpha}\left(X_t, Z_t^\star\right)\right) dt}_{\textit{Producer's costs}} = \underbrace{\int_0^T \kappa_1 \hat{\alpha}\left(X_t, Z_t^\star\right) X_t^C + \kappa_2 \frac{\left(\hat{\alpha}\left(X_t, Z_t^\star\right) X_t^C\right)^2}{2} dt}_{\textit{Construction costs}} \\ + \underbrace{\int_0^T a X_t^C dt}_{\textit{Maintenance costs}} + \underbrace{\int_0^T b(X_t^C \wedge X_t^D) dt}_{\textit{Production costs}}, \end{aligned}$$

(ii) $Y_T^{\mathcal{R},Z^\star}$ *shares the risk (realized uncertainties on demand and capacity) between producers and consumers, by transferring part of the randomness to the Agent, while providing him with a risk compensation at the*

[15] We omit the indicator function for the sake of clarity.

same time, to overcome his risk-aversion:

$$\textit{Risk part} = \underbrace{\int_0^T Z_t^\star \cdot \sigma(X_t)\, dW_t^{\hat{\alpha}}}_{\textit{Risk shared}} + \underbrace{\frac{\eta_A}{2}\int_0^T |\sigma(X_t) Z_t^\star|^2 dt}_{\textit{Risk compensation}},$$

(iii) $Y_T^{\mathcal{R},Z^\star}$ *is a random variable which depends on the scenario. In particular, its value changes as the uncertainties change, and might even become negative. This means that Agent might earn less or more than the total costs he has to bear, depending on the outcome of uncertainties (for example very sunny or windy years might lead to low spot prices and therefore to a higher capacity remuneration). Nevertheless, in expectation, he is guaranteed to earn* $\mathcal{R}$*; the cash equivalent of his reservation utility.* (iv) *We can rewrite the decomposition* (2.17) *as follow:*

$$\textit{Capacity remuneration} + \textit{Spot compensation} = \mathcal{R} + \textit{Producer's costs} + \textit{Risk shared} + \textit{Risk compensation}. \tag{2.18}$$

2.7 Producer's participation constraint: the problem without capacity payment

In absence of a capacity payment, producer's only income is the spot compensation and therefore his problem is a standard Markovian stochastic control problem:

$$\hat{V}_0^A := V_0^A(0) = \sup_{\mathbb{P}^\alpha \in \mathcal{P}} \mathbb{E}^{\mathbb{P}^\alpha}\left[U_A\left(\int_0^T P(X_t)\, X_t^C \wedge X_t^D dt - \int_0^T c^A(X_t, \alpha_t)\, dt\right)\right], \text{ for } \mathbb{P}^\alpha \in \mathcal{P}. \tag{2.19}$$

In this case, the consumer has no bargaining power and no control on investment decisions on capacity, and is then subject to shortage risk. The producer does not care anymore about consumer's value function, and there is no guarantee that a criteria such as the LoLE[16] constraint is satisfied.

Because of the structure of the spot function (decreasing in $x^C - x^D$), it turns out that the producer makes a compromise between having few installed capacities (less than in the case with a capacity payment) to save maintenance costs and increase the spot prices, and enough capacities to satisfy to earn more on the spot market (since he can only sell the energy he produces). Remark that this kind of arbitrage can be seen in practice even among producers in perfect competition.

The problem (2.19) is solved in Appendix 5.6 using standard stochastic control tools, and provides a good proxy for producer's participation constraint, which we define as follow :

$$\mathcal{R} := U_A^{-1}\left(\hat{V}_0^A\right) \vee 0. \tag{2.20}$$

The maximum is taken in the previous equation between 0 and $U_A^{-1}\left(\hat{V}_0^A\right)$ as the producer has two choices: he can decide whether to operate his power plants and earn the spot price which provides him with a utility of $\hat{V}_0^A$, or if he sees that $U_A^{-1}\left(\hat{V}_0^A\right) < 0$, then he would stop all his activity which would lead him to 0 earnings (assuming that we neglect any agency costs related to bankruptcy).

[16]Recall that the Loss of Load Expectation (LoLE) is the targeted maximum number of hours of shortage per year, set at 3 hours per year for most European countries.

3 Numerical results and interpretations

In this section our model is numerically solved for a stylized system, based on the French electricity power system. We implement the optimal capacity contract and optimal policy, by numerically solving the PDE (5.15) describing consumer's value function, with parameters calibrated on the French power system. Then we observe multiple scenarios and the evolution of state variables under this policy.

3.1 Case study: the French power system

We consider a time horizon $T = 5\,$[Years], and we discretize it with a time step $\Delta t = \frac{1}{400}\,$[Years] for the diffusion of state variables, roughly speaking, over one time step per day.
The state variables X^C and X^D are expressed in GW, and the contract and costs (quantities inside of the utility function) are in 10^6€, [M€] in the sequel. As stated earlier, X^C is the instantaneous overall available capacity, and the control is only on the peak power plants which are assumed to be gas turbine.

3.1.1 Capacity and demand

We use the generation capacity, demand and spot prices available online[17] for our calibration. We start by calibrating the parameters of SDE (2.3) modeling the dynamics of X^C and X^D. As mentioned in Remark 2.1, we only consider deseasonalized state variables in our model. Therefore, to deseasonalize the input data, we use a locally weighted scatterplot smoothing algorithm implemented in the software R; the function "STL" which decomposes time series into three components: a trend, a seasonal component and a residual noise. This algorithm extracts the trend by averaging locally, then computes the seasonality on residuals by averaging across a given frequency. Once the seasonal component is computed, it is substracted from the original time series to get the deseasonalized data. We apply this procedure twice; once for the annual seasonality and another time for the weekly seasonality.

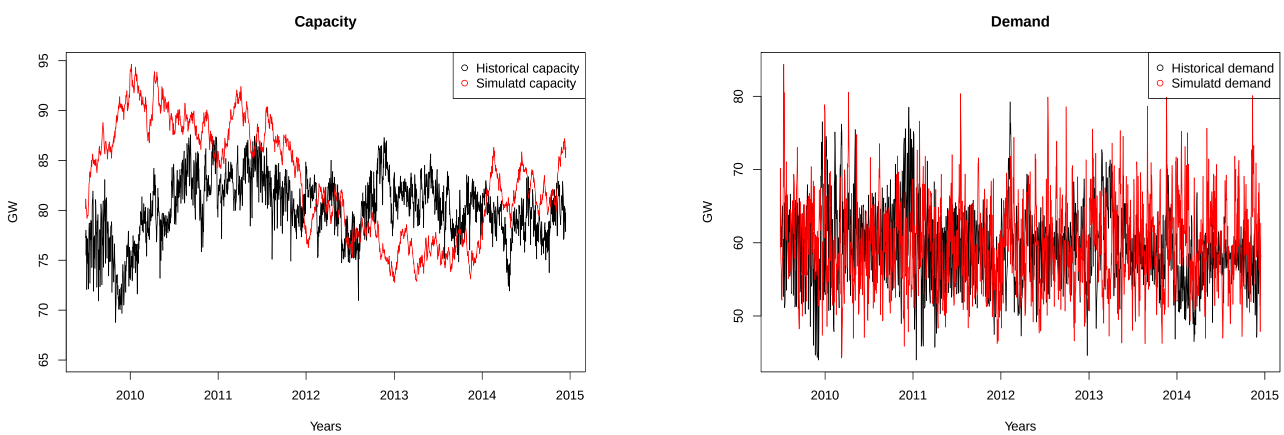
The demand X^D modeled as the exponential of an Orstein–Uhlenbeck process is calibrated by linear regression of the returns of daily data fixed at 7 p.m., the hour of the day with the highest demand.
As for the capacity X^C, we take the daily sum of the different generation technology capacities: nuclear, gas, coal, fuel, hydro-power (reservoir and run-of-the-river) and then σ^C is calibrated as to have simulated trajectories with similar behaviour with historical (observed) capacity data.

Table 1 summarizes our estimated parameters for capacity and demand processes, and we can see in figure 1 a comparison between historical (deseasonalized) data with generated scenarios of demand and capacity with our calibrated parameters.

[17]For the time period 29/06/2009-15/12/2014, available in the French TSO RTE website for capacity and demand `https://clients.rte-france.com`, and the EPEX SPOT website `https://www.epexspot.com` for spot prices. Remark that we stop at 2014 because the latest available capacity data is provided in that year, as the French TSO stopped publishing available capacity records on an aggregated basis per technology. Nevertheless, we assume that the uncertainties on capacity remain unchanged and use this data for our current calibration as the French production mix did not change a lot in that period.

	Parameter	Value	Unit
Available generation capacity	x_0^C	90	[GW]
	σ^C	0.1	$[\text{Year}]^{-\frac{1}{2}}$
Demand	x_0^D	60	[GW]
	μ^D	61.92	$[\text{Year}]^{-1}$
	$\exp\left(m^D\right)$	60	[GW]
	σ^D	0.86	$[\text{Year}]^{-\frac{1}{2}}$

Table 1: Calibrated parameters for capacity and demand.



(a) Capacity process

(b) Demand process

Figure 1: Comparison between historical data and simulated processes.

3.1.2 Spot price function

We calibrate our spot price function P defined in (2.4) using historical data, and taking one price per day, at 7 p.m.; the same as for demand data (the hour of daily demand peak). The calibration is simply done by taking the log of the time series, and then applying a linear regression. Remark that the spot price function P is completely characterized by the capacity and demand, so the seasonality is naturally accounted for. Table 2 summarizes our choice of the spot price function P with its calibrated parameters, and the figure 2 represents a comparison between historical and simulated spot prices.

	Model: $\frac{[\text{M€}]}{[\text{GW}][\text{Year}]}$	Parameter	Value	Unity
Spot price function	$P(x) = \beta_0 e^{-\beta_1 \left(x^C - x^D\right)}$	β_0	102.8	[€/MWh]
		β_1	335.3×10^{-4}	$[\text{GW}]^{-1}$

Table 2: Spot price model and its calibrated parameters.

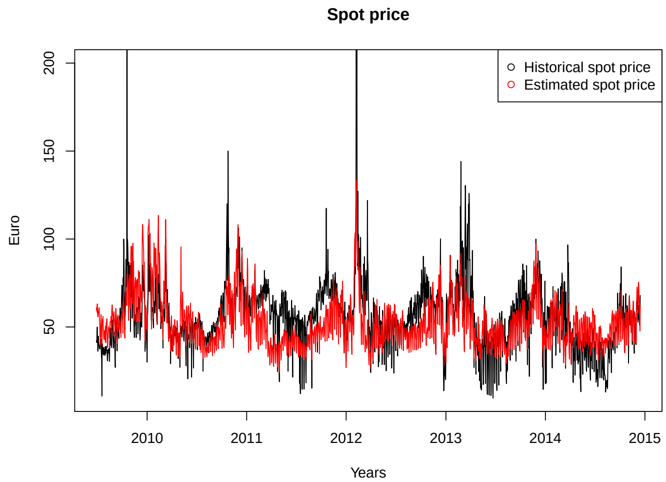


Figure 2: Historical and estimated spot prices.

Remark that our model reproduces quite well the behaviour of spot prices but without the largest peaks. This is coherent with a market with a low price cap (a price cap which can be seen for example in many European countries).

3.1.3 Costs

Electricity supplier has to take into account the construction, maintenance and production costs of different power plants. These costs are provided by the French TSO and WEO 2018[18]. The cost of maintenance a and the cost of production b in [€/MWh] are estimated as weighted averages between the different costs of technologies, where the weights used for maintenance are the same as for the installed capacities, and those for production costs are taken as the proportions of production; cf Table 3.

Technology	Installed capacities	Percentage of production
Nuclear	48 %	72 %
Coal	2 %	1 %
Gaz and Fioul	8 %	4 %
Wind turbines and Photovoltaic	18 %	7 %
Hydropower	19 %	12 %
Bioenergy	6 %	4 %

Table 3: Different technologies and their wheights

The cost of construction per unit κ_1 in [€/MWh] is taken as the Equivalent Annual Cost of a gas

[18]See the *"WEO 2018 report"* and *"Impact assessment of the French Capacity Market, 2018."* by the French TSO RTE.

turbine power plant[19] instead of a weighted average since only peak power plants are used for the control (construction or dismantling), and the adjustment coefficient κ_2 is taken as $\kappa_2 = 2 \times \kappa_1$ where κ_2 is in [€/(MWh×MW)]. Different sensitivities with respect to this parameter are then performed and ensure that results are stable within a reasonable range of parameters.

A summary of our calibrated parameters for producer's costs can be found in table 4.

	Parameter	Value	Unity
Producer's costs	κ_1	13.9	[€/MWh]
	κ_2	31.8×10^{-4}	[€/(MWh×MW)]
	a	8.6	[€/MWh]
	b	17.6	[€/MWh]

Table 4: Calibrated parameters for construction, maintenance and production costs.

3.1.4 Utility, disutility and risk aversion

Risk aversions and utility preferences, θ, k, η_A, η_P and the participation constraint $\mathcal{R}$, are parameters less straightforward to calibrate.

The participation constraint (or cash equivalent of the reservation utility) $\mathcal{R}$ is defined as a function of the solution to producer's problem in absence of capacity payment, and given by 2.20. $\mathcal{R}$ is computed by numerically solving the PDE (5.22).

The parameter θ is expressed in [€/MWh], and reflects consumers' satisfaction in consumption per GW over time, or how much they are willing to pay for the electricity. This parameter is calibrated as the *Value of Lost Load* (VoLL)[20].

We assume that producer should be more risk averse than consumer. The reason of this assumption is that consumer's risk aversion embeds the aversion to shortage represented by the term in k. Roughly speaking, as soon as there is a shortage, consumer's utility starts to decrease because of the term $\eta_P \times k(X^D - X^C)^+$. We can interpret it by saying that consumer is willing to accept more financial risk than producer, in exchange of offsetting her (the consumer) risk of having a shortage. We choose therefore $\eta_A > \eta_P$ and we take values inspired by the calibration in [APT16]. A sensitivity analysis is then performed on these parameters and we find that our results are not affected if producer is more risk averse than the consumer or the contrary.

Regarding the parameter k, we use a further constraint which is that the average number of shortage hours per year should be reasonable. Indeed, for most European electricity systems, the targeted maximum number of hours of shortage, also called the Loss of Load Expectation (LoLE), is 3 hours (see [New16]).

