Coverage for fuel poverty*

Corinne Chaton† Marie-Laure Guillerminet‡

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Abstract

To cope with rising energy prices, the French government has implemented an energy price shield in the fall of 2021 in order to "*continue to protect the French households and their purchasing power*" (sic Gabriel Attal, September 3rd, 2022). Following the "roadmap"[1] of 20/21 October 2022, several European Union countries such as Germany or Autria are taking up this idea of a tariff shield to contain energy prices. The United Kingdom also presented an amending budget from October 31 including the tariff shield promised by Lizz Truss. This tariff shield is a short-term and emergency measure that benefits all French economic agents who benefit from regulated energy tariffs. We will focus here on households. Among other things, it helps limit the expenses of precarious households that also receive other forms of assistance and prevents some of them from falling into energy insecurity. We propose another measure to cover households against fuel poverty: an insurance that guarantees a minimum level of energy to households. It applies to non fuel-poor households and is differentiated over income and housing characteristics. Indeed, guaranteed minimum levels of energy supply and discounts are household specific.

To illustrate our results, we focus on non-fuel poor households living in all-electric dwellings and whose electricity price is EDF's Blue peak/off-peak pricing scheme. We estimate that 2.06 TWh are needed to guarantee the minimum level of electricity to these households. This measure will be less expensive than the cost of the tariff shield for these households as long as the sourcing electricity price stays under 803€/MWh!

Key words: Fuel Poverty, Energy Insurance, Tariff Shield.

JEL Codes: D43, L13, Q2.

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†Contact: corinne.chaton@dauphine.psl.eu, Finance for Energy Market Research Centre (FIME), Paris, France.

‡The French Agency for Ecological Transition (ADEME), Valbonne Sophia-Antipolis, France

[1] see https://www.consilium.europa.eu/fr/press/press-releases/

1 Introduction

This study is devoted to a short-term solution, a cover, to protect households from the risk of fuel poverty. This coverage is subscribed by non-precarious households that hedge against fuel poverty that could occur following a drop in their income or a rise in energy prices[2], i.e. a decline in purchasing power. This energy (not monetary) insurance allows households to maintain a decent minimum energy consumption. This aid is tuned to the beneficiary's individual situation, including in terms of energy. It then differs from those already offered by some energy suppliers (Electricité de France - also called EDF, ENGIE and Iberdrola): the latter help pay energy bills by providing insurance payment to households that meet criteria such as loss of employment, cessation of employment (total temporary incapacity), hospitalization, disability (total and irreversible loss of autonomy), accidental death but but not energy price increases.

What is a fuel-poor household? Fuel poverty is defined as follows in the French legislation (cf. the July 12, 2010 law, also named Grenelle II): *"a household that has difficulties obtaining the necessary energy to satisfy his basic needs due to the inadequacy of his resources or his living conditions is in fuel poverty under this Act"*. This definition takes three aspects of fuel poverty into account: energy prices, household resources and housing conditions. In other words, a household is fuel poor when it does not have enough resources to satisfy its basic energy needs, such as proper heating, or when it endures bad living conditions (such as bad insulation against humidity/cold). Of course, some households experience accumulated difficulties: they live in housing with very bad energetic performance (which we literally call "energetic strainers" in French), and they have low revenues. Fuel poverty is a serious issue that affects fairly large populations, even in a developed country such as France, in larger proportions than the famous poverty rate that defines precarious households. Chaton and Gouraud [2020] estimated that 17% of French households were in fuel poverty in 2018. In absolute value, this corresponds approximately to 5 million households and 11 million people. Whereas, according to Delmas and Guillaneuf [2020], in France in 2018, poverty (defined with a 60% threshold of the median revenue of the population) affected 9.3 million people, or almost 15% of the population. Moreover, according to Chaton and Lacroix [2018], 20% of non–fuel–poor households will fall into fuel poverty although almost 12% will not remain in it.

[2]Since the beginning of 2019, an increase in energy prices has been observed in all European and Asian countries. It is explained by the global economic recovery observed following the covid crisis: pending the restart of production, demand is therefore greater than supply. This phenomenon is reinforced for natural gas by "exceptional context" (tight liquefied natural gas market, storage levels at their lowest in years and moderation of Russian exports due to the fire of the Yamal-Europe gas pipeline). Finally the same goes for electricity prices because in Europe, they are established by taking into account the production cost of the last power station called for a production peak, which is a thermal power station that runs on natural gas or fuel oil. This price increase has been accentuated since March 2022 with the war in Ukraine: price movements have nothing to do with fundamentals, but the operators integrate the increased risk of sanctions against the Russian gas.

This insurance is an ex ante measure to prevent energy poverty, which is costly for society. First of all, fuel poverty affects health[3] and the induced costs. According to Eurofund [2016], in France, the direct medical costs related to poor housing are estimated at 930 million euros, and the indirect annual costs (e.g. absenteeism at work and loss of productivity) are estimated at 20.3 billion euros[4]. In addition, late payments of electricity and/or gas bills can create liquidity problems for energy suppliers, and for social and private landlords. Thereby, Sharam [2007], who is interested in the financial stress of low–income households in Australia, specifically in Victoria, finds that overdue rent from these households is used as an overdraft facility to manage cash flow and pay for electricity and/or gas bills. Indeed, electricity and/or gas bills are the main causes of rent arrears (63%), followed by food (34%), because households want at all costs to avoid power and/or gas cuts. At last, fuel poverty and more generally the use of poor energy performance houses have negative consequences on the environment (energy waste and large CO_2 emissions). Consequently, acting against fuel poverty can generate gains for society, especially if this compensating action is more focused on households in fuel poverty. This makes it more effective (in terms of increasing the well-being of individuals), according to Rodriguez-Alvarez et al. [2019].

This insurance complete the panel of existing measures put in place by the French government the energy voucher for households in fuel poverty[5] (curative measure), and the incentive to insulate buildings (long–term measure). First, the energy voucher is a means-tested assistance dedicated to energy: it can be used to pay gas and electricity bills but also wood and heating oil bills (90% of these vouchers were used to pay electricity and/or natural gas suppliers in the 2016 and 2017 trials). It has been available in all French departments since January 1, 2018. In 2021, this aid stretched between 48 and 277 euros per year (the mean amount was 200 euros). This voucher can also be used to help pay for energetic refurbishments in houses, such as new boilers or window insulation against humidity. However, due to the lack of vouchers used for energy efficiency home improvement (63 out of 170,000 in 2016)[6], this aid can be described as a short term measure. The second type of measure aims to save energy, which is a major issue in the fight against fuel poverty. So in the long term, and accessible to all according to household income and composition, numerous public policies are in effect that provide fiscal or financial

[3]Householders in fuel poverty are often forced to make choices with harmful consequences for their health: eating or heating, giving up care or giving up going out. The effects of fuel poverty on the physical and mental health of individuals are not questionable (see, for example, Lacroix and Chaton [2015], Kahouli [2020] and Awaworyi Churchill and Smyth [2021]).