[19]The Equivalent Annual Cost of a gas turbine power plant with a total cost of investment $C_{\text{Total cost}} = 62.7$ [€/MWh]– which should not be confused with the Levelized Cost Of Energy, and is computed by dividing the annual cost of investment by the total number of hours per year (8760 hours)– , a lifetime $T_{\text{Gas Turbine}} = 30$ [Years], and a discount rate $r = 8\%$ is computed as $\kappa_1 = \frac{nrC_{\text{Total cost}}}{1-(1+r)^{T_{\text{Gas Turbine}}}} = 13.9$ [€/MWh], where n is the number of upcoming annuities approximated by $n \simeq 2.5$. A more precise computation requires to take n as the number of annuities left to pay during the contract time (between 1 and 5 years), but we chose to simplify and take an average value $n = 2.5$ to avoid technical issues.

[20]See again *"Impact assessment of the French Capacity Market, 2018."* by RTE.

Therefore we calibrate these parameters by an iterative procedure, i.e., by repeatedly solving the problem, diffusing the state variables, computing the total period of shortage and adjusting the parameters until we attain a reasonable number of shortage hours per year. A possible set of consumer's preferences and risk aversions of both parties is given in the table 5. Of course, one could always argue that this set of parameters is not unique because of the degrees of liberties compared to the number of constraints, but this set seems quite reasonable and produces stable numerical results, which we present in the next section along with a sensitivity analysis.

	Parameter	Value	Unity
Consumer's preferences	θ	20000	[€/MWh]
	k	200000	[€/MWh]
Risk aversions	η_A	0.852×10^{-4}	$[\text{M€}]^{-1}$
	η_P	0.8094×10^{-5}	$[\text{M€}]^{-1}$
Participation constraint	$\mathcal{R}$	2.8	[€/MWh]

Table 5: Choice of risk aversions, and calibrated preferences.

3.2 Numerical results : Comparison between the system with and without a CRM

Once the parameters are fixed, we simulate $N = 5000$ scenarios and compare between three different cases; one "without a Capacity Remuneration Mechanism", where producer adjust their capacities to maximize their utility, another "with a CRM"; using the optimal policy for both consumers (optimal compensation (2.17)) and producers (recommended effort (2.14)), and a third one with no capacity adjustment ("No adjustment"), i.e., no building or dismantling of capacities; leaving them subject to external uncertainties.Table 6 summarizes the results of our simulations.[21]

[21]In the sequel, when dealing with a random variable W, we will denote by $\overline{W}$ its empirical average estimated with our scenarios, i.e., $\overline{W}$ is the Monte-Carlo estimator of $\mathbb{E}[W]$, which we will provide with the corresponding 95% confidence interval.

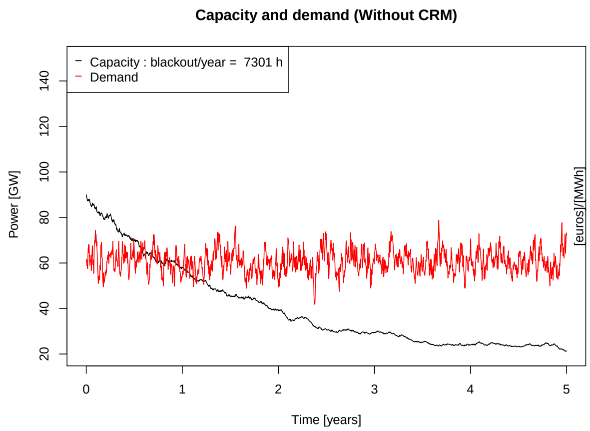
	Without CRM		With a CRM		No adjustment	
	mean	sd	mean	sd	mean	sd
Shortage hours per year [Hours]	6165.4	1459.1	2.2	7.4	178.4	594.3
Average Spot price [euro/MWh]	146.5	30	37.6	9.6	43	16.4
Average Margin [GW]	-6	8.2	32.4	9.0	28.1	11.4
Spot revenues [euro/MWh]	134.6	25.6	37.8	9.6	43.1	16.2
Capacity payment [euro/MWh]	NA	NA	12.3	13.2	NA	NA
Spot + Capacity payment[euro/MWh]	134.6	25.6	50.1	10.9	43.1	16.2
Participation constraint [euro/MWh]	NA	NA	2.8	0.0	NA	NA
Risk shared [euro/MWh]	NA	NA	-0.2	11.7	NA	NA
Risk compensation [euro/MWh]	NA	NA	14.9	3.9	NA	NA
Total costs [euro/MWh]	77.4	9.3	32.7	3.2	30.5	1.6
Construction and dismantling [euro/MWh]	50.3	9.7	1.8	3.7	0	0
Maintenance [euro/MWh]	9.5	0.7	13.2	1.3	12.9	1.6
Production [euro/MWh]	17.6	0	17.6	0.0	17.6	0

Table 6: Comparison between different policies

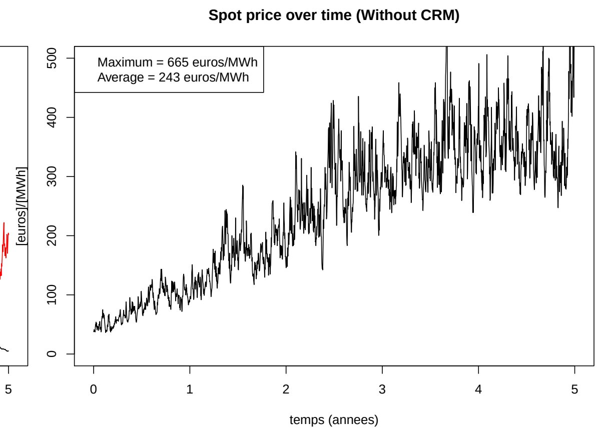
3.2.1 The system evolution without a capacity payment

We start by analyzing the system without a capacity payment. As mentioned before, the producer has market power and no incentive to satisfy the LoLE constraint, since his goal is to maximize his utility function instead of just offsetting his marginal costs. On the contrary, as his only compensation is from the spot market, his optimal strategy consists in finding the equilibrium between high enough spot prices (corresponding to low or even negative capacity margins) and high enough available capacity as spot compensation is $\int_0^T P(X_t)X_t^C \wedge X_t^D dt$. We can see from table 6 that he settles for a -6 GW average, which corresponds to an average spot price of 146 euros/MWh. It follows from this negative equilibrium average margin that the system is in shortage situation most of the period $[0, T]$, which is confirmed by the numerical results (6165 shortage hours per year).

In figure 3a, we can observe one scenario without a CRM. As stated before, producer's optimal strategy is to decrease the capacity level which leads to the spot prices increasing, see figure 3b. This decrease in capacity continues even after reaching the average demand level (about 60 GW) and attains an equilibrium (around 20 GW in the scenario which is more severe than average). This explains the high construction and dismantling costs mainly due to dismantling actions.



(a) Capacity and demand



(b) Spot price

Figure 3: State variables without a capacity payment policy

The figures 3a and 3b illustrate that without a Capacity Remuneration Mechanism, producer will be better off with low capacity and high spot revenues, and therefore if we aim at keeping a reasonable level of available capacity (which implies lower spot prices), it is necessary to provide him with a complementary compensation to replace his losses in spot revenues. We highlight the fact that the absence of a capacity payment would have much less drastic impacts in the real life than what we observe in our numerical simulations. This is due from one side to the regulation authorities which would not allow for such levels of shortage to occur, and from the other side because of the presence of multiple producers in competition who might decide to invest more –breaking the market power–, or even new actors (producers) willing to invest and enter the market in such favorable conditions (i.e., with a spot price much higher than costs.)

The loss of spot revenues incurred by producer when keeping high capacity levels is partly captured by his participation constraint, since $\mathcal{R}$ also accounts for the change in construction, maintenance and production costs. In fact, the more profitable the system without a CRM to the producer, the higher $\mathcal{R}$, and the more inciting the contract (CRM) needs to be. In our setting $\mathcal{R} = 2.8€/\text{MWh}$ [22], and is decreasing in σ^C and increasing in σ^D. When the volatility of capacity is high, the efforts of producer have less and less impact on X^C -and therefore on the system- and his utility (or its cash equivalent $\mathcal{R}$) is lower. On the other side, whenever the volatility of demand is high, the probability of shortage increases and when capacity margin becomes low this drives the spot prices up in an amplified manner which gives the producer more utility.

Similar to the volatility of capacity σ^C, the parameters κ_2 and x_0^C have the same impact on $\mathcal{R}$; the cost of control (in this case, the cost of shutting down powerplants) becomes higher with κ_2 which lowers producer's utility, while a positive change in the initial value x_0^C increases the capacity margin and decreases the spot prices and $\mathcal{R}$ as a result. Finally, and obviously, higher spot prices (because of higher spot levels β_0) increase $\mathcal{R}$.

[22] Note that it is not possible to infer from the first column of table 6 the participation constraint $\mathcal{R}$. Indeed, as the utility fonction is concave, $\mathcal{R} = U_A^{-1}\left(\mathbb{E}\left[U_A\left(\text{Spot} + \text{Capacity payment} - \text{Total Costs}\right)\right]\right)$ is different (lower) from what can be read from the table which corresponds to $\mathbb{E}[\text{Spot}] + \mathbb{E}[\text{Capacity payment}] - \mathbb{E}[\text{Total Costs}]$.

3.2.2 Analysis of the system evolution under optimal policy

Coming back to table 6, we can see that introducing the CRM drastically improves the security of the system; (an average of 2 hours shortage per year–*respecting the LoLE constraint*– compared to 178 hours per year when there is no capacity adjustment and 6165 hours per year when producer has market power.) Remark that this also reduces consumer's payments: it is less costly for the consumer to pay for capacity and the spot prices (which is in average rather low because the system margin is high) than paying only the spot prices "without CRM" where spot prices are very high.

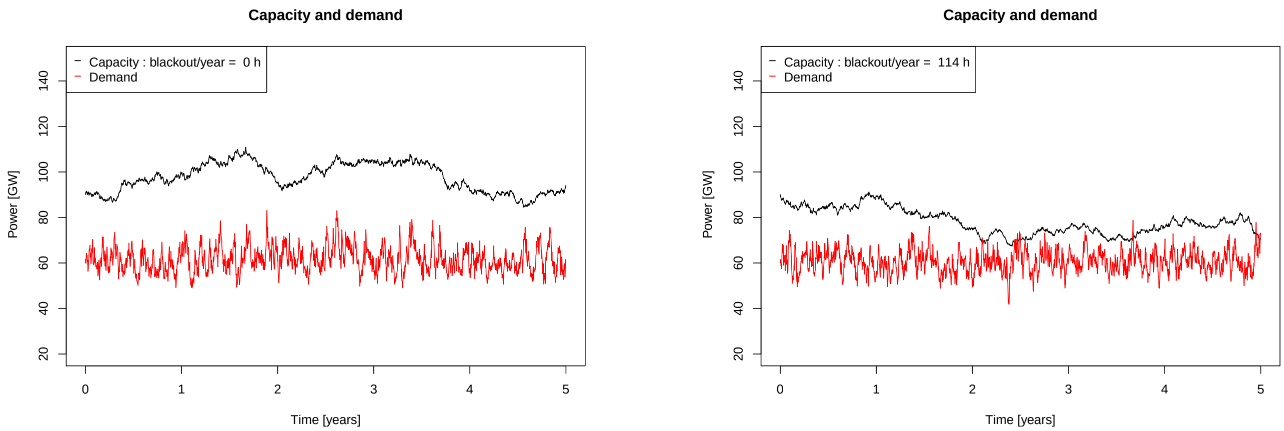
Observe also that when comparing the system with a CRM and without capacity adjustment, we see that the average margin is positive in both cases and quite high (32 GW and 28 GW) so one would expect that these two settings would be quite similar. However, we see that we obtain a substantial gain in the average number of shortage hours per year when using following the dictated policy (with 32 GW capacity margin), going from 178 hours per year to only 2 hours per year. This owes to the design of the contract in the CRM taking into account the magnitude of uncertainties and other characteristics of the system.

In order to better interpret producer's optimal policy, we select and analyze two of the 5000 simulated scenarios; a severe scenario –the one with the highest number of shortage hours over the period of simulation, selected a posteriori– and a Favorable scenario. Table 7 provides the outcomes of these scenarios compared with the average scenario.

	Average scenario	Favorable scenario	Severe scenario
Shortage hours per year [Hours]	2.2	0.0	113.9
Average Spot price [euro/MWh]	37.6	31.9	59.4
Average Margin [GW]	32.4	36.0	17.3
Spot revenues [euro/MWh]	37.8	32.1	59.7
Capacity payment [euro/MWh]	12.3	57.9	21.7
Spot + Capacity payment[euro/MWh]	50.1	90.0	81.4
Participation constraint [euro/MWh]	2.8	2.7	2.7
Risk shared [euro/MWh]	-0.2	43.4	-40.7
Risk compensation [euro/MWh]	14.9	12.7	52.1
Total costs [euro/MWh]	32.7	31.2	67.3
Construction and dismantling [euro/MWh]	1.8	0.0	38.6
Maintenance [euro/MWh]	13.2	13.6	11.1
Production [euro/MWh]	17.6	17.6	17.6

Table 7: Different scenarios outcomes

We plot first the evolution of state variables in figure 4. We can see in red the evolution of demand, and in black the available capacity. The demand process is by construction a mean-reverting process. However, the capacity is a geometric Brownian motion. So the capacity has a priori no reason to exhibit a mean-reverting behavior which is nevertheless observed in 4 on the right figure (b). This mean-reversion can be explained by the effort rate $\hat{\alpha}$ which readjusts the capacity depending on the randomness, and the level of security fixed by consumers' preferences. This readjustment can also be seen in the difference between construction costs in table 7; the severe scenario having the highest cost suggesting a policy with intensive construction.

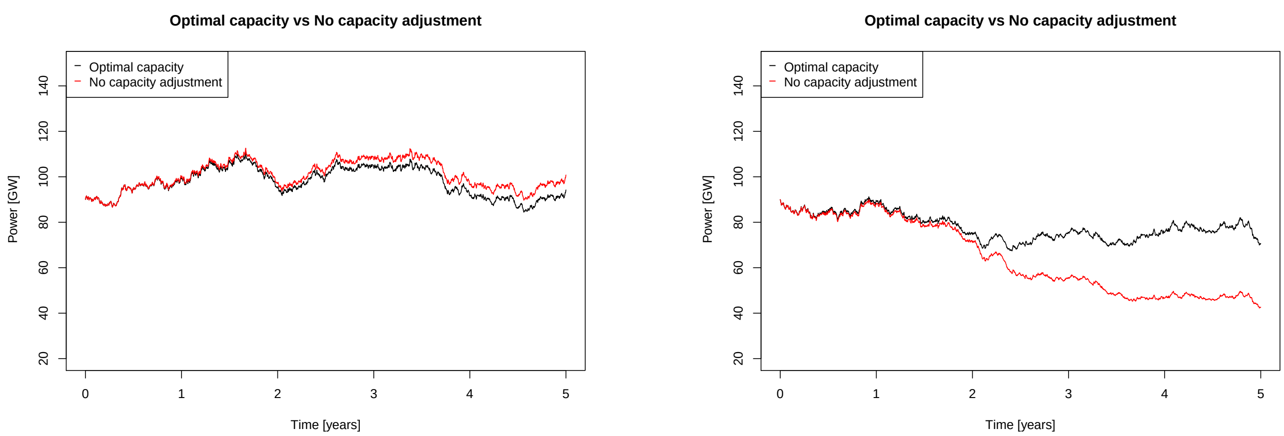


(a) Favorable scenario

(b) Severe scenario

Figure 4: Evolution of capacity and demand under optimal policy $\hat{\alpha}_t$.

Figure 5 provides an interpretation of producer's optimal control, by comparing two policies. In black, we see the capacity evolution obtained with producer's optimal policy, and in red the capacity without adjustment, i.e., $\alpha_t = 0$ for $t \in [0, T]$. This is interpreted as a comparison between our model and a the "No adjustment" policy model in which producer sets an initial capacity margin (30 GW in this case) and the system is then only impacted by the uncertainties.



(a) Favorable scenario

(b) Severe scenario

Figure 5: Comparison between Agent optimal policy and constant policy.

The figure shows that the Favorable scenario is a scenario where the capacities naturally experience favorable outcomes. For example, the inputs of hydro-powerplants, the load factor of wind and photovoltaic should have been very high, or no major failure of power plant should have been observed. On the contrary, the severe scenario is a scenario where capacities uncertainties are very unfavorable (strong drop of capacity after year 2, which can be seen in the "no capacity adjustment policy"). We can see that optimal policy absorbs this shock, at least partially because of the costs of construction preventing producer from restoring a higher capacity margin.

In fact, whenever the capacity margin is tight (when $x^C - x^D$ is small), the control $\hat{\alpha}$ takes higher values, and pushes the capacity process up. This is what is observed at year 2 and 2.5 in the severe scenario where the producer invests to counteract a negative shock in capacity. However, between year 3 and 4.5, as it becomes very expensive to keep a positive capacity margin, the optimal control does not follow the shock and a serie of shortages occurs. This implies that starting from some threshold consumers are willing to accept a shortage instead of paying a very high price to avoid it.

Finally, remark that from consumer's perspective the total payments (capacity remuneration + spot) are higher than average in the extreme scenarios, whether favorable or severe. Indeed, when the scenario is severe, the consumers is going to pay a lot in terms of spot prices but the capacity compensation is also high because of construction costs. When the scenario is very favorable, the spot prices are rather low but the consumer need to incentive the producer with a high capacity compensation and then share the positive risks.

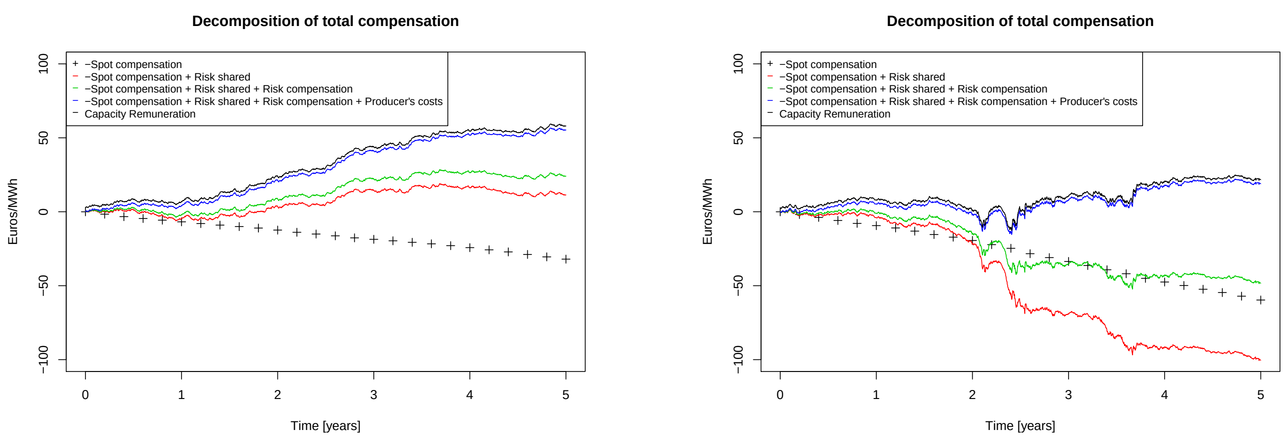
3.3 Analysis of the optimal contract

3.3.1 Decomposition of capacity payment

To understand how the contract is designed, we use the decomposition (2.18) suggested in section 2.6, recalled below:

Capacity remuneration ($\xi^\star$) + Spot compensation = $\mathcal{R}$ + Producer's costs + Risk shared + Risk compensation.

This decomposition is represented for the favorable and severe scenarios on figure 6. The capacity payment is represented in Black, constituted gradually by adding one by one the terms in the decomposition (2.18). We stress the fact that the contract is a lump-sum payment at maturity T and that its evolution over time is only informative.



(a) Favorable scenario

(b) Severe scenario

Figure 6: Decomposition of $Y_T^{\mathcal{R},Z^\star}$ in terms of costs, risk shared and risk compensation.

We start by taking the spot revenues (black marks "+") from producer (in reality the decomposition is for total compensation not only capacity, so spot revenues appear as a negative term in the decomposition of capacity payment). Then we add to that the Risk shared (whether positive as in the favorable scenario or negative as in the severe one) to obtain the red line. Then we add the risk compensation (always positive) to have the green line, the costs compensation to have the blue one, and finally the Participation constraint to obtain capacity compensation (black line).

In the severe scenario –figure 6b– the realized randomness is very unprofitable to the system. Our contract automatically shares this negative randomness with the producer (negative "risk shared"). On the contrary, in the favorable scenario –figure 6a– where capacity outcomes are naturally high and profitable, this positive randomness is also shared with the producer but positively.

In addition, the contract accounts for the costs needed to implement the optimal policy, the remuneration from spot and the risk compensation. Remark that, as expected, the risk compensation is positive in both cases, even when the shared randomness is positive. This helps to offset the impact of the risk shared, for example in the severe scenario the negative risk shared (-40 euros/MWh) is completely canceled by the risk compensation (52 euros/MWh), see Table 7.

In the severe scenario the compensation for costs (the difference between blue and green) is quite high (this is mainly linked to the high costs needed to follow the optimal policy, consisting in investing a lot), whereas under the favorable scenario, this part is much limited (investment to be made are small). However, the remuneration obtained from the spot market in the favorable scenario (with a high capacity margin) is much less than in the severe scenario (with a low capacity margin). This leads to a low capacity payment under the severe scenario compared to the favorable scenario.

3.3.2 Link between capacity payment and spot compensation

Table 8 provides another comparison between the system without a CRM, with a CRM and with no adjustment, but this time with regard to the occurrences of missing money (when total costs are more than spot revenues) and the scenarios with negative net revenues for producer, along with the role played by spot in total compensation in the case of a CRM, and the percentage of scenarios with a negative capacity remuneration.

	Without CRM	With a CRM	No adjustment
Spot revenues/Total revenues [%]	100	70	100
Scenarios with missing money [%]	2	28	26
Scenarios with negative capacity remuneration [%]	NA	18	NA
Scenarios with negative total compensation [%]	NA	0	NA
Scenarios with negative net revenues [%]	2	5	26

Table 8: Comparison between policies with and without CRM

Regarding the system with a CRM, we obtain an average number of shortage hours per year less than three hours, which satisfies the LoLE constraint. The revenue provided by the capacity payment is about 30% of the total compensation (compared to 70% for spot market).

In our simulations $\mathbb{E}^{\mathbb{P}^{\alpha^\star}}\left[\frac{S_T+\xi^\star}{S_T}\right] \in [1.42, 1.43]$ with a 95% confidence level and a standard deviation of 0.53. This corresponds roughly to a partition of total revenues into 70% from spot market and 30% from capacity compensation. However, as we can see in figure 7, the distribution of $\frac{S_T+\xi^\star}{S_T}$ can take values less than 1 (even negative theoretically, but not observed in practice (cf. Table 8)) when $\xi^\star$ is negative. Whenever they occur, the negative capacity prices could be interpreted as a reimbursement from producers when their revenues from the spot market are high.

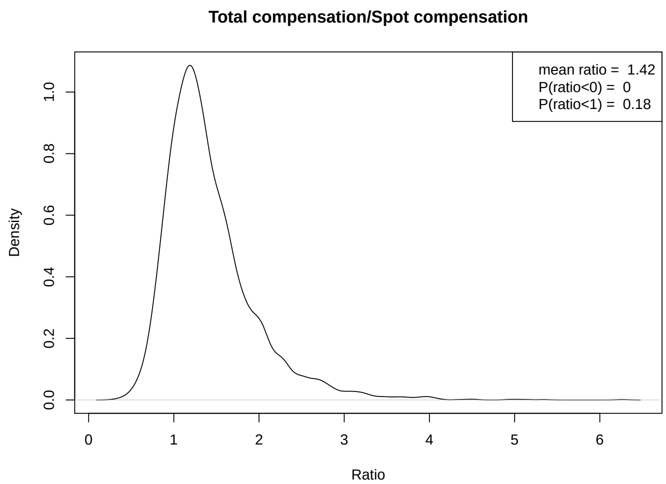
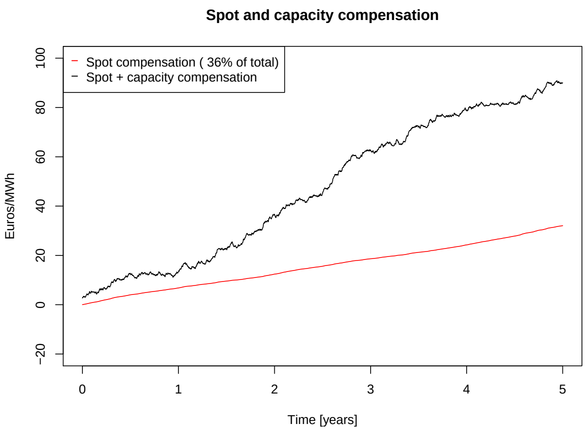
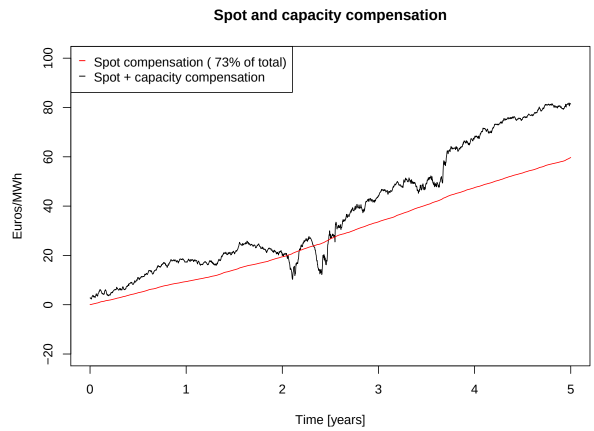


Figure 7: Distribution of $\frac{S_T+\xi^\star}{S_T}$ with a CRM.

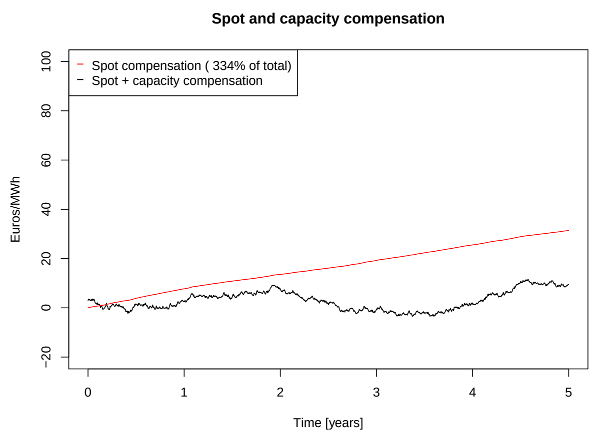
We investigate more this ratio by observing the evolution of the spot compensation and total payment (what the consumer pays; the sum of spot and capacity remunerations). In figures in 8, we plot for each scenario the spot compensation (in red), and then we add the capacity payment (whether positive or negative) to obtain the total compensation (in black).



(a) Favorable scenario



(b) Severe scenario



(c) Scenario with the lowest $\frac{S_T+\xi^\star}{S_T}$ ratio

Figure 8: Decomposition of $Y_T^{\mathcal{R},Z^\star}$ in terms of spot and capacity payment.

In the favorable scenario previously analyzed, figure 8a shows that the capacity payment is going to be more active than in the severe scenario 8b to complement spot remuneration. For the favorable scenario, the capacity payment represents 64% of the total remuneration of the producers, compared to 27% in the severe scenario. This is explained by the capacity margin, which dictates the level of spot prices and the need for a complementary compensation. Figure 8c illustrates a scenario where the capacity margin is very little during all the period leading to high spot prices, accompanied by a negative capacity remuneration.

Observe finally that the probability of getting negative net revenues drops from 26% in the case of no adjustment (very uncertain for producer) to 5% in the case with a CRM. It remains howerver slightly more than in the case without a CRM (2%) which is natural since in this scenario the producer's only goal is to maximize his utility.

3.3.3 When is there missing money or a negative capacity payment

We investigate in this section "unfavorable" scenarios. Over the 5000 scenarios simulated earlier with a capacity remuneration mechanism, we select the ones where there is a missing money (MM) (where the spot remuneration is less than producer's total costs (28% of scenarios)), and the scenarios where the capacity remuneration is negative (NCR); (18% of scenarios), and the ones where there is a missing money and the capacity remuneration is negative (MM and NCR); (1% of scenarios).

	MM	NCR	MM and NCR
Percentage [%]	28	18	1

Table 9: Missing money and negative capacity payment

We compute the same indicators as before for the two groups of the selected scenarios (the ones with missing money "MM" and the ones with a negative capacity remuneration "NCR"), and we discard the third one (with both "MM and NCR") since they are happen in only 1% of the scenarios (average quantities are indeed meaningless in that case as the number of scenarios are very low). We summarize the results in table 10[23].