[4]Roys et al. [2010] estimate that in Great Britain, the health expenses due to poor living accommodations are around 516 million euros per year. Taking into account the indirect costs for society, the bill amounts to almost 1.3 billion euros per year.

[5]The voucher first targeted households that devote more than 10% of their budget to paying their energy bills, then households whose income per consumption unit is less than 10,800 euros per year.

[6]This can be explained by the small amount of assistance.

incentives to households to renovate their home in order to improve energy efficiency[7].

Moreover, to cope with the rising energy prices, French Prime Minister Jean Castex has announced, at the end of September 2021, an energy tariff shield for 2022 which was voted in the 2022 Finance Act[8]. This cap on price increases benefits all households and limits the increase in their energy bill to the same percentage, regardless, among other factors, of the size, location and quality of the thermal insulation of the home or the composition of the household (e.g. the size and the number of hours the family members are present at home during the day). So, at the beginning of September 2022, the Deputy to the Budget Minister, Gabriel Attal, announced that this energy price cap would carry over in 2023 in order to "*continue to protect the French households and their purchasing power*". This measure is not differentiated according to households and it is costly. Hence, since the fall of 2021, this policy cost 24 billion euros, including 10.5 billion euros to cap the increase of electricity rates at 4% according to the Ministry of Economy and Finance. In view of the cost for the State and for national energy utilities, French Prime Minister Elisabeth Borne announced on July 9, 2022 that the public authorities also planned to switch in 2023 from general measures to more targeted measures for low–income households. Faced with fuel poverty, Villeneuve [2012] also recommends targeted policies. He thus sees aid as preservation of purchasing power, macroeconomic insurance for the poorest. This aid is based on the individual diagnosis of the beneficiary's energy situation. Due to this, individual needs are better identified and aid depends on actual expenditure.

The aim of this paper is to design an insurance that allows non–fuel poor households to avoid this precariousness due to a drop in its purchasing power linked to a drop in income and/or a rise in energy prices. In Section 2, we develop a two period energy insurance model as in Schlesinger and Zhuang [2014, 2019], Alasseur et al. [2022]. We propose as in Alasseur et al. [2022] a consumption model in which each household consumes two goods, a composite one (which includes at least food and housing) and energy, under a budget constraint. During the first period, the standard of living in the second period is uncertain and households can subscribe to an insurance to be sure to have a minimum amount of energy at their disposal during the following period. During the second period, households that have purchased insurance are guaranteed to be able to consume at least that amount of energy. This model is based on household behavior in the face of risk of variation in purchasing power linked to variations in the price of goods (energy or the composite good) or loss of income. This behavior is represented by a utility function which is based on the work of Kahneman and Tversky [1979] but which

[7] In France, domestic energy efficiency refurbishment programmes can be financed, for examples, by eco-loans at zero rates or tax credits (such as "MaPrimeRénov' "). The "Living better" ("Habiter mieux") program complements these measures for precarious households.

[8] Note that this reflection can be extended to transportation. Since, on March 12, 2022, following the war in Ukraine, it has also announced a discount on fuel prices of 15 euro cents, or even 20 euro cents with the contribution of these distributors, from April 1, 2022.

integrates "essential baskets", i.e. the baskets of goods ("minimum energy" and an "essential composite good") necessary to live decently. To our knowledge, no such utility function has ever been specified. As indicated above, we are considering a differentiated insurance system. "Essential baskets" are therefore set individually for each household. The value of the "essential composite good" depends, for example, on the composition of the family. The value of the "minimum energy" depends on how the home is heated, on its insulation or on the sensitivity of the household to cold. This level can also be chosen and declared by the household.

In Section 3, we calibrate this model with the 2018 and 2019 waves of France's Statistics on Resources and Living Conditions (2018SRCV - 2019SRCV)[9]. We do not take data after 2019, as this would distort calculations of essential levels of goods due to, for example, restraint behavior during the Covid containment periods and that of inflation. We have chosen to focus on households whose housing is all-electric and that have subscribed to the regulated tariff. They are also "well-off" households in the sense that they have sufficient means to pay the minimum level of both goods and this insurance. Since the price of electricity for the households that we consider is regulated, in our application we will assume that the uncertainty will carry on the household's future income[10]. We also illustrate the behavior of a median household, then we observe the results obtained for all the selected households. We proceed as follows. For each household, we determine the value of the "minimum energy" and "essential composite good" and we calibrate the utility function. Then for each "well-off" household, we calculate the probability of having an income in the second period that drops under the insurance trigger threshold.[11] We determine the cost of coverage to protect these households from fuel poverty. We compare the results obtained under our insurance system with that of the tariff shield. We find that such a targeted policy is less costly for the society given a sourcing price that we have estimated. It benefits those who would otherwise fall into precariousness. We determine the amount of money each "well-off" household should save to cover their fuel poverty risk. This amount is referred to hereafter as the *minimum price*. At this price all "well-off" households should agree to cover themselves. However, this may not guarantee the financial equilibrium of the insurer if it has to source its energy on the wholesale market.

The solution would be to make households pay the maximum price they would be willing to pay. This is determined from the utility function as mentioned in 2.5. But for this to

[9]This survey, published by the French National Institute of Statistics and Economic Studies (INSEE), is a part of the European Union Statistics on Income and Living Conditions (EU-SILC), which uses face-to-face interviews to collect information on income distribution, poverty, social exclusion and living conditions. It is used as a reference for income distribution comparisons among European Union member states and for Community actions against social exclusion.

[10]The purpose of these mechanisms (insurance or tariff shield) is to limit the impact of uncertainty on households. As an extension of this paper in the conclusion, we propose to include this type of mechanism in the Universal Service Obligation which also extends to the market price. We will also discuss the pricing structures.

[11]In the illustration, we determine the probability of the household's income loss from 2018-2019 data.

happen, it is necessary to integrate all the aid that "well-off" households likely to receive if they fall into precariousness. Insofar as this consent depends on the desired minimum of energy, it is not obvious that this consent is decreasing with income. Nevertheless, for the same level of insurance, this consent will be higher for the lowest incomes (higher probability of losing purchasing power). It is not certain that "well-off" households will accept to be covered. And even if they all agree to take out insurance, the amount of the premiums may not cover all the costs borne by the insurer, which is dependent on prices on the wholesale market. Thus, this system must be seen as a public-private risk-sharing mechanism: the insurer who has to obtain sourcing from the electricity markets may be in deficit, and it is up to the State to make up for this deficit.