	Reference		MM		NCR	
	mean	sd	mean	sd	mean	sd
Shortage hours per year [Hours]	2.2	7.4	0.4	5.1	3.5	8.7
Average Spot price [euro/MWh]	37.6	9.6	25.8	4.9	44.6	6.5
Average Margin [GW]	32.4	9.0	43.9	6.9	26.2	4.7
Spot revenues [euro/MWh]	37.8	9.6	26.0	4.9	44.8	6.5
Capacity payment [euro/MWh]	12.3	13.2	22.7	11.6	-6.4	5.3
Spot + Capacity payment[euro/MWh]	50.1	10.9	48.7	10.4	38.4	7.7
Participation constraint [euro/MWh]	2.8	0.0	2.8	0.0	2.8	0.0
Risk shared [euro/MWh]	-0.2	11.7	0.1	11.2	-14.6	8.2
Risk compensation [euro/MWh]	14.9	3.9	13.1	2.8	16.7	4.7
Total costs [euro/MWh]	32.7	3.2	32.8	2.7	33.5	4.1
Construction and dismantling [euro/MWh]	1.8	3.7	0.3	2.8	3.5	4.5
Maintenance [euro/MWh]	13.2	1.3	14.9	1.0	12.4	0.7
Production [euro/MWh]	17.6	0.0	17.6	0.0	17.6	0.0

Table 10: Missing money and negative capacity remuneration

The major effect which explains Missing Money and Negative Capacity remuneration is the margin of the system. Missing Money is often coming with high margin while a negative capacity remuneration happens mostly with a low margin.

A close look at table 10 shows that there is missing money whenever the average margin is high (43 GW) in average, and so spot revenues are low (26 euros/MWh), and are not enough to cover total costs especially

[23] Remark that these indicators are computed with a Monte Carlo method using different sizes of samples, and so do not have the same accuracy.

since maintenance costs are higher than average (14.9 euros/MWh compared to 13.2 euros/MWh) which overrules the fact that construction costs are close to zero. The capacity remuneration mechanism completes producers earnings on spot market since it compensates him systematically for his costs (whether high or low), and this can be seen in table 10 by the fact that average total compensation is above costs.

The scenarios with negative capacity remuneration are more subtle to understand. In fact, they occur because our CRM is designed to take into account the spot compensation and complement it only when needed. In other words, if we recall again the decomposition of total compensation (2.18):

$$\text{Capacity remuneration} + \text{Spot compensation} = \mathcal{R} + \text{Producer's costs} + \text{Risk shared} + \text{Risk compensation},$$

we can see that negative capacity remuneration is equivalent to

$$\text{Spot compensation} > \mathcal{R} + \text{Producer's costs} + \text{Risk shared} + \text{Risk compensation}. \tag{3.1}$$

This happens whenever the risk shared is negative (as confirmed by table 10 : -14.6 euros/MWh), suggesting unfavorable uncertainties and low capacity margins (26 GW compared to 32 GW in average), which increases spot revenues (44.8 euros/MWh) and consolidates the inequality (3.1). A typical scenario with a negative capacity remuneration would be one with a consistent low demand and high capacity without much realised volatility. This would keep the risk compensation low, with negative risk shared and high spot compensation. Nevertheless, observe that even in such conditions (unfavorable uncertainties), producer manages to keep an acceptable number of shortage hours per year (3.5 hours per year).

3.4 The optimal capacity payment in other setups

We test our system under different conditions, in the case where there are more renewable energies; and thus more uncertainties in the system (associated with higher σ^C (50% higher)), or when there is a demand response program applied on consumers, i.e., assuming demand volatility σ^D is lower (50% lower).
We also test different risk aversions; with more risk averse consumers or more risk averse producers (multiplying by 10 the risk aversion parameter each time).

3.4.1 More renewable energies or a demand response program (σ^C and σ^D)

A brief summary of the numerical results with perturbations of volatilites can be found in table 11.

	Reference	More renewables	Demand response program
Spot revenues/Total revenues [%]	70	41	98
Scenarios with missing money [%]	28	74	25
Scenarios with negative capacity remuneration [%]	18	0	60
Scenarios with negative total compensation [%]	0	0	0
Scenarios with negative net revenues [%]	5	0	13

Table 11: Different volatilities ratios

We can see that introducing more renewable energies in the system yields a higher percentage of scenarios with missing money (74% as opposed to 28%), and to positive net revenues all the time. In this context, the capacity compensation is never negative (in the observed scenarios), and plays a much more important role in complementing producer revenues (59% of total revenues instead of only 30%). This is similar to what we observed in section 3.3.2, when comparing a severe and favorable scenario.

By looking further into table 12, we can explain the missing money by the high costs of construction and dismantling (7.6 euros/MWh compared to 1.8 euros/MWh), and the role of capacity payment by the considerable amount of risk compensation (34.5 euros/MWh compared to 14.9 euros/MWh).

So in summary the more uncertainties in production, the more the need for a capacity mechanism in order to cope with the random electricity demand and the random available capacity, since otherwise the number of shortage hours would increase. The capacity mechanism results in increasing the capacity margin of the system. As the system is longer in terms of capacity, spot prices are lower. Therefore, in average, consumers have to pay a higher capacity remuneration when the capacity volatility increases, and producers receive a higher total compensation, and higher earnings even though the spot compensation decreases.

	Reference	More renewables	Demand response program
Shortage hours per year [Hours]	2.2	2.8	0.7
Average Spot price [euro/MWh]	37.6	32.5	40.9
Average Margin [GW]	32.4	39.1	29.9
Spot revenues [euro/MWh]	37.8	32.6	40.7
Capacity payment [euro/MWh]	12.3	41.0	-3.2
Spot + Capacity payment[euro/MWh]	50.1	73.6	37.4
Participation constraint [euro/MWh]	2.8	0.0	2.2
Risk shared [euro/MWh]	-0.2	-0.4	-0.1
Risk compensation [euro/MWh]	14.9	34.5	4.0
Total costs [euro/MWh]	32.7	39.4	31.3
Construction and dismantling [euro/MWh]	1.8	7.6	0.8
Maintenance [euro/MWh]	13.2	14.2	12.9
Production [euro/MWh]	17.6	17.6	17.6

Table 12: Results with different volatilities

The third column of tables 11 and 12 summarizes numerical results when the demand volatility is lower, which is a model for a demand response program; i.e., we assume that consumer is somehow incentivized to behave in a more predictable manner, so that the uncertainty on demand fluctuations (σ^D) is lower.

In this case, the security of the system can be ensured with a lower capacity margin (29.9 GW instead of 32.4 GW) since there are less uncertainties. Therefore the average spot price and the spot revenues are higher (40.9 euros/MWh instead of 37.6 euros/MWh), so we are likely to obtain the inequality (3.1)

$$\text{Spot compensation} > \mathcal{R} + \text{Producer's costs} + \text{Risk shared} + \text{Risk compensation},$$

which is equivalent to having a negative capacity remuneration especially since none of the terms on the right hand side should be high. This explains the high percentage of scenarios with negative capacity remuneration (60%).

These conditions suggest an auto-regulated market and less need for a CRM, since spot revenues represent 98% of total compensation, even though there is missing money 25% percent of the time, and often a negative capacity remuneration (60% of the time). We can guess that taking away the capacity remuneration would lead to an equilibrium situation with more uncertainties for producer: he incurs losses when there is a missing money, but these losses are balanced by the scenarios where the spot price is high, and he does not have to "pay" for capacity remuneration which is now a cost for him rather than a revenue, making his net revenues negative in 13% of the scenarios. Nevertheless, the total remuneration of producer (spot + capacity payment) is still above in average the total costs: in average with the CRM, the producer is going to earn money.

3.4.2 More risk aversion producer or consumer (η_A and η_P)

We analyze the impacts of the CRM when producer or consumer is more risk averse. The numerical results are summarized in tables 13 and 14.

	Reference	Risk averse producer	Risk averse consumer
Shortage hours per year [Hours]	2.2	0.8	0.5
Average Spot price [euro/MWh]	37.6	32.8	34.4
Average Margin [GW]	32.4	36.6	34.9
Spot revenues [euro/MWh]	37.8	33.0	34.6
Capacity payment [euro/MWh]	12.3	51.2	380.6
Spot + Capacity payment[euro/MWh]	50.1	84.2	415.3
Participation constraint [euro/MWh]	2.8	0.0	2.7
Risk shared [euro/MWh]	-0.2	-0.1	-0.3
Risk compensation [euro/MWh]	14.9	49.7	378.7
Total costs [euro/MWh]	32.7	34.6	34.1
Construction and dismantling [euro/MWh]	1.8	3.2	2.9
Maintenance [euro/MWh]	13.2	13.8	13.6
Production [euro/MWh]	17.6	17.6	17.6

Table 13: More risk aversions

The first observation is that more risk averse producer has a zero participation constraint, which means that the utility he would gain from a system without a CRM is negative.

Apart from this observation, we have in both cases the same impacts but with different magnitudes. We can see that the costs of construction are slightly higher, with lower spot revenues. This leads to an increase in occurrences of missing money scenarios, and a larger part of total revenues coming from capacity compensation (always positive, and representing 41% and 92% from total compared to 30%). This capacity compensation comes mainly from the risk compensation (51.2 euros/MWh and 380 euros/MWh compared to 15 euros/MWh in the reference case).

	Reference	Risk averse producer	Risk averse consumer
Spot revenues/Total revenues [%]	70	39	8
Scenarios with missing money [%]	28	53	41
Scenarios with negative capacity remuneration [%]	18	0	0
Scenarios with negative total compensation [%]	0	0	0
Scenarios with negative net revenues [%]	5	0	0

Table 14: More risk aversions ratios

To summarize, whenever one of the two parties is more risk averse, it becomes very costly for consumer to pay for capacity remuneration. A risk averse producer would require more risk compensation (the risk compensation is proportional to producer's risk aversion by definition). From the other hand, a risk averse consumer would be ready to spend a lot to avoid potential shortage or blackout. The consequence is that the producer gets positive total compensation and positive net revenues 100% of the time.

4 Conclusion

In this paper, we provide some insights on how electricity producers and consumers could share the financial and physical risks (and uncertainties) to ensure the security of the system. We propose a Capacity Remuneration Mechanism based on contract theory, which incentivizes producers to perform an optimal level of effort to maintain and develop new power-plants. It takes the form of a contract and a recommended effort, with payment adjusted to the uncertainty of outcomes (weather, outages...) ensuring to producers the right level of average earnings and financial risks while accounting for the spot revenues, as long as they follow the recommended effort.

This means that the CRM does not disrupt the spot market operations, which remains therefore able to ensure the short-term optimal economical dispatch. Given a predefined level of security for the system, the capacity mechanism we propose provides the right level of investment needed to insure it, and gives us insights to challenge real implementations of capacity markets.

One of our main results is that we point out the necessity of a capacity remuneration mechanism. This is reinforced with the level of randomness of the capacity and demand. Higher share of random renewable production in the electrical system means that a higher fraction of the compensation of the producers needs to come from a capacity remuneration system. As a matter of fact, in that case the volume of installed capacities should be more important to ensure the security of the system. Meanwhile, the spot price decreases (the spot price is decreasing with respect to installed capacities, which is consistent with the increase of the supply curve), and so it is essential to support installed capacities.

In the meantime, the higher the volatility of consumption or supply, the higher the volatility of the capacity market. The mechanism we proposed also enables to study how risks should be share between the producers and the consumers. The increase of the financial risk is principally supported by the producers who are then compensated by a higher average revenue. However, even when there is a capacity payment, the number of hours of shortage increases when consumption or production become more volatile. In fact, if consumers do not modify the virtual value they associate to shortage, it is economically optimal for them to accept more hours of shortage instead of increasing suppliers' compensation. The capacity payment enables to share the financial risk between the producers and consumers depending on their risk aversion. The producers accept to take more financial risk if it comes with an increase of their average revenue. This is also what happens when the consumers want to reduce the physical risk.

Finally, we propose some variants for further research. It would be interesting to challenge our results by using other spot functions, especially ones that could reach higher peaks than the function (2.4). Furthermore, we assumed that the demand process is completely exogenous and uncontrolled. We explored this aspect of the model to a certain extent in section 3.4 by studying the sensitivities of our results with respect to the volatility of demand. Adding a direct control on the demand process would be relevant but would also change drastically the resolution methodology and is therefore an open research question. The same can be said about the assumption that dismantling power plants has the same cost as building new ones.

In addition, representing several technologies to produce electricity would also make a lot of sense. This could be the starting point for a future work by considering for example the possibility to control the volatility of the capacity process or to have several producers with different technologies instead of one. We also used a continuous time setting, which is convenient for modeling and computations, but requires using small time steps to discretize and approximate the optimal control (in our simulations we used a daily time step). This is equivalent to ignoring the delay needed to build new power plants which is not completely realistic, but remains quite common in such models because decision-making delay is hard to capture. Finally, our model only represents one design of CRM (a kind of sophisticated capacity payment). Further work may include other possible market designs.

5 Appendix

5.1 Rigorous mathematical framework and weak formulation

This section is devoted to the rigorous mathematical formulation of the problem. For $T \in (0, +\infty)$ a fixed maturity, we denote by $\Omega := \mathcal{C}\left([0,T], \mathbb{R}^2\right)$ the space of continuous functions from $[0, T]$ to $\mathbb{R}^2$. The system is described by the state variable $X := \left(X^C, X^D\right)^T$ which is the canonical process on Ω. Finally we endow Ω with its Borel σ-algebra $\mathcal{F}_T$ and define the completed filtration $\mathbb{F}$ generated by the process X.

We define the reference probability measure as the weak solution of the controlled equation (2.3) with a constant control set to zero, $\alpha_t = 0,\ \forall t \in [0,T]$, on the space $(\Omega, \mathcal{F}_T)$ and we denote it $\mathbb{P}^0$. It is characterized as the unique probability measure such that $\mathbb{P}^0 \circ (X_0)^{-1} = \delta_{x_0}$ for some $x_0 \in \mathbb{R}^2_+$ and the processes $\left(X_t - \int_0^t \mu(X_s, 0)\, ds\right)_{t\in[0,T]}$ is a $\left(\mathbb{P}^0, \mathbb{F}\right)$–martingale with $\langle X \rangle_t = \int_0^t \sigma(X_s)\,\sigma^T(X_s)\, ds$ for $t \in [0,T]$. Note that existence and uniqueness of this measure are insured by the existence of a unique strong solution to the corresponding SDE. Therefore there exists a 2–dimensional $\mathbb{P}^0$–Brownian motion $W^{\mathbb{P}^0}$ such that we can write

$$X_t = X_0 + \int_0^t \mu(X_s, 0)\, ds + \int_0^t \sigma(X_s)\, dW_s^{\mathbb{P}^0},\ \text{for } t \in [0,T].$$

Notice that under $\mathbb{P}^0$ the component X^C is a martingale (the drift part of X^C is zero when the control is constantly equal to 0).
An admissible control $(\alpha_t)_{t\in[0,T]}$ is an $\mathbb{F}$–predictable real valued process such that:

$$\mathbb{E}^{\mathbb{P}^0}\left[\mathcal{E}\left(\int_0^T \frac{\alpha_s}{\sigma^C} dW_s^{C,\mathbb{P}^0}\right)\right] = 1, \tag{5.1}$$

where $W^{C,\mathbb{P}^0}$ is the capacity component of the Brownian motion.

We can then define $\mathbb{P}^\alpha$ on $(\Omega, \mathcal{F}_T)$ as the equivalent measure to $\mathbb{P}^0$ with its Radon–Nikodym derivative, which is a real martingale by virtue of (5.1)

$$\frac{\mathrm{d}\mathbb{P}^\alpha}{\mathrm{d}\mathbb{P}^0} := \mathcal{E}\left(\int_0^T \frac{\alpha_s}{\sigma^C} dW_s^{C,\mathbb{P}^0}\right). \tag{5.2}$$

Applying Girsanov theorem, we get that for an admissible control process α, the process X has the law of SDE (2.3) under $\mathbb{P}^\alpha$. Remark that the demand component is not affected by this change of probability, i.e., $\mathbb{P}^0 \circ \left(X^D\right)^{-1} = \mathbb{P}^\alpha \circ \left(X^D\right)^{-1}$, for every admissible α, which is consistent with our model: the only possible control is on (the drift) of the capacity. We denote by $\mathcal{U}$ the set of admissible controls α, and by $\mathcal{P}$ the corresponding set of admissible probability measures.

To ease notations we will denote for an admissible control α, the (conditional) change of measure as

$$L_t^\alpha := \left.\frac{\mathrm{d}\mathbb{P}^\alpha}{\mathrm{d}\mathbb{P}^0}\right|_{\mathcal{F}_t}.$$

5.2 Non degeneracy of Principal's and Agent's problems

We have for all $(\xi, \mathbb{P}^\alpha) \in \Xi \times \mathcal{P}$

$$\begin{aligned}
\left|J_0^A(\xi,\mathbb{P}^\alpha)\right| &= \mathbb{E}^{\mathbb{P}^\alpha}\left[e^{-\eta_A\xi+\eta_A\int_0^T (c^A(X_t,\alpha_t)-P(X_t)X_t^C\wedge X_t^D)dt}\right]\\
&= E^{\mathbb{P}^0}\left[\frac{\mathrm{d}\mathbb{P}^\alpha}{\mathrm{d}\mathbb{P}^0}e^{-\eta_A\xi+\eta_A\int_0^T (c^A(X_t,0)-P(X_t)X_t^C\wedge X_t^D)dt}\right]\\
&\le K_1(T)\,\mathbb{E}^{\mathbb{P}^0}\left[\frac{\mathrm{d}\mathbb{P}^\alpha}{\mathrm{d}\mathbb{P}^0}e^{-\eta_A\xi}\right]\\
&= K_1(T)\,\mathbb{E}^{\mathbb{P}^\alpha}\left[e^{-\eta_A\xi}\right] \le K_1(T)\sup_{\mathbb{P}^\alpha\in\mathcal{P}}\mathbb{E}^{\mathbb{P}^\alpha}\left[e^{-\eta_A\xi}\right] < \infty,
\end{aligned}$$

and similarly

$$\begin{aligned}
\left|J_0^P(\xi,\mathbb{P}^\alpha)\right| &\le \mathbb{E}^{\mathbb{P}^\alpha}\left[e^{\eta_P\xi-\eta_P\int_0^T (c^P(X_t)-P(X_t)X_t^C\wedge X_t^D)dt}\right]\\
&\le K_2(T)\,\mathbb{E}^{\mathbb{P}^\alpha}\left[e^{\eta_P\xi}\right] \le K_2(T)\sup_{\mathbb{P}^\alpha\in\mathcal{P}}\mathbb{E}^{\mathbb{P}^\alpha}\left[e^{\eta_P\xi}\right] < \infty,
\end{aligned}$$

where we used the boundedness of the functions $\mathbb{R}^2 \ni x \mapsto c^A(x,0)$, and $\mathbb{R}^2 \ni x \mapsto P(x)\,x^C \wedge x^D$, and $\mathbb{R}^2 \ni x \mapsto c^P(x)$, –insured by the truncation function in P and c^A and c^P– along with the assumptions on the admissible contracts.

5.3 Definition and formal derivation of the class of revealing contracts $\mathcal{Z}$

5.3.1 Definition

We introduce then two control variables $(Y_0, Z) \in (\mathbb{R}\times\mathcal{V})$, where $\mathcal{V}$ is the set of $\mathbb{F}$-predictable processes Z valued in $\mathbb{R}^2$ such that for all $\mathbb{P}^\alpha$ in $\mathcal{P}$ such that

$$\mathbb{E}^{\mathbb{P}^\alpha}\left[\mathcal{E}\left(-\eta_A\int_0^T Z_t\cdot\sigma(X_t)\,dW_t^\alpha\right)\right] = 1,$$

and the process $Y^{Y_0,Z}$ is defined using the *Hamiltonian* (recall (2.12)):

$$Y_t^{Y_0,Z} = Y_0 + \int_0^t Z_s\cdot dX_s - \int_0^t H(X_s,Z_s)\,ds,\ \text{for all } t\in[0,T].$$

The class of *revealing contracts* $\mathcal{Z}$ is then defined as

$$\mathcal{Z} := \left\{Y_T^{Y_0,Z} \text{ for some } (Y_0,Z)\in\mathbb{R}\times\mathcal{V} \text{ with } Y_T^{Y_0,Z}\in\Xi\right\}, \tag{5.3}$$

and we will sometimes abuse notations and say that $(Y_0, Z)\in\mathcal{Z}$, meaning that $Y_T^{Y_0,Z}\in\mathcal{Z}$.

5.3.2 Formal derivation

The aim of this subsection is to provide a formal intuition about the derivation of the class of revealing contracts $\mathcal{Z}$ in the spirit of [San08], which is a key ingredient in the resolution of our problem. Rather than providing a rigorous treatment of this question (for which we refer the reader to [CPT18]), we only present here the main ideas behind it.

The main goal of introducing the class $\mathcal{Z}$ which–we recall–plays the role of a *performance index*, is to overcome the non-markovianity of the problem, and to make Agent's response "predictable" by principle. In order to do so, we rely on the *Martingale Optimality Principle*.

We start by recalling Agent's value function:

$$V_0^A(\xi) = \sup_{\mathbb{P}^\alpha \in \mathcal{P}} \mathbb{E}^{\mathbb{P}^\alpha}\left[U_A\left(\xi + \int_0^T P(X_t)\, X_t^C \wedge X_t^D dt - \int_0^T c^A(X_t, \alpha_t)\, dt\right)\right].$$

The first idea is to restrict ourselves to contracts ξ which are terminal values of some diffusion process,i.e., of the form $\xi = Y_T^{Y_0,Z}$ where Y_0 and Z are new control variables, and $Y_T^{Y_0,Z} = Y_0 + \int_0^T Z_t \cdot dX_t + \int_0^T g(t, X_t, Y_t, Z_t)\, dt$, where g is a deterministic function to be determined later.

Roughly speaking, this allows us to reduce to the markovian case by recapturing Principal's "missing" information and plug it into the new process $Y_T^{Y_0,Z}$. This is done through the change of control variables from ξ to Y_0 and Z. We underline here the fact that this is a formal reasoning since $Y_T^{Y_0,Z}$ is a solution to some SDE and has a priori no reason to exist (so far we didn't impose any restrictions on the function g).

Now that we are in a markovian setting, we want to get rid of the moral-hazard, i.e., to have a predicable response from the Agent. This is possible by a careful choice of the function g. In fact, by the structure of the exponential utility $U_A(x) = -e^{-\eta_A x}$, we have by a simple application of Itô formula that the process defined by $U_A\left(Y_t^{Y_0,Z} - \int_0^t c^A(X_s, \alpha_s)\, ds\right)_{t \in [0,T]}$, will be a $(\mathbb{P}^\alpha, \mathbb{F})$-supermartingale and a martingale only for some $\hat{\alpha}$ if we choose g such that

$$g(t, x, y, z) = -H(x, z) \text{ for } (t, x, y, z) \in [0, T] \times \mathbb{R}_+^2 \times \mathbb{R} \times \mathbb{R}^2,$$

with H the hamiltonian defined in (2.12). This gives us first an upper bound on Agent's optimal control $V_0^A\left(Y_T^{Y_0,Z}\right) \leq U_A(Y_0)$ by the supermartingale property, and we have that this bound is attained for the control induced by $\mathbb{P}^{\hat{\alpha}}$ by construction.

Therefore, since Agent is rational and aims at maximizing his utility, he will choose the control $\hat{\alpha}$, which is a deterministic function of the pair $(X_t, Z_t)_{t \in [0,T]}$, both observable by Principal.

5.4 Solving Producer's problem: proof of Proposition 2.3

By definition

$$\begin{aligned} J_0^A\left(Y_T^{Y_0,Z}, \mathbb{P}^\alpha\right) &= \mathbb{E}^{\mathbb{P}^\alpha}\left[U_A\left(Y_T^{Y_0,Z} + \int_0^T P(X_t)\, X_t^C \wedge X_t^D dt - \int_0^T c^A(X_t, \alpha_t)\, dt\right)\right], \\ &= \mathbb{E}^{\mathbb{P}^\alpha}\left[U_A\left(Y_0 + \int_0^T Z_t \cdot dX_t - \int_0^T H(X_t, Z_t)\, dt - \int_0^T c^A(X_t, \alpha_t)\, dt\right)\right]. \end{aligned}$$

we use (2.13) :

$$\int_0^T Z_s \cdot dX_s = \int_0^T h(X_t, Z_t, \alpha_t)\, dt + \int_0^T \sigma(X_t) Z_t \cdot dW_t^\alpha$$
$$- \int_0^T P(X_t) X_t^C \wedge X_t^D dt + \int_0^T c^A(X_t, \alpha_t) + \frac{\eta_A}{2} \int_0^T |\sigma(X_t) Z_t|^2 dt, \tag{5.4}$$

and therefore replacing U_A with its expression and injecting (5.4) we get

$$J_0^A\left(Y_T^{Y_0,Z}, \mathbb{P}^\alpha\right) = \mathbb{E}^{\mathbb{P}^\alpha}\left[-e^{-\eta_A\left(Y_0 + \int_0^T h(X_t,Z_t,\alpha_t) - H(X_t,Z_t)dt + \int_0^T \sigma(X_t)Z_t \cdot dW_t^\alpha + \frac{\eta_A}{2}\int_0^T |\sigma(X_t)Z_t|^2 dt\right)}\right]$$
$$= U_A(Y_0)\, \mathbb{E}^{\mathbb{P}^\alpha}\left[e^{\eta_A \int_0^T \{H(X_t,Z_t) - h(X_t,Z_t,\alpha_t)\}dt}\mathcal{E}\left(-\eta_A \int_0^T Z_t \cdot \sigma(X_t)\, dW_t^\alpha\right)\right].$$

Since $Z \in \mathcal{V}$ we can define the the probability measure $\widetilde{\mathbb{P}^\alpha}$ equivalent to $\mathbb{P}^\alpha$ with the Radon-Nikodym derivative

$$\frac{\mathrm{d}\widetilde{\mathbb{P}^\alpha}}{\mathrm{d}\mathbb{P}^\alpha} := \mathcal{E}\left(-\eta_A \int_0^T Z_t \cdot \sigma(X_t)\, dW_t^\alpha\right),$$

and so

$$J_0^A\left(Y_T^{Y_0,Z}, \mathbb{P}^\alpha\right) = U_A(Y_0)\, \mathbb{E}^{\widetilde{\mathbb{P}^\alpha}}\left[e^{\eta_A \int_0^T \{H(X_t,Z_t) - h(X_t,Z_t,\alpha_t)\}dt}\right].$$

Recalling that $U_A(Y_0) < 0$, and $H(X_t, Z_t) - h(X_t, Z_t, \alpha_t) \geq 0$ with equality if and only if $\alpha_t = \hat{\alpha}_t(X_t, Z_t)$ for all t in $[0, T]$, we obtain that

$$J_0^A\left(Y_T^{Y_0,Z}, \mathbb{P}^\alpha\right) \leq U_A(Y_0) \text{ for all } \mathbb{P}^\alpha \in \mathcal{P},$$

and

$$J_0^A\left(Y_T^{Y_0,Z}, \mathbb{P}^{\hat{\alpha}}\right) = U_A(Y_0),$$

which yields

$$V_0^A\left(Y_T^{Y_0,Z}\right) = U_A(Y_0), \text{ and } \alpha_t^\star = \hat{\alpha}_t(X_t, Z_t) = \frac{Z_t^c - \kappa_1}{\kappa_2 X_t^c}, \ \forall t \in [0, T].$$

□

5.5 Optimal contract and capacity payment: proof of Proposition 2.5

To prove this result, we use first the previous characterization of the Agent's optimal response, given by Proposition 2.3, for any admissible contract by virtue of Theorem 2.4. As a consequence, Principal needs only to solve her problem on the subset of revealing contracts $\mathcal{Z}$ since she is able to control Agent's value function by controlling the dynamics of the contract. We can then rewrite the Principal's problem as follows

$$V_0^P = \sup_{Y_0 \geq \mathcal{R}} \sup_{Z \in \mathcal{V}} \sup_{\mathbb{P}^\alpha \in \mathcal{P}^\star\left(Y_T^{Y_0,Z}\right)} J_0^P\left(Y_T^{Y_0,Z}, \mathbb{P}^\alpha\right), \tag{5.5}$$

and we have that every maximizer $(Y_0^\star, Z^\star)$ of (5.5) induces an optimal contract $\xi^\star := Y_T^{Y_0^\star, Z^\star}$. In order to compute $Y_0^\star$ we define

$$\begin{aligned}\underline{V}^P(Y_0) &:= \sup_{Z\in\mathcal{V}} \sup_{\mathbb{P}^\alpha\in\mathcal{P}^\star\left(Y_T^{Y_0,Z}\right)} J_0^P\left(Y_T^{Y_0,Z}, \mathbb{P}^\alpha\right),\\ &= \sup_{Z\in\mathcal{V}} J_0^P\left(Y_T^{Y_0,Z}, \mathbb{P}^{\hat\alpha}\right),\end{aligned} \tag{5.6}$$

where we used the incentive compatibility property of the contract $Y_T^{Y_0,Z}$ to identify the optimal control $\mathbb{P}^\alpha$ as $\mathbb{P}^{\hat\alpha}$. So we have the identity

$$V_0^P = \sup_{Y_0\geq\mathcal{R}} \underline{V}^P(Y_0). \tag{5.7}$$

Observing that $U_P(x+y) = \exp(-\eta_P x)\, U_P(y), \forall (x,y)\in\mathbb{R}^2$, and from the homogeneity[24] of $Y^{Y_0,Z}$, we get $\underline{V}^P(Y_0) = \exp(-\eta_P Y_0)\, \underline{V}^P(0)$, and therefore $Y_0^\star = \mathcal{R}$.