2 Modelling strategy

2.1 General assumptions and notation

We consider that a household makes a decision regarding the consumption of two goods: a composite good X, and energy E during two periods (years) $(t = 0, 1)$ and we denote the consumption in period t by $x_t \geqslant 0$ and $e_t \geqslant 0$, and their prices by p_t^x and p_t^e. The subjective discount factor between the two periods is β $(0 < \beta \leqslant 1)$. We are interested in non–fuel poor household in $t = 0$. More exactly, in $t = 0$, the household income (w_0) is sufficient to consume at least the energy level, denoted $\underline{e}$, needed to heat the home to an adequate standard and to meet the needs for lighting, cooking and running domestic appliances. This non–fuel poor household also consumes at least a level of the composite good $\underline{x}$. This level $\underline{x}$ can be seen as the household's "essential composite good", the minimum that allows it to eat and live. Therefore, in $t = 0$, the income w_0 of this household must be at least equal to $p_0^x\underline{x} + p_0^e\underline{e}$.

The household does not use its savings to pay its energy bills as well as expenses on composite good, which we can justify in different ways: the savings are not available from one year to the next one, the energy price increases are higher than those in savings interest rates[12].

Utility function. We represent the preferences of a household over the goods' consumption at time t by using a separable utility function:

$$U(x_t, e_t) = u(x_t) + \alpha v(e_t), \tag{2.1}$$

where $u(.)$ and $v(.)$ are continuous (on the domain of definition $\mathbb{R}_+$) and increasing functions, and $\alpha \in \mathbb{R}_+$ parameterizes the elasticity of substitution between energy and the composite good.

[12] We would take savings into account in household renovation decisions, but that is not the purpose of this work.

A decent level of energy: $\underline{e}$. This level of energy $\underline{e}$ depends on the home's characteristics (number of rooms occupied, energy efficiency and type of heating), its location, the composition of the family and the number of hours spent in the dwelling during the year. We assume that $\underline{e}$ is observable and chosen by the household. Any consumption e below the necessary level $\underline{e}$ generates disutility for the household. This disutility represents the inconvenience caused by living in poorly heated housing (described in the introduction). As a result, for all $e < \underline{e}$, $v(e) < 0$.

A decent level of composite good: $\underline{x}$. Based on the work of the "decent minimum income" of the French Observatory of Poverty and Social Exclusion[13] which details composition of the baskets of goods and services by type of household (see ONPES [2014-2015]), we assume that the quantity $\underline{x}$, just like $\underline{e}$, is observable and depends on the household. As for energy, any consumption of the composite good x below $\underline{x}$ leads to disutility, the household restricting itself. As a result, for all $x < \underline{x}$, $u(x) < 0$.

Loss of income and fuel poverty insurance. Future household income is uncertain. We denote $w_1 = \widetilde{\omega} w_0$ as the household income in $t = 1$, with $f_{w_0}(\widetilde{\omega})$ being the probability density function of $\widetilde{\omega}$ knowing w_0. To hedge oneself against the effects of a significant loss of income or purchasing power on its energy consumption[14], the household, in the first period, can take out insurance at the price denoted p^I. We denote the decision variable for insurance as I, where I equals 1 when the household contracts insurance and 0 otherwise. In the second period, the household that has purchased the insurance will have a guarantee to consume $\underline{e}$ when its income in $t = 1$ is below a certain threshold, specifically if $\omega \leqslant \underline{\omega}$. Throughout this study, this insurance is named insurance against fuel poverty whose contract is characterized by $(p^I, \underline{e}, \underline{\omega})$. Given the existence of this insurance, the energy consumption of the household in period 1 is

$$e_1^I(\omega, w_0) = e_1^{'}(\omega, w_0) + \underline{e} 1_{I=1, \omega \leqslant \underline{\omega}} \tag{2.2}$$

where $e_1^{'}$ is the amount of energy purchased by the household in $t = 1$ and

$$1_{I=1,\omega\leqslant\underline{\omega}} = \begin{cases} 1 & \text{if} \quad I = 1 \ \text{ and } \ \omega \leqslant \underline{\omega}, \\ 0 & \text{otherwise.} \end{cases}$$

Remark 2.1. *If $\underline{\omega} \to \infty$, then the insurance against fuel poverty is equivalent to prepayment.*

[13] ONPES, Observatoire National de la Pauvreté et de l'Exclusion Sociale.

[14] We could also be interested in insurances or mechanisms that guarantee to households the level of $\underline{x}$ but this is not our subject, and they already exist through for example the unemployment insurance, the income of active solidarity, the retirement insurance, ...

2.2 Household's problem

At time $t = 0$, the household is likely to accept the insurance contract ($I = 1$) only if it provides it with a level of expected utility (EU) at least equal to its expected utility without it ($\mathrm{EU}^{\varnothing}$).

Without insurance, the household maximises independently in each period t his utility (defined by 2.1), under budget constraint:

$$V^0(W_t) := \max_{(x_t,e_t)\in\mathbb{R}^2_+} U(x_t, e_t), \quad \text{u.c.} \quad p^x_t x_t + p^e_t e_t \leqslant w_t. \tag{2.3}$$

Therefore, the expected intertemporal utility of a household without fuel poverty insurance, denoted by $\mathrm{EU}^{\varnothing}$ is

$$\mathrm{EU}^{\varnothing}(w_0, \omega) := V^0(w_0) + \beta\mathbb{E}[V^0(\omega w_0)]. \tag{2.4}$$

The equation (2.4) defines the reservation utility.

The payment of **the fuel poverty insurance** premium p^I only impacts budget constraint household at time $t = 0$, and his maximum utility is thus given by

$$V_0(w_0, p^I) := V^0(w_0 - p^I). \tag{2.5}$$

At time $t = 1$, the fuel poverty insurance ensures the household a fixed non-negative amount $\underline{e}$ of energy if it suffers a sufficient loss of income, i.e. if $\omega < \bar{\omega}$. Therefore, its maximisation problem is

$$V_1(W_0, \omega, \bar{\omega}) := \max_{(x_1,e_1^{'})\in\mathbb{R}^2_+} U(x_1, e_1^{'} + \underline{e}1_{I=1,\omega\leqslant\bar{\omega}}), \quad \text{u.c.} \quad p^x_1 x_1 + p^e_1 e_1^{'} \leqslant \omega w_0. \tag{2.6}$$

Therefore, the intertemporal expected utility of a household with an insurance contract is defined by

$$\mathrm{EU}(w_0, p^I, \omega, \bar{\omega}) := V_0(w_0, p^I) + \beta\mathbb{E}[V_1(w_0, \omega, \bar{\omega})]. \tag{2.7}$$

2.3 Household Preferences: First assumptions

With the aim of obtaining the most explicit results possible, we choose to represent the preferences of the household towards the consumption of goods, by a separable utility function and based on the representation of utilities in prospect theory (see Kahneman and Tversky [1979]). More specifically the functions u and v in (2.1) are specified as follows

$$u\,(y) := v\,(y) := \begin{cases} \nu_m(y) := -\ln\left(1 - y + \underline{y}\right) & \text{if} \quad 0 \leqslant y < \underline{y}, \\ \nu_M(y) := \ln(1 + y - \underline{y}) & \text{if} \quad y \geqslant \underline{y}. \end{cases} \tag{2.8}$$

We also make assumptions about the order of household preferences. As we are only considering two goods, one of which is specific to energy for housing, it goes without saying that if the consumption of the composite good is zero then that of the energy good will be too.

Assumption 2.2. *We assume that the household prefers first to consume a certain quantity of food, then to afford housing (which thus determine $\underline{x}$) and finally to heat its housing. In other words, the household will never stop consuming the composite good, regardless of its income, and will not consume any energy good until before the consumption of the composite good reaches the minimum level $\underline{x}$, i.e. $\forall x < \underline{x}$, $e_t = 0$.*

Assumption 2.2 is plausible insofar as the basket of the composite good includes food and shelter. It should be noted that the value of the minimum level of the composite good, $\bar{x}$, does not depend solely on the composition of the household. The ONPES [2014-2015] report provides us with details on the composition of the baskets of goods that guarantee households a subsistence minimum. This depends in particular on income, e.g. a higher income can be linked to a job that involves more transport expenses. In this case, the minimum value of the composite good $p_t^x \underline{x}$ must include these additional expenses. For some households, health care expenses take priority over heating expenses. Other households will forego health care in order to heat their homes. In the latter case, unlike the former, $p_t^x \underline{x}$ does not include health expenses.

Under Assumption 2.2 and with the specification of $u(.)$ and $v(.)$ defined by (2.8), we consider that the household utility function is

$$U(x_t, e_t) = \begin{cases} U_{m,0}(x_t, e_t) = \nu_m(x) - \alpha_m \ln(1 + \underline{e}) & \text{if} \quad 0 \leqslant x_t < \underline{x} \quad (\forall e_t), \\ U_{M,m}(x_t, e_t) = \nu_M(x) + \alpha_m \nu_m(e) & \text{if} \quad x_t \geqslant \underline{x} \quad \text{and} \quad 0 \leqslant e_t < \underline{e}, \\ U_{M,M}(x_t, e_t) = \nu_M(x) + \alpha_M \nu_M(e) & \text{if} \; x_t \geqslant \underline{x} \quad \text{and} \quad e_t \geqslant \underline{e}. \end{cases} \tag{2.9}$$

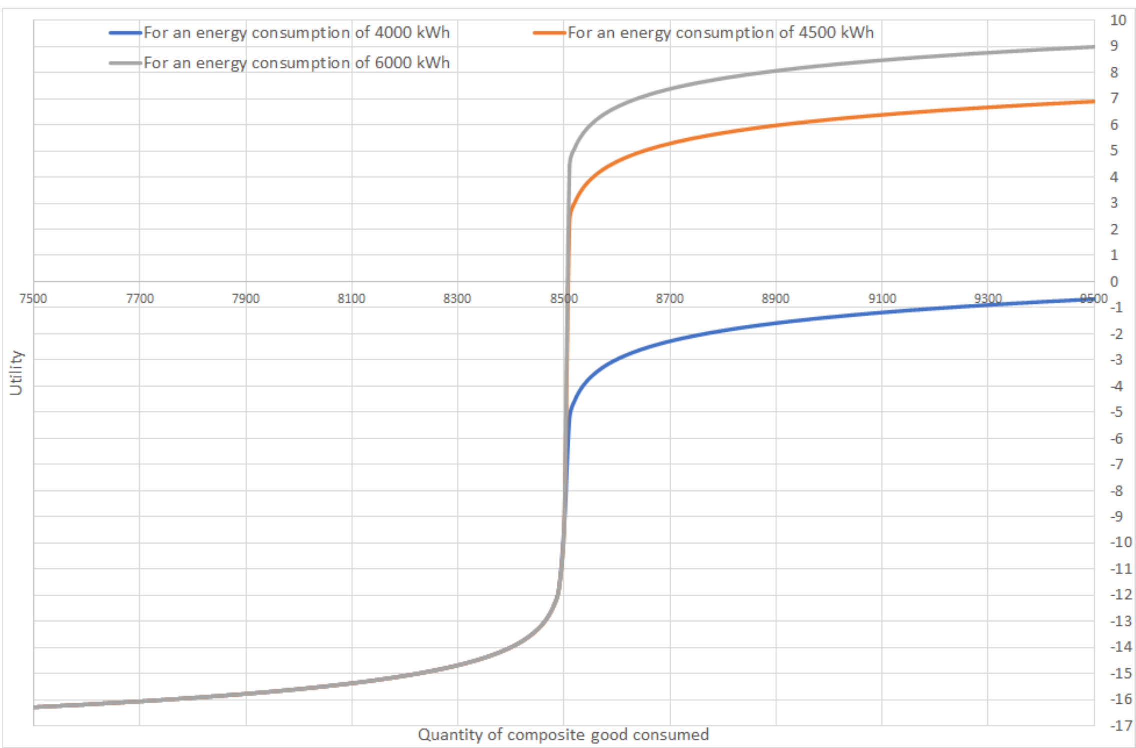
Thus, it is continuous on $\mathbb{R}_+^2$ and it is inspired by the one proposed by Kahneman and Tversky [1979] to evaluate risky projects which reflects the following characteristics of risk behavior. While classical economic theory postulates that individuals evaluate different states of the world absolutely and objectively, Kahneman and Tversky [1979] propose that individuals evaluate situations relatively, with respect to a reference point which may be subjective. In our analysis the reference point is defined by $\underline{x}$ and $\underline{e}$. The utility function $u(.)$ (respectively $v(.)$) is defined around a reference point, $\underline{x}$ (respectively $\underline{e}$), which determines the area of losses and the area of gains. The shape of the function in the loss area (below the reference point) is not the same as in the gain area. Specifically, the shape of the value function above the reference point in the gain region is similar to that of the expected utility function, namely concave. In contrast, it is convex and steeper below the reference point (loss area), reflecting both risk-taking behavior in this zone and loss aversion (see Figure 1 for illustration).

The elasticity of substitution between the two goods, the composite good and energy, represented through the parameters α_m and α_M is therefore not the same depending on whether one is below or above the reference point. The parameter α_M characterizes **the energy sobriety of the household**. Indeed, the lower α_M is, the more energy efficient the household is. It is not the same for α_m, which represents **the effort that the household is forced to make** due to its lack of money to access the decent level of energy.