Now we can rewrite Principal's problem as a Markovian stochastic control problem :

$$V_0^P = \sup_{Z\in\mathcal{V}} \mathbb{E}^{\mathbb{P}^{\hat\alpha}}\left[U_P\left(-Y_T^{\mathcal{R},Z} - \int_0^T P(X_t)\, X_t^C \wedge X_t^D dt + \int_0^T c^P(X_t)\, dt\right)\right], \tag{5.8}$$

with the state variables following Agent's optimal response (which we recall is the same as Principal's recommendation) i.e., $\alpha_t^\star = \hat\alpha(X_t, Z_t)\,,\ \forall t\in[0,T]$:

$$\begin{cases} X_t = \begin{pmatrix} x_0^C \\ x_0^D \end{pmatrix} + \int_0^t \left(\tilde\mu(X_s) + \begin{pmatrix} \frac{Z_s^C - \kappa_1}{\kappa_2} \\ 0 \end{pmatrix}\right) ds + \int_0^t \sigma(X_s)\, dW_s^{\hat\alpha}, \\ Y_t^{\mathcal{R},Z} = \mathcal{R} + \int_0^t \left(\tilde c^A(X_s) + \frac{\left(Z_s^C\right)^2 - (\kappa_1)^2}{2\kappa_2} + \frac{\eta_A}{2}|\sigma(X_s)\, Z_s|^2 - P(X_s)\, X_s^C \wedge X_s^D\right) ds + \int_0^t Z_s\cdot\sigma(X_s)\, dW_s^{\hat\alpha}. \end{cases} \tag{5.9}$$

We define then the dynamic version of the problem

$$V^P(t,x,y) := \sup_{Z\in\mathcal{V}_t} \mathbb{E}^{\mathbb{P}^{\hat\alpha}}\left[U_P\left(-Y_T^{\mathcal{R},Z} - \int_t^T P(X_s)\, X_s^C\wedge X_s^D ds + \int_t^T c^P(X_s)\, ds\right)\right], \tag{5.10}$$

and by standard stochastic control theory (see for instance [Tou12]), V^P defined in (5.10) is the unique viscosity solution of the following HJB equation :

$$\begin{cases} -\partial_t V^P - G\left(x, V^P, DV^P, D^2V^P\right) = 0, \\ V^P(T,x,y) = U_P(-y) = -\exp(\eta_P y), \end{cases} \quad \forall (t,x,y)\in[0,T)\times\mathbb{R}_+^2\times\mathbb{R}, \tag{5.11}$$

where we used the notations $\partial_t V^P$, DV^P and D^2V^P for the derivative w.r.t time, the gradient and the hessian of the function V^P whenever they are defined, otherwise (5.11) should be understood in the weak sense-, and $G : \mathbb{R}_+^2\times\mathbb{R}\times\mathbb{R}^3\times\mathcal{M}_3(\mathbb{R})\to\mathbb{R}$ is the Hamiltonian of (5.10) defined as follow :

$$\begin{aligned} G(x,r,p,\gamma) := \sup_{z\in\mathbb{R}^2} \Bigg\{ & \tilde\mu(x)\cdot p_x + (\tilde c^A(x) - P(x)x^C\wedge x^D - \frac{\kappa_1^2}{2\kappa_2})p_y + \frac{1}{2}\sigma\sigma^T(x) : \gamma_{xx} - \eta_P c^P(x)\, r + \frac{z^C - \kappa_1}{\kappa_2} p_{x^C} \\ & + \eta_P P(x) x^C\wedge x^D(x)\, r + \frac{\left(z^C\right)^2}{2\kappa_2} p_y + \frac{\eta_A}{2} z^T\sigma\sigma^T(x)\, z p_y + \frac{1}{2} z^T\sigma\sigma^T(x)\, z\gamma_{yy} + z^T\sigma\sigma^T(x)\, \gamma_{xy} \Bigg\}. \end{aligned}$$

[24] Which implies that the flow of $Y^{Y_0,Z}$ is positive.

The maximizer in G can be expressed as $\hat{z}_v(x,p,\gamma) := \mathcal{A}^{-1}l$, with

$$\mathcal{A} := (\eta_A p_y + \gamma_{yy})\,\sigma\sigma^T(x) + \frac{1}{\kappa}\begin{pmatrix} p_y & 0 \\ 0 & 0 \end{pmatrix} \text{ and } l := -\sigma\sigma^T(x)\,\gamma_{xy} - \frac{1}{\kappa}\begin{pmatrix} p_{x^C} \\ 0 \end{pmatrix}, \tag{5.12}$$

so that

$$\begin{aligned} G(x,r,p,\gamma) \;&=\; \tilde{\mu}(x)\cdot p_x - \frac{\kappa_1}{\kappa_2}p_x^C + (\tilde{c}^A(x) - P(x)x^C \wedge x^D - \frac{\kappa_1^2}{2\kappa_2})p_y \\ &\quad + \frac{1}{2}\sigma\sigma^T : \gamma_{xx} - \eta_P\left(c^P(x) - P(x)x^C \wedge x^D\right) r - \frac{1}{2}l^T\mathcal{A}^{-1}l. \end{aligned} \tag{5.13}$$

Whenever V^P is a classical solution of PDE (5.10), the optimal control $Z_t^\star$ of Principal on the contract can be written as

$$Z_t^\star = \hat{z}_v\left(X_t, DV^P(t,X_t,Y_t), D^2V^P(t,X_t,Y_t)\right), \forall t \in [0,T]. \tag{5.14}$$

We can see that (5.10) might be simplified further by a change of variable $V^P(t,x,y) = -\exp(\eta_p y)\exp(-\eta_P u(t,x))$, thanks to the choice of the exponential utility function. This allows us to express Principal's value function in terms of the viscosity solution of a 2-dimensional PDE (instead of 3). In fact, $u : [0,T]\times\mathbb{R}_+^2 \to \mathbb{R}$ is the unique viscosity solution to the PDE :

$$\begin{cases} \partial_t u + \left(\tilde{\mu}(x) - \begin{pmatrix} \frac{\kappa_1}{\kappa_2} \\ 0 \end{pmatrix}\right)\cdot Du + \frac{1}{2}\sigma\sigma^T(x) : D^2u + f(x) - \frac{\kappa_1^2}{2\kappa_2} + \frac{1}{2}\rho(x)\cdot\begin{pmatrix} (\partial_{x^C}u)^2 \\ (\partial_{x^D}u)^2 \end{pmatrix} = 0 \\ u(T,x) = 0 \end{cases} ,\forall (t,x) \in [0,T]\times\mathbb{R}_+^2, \tag{5.15}$$

where $f : \mathbb{R}^2 \to \mathbb{R}$ is defined as $f := \tilde{c}^A - c^P$, and

$$\rho(x) := \begin{pmatrix} \frac{\frac{1}{\kappa_2^2} + \frac{1}{\kappa_2}\eta_P\left(\sigma^C x^C\right)^2 - \eta_A\eta_P\left(\left(\sigma^C x^C\right)^2\right)^2}{(\eta_A+\eta_P)(\sigma^C x^C)^2 + \frac{1}{\kappa_2}} \\ \frac{-\eta_A\eta_P\left(\sigma^D x^D\right)^2}{(\eta_A+\eta_P)} \end{pmatrix}. \tag{5.16}$$

Similarly, whenever u is a classical solution of (5.15), $Z_t^\star$ can be computed in terms of u, as

$$Z_t^\star = \hat{z}_u(X_t, Du(t,X_t)),\ \forall t \in [0,T],$$

where $\hat{z}_u : \mathbb{R}_+^2 \times \mathbb{R}^2 \to \mathbb{R}$ is defined as follow

$$\hat{z}_u(x,p) := \begin{pmatrix} \frac{\eta_P\left(\sigma^C x^C\right)^2 + \frac{1}{\kappa_2}}{(\eta_A+\eta_P)(\sigma^C x^C)^2 + \frac{1}{\kappa_2}} & 0 \\ 0 & \frac{\eta_P}{(\eta_A+\eta_P)} \end{pmatrix} p. \tag{5.17}$$

□

5.6 Producer's participation constraint: the problem without capacity payment

Using standard stochastic control tools, we can solve (2.19) by characterizing Producer's continuation utility function $\hat{V}^A : [0,T]\times\mathbb{R}_+^2 \to \mathbb{R}$ defined by

$$\hat{V}^A(t,x) := \sup_{\mathbb{P}^\alpha\in\mathcal{P}} \mathbb{E}^{\mathbb{P}^\alpha}\left[U_A\left(\int_t^T P(X_s)\,X_s^C \wedge X_s^D ds - \int_t^T c^A(X_s,\alpha_s)\,ds\right)\right], \tag{5.18}$$

as the unique viscosity solution to the HJB equation :

$$\begin{cases} -\partial_t \hat{V}^A - \hat{H}\left(x, \hat{V}^A, D\hat{V}^A, D^2\hat{V}^A\right) = 0, \\ \hat{V}^A(T,.) = -1. \end{cases} \tag{5.19}$$

Where $\hat{H} : \mathbb{R}_+^2 \times \mathbb{R}^2 \times \mathcal{M}_2(\mathbb{R}) \to \mathbb{R}$ is the Hamiltonian of the producer (acting on his own, by opposition to the previous Hamiltonian H) defined as

$$\hat{H}\left(x, \hat{V}^A, D\hat{V}^A, D^2\hat{V}^A\right) := \sup_{\alpha\in\mathbb{R}} \left\{ \mu(x,\alpha) \cdot D\hat{V}^A + \frac{1}{2}\sigma\sigma^T : D^2\hat{V}^A + \eta_A(c^A(x,\alpha) - P(x)x^C \wedge x^D)\hat{V}^A \right\}. \tag{5.20}$$

The maximizer is a function of the state variable and the value function; $\hat{\alpha} = -\frac{\partial_{x^C}\hat{V}^A + \eta_A\kappa_1\hat{V}^A}{\eta_A\kappa_2 x^C\hat{V}^A}$, and the PDE (5.19) can be written as

$$\begin{cases} \partial_t \hat{V}^A + \tilde{\mu}\cdot D\hat{V}^A + \frac{1}{2}\sigma\sigma^T : D^2\hat{V}^A + \eta_A\left(\tilde{c}^A(x) - P(x)x^C\wedge x^D\right)\hat{V}^A - \frac{\left(\hat{V}^A_{x^C} + \eta_A\kappa_1\hat{V}^A\right)^2}{2\eta_A\kappa_2\hat{V}^A} = 0, \\ \hat{V}^A(T,.) = -1. \end{cases} \tag{5.21}$$

Using again the change variable $\hat{V}^A(t,x) = -e^{-\eta_A\hat{u}^A(t,x)}$, where $\hat{u}^A : [0,T]\times\mathbb{R}_+^2 \to \mathbb{R}$, we have $\hat{\alpha} = \frac{\partial_{x^C}\hat{u}^A - \kappa_1}{x^C\kappa_2}$ with $\hat{u}^A$ is the unique viscosity solution of the PDE :

$$\begin{cases} \partial_t \hat{u}^A + \tilde{\mu}\cdot D\hat{u}^A + \frac{1}{2}\sigma\sigma^T : D^2\hat{u}^A + \frac{1}{2}\frac{\left(\hat{u}^A_{x^C} - \kappa_1\right)^2}{\kappa_2} - \frac{\eta_A}{2}\sigma\sigma^T : D\hat{u}^A\left(D\hat{u}^A\right)^T - \left(\tilde{c}^A(x) - P(x)x^C\wedge x^D\right) = 0, \\ \hat{u}^A(T,.) = 0. \end{cases} \tag{5.22}$$

5.7 Proof of Theorem 2.4

For readers familiar with BSDE theory, Theorem 2.4 can be seen as an existence result for BSDEs with a quadratic generator. The following proof is largely inspired by [ED16] and [EE+18], is classical in the non-Markovian stochastic control theory and relies on the *Agent's continuation utility* as a natural candidate for the solution of the BSDE.

We start by defining Agent's continuation utility, and prove that it satisfies the Dynamic Programming Principle. Then we use the assumptions on the set of admissible contracts and the properties of Agent's continuation utility to conclude the proof of Theorem 2.4.

Definition 5.1. *Let τ be a stopping time valued in $[t,T]$. We denote by $\mathcal{U}_\tau$ the restriction of (Agent's) controls to $[\tau,T]$. We define the dynamic version of Agent's objective function for a given $\xi\in\Xi$ as*

$$J^A_\tau(\xi,\mathbb{P}^\alpha) := \mathbb{E}^{\mathbb{P}^\alpha}_\tau\left[U_A\left(\xi + \int_\tau^T P(X_s)X^C_s\wedge X^D_s ds - \int_\tau^T c^A(X_s,\alpha_s)ds\right)\right] \text{ and } \mathcal{J}^A_\tau(\xi) := \left(J^A_\tau(\xi,\mathbb{P}^\alpha)\right)_{\alpha\in\mathcal{U}_\tau}, \tag{5.23}$$

and his continuation utility

$$V^A_\tau(\xi) := \operatorname*{ess\,sup}_{\alpha\in\mathcal{U}_\tau} J^A_\tau(\xi,\mathbb{P}^\alpha). \tag{5.24}$$

Remark that for any $\mathbb{P}^\alpha \in \mathcal{P}$, the conditional expectation $\mathbb{E}_\tau^{\mathbb{P}^\alpha}$ depends only on the restriction of α on $[\tau, T]$. It is then defined without ambiguity for $\alpha \in \mathcal{U}_\tau$.