Proposition 2.3. *If $e_t \geqslant \underline{e}$ and $x_t \geqslant \underline{x}$ the indifference curves are convex (the Marginal Rate of Substitution decreasing); if $e_t < \underline{e}$ and $x_t \geqslant \underline{x}$ it depends on the values of the parameter α_m.*

Proof. See Appendix A.1.2. □

Lemma 2.4. *According Assumption 2.2, if the household income allows it to consume at least $\underline{x}$ then it consumes $\underline{x}$ and can also consume energy. Therefore if $x_t \geqslant \underline{x}$, $\forall e_t$ the indifference curve must be convex. So $\alpha_m > 1$.*



Interpretation: The figure shows the utility as a function of the quantity of the composite good (x_t) for three values of energy consumption (e_t) when the parameter values are $\underline{x} = 8,500$; $\underline{e} = 4,500$; $\alpha_m = 1.1$ and $\alpha_M = 0.3$. The orange curve represents the case where $e = \underline{e}$. It intersects the x-axis when $x = \underline{x}$. If the quantity consumed of the composite good is higher than its decent level i.e. $\underline{x}$, an increase (respectively decrease) of e generates an upward (respectively downward) translation of the utility curve as for the grey curve (respectively blue curve). The three curves are merged if $x_t < \underline{x} = 8,500$. Indeed, in this case, the consumption of the energy good does not increase utility: the household seeks to consume the composite good as a priority.

Figure 1: Representation of the household utility function

Remarks: The lowest utility level (obtained when $x_t = 0$ and $e_t = 0$) is $-(\ln(1+\underline{x}) + \alpha_m \ln(1+\underline{e}))$. The highest utility level when $e_t = 0$ (obtained when $x_t = \underline{x}$ and $e_t = 0$) is equal to $-\alpha_m \ln(1+\underline{e})$. When the household consumes exactly the decent level of both goods its utility is zero.

Notation: In the following, the optimal quantities of the composite good and the energy good are denoted with the exponent (k,l), with k and $l \in \{m, M\}$, that characterizes three states. Thus,

- $(m,0)$ corresponds to the case: $x_t < \underline{x}$ and $e_t = 0$, which characterizes a situation of great poverty. In other words, the household is precarious.
- (M,m) corresponds to the case: $x_t \geqslant \underline{x}$ and $e_t < \underline{e}$, which characterizes a situation of poverty, where households give up heating their dwellings in order to have the minimum baskets of composite goods (for example, giving up heating its dwelling in order to feed itself). Depending on the value of $\underline{x}$ considered, some consider that in this case, these households are not precarious but fuel poor ($e_t < \underline{e}$).
- (M,M) corresponds to the case $x_t \geqslant \underline{x}$ and $e_t \geqslant \underline{e}$ which characterizes the consumption of non-poor households.

2.4 Household choice

2.4.1 Financial constraint, remaining or missing income

Depending on its income, a household does not necessarily have the same preferences. The utility function specification partly takes this into account, through the financial capacity to purchase $\underline{e}$ and/or $\underline{x}$. There are therefore the following three possibilities.

1. The income of the household does not allow it to buy $\underline{\mathrm{x}}$, i.e. $w_t < p_t^x \underline{x}$. The household is necessarily in state $(m,0)$.
2. The income of the household allows it to buy $\underline{x}$ but not $\underline{x}$ plus $\underline{e}$, i.e. $p_t^x \underline{x} \leqslant w_t < p_t^e \underline{e} + p_t^x \underline{x}$. The household is then in state (M,m).
   Reminder: it satisfies his essential needs by consuming the composite good x until the threshold $\underline{x}$ before buying the energy good (see Assumption 2.2). Without additional assumption, it is possible that the household consumes more of the composite good than the essential level, i.e. $x_t > \underline{x}$, but does not consume the energy good, $e_t = 0$. To avoid this, in 2.4.3, α_m is defined as a function of income.

3. The income of the household allows it to buy $\underline{e}$ and $\underline{x}$, i.e. $w_t \geqslant p_t^e \underline{e} + p_t^x \underline{x}$. The household consumes at least $\underline{x}$ of the composite good and is in state (M, M): it also consumes at least $\underline{e}$.

Definition 2.5. *Let us define the disposable (or missing) income at period t of the household after (for) the purchase $\underline{x}$ and $\underline{e}$, i.e.*

$$W_t^d(w_t) = w_t - (p_t^x \underline{x} + p_t^e \underline{e}). \tag{2.10}$$

We consider that the household is precarious in period t if $W_t^d < 0$.

2.4.2 Consumption when the household does not take out fuel poverty insurance

Given the specification of the utility function, the consumption of a household that has not taken out fuel poverty insurance (i.e. $I = 0$) obviously depends on the state in which it is. Optimal consumption without insurance is defined in Proposition 2.6.

Proposition 2.6. *The solutions of the following maximization program*

$$\max_{(x_t, e_t) \in \mathbb{R}_+^2} U(x_t, e_t)$$

u.c. $p_t^x x_t + p_t^e e_t \leqslant w_t$, where $U(.)$ is defined by 2.9 are

- ***in the state*** $(m, 0)$*, i.e.* $U(.) = U_{m,0}(.)$*,*

$$x_t^{(m,0)} = \frac{w_t}{p_t^x} \text{ and } e_t^{(m,0)} = 0, \tag{2.11}$$

- ***in the state*** (M, m)*, i.e.* $U(.) = U_{M,m}(.)$*,*

$$x_t^{(M,m)}(w_t) = \underline{x} + \frac{p_t^e - p_t^x \alpha_m}{p_t^x(\alpha_m - 1)} + \frac{-W_t^d(w_t)}{p_t^x(\alpha_m - 1)}, \tag{2.12}$$

$$e_t^{(M,m)}(w_t) = \underline{e} + \frac{p_t^x \alpha_m - p_t^e}{p_t^e(\alpha_m - 1)} - \frac{-\alpha_m W_t^d(w_t)}{p_t^e(1 - \alpha_m)}; \tag{2.13}$$

- *and* ***in the state*** (M, M)*, i.e.* $U(.) = U_{M,M}(.)$*,*

$$x_t^{(M,M)}(w_t) = \underline{x} - \frac{p_t^x \alpha_M - p_t^e}{p_t^x(\alpha_M + 1)} + \frac{W_t^d(w_t)}{p_t^x(1 + \alpha_M)}, \tag{2.14}$$

$$e_t^{(M,M)}(w_t) = \underline{e} + \frac{p_t^x \alpha_M - p_t^e}{p_t^e(\alpha_M + 1)} + \frac{\alpha_M W_t^d(w_t)}{p_t^e(1 + \alpha_M)}. \tag{2.15}$$

Optimality in these states requires certain conditions on income but also on parameters. Indeed, as mentioned before, to be in the state (M, m) (respectively (M, M)) it is necessary that $w_t > p_t^x \underline{x}$ (respectively $w_t > p_t^x \underline{x} + p_t^e \underline{e}$). These conditions are not sufficient.

Proposition 2.7. *For there to be a state (M, m) when $w_t \in [p_t^x \underline{x}; p_t^e \underline{e} + p_t^x \underline{x}[\Leftrightarrow W_t^d(w_t) \in [-p_t^e \underline{e}; 0[$, the following condition must be verified*

$$W_t^d(w_t) < \min\left(p_t^e - p_t^x \alpha_m; \frac{p_t^e}{\alpha_m} - p_t^x\right) \Leftrightarrow$$

$$w_t < W_t^m = \min\left(p_t^e(\underline{e}+1) + p_t^x(\underline{x} - \alpha_m); p_t^e\left(\underline{e} + \frac{1}{\alpha_m}\right) + p_t^x(\underline{x} - 1)\right). \tag{2.16}$$

Proof. See Appendix A.2.1. □

Proposition 2.8. *For there to be a state (M, M) for all $W_t^d \geqslant 0$, α_M must be in the interval $\left[\frac{p_t^e}{p_t^x + W_t^d}; \frac{p_t^e + W_t^d}{p_t^x}\right]$. Assuming that $\alpha_M < 1$[15], then it is necessary that $p_t^e < p_t^x + W_t^d$.*

Proof. See Appendix A.2.2. □

Since W_t^d is a function of household income, the interval in which α_M must be is a function of income. As a result, the following assumption is specific to the household (via its income).

Assumption 2.9. *In the following we assume that $p_t^e < p_t^x + \max(W_t^d; 0)$ and $\alpha_M \in \left[\frac{p_t^e}{p_t^x + W_t^d}; 1\right]$.*[16]

Under Assumption 2.9, all households whose income w_t is such that $W_t^d(w_t) \geqslant 0$, consume at least the two decent levels of goods, i.e. $\underline{x}$ and $\underline{e}$.

Proposition 2.10. *Under Assumption 2.9,*

- *$W_t^m = p_t^e(\underline{e}+1) + p_t^x(\underline{x} - \alpha_m)$ (W_t^m is defined by (2.16));*
- *if $\alpha_m < \frac{p_t^e(\underline{e}+1)}{p_t^x}$ then $p_t^x \underline{x} < p_t^e(\underline{e}+1) + p_t^x(\underline{x} - \alpha_m)$, otherwise $p_t^x \underline{x} \geqslant p_t^e(\underline{e}+1) + p_t^x(\underline{x} - \alpha_m)$;*
- *$p_t^e(\underline{e} + \frac{1}{\alpha_m}) + p_t^x(\underline{x} - 1) < p_t^e \underline{e} + p_t^x \underline{x}$.*

Proof. See Appendix A.2.3. □

[15] This hypothesis is not strong. Indeed, α_M, a parameter associated with non-fuel poor households, represents the share of energy in relation to other goods in the utility of households. As previously mentioned, the lower α_M is, the more sober the household will be. An α_M greater than 1 could indicate wasteful behavior. The study of the utility function (subsection 3.1) on French households confirms that $\alpha_M < 1$ (see Figure 3).

[16] It should be noted that as in the state (M, M), $W_t^d > 0$, it follows from Proposition 2.8 that if $p_t^e < p_t^x$ and $W_t^d \geqslant 0$ for any α_M included in the interval $\left[\frac{p_t^e}{p_t^x}; 1\right]$, the household is in state (M, M).

Theorem 2.11. *When* $\alpha_m \leqslant \frac{p_t^e(\underline{e}+1)}{p_t^x}$ *(the opposite case is dealt with in Appendix A.3), the optimal consumption of a household that has not taken out fuel poverty insurance is*

$$x_t^*(w_t) := \begin{cases} x_t^{(m,0)} & \text{if } w_t < p_t^x \underline{x}, \\ x_t^{(M,m)} & \text{if } p_t^x \underline{x} \leqslant w_t \leqslant p_t^e(\underline{e}+1) + p_t^x(\underline{x} - \alpha_m), \\ x_t^{(M,M)} & \text{if } w_t \geqslant p_t^e \underline{e} + p_t^x \underline{x}, \end{cases} \tag{2.17}$$

$$e_t^*(w_t) := \begin{cases} 0 & \text{if } w_t < p_t^x \underline{x}, \\ e_t^{(M,m)} & \text{if } p_t^x \underline{x} \leqslant w_t \leqslant p_t^e(\underline{e}+1) + p_t^x(\underline{x} - \alpha_m), \\ e_t^{(M,M)} & \text{if } w_t \geqslant p_t^e \underline{e} + p_t^x \underline{x}, \end{cases} \tag{2.18}$$

where $x^{(m,0)}$, $x^{(M,m)}$, $e^{(M,m)}$, $x^{(M,M)}$ *and* $e^{(M,M)}$ *are defined in (2.11)–(2.15). Given the specification of the utility function defined by (2.9), there is admittedly a small income interval,* $\mathbb{I} = [p_t^e \underline{e} + p_t^x \underline{x} + p_t^e - \alpha_m p_t^x; p_t^e \underline{e} + p_t^x \underline{x}]$*, for which there is no equilibrium.*

To compensate for the non-existence of equilibrium when incomes belong to $\mathbb{I}$, in the following we add in what follows some assumptions about the behavior of the household, more precisely on the effort parameter, α_m.

2.4.3 Household preferences: income-dependent parameters

The change in household behavior can be result in α parameters that vary with income. We now assume that this is the case for α_m. More specifically, we make the following hypothesis.

Assumption 2.12. *As long as a poor household does not consume* $\underline{e}$*, its consumption of the composite good will not exceed* $\underline{x}$*.*

Assumption 2.12 is justified insofar as energy comes last in the order of preference for basic necessities. If this were not the case, we would simply have to split the composite good $\underline{x}$ in two and thus have two minimum levels $\underline{x}_1$ and $\underline{x}_2$ and consider not three but four cases[17]. Therefore, under assumption 2.12, from (2.12) or (2.13) we deduce

$$\alpha_m(w_t) = \underline{x} + \frac{p_t^e(1+\underline{e}) - w_t}{p_t^x}. \tag{2.19}$$

So, the maximum value of the parameter α_m is obtained when $w_t = p_t^x \underline{x}$ and is denoted $\bar{\alpha}_m$. Then

$$\bar{\alpha}_m = \frac{p_t^e}{p_t^x}(1+\underline{e}). \tag{2.20}$$

[17] $(m,0,0)$ corresponds to the case: $x_t^1 < \underline{x}^1$, $e_t = 0$ and $x_t^2 = 0$; $(M,m,0)$ corresponds to the case: $x_t^1 \geqslant \underline{x}^1$, $e_t < \underline{e}$ and $x_t^2 = 0$; (M,M,m) corresponds to the case: $x_t^1 \geqslant \underline{x}^1$, $e_t \geqslant \underline{e}$ and $x_t^2 < \underline{x}^2$; (M,M,M) corresponds to the case: $x_t^1 \geqslant \underline{x}^1$, $e_t \geqslant \underline{e}$ and $x_t^2 \geqslant \underline{x}^2$.