Lemma 5.2. *For $\xi \in \Xi$, $t \in [0,T]$, and τ an $\mathbb{F}$-stopping time in $[t,T]$, we have that*
(i) *The family $\mathcal{J}_\tau^A(\xi)$ satisfies the lattice property, therefore the limiting sequence approaching $V_\tau^A(\xi)$ can be chosen to be non-decreasing, i.e., there exists a sequence of $\left(\mathbb{P}^{\alpha^n}\right)_{n\geq 0}$ such that*

$$V_\tau^A(\xi) = \lim_{n\to+\infty} \uparrow J_\tau^A\left(\xi, \mathbb{P}^{\alpha^n}\right). \tag{5.25}$$

(ii) *The dynamic programming principle for Agent's value function holds, i.e. :*

$$V_t^A(\xi) = \operatorname*{ess\,sup}_{\mathbb{P}^\alpha\in\mathcal{P}} \mathbb{E}_t^{\mathbb{P}^\alpha}\left[V_\tau^A(\xi)\, e^{\eta_A \int_t^\tau c^A(X_s,\alpha_s) - P(X_s)X_s^C\wedge X_s^D ds}\right]. \tag{5.26}$$

Proof. (i) We consider two controls α and α' in $\mathcal{U}_\tau$. We define then

$$\tilde{\alpha} := \alpha \mathbb{1}_{\left\{J_\tau^A(\xi,\mathbb{P}^\alpha)\geq J_\tau^A\left(\xi,\mathbb{P}^{\alpha'}\right)\right\}} + \alpha' \mathbb{1}_{\left\{J_\tau^A(\xi,\mathbb{P}^\alpha)< J_\tau^A\left(\xi,\mathbb{P}^{\alpha'}\right)\right\}}$$

Then $\tilde{\alpha} \in \mathcal{U}_\tau$ and from the definition of $\tilde{\alpha}$ we have the inequality

$$J_\tau^A\left(\xi, \mathbb{P}^{\tilde{\alpha}}\right) \geq \max\left(J_\tau^A\left(\xi, \mathbb{P}^\alpha\right), J_\tau^A(\xi, \mathbb{P}^{\alpha'})\right),$$

which proves the lattice property, implying (i) (see for instance [Nev72, Proposition VI.I.I, p121]).

(ii) The proof of this part is similar to the one in [CK93, Proposition 6.2]. We proceed in two steps proving each of the two inequalities. The first inequality is a direct consequence of the tower property. In fact, for $t \in [0,T]$ and $\tau \in \mathcal{T}_{t,T}$, we have by definition

$$\begin{aligned} V_t^A(\xi) &= \operatorname*{ess\,sup}_{\alpha\in\mathcal{U}_t} \mathbb{E}_t^{\mathbb{P}^\alpha}\left[-e^{-\eta_A\left(\xi - \int_t^T (c^A(X_s,\alpha_s) - P(X_s)X_s^C\wedge X_s^D)ds\right)}\right] \\ &= \operatorname*{ess\,sup}_{\alpha\in\mathcal{U}_t} \mathbb{E}_t^{\mathbb{P}^\alpha}\left[-e^{-\eta_A\left(\xi - \int_t^\tau (c^A(X_s,\alpha_s) - P(X_s)X_s^C\wedge X_s^D)ds - \int_\tau^T (c^A(X_s,\alpha_s) - P(X_s)X_s^C\wedge X_s^D)ds\right)}\right] \end{aligned}$$

By the tower property of the expectation we write

$$V_t^A(\xi) = \operatorname*{ess\,sup}_{\alpha\in\mathcal{U}_t} \mathbb{E}_t^{\mathbb{P}^\alpha}\left[e^{\eta_A \int_t^\tau (c^A(X_s,\alpha_s) - P(X_s)X_s^C\wedge X_s^D)ds} \mathbb{E}_\tau^{\mathbb{P}^\alpha}\left[-e^{-\eta_A\left(\xi - \int_\tau^T (c^A(X_s,\alpha_s) - P(X_s)X_s^C\wedge X_s^D)ds\right)}\right]\right]$$

Using Bayes rule and remarking that $\mathbb{E}_\tau^{\mathbb{P}^\alpha}\left[-e^{-\eta_A\left(\xi - \int_\tau^T (c^A(X_s,\alpha_s) - P(X_s)X_s^C\wedge X_s^D)ds\right)}\right]$ depends only on values of α after τ, we have that for an arbitrary $\alpha \in \mathcal{U}$

$$\begin{aligned} \mathbb{E}_\tau^{\mathbb{P}^\alpha}\left[-e^{-\eta_A\left(\xi - \int_\tau^T (c^A(X_s,\alpha_s) - P(X_s)X_s^C\wedge X_s^D)ds\right)}\right] &\leq \operatorname*{ess\,sup}_{\alpha\in\mathcal{U}} \mathbb{E}_\tau^{\mathbb{P}^\alpha}\left[U_A\left(\xi - \int_\tau^T (c^A(X_s,\alpha_s) - P(X_s)X_s^C\wedge X_s^D)ds\right)\right] \\ &= \operatorname*{ess\,sup}_{\alpha\in\mathcal{U}_\tau} \mathbb{E}_\tau^{\mathbb{P}^\alpha}\left[U_A\left(\xi - \int_\tau^T (c^A(X_s,\alpha_s) - P(X_s)X_s^C\wedge X_s^D)ds\right)\right] \\ &= V_\tau^A(\xi), \end{aligned}$$

and then

$$V_t^A(\xi) \leq \operatorname*{ess\,sup}_{\mathbb{P}^\alpha \in \mathcal{P}} \mathbb{E}_t^{\mathbb{P}^\alpha}\left[V_\tau^A(\xi)\, e^{\eta_A \int_t^\tau (c^A(X_s,\alpha_s) - P(X_s) X_s^C \wedge X_s^D) ds}\right]. \tag{5.27}$$

We proceed next to prove the second inequality. Consider $\alpha \in \mathcal{U}$ and $\nu \in \mathcal{U}_\tau$. Define then the concatenation of the two controls for $0 \leq s \leq T$ as $(\alpha \otimes_\tau \nu)_s := \alpha_s \mathbb{1}_{0 \leq s \leq \tau} + \nu_s \mathbb{1}_{\tau \leq s \leq T}$, where τ is an $\mathbb{F}$–stopping time.

We have then $(\alpha \otimes_\tau \nu) \in \mathcal{U}$ and by definition of the essential supremum (where we denote $\mathbb{E}_t^{\alpha \otimes_\tau \nu}$ instead of $\mathbb{E}_t^{\mathbb{P}^{\alpha \otimes_\tau \nu}}$):

$$\begin{aligned}
V_t^A(\xi) &\geq \mathbb{E}_t^{\alpha \otimes_\tau \nu}\left[-e^{-\eta_A\left(\xi - \int_t^T c^A(X_s,(\alpha\otimes_\tau \nu)_s) - P(X_s) X_s^C \wedge X_s^D ds\right)}\right] \\
&= \mathbb{E}_t^{\alpha \otimes_\tau \nu}\left[-e^{-\eta_A\left(-\int_t^\tau (c^A(X_s,\alpha_s) - P(X_s) X_s^C \wedge X_s^D) ds - \int_\tau^T (c^A(X_s,\nu_s) - P(X_s) X_s^C \wedge X_s^D) ds\right)} e^{-\eta_A \xi}\right] \\
&= \mathbb{E}_t^{\alpha \otimes_\tau \nu}\left[e^{\eta_A \int_t^\tau (c^A(X_s,\alpha_s) - P(X_s) X_s^C \wedge X_s^D) ds} \mathbb{E}_\tau^{\alpha \otimes_\tau \nu}\left[-e^{-\eta_A\left(\xi - \int_\tau^T (c^A(X_s,\nu_s) - P(X_s) X_s^C \wedge X_s^D) ds\right)}\right]\right]
\end{aligned}$$

Using again Bayes formula on the conditional expectation w.r.t $\mathcal{F}_\tau$, we have that

$$\mathbb{E}_\tau^{\alpha \otimes_\tau \nu}\left[-e^{-\eta_A\left(\xi - \int_\tau^T (c^A(X_s,\nu_s) - P(X_s) X_s^C \wedge X_s^D) ds\right)}\right] = \mathbb{E}_\tau^0\left[-\frac{L_T^{\alpha \otimes_\tau \nu}}{L_\tau^{\alpha \otimes_\tau \nu}} e^{-\eta_A\left(\xi - \int_\tau^T (c^A(X_s,\nu_s) - P(X_s) X_s^C \wedge X_s^D) ds\right)}\right]$$

Now notice that $\frac{L_T^{\alpha \otimes_\tau \nu}}{L_\tau^{\alpha \otimes_\tau \nu}} = \frac{L_T^\nu}{L_\tau^\nu}$ (as stated earlier the change of measure applied to the conditional expectation depends only on the control after τ). We have therefore

$$\begin{aligned}
\mathbb{E}_\tau^{\alpha \otimes_\tau \nu}\left[-e^{-\eta_A\left(\xi - \int_\tau^T (c^A(X_s,\nu_s) - P(X_s) X_s^C \wedge X_s^D) ds\right)}\right] &= \mathbb{E}_\tau^0\left[-\frac{L_T^\nu}{L_\tau^\nu} e^{-\eta_A\left(\xi - \int_\tau^T (c^A(X_s,\nu_s) - P(X_s) X_s^C \wedge X_s^D) ds\right)}\right] \\
&= J_\tau^A(\xi, \mathbb{P}^\nu).
\end{aligned}$$

Thus we obtain the following inequality

$$V_t^A(\xi) \geq \mathbb{E}_t^{\alpha \otimes_\tau \nu}\left[e^{\eta_A \int_t^\tau (c^A(X_s,\alpha_s) - P(X_s) X_s^C \wedge X_s^D) ds} J_\tau^A(\xi, \mathbb{P}^\nu)\right].$$

We use again Bayes Formula for the change of measure and the tower property of condtionional expectation leading to

$$\begin{aligned}
V_t^A(\xi) &\geq \mathbb{E}_t^0\left[\frac{L_T^{\alpha \otimes_\tau \nu}}{L_t^{\alpha \otimes_\tau \nu}} e^{\eta_A \int_t^\tau (c^A(X_s,\alpha_s) - P(X_s) X_s^C \wedge X_s^D) ds} J_\tau^A(\xi, \mathbb{P}^\nu)\right] \\
&= \mathbb{E}_t^0\left[\mathbb{E}_\tau^0\left[\frac{L_T^{\alpha \otimes_\tau \nu}}{L_t^{\alpha \otimes_\tau \nu}} \frac{L_\tau^{\alpha \otimes_\tau \nu}}{L_\tau^{\alpha \otimes_\tau \nu}} e^{\eta_A \int_t^\tau (c^A(X_s,\alpha_s) - P(X_s) X_s^C \wedge X_s^D) ds} J_\tau^A(\xi, \mathbb{P}^\nu)\right]\right] \\
&= \mathbb{E}_t^0\left[\mathbb{E}_\tau^0\left[\frac{L_T^{\alpha \otimes_\tau \nu}}{L_\tau^{\alpha \otimes_\tau \nu}}\right] \frac{L_\tau^{\alpha \otimes_\tau \nu}}{L_t^{\alpha \otimes_\tau \nu}} e^{\eta_A \int_t^\tau (c^A(X_s,\alpha_s) - P(X_s) X_s^C \wedge X_s^D) ds} J_\tau^A(\xi, \mathbb{P}^\nu)\right] \\
&= \mathbb{E}_t^0\left[\frac{L_\tau^{\alpha \otimes_\tau \nu}}{L_t^{\alpha \otimes_\tau \nu}} e^{\eta_A \int_t^\tau (c^A(X_s,\alpha_s) - P(X_s) X_s^C \wedge X_s^D) ds} J_\tau^A(\xi, \mathbb{P}^\nu)\right]
\end{aligned}$$

Now recall that for $0 \leq s \leq \tau$ we have by definition $(\alpha \otimes_\tau \nu)_s = \alpha_s$, and therefore $\frac{L_\tau^{\alpha \otimes_\tau \nu}}{L_t^{\alpha \otimes_\tau \nu}} = \frac{L_\tau^\alpha}{L_t^\alpha}$ leading to

$$V_t^A(\xi) \geq \mathbb{E}_t^0\left[\frac{L_\tau^\alpha}{L_t^\alpha} e^{\eta_A \int_t^\tau (c^A(X_s,\alpha_s) - P(X_s) X_s^C \wedge X_s^D) ds} J_\tau^A(\xi, \mathbb{P}^\nu)\right]$$
$$= \mathbb{E}_t^\alpha\left[e^{\eta_A \int_t^\tau (c^A(X_s,\alpha_s) - P(X_s) X_s^C \wedge X_s^D) ds} J_\tau^A(\xi, \mathbb{P}^\nu)\right]. \tag{5.28}$$

The inequality (5.28) holds for $\alpha \in \mathcal{U}$ and $\nu \in \mathcal{U}_\tau$, we can then by virtue of (i) choose a sequence $(\nu^n)_{n\in\mathbb{N}}$ of controls in $\mathcal{U}_\tau$ such that

$$V_\tau^A(\xi) = \lim_{n\to+\infty} \uparrow J_\tau^A\left(\xi, \mathbb{P}^{\nu^n}\right),$$

then we have by the monotone convergence theorem that for $\alpha \in \mathcal{U}$

$$V_t^A(\xi) \geq \lim_{n\to+\infty} \uparrow \mathbb{E}_t^\alpha\left[e^{\eta_A \int_t^\tau (c^A(X_s,\alpha_s) - P(X_s) X_s^C \wedge X_s^D) ds} J_\tau^A\left(\xi, \mathbb{P}^{\nu^n}\right)\right]$$
$$= \mathbb{E}_t^\alpha\left[e^{\eta_A \int_t^\tau (c^A(X_s,\alpha_s) - P(X_s) X_s^C \wedge X_s^D) ds} \lim_{n\to+\infty} \uparrow J_\tau^A\left(\xi, \mathbb{P}^{\nu^n}\right)\right]$$
$$= \mathbb{E}_t^\alpha\left[e^{\eta_A \int_t^\tau (c^A(X_s,\alpha_s) - P(X_s) X_s^C \wedge X_s^D) ds} V_\tau^A(\xi)\right],$$

concluding the proof of Lemma 5.2. □

Proof of Theorem 2.4

No that we proved the Dynamic Programming Principle, we move to the existence of the BSDE. The set $\mathcal{Z}$ is defined a subset of Ξ, so the first inclusion $\mathcal{Z} \subset \Xi$ is trivial. To prove the second inclusion, we fix some $\xi \in \Xi$, and define Agent's continuation utility as in (5.24).

By virtue of Lemma 5.2, the family $\left(V_\tau^A(\xi) e^{\eta_A \int_0^\tau (c^A(X_s,\alpha_s) - P(X_s) X_s^C \wedge X_s^D) ds}\right)_{\tau\in\mathcal{T}_{0,T}}$ is a $(\mathbb{P}^\alpha, \mathbb{F})$–supermartingale system. Therefore, by the results of [LD81], it can be aggregated by a unique $\mathbb{F}$-optional process up to indistinguishability, which coincides with $\left(V_t^A(\xi) e^{\eta_A \int_0^t (c^A(X_s,\alpha_s) - P(X_s) X_s^C \wedge X_s^D) ds}\right)_{t\in[0,T]}$, and remains a $(\mathbb{P}^\alpha, \mathbb{F})$–supermartingale, which then admits a càd-làg modification since the filtration considered satisfies the usual conditions.