Note that $\lim_{w_t \to p_t^e \underline{e} + p_t^x \underline{x}} \alpha_m(w_t) = \frac{p_t^e}{p_t^x}$. But according to Assumption 2.9, $\frac{p_t^e}{p_t^x} < \frac{p_t^x + \max(W_t^d;0)}{p_t^x}$. So with the addition of Assumption 2.4, $\alpha_m(w_t)$ converges to 1 when income tends to $p_t^e \underline{e} + p_t^x \underline{x}$ and the interval $\mathbb{I}$ defined in theorem 2.11 is empty. Therefore, we now consider the following utility function

$$U(x_t, e_t) = \begin{cases} U_{m,0}(x_t, e_t) = \nu_m(x) - \bar{\alpha}_m \ln(1 + \underline{e}) & \text{if} \quad 0 \leqslant x_t < \underline{x} \quad (\forall e_t), \\ U_{M,m}(x_t, e_t) = \nu_M(x) + \alpha_m(w_t)\nu_m(e) & \text{if} \quad x_t \geqslant \underline{x} \quad \text{and} \quad 0 \leqslant e_t < \underline{e}, \\ U_{M,M}(x_t, e_t) = \nu_M(x) + \alpha_M \nu_M(e) & \text{if}\ x_t \geqslant \underline{x} \quad \text{and} \quad e_t \geqslant \underline{e}. \end{cases} \tag{2.21}$$

This results in the optimal consumptions given in the following theorem.

Theorem 2.13. *Under Assumption 2.9 and Assumption 2.12, the optimal consumption of a household whose utility function verifies (2.9) is*

$$x_t^{\varnothing}(w_t) \quad := \quad \begin{cases} x_t^{(m,0)} & \text{if } w_t < p_t^x \underline{x}, \\ \underline{x} & \text{if } p_t^x \underline{x} \leqslant w_t < p_t^e \underline{e} + p_t^x \underline{x}, \\ x_t^{(M,M)} & \text{if } w_t \geqslant p_t^e \underline{e} + p_t^x \underline{x}, \end{cases} \tag{2.22}$$

$$e_t^{\varnothing}(w_t) \quad := \quad \begin{cases} 0 & \text{if } w_t < p_t^x \underline{x}, \\ w_t - p_t^x \underline{x} & \text{if } p_t^x \underline{x} \leqslant w_t \leqslant p_t^e \underline{e} + p_t^x \underline{x}, \\ e_t^{(M,M)} & \text{if } w_t \geqslant p_t^e \underline{e} + p_t^x \underline{x}, \end{cases} \tag{2.23}$$

where $x^{(m,0)}$, $x^{(M,M)}$ and $e^{(M,M)}$ are respectively defined by (2.11), (2.14) and (2.15).

The solutions are obvious given the assumptions we have made. We have chosen not to impose them from the beginning because, on the one hand, we had to justify them and, on the other hand, our approach allows us to determine endogenously the effort parameters α_m and to give an interval for the sobriety parameters. Let us recall that these parameters depend on the characteristics of the household and those of their housing. When $W_t^d > 0$, the optimal quantities of the two goods are increasing with income and decreasing with the prices of the goods. When in addition α_M increases, the quantity of energy increases while the quantity of the composite good decreases (see Appendix A.4).

2.4.4 Consumption when the household has taken out insurance

Households likely to be covered. The insurance against fuel poverty is aimed at those who are neither poor nor fuel poor in order to prevent them from becoming so. According to the proposition 2.8, the households that can be insured are those whose income in $t = 0$ is such that $W_0^d(w_0)$ is greater than $p^I + \max\left(\frac{p_t^e}{\alpha_M} - p_t^x; -p_t^e + p_t^x \alpha_M\right)$ where $W_0^d(.)$ is defined by (2.10) and p^I is the price of the insurance

The consumption in $t = 0$ of a household that has taken out the insurance is $(x_0^{(M,M)}(w_0 - p^I), e_0^{(M,M)}(w_0 - p^I))$ where $x_0^{(M,M)}$ and $e_0^{(M,M)}$ are respectively defined by 2.14.

Assumption 2.14. ***Insurance trigger threshold****.* *A household that has taken out fuel poverty insurance receives* $\underline{e}$ *in* $t = 1$*, only if it is precarious, i.e. according to Definition 2.5 if it does not have sufficient income to buy* $p_t^x \underline{x} + p_t^e \underline{e}$*, i.e.* $W_1^d(w_1) < 0$ *or equivalently* $\omega < \underline{\omega}(w_0)$ *where*

$$\underline{\omega}(w_0) = \frac{p_1^e \underline{e} + p_1^x \underline{x}}{w_0}. \tag{2.24}$$

Under Assumption 2.14, households hedge against the risk of loss of income but also against the price variations of energy and composite good, i.e. against the risks of loss of purchasing power. We will not explore further what would happen if the insurance only covered possible loss of income and not price variation, which could lead to windfall effects and selection biases.

In $t = 1$, if the household is not precarious, i.e. $W_1^d(w_1) > 0$, then its income is sufficient to prevent it from falling into fuel poverty ($\omega > \underline{\omega}$). It does not receive any indemnity and consequently its consumption is the same as in the case without insurance, i.e. $(x_1^{(M,M)}(\omega w_0), e_1^{(M,M)}(\omega w_0))$ where $x_0^{(M,M)}(.)$ and $e_0^{(M,M)}(.)$ are respectively defined by (2.14) and (2.15). If the loss of purchasing power is significant, i.e. $\omega \leqslant \underline{\omega}(w_0)$, then it receives $\underline{e}$ and two cases must be considered:

- either its income is sufficient to buy $\underline{x}$ then the insured household will be in the state (M, M);
- or it is not and the household will be in the state $(m, \underline{e})$.

This leads to Proposition 2.16 which expresses the optimal consumption in $t = 1$ of a household having subscribed to the insurance in $t = 0$.