Now we use a key-ingredient in our proof, which is an admissibility constraint of the contract ξ; that $\mathcal{P}^\star(\xi) \neq \emptyset$ and therefore there exists $\mathbb{P}^{\alpha^\star}$[25] such that $V_t^A(\xi) = J_t^A\left(\xi,\ \mathbb{P}^{\alpha^\star}\right)$, for $t \in [0,T]$, and so the process $\left(J_t^A\left(\xi,\ \mathbb{P}^{\alpha^\star}\right) e^{\eta_A \int_0^t (c^A(X_s,\alpha_s) - P(X_s) X_s^C \wedge X_s^D) ds}\right)_{t\in[0,T]}$ is a $(\mathbb{P}^\alpha, \mathbb{F})$–supermartingale, for $\mathbb{P}^\alpha \in \mathcal{P}$.

Furthermore, the processes $\left(J_t^A\left(\xi,\ \mathbb{P}^{\alpha^\star}\right) e^{\eta_A \int_0^t (c^A(X_s,\alpha_s^\star) - P(X_s) X_s^C \wedge X_s^D) ds}\right)_{t\in[0,T]}$ is a $\left(\mathbb{P}^{\alpha^\star}, \mathbb{F}\right)$–UI martingale, since by the super-martingale inequality and the tower property of conditional expectations, we have for every $t_1 \leq t_2 \in [0,T]$:

[25]Note here that we wrote $\mathbb{P}^{\alpha^\star}$ instead of $\mathbb{P}^{\alpha^\star(\xi)}$ to ease notations.

$$
\begin{aligned}
J^A_{t_1}\left(\xi,\mathbb{P}^{\alpha^\star}\right)e^{\eta_A\int_0^{t_1}(c^A(X_s,\alpha_s^\star)-P(X_s)X_s^C\wedge X_s^D)ds} &\geq \mathbb{E}^{\mathbb{P}^{\alpha^\star}}_{t_1}\left[J^A_{t_2}\left(\xi,\mathbb{P}^{\alpha^\star}\right)e^{\eta_A\int_0^{t_2}(c^A(X_s,\alpha_s^\star)-P(X_s)X_s^C\wedge X_s^D))ds}\right],\\
&= \mathbb{E}^{\mathbb{P}^{\alpha^\star}}_{t_1}\left[\mathbb{E}^{\mathbb{P}^{\alpha^\star}}_{t_2}\left[U_A\left(\xi-\int_0^T(c^A\left(X_s,\alpha_s^\star\right)-P\left(X_s\right)X_s^C\wedge X_s^D)ds\right)\right]\right],\\
&= \mathbb{E}^{\mathbb{P}^{\alpha^\star}}_{t_1}\left[U_A\left(\xi-\int_0^T(c^A\left(X_s,\alpha_s^\star\right)-P\left(X_s\right)X_s^C\wedge X_s^D)ds\right)\right],\\
&= J^A_{t_1}\left(\xi,\mathbb{P}^{\alpha^\star}\right)e^{\eta_A\int_0^{t_1}(c^A(X_s,\alpha_s^\star)-P(X_s)X_s^C\wedge X_s^D)ds}.
\end{aligned}
$$

Therefore all the previous terms are equal a.s., in particular, for $t\in[0,T]$

$$
J^A_t\left(\xi,\mathbb{P}^{\alpha^\star}\right)e^{\eta_A\int_0^t(c^A(X_s,\alpha_s^\star)-P(X_s)X_s^C\wedge X_s^D)ds} = \mathbb{E}^{\mathbb{P}^{\alpha^\star}}_t\left[U_A\left(\xi+\int_0^T P\left(X_s\right)X_s^C\wedge X_s^D ds-\int_0^T c^A\left(X_s,\alpha_s^\star\right)ds\right)\right], \tag{5.29}
$$

which proves that $\left(J^A_t\left(\xi,\mathbb{P}^{\alpha^\star}\right)e^{\eta_A\int_0^t(c^A(X_s,\alpha_s^\star)-P(X_s)X_s^C\wedge X_s^D)ds}\right)_{t\in[0,T]}$ is a $\mathbb{P}^{\alpha^\star}$–closed martingale. We can then apply the martingale representation theorem and Itô formula to prove that there exists a predictable process $\tilde{Z}\in\mathbb{H}^2_{loc}$ valued in $\mathbb{R}^2$ such that the following representation holds

$$
\begin{aligned}
J^A_t\left(\xi,\mathbb{P}^{\alpha^\star}\right)e^{\eta_A\int_0^t(c^A(X_s,\alpha_s^\star)d-P(X_s)X_s^C\wedge X_s^D)ds} &= J^A_0\left(\xi,\mathbb{P}^{\alpha^\star}\right)+\int_0^t\tilde{Z}_s dW_s^{\alpha^\star}\\
&= J^A_0\left(\xi,\mathbb{P}^{\alpha^\star}\right)\mathcal{E}\left(-\eta_A\int_0^t Z_s\cdot\sigma\left(X_s\right)dW_s^{\alpha^\star}\right),
\end{aligned}
$$

where

$$
Z_t := -\frac{\sigma^{-1}(X_t)\tilde{Z}_t}{\eta_A J^A_t\left(\xi,\mathbb{P}^{\alpha^\star}\right)e^{\eta_A\int_0^t(c^A(X_s,\alpha_s^\star)-P(X_s)X_s^C\wedge X_s^D)ds}}.
$$

Now, for an arbitrary $\alpha\in\mathcal{U}$ we recall that $\tilde{Y}^\alpha_t := J^A_t\left(\xi,\mathbb{P}^{\alpha^\star}\right)e^{\eta_A\int_0^t(c^A(X_s,\alpha_s)-P(X_s)X_s^C\wedge X_s^D)ds}$ is a $(\mathbb{P}^\alpha,\mathbb{F})$–supermartingale and we compute

$$
\frac{\tilde{Y}^\alpha_t}{\eta_A} = \frac{1}{\eta_A}J^A_0\left(\xi,\mathbb{P}^{\alpha^\star}\right)\mathcal{E}\left(-\eta_A\int_0^t Z_s\cdot\sigma\left(X_s\right)dW_s^{\alpha^\star}\right)e^{\eta_A\int_0^t c^A(X_s,\alpha_s)-c^A(X_s,\alpha_s^\star)ds}.
$$

We apply then Itô formula and Girsanov Theorem, therefore

$$
\begin{aligned}
\frac{d\tilde{Y}^\alpha_t}{\eta_A\tilde{Y}^\alpha_t} &= \left(c^A\left(X_t,\alpha_t\right)-c^A\left(X_t,\alpha_t^\star\right)\right)dt-Z_t\cdot\sigma\left(X_t\right)dW_t^{\alpha^\star},\\
&= -Z_t\cdot\sigma\left(X_t\right)dW_t^\alpha-\left\{\left(Z_t\cdot\mu\left(X_t,\alpha_t\right)-c^A\left(X_t,\alpha_t\right)\right)-\left(Z_t\cdot\mu\left(X_t,\alpha_t^\star\right)-c^A\left(X_t,\alpha_t^\star\right)\right)\right\}dt,
\end{aligned}
$$

and by the supermartingale propoerty and the sign of $\tilde{Y}^\alpha_t$ we conclude that

$$
\alpha^\star\in\operatorname{argmax}\left(Z_t\cdot\mu\left(X_t,\alpha_t\right)-c^A\left(X_t,\alpha_t\right)\right).
$$

Finally, we define the process $Y_t := U_A^{-1}\left(J^A_t\left(\xi,\mathbb{P}^{\alpha^\star}\right)\right)$ which we compute by Itô Formula

$$
\begin{aligned}
Y_t &= U_A^{-1}\left(J_0^A\left(\xi,\mathbb{P}^{\alpha^\star}\right)\right)+\int_0^t Z_s\cdot\sigma\left(X_s\right)dW_s^{\alpha^\star}+(c^A\left(X_s,\alpha_s^\star\right)-P\left(X_s\right)X_s^C\wedge X_s^D)ds\\
&= U_A^{-1}\left(J_0^A\left(\xi,\mathbb{P}^{\alpha^\star}\right)\right)+\int_0^t Z_s\cdot dX_s+\eta_A\int_0^t|\sigma(X_s)Z_s|^2ds\\
&-\int_0^t\left(Z_s\cdot\mu\left(X_s,\alpha_s^\star\right)-c^A\left(X_s,\alpha_s^\star\right)+P\left(X_s\right)X_s^C\wedge X_s^D\right)ds,
\end{aligned}
$$

which concludes the proof of the theorem; as we just proved the existence of a pair (Y,Z), satisfying (2.16), with $Y_t:=U_A^{-1}\left(V_t^A\left(\xi\right)\right)$ and $Z_t:=-\frac{\sigma^{-1}(X_t)\tilde{Z}_t}{\eta_A V_t^A(\xi)e^{\eta_A\int_0^t(c^A(X_s,\alpha_s^\star)-P(X_s)X_s^C\wedge X_s^D)ds}}$, such that $\mathcal{E}\left(-\eta_A\int_0^{\cdot}Z_t\cdot\sigma\left(X_t\right)dW_t^{\alpha}\right)$ is a UI martingale. ∎

References

[Aïd15] René Aïd. *Electricity derivatives.* Springer, 2015.

[APT16] René Aïd, Dylan Possamaï, and Nizar Touzi. "A principal–agent model for pricing electricity volatility demand". In: *preprint* (2016).

[Aïd+09] René Aïd, Luciano Campi, Adrien Nguyen Huu, and Nizar Touzi. "A structural risk-neutral model of electricity prices". In: *International Journal of Theoretical and Applied Finance* 12.07 (2009), pp. 925–947.

[BB17] Raúl Bajo-Buenestado. "Welfare implications of capacity payments in a price-capped electricity sector: A case study of the Texas market (ERCOT)". In: *Energy Economics* 64 (2017), pp. 272–285.

[Bha+17] Pradyumna C Bhagwat, Anna Marcheselli, Jörn C Richstein, Emile JL Chappin, and Laurens J De Vries. "An analysis of a forward capacity market with long-term contracts". In: *Energy Policy* 111 (2017), pp. 255–267.

[BK13] RJ Briggs and Andrew Kleit. "Resource adequacy reliability and the impacts of capacity subsidies in competitive electricity markets". In: *Energy Economics* 40 (2013), pp. 297–305.

[Bro18] David P Brown. "Capacity payment mechanisms and investment incentives in restructured electricity markets". In: *Energy Economics* 74 (2018), pp. 131–142.

[Bub+19] Andreas Bublitz, Dogan Keles, Florian Zimmermann, Christoph Fraunholz, and Wolf Fichtner. "A survey on electricity market design: Insights from theory and real-world implementations of capacity remuneration mechanisms". In: *Energy Economics* (2019).

[Cou04] Electricity Consumers Resource Council. "The economic impacts of the August 2003 blackout". In: *Washington, DC* (2004).

[CK93] Jakša Cvitanić and Ioannis Karatzas. "Hedging contingent claims with constrained portfolios". In: *The Annals of Applied Probability* (1993), pp. 652–681.

[CPT18] Jakša Cvitanić, Dylan Possamaï, and Nizar Touzi. "Dynamic programming approach to principal–agent problems". In: *Finance and Stochastics* 22.1 (2018), pp. 1–37.

[CZ13] Jakša Cvitanić and Jianfeng Zhang. *Contract theory in continuous-time models.* Springer, 2013.

[EE+18] Omar El Euch, Thibaut Mastrolia, Mathieu Rosenbaum, and Nizar Touzi. "Optimal make-take fees for market making regulation". In: *Available at SSRN 3174933* (2018).

[ED16] Romuald Elie and Possamaï Dylan. "Contracting theory with competitive interacting agents". In: *arXiv preprint* (2016).

[Fab18] Natalia Fabra. "A primer on capacity mechanisms". In: *Energy Economics* 75 (2018), pp. 323–335.

[HCS16] Daniel Hach, Chi Kong Chyong, and Stefan Spinler. "Capacity market design options: A dynamic capacity investment model and a GB case study". In: *European Journal of Operational Research* 249 (2016), pp. 691–705.

[HRS16] Nicolas Hary, Vincent Rious, and Marcelo Saguan. "The electricity generation adequacy problem: Assessing dynamic effects of capacity remuneration mechanisms". In: *Energy Policy* 91 (Apr. 2016), pp. 113–127.

[HPP07] Florence Hermon, Delphine Perrot, and Virginie Pignon. "Asymmetric Information and Security of Supply: An application of Agency theory to electricity markets". In: *Energy Studies Review* 15.2 (2007).

[Hob+07] Benjamin F. Hobbs, Ming-Che Hu, Javier G. Iñón, Steven E. Stoft, and Murty P. Bhavaraju. "A Dynamic Analysis of a Demand Curve-Based Capacity Market Proposal: The PJM Reliability Pricing Model". In: *IEEE Transactions on Power Systems* 22 (2007), pp. 3 –14.

[HM87] Bengt Holmström and Paul Milgrom. "Aggregation and linearity in the provision of intertemporal incentives". In: *Econometrica: Journal of the Econometric Society* (1987), pp. 303–328.

[Hös+17] Hanspeter Höschle, Cedric De Jonghe, Hélène Le Cadre, and Ronnie Belmans. "Electricity markets for energy, flexibility and availability—Impact of capacity mechanisms on the remuneration of generation technologies". In: *Energy Economics* 66 (2017), pp. 372–383.

[Jos06] Paul Lewis Joskow. "Competitive electricity markets and investment in new generating capacity". In: *Massachusetts Institute of Technology (MIT)-Department of Economics* (2006).

[Léa16] Thomas-Olivier Léautier. "The visible hand: ensuring optimal investment in electric power generation". In: *The Energy Journal* 37.2 (2016).

[LD81] Érik Lenglart and Claude Dellacherie. "Sur des problèmes de régularisation, de recollement et d'interpolation en théorie des martingales". In: *Séminaire de probabilités (Strasbourg)* 15 (1981), pp. 328–346.

[LB15] Todd Levin and Audun Botterud. "Capacity Adequacy and Revenue Sufficiency in Electricity Markets With Wind Power". In: *IEEE Trans. On Power Systems* 30 (2015), pp. 1644 –1653.

[Nev72] Jacques Neveu. "Martingales à temps discret". In: *Masson et CIE* (1972).

[New16] David Newbery. "Missing money and missing markets: Reliability, capacity auctions and interconnectors". In: *Energy Policy* 94 (2016), pp. 401–410.

[San08] Yuliy Sannikov. "A continuous-time version of the principal-agent problem". In: *The Review of Economic Studies* 75.3 (2008), pp. 957–984.

[Sco18] Charlotte Scouflaire. "Capacity remuneration in power markets : an empirical assessment of the cost of precaution". In: *CEEM Working Paper 31* (2018).

[Tou12] Nizar Touzi. *Optimal stochastic control, stochastic target problems, and backward SDE*. Vol. 29. Springer Science & Business Media, 2012.

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