Assumption 2.15. *The utility of the household in the state* $(m, \underline{e})$ *is that of the state* $(m, 0)$ *plus* $\alpha_m \ln(1 + \underline{e})$*, then*

$$U^I_{m,\underline{e}}(x_1, e_1) = \nu_m(x) \quad \textit{if} \quad 0 \leqslant x_1 < \underline{x} \quad (\forall e_1). \tag{2.25}$$

Proposition 2.16. *Let us define* $W^{d_x}(\omega) = \omega w_0 - p_1^x \underline{x}$*, then the optimal consumption of insured* *household is*

$$x_1^1(w_0,\omega) \quad := \quad \begin{cases} x_1^{(m,\underline{e})} = x_1^{(m,0)} & \textit{if } \omega < \frac{p_1^x \underline{x}}{w_0}, \\ x_1^{(M,\underline{e}^+)} = \underline{x} + \frac{W^{d_x}(\omega) - p_1^x \alpha_M + p_1^e}{p_1^x(\alpha_M+1)} & \textit{if } \frac{p_1^x \underline{x}}{w_0} \leqslant \omega < \underline{\omega}(w_0), \\ x_1^{(M,M)} & \textit{otherwise}, \end{cases} \tag{2.26}$$

$$e_1^1(w_0,\omega) \quad := \quad \begin{cases} \underline{e} & \textit{if } \omega < \frac{p_1^x \underline{x}}{w_0}, \\ e_1^{(M,\underline{e}^+)} = \underline{e} + \frac{(W^{d_x}(\omega) + p_1^x)\alpha_M - p_1^e}{p_1^e(\alpha_M+1)} & \textit{if } \frac{p_1^x \underline{x}}{w_0} \leqslant \omega < \underline{\omega}(w_0), \\ e_1^{(M,M)} & \textit{otherwise}, \end{cases} \tag{2.27}$$

where $x^{(m,0)}$, $x^{(M,M)}$ and $e^{(M,M)}$ are respectively defined by (2.11), (2.14) and (2.15).

Thus, if the household falls into severe poverty, the amount consumed of the composite good will be the same as it would have been without insurance, but it will consume the minimum level of energy $\underline{e}$. Of course, the household may also receive food and housing assistance. Analytically, these aids for the consumption of the composite good are similar to a decrease in $\underline{x}$.

2.5 Should The Household Be Insured Against Fuel Poverty?

As mentioned above, a household will take out fuel poverty insurance if it is not poor and if the difference between its expected intertemporal utility obtained with insurance $(\mathrm{EU}(w_o, p^I, \omega, \bar{\omega}))$ and that obtained without insurance $(\mathrm{EU}^{\varnothing}(w_0,\omega))$ is positive, i.e

$$\Delta \mathrm{U} = \mathrm{EU}(w_o, p^I, \omega, \bar{\omega}) - \mathrm{EU}^{\varnothing}(w_0,\omega). \tag{2.28}$$

2.5.1 Utility at $t = 0$

From the optimal consumption levels defined in Proposition 2.6 (respectively in Proposition 2.16), we deduce[18] the utility at time $t = 0$ of a non-poor household that has not taken out fuel poverty insurance, $V_0(w_0)$ (resp. the utility of an insured household, $V_0(w_0 - p^I)$) where

$$V_0(X) := \alpha_M \ln\left(\frac{\alpha_M}{p_0^e}\right) + \ln\left(\frac{1}{p_0^x}\right) + (1+\alpha_M)\ln\left(\frac{p_0^e(1-\underline{e}) + p_0^x(1-\underline{x}) + X}{1+\alpha_M}\right) \tag{2.29}$$

and $X = w_0$ without insurance, or resp. $X = w_0 - p^I$ with insurance.

Let $\Delta V_0 = V_0(W_0 - p^I) - V^0(W_0)$. Then the loss of utility at $t = 0$ generated by taking out

[18] See (2.3).

fuel poverty insurance is

$$\begin{aligned}\Delta V_0 &= (1+\alpha_M)\ln\left(1-\frac{p^I}{w_0+p_0^e(1-\underline{e})+p_0^x(1-\underline{x})}\right)\\ &= (1+\alpha_M)\ln\left(1-\frac{p^I}{W_0^d(w_0)+p_0^e+p_0^x}\right).\end{aligned} \tag{2.30}$$

Remark 2.17. ΔV_0 *is defined and negative since by hypothesis the household which subscribes to the insurance has an income* w_0 *such that* $W_0^d(w_0) > p^I$*. The presence of prices (*p_0^e *and* p_0^x*) in the denominator of 2.30 is due to the number 1 in the logarithm of the utility function 2.8.*

2.5.2 Utility at $t = 1$

Given Theorem 2.13, the utility of a household without fuel poverty insurance is in

- the state $(m, 0)$, i.e. $\omega w_0 < p_1^x \underline{x}$

$$V_{m,0}^{\varnothing}(\omega, w_0) = -\ln\left(1+\underline{x}-\frac{\omega w_0}{p_1^x}\right)-\frac{p_1^e}{p_1^x}(1+\underline{e})\ln(\underline{e}+1). \tag{2.31}$$

- the state (M, m), i.e. $p_1^x\underline{x} \leqslant \omega w_0 \leqslant p_t^e\underline{e}+p_1^x\underline{x}$

$$V_{M,m}^{\varnothing}(\omega, w_0) = -\frac{p_1^e-W_1^d(\omega)}{p_1^x}\ln\left(\frac{p_1^e-W_1^d(\omega)}{p_1^e}\right). \tag{2.32}$$

- the state (M, M), i.e. $\omega w_0 \geqslant p_t^e\underline{e}+p_1^x\underline{x}$

$$V_{M,M}^{\varnothing}(\omega, w_0) = \ln\left(\frac{W_1^d(\omega)+p_1^e-p_1^x}{p_1^x(\alpha_M+1)}\right)+\alpha_M\ln\left(\frac{(W_1^d(\omega)+p_1^e-p_1^x)\alpha_m}{p_1^e(\alpha_M+1)}\right). \tag{2.33}$$

The insured household will receive at $t = 1$ the energy power $\underline{e}$ only if its loss of income is significant, more precisely only if $\omega < \underline{\omega}(w_0)$ where $\omega < \underline{\omega}(w_0)$ is defined by 2.24. Therefore, given Proposition 2.16, the utility at $t = 1$ of an insured household is

$$V_1(\omega, \underline{\omega}(w_0)) := \begin{cases} V_{m,0}^{\varnothing}(\omega, w_0)+\bar{\alpha}_m\ln(\underline{e}+1) & \text{if } \omega < \frac{p_1^x\underline{x}}{w_0},\\ V_{\underline{\omega}}(\omega, w_0) & \text{if } \frac{p_1^x\underline{x}}{w_0} \leqslant \omega < \underline{\omega}(w_0),\\ V_{M,M}^{\varnothing}(\omega, w_0) & \text{if } \omega \geqslant \underline{\omega}(w_0),\end{cases} \tag{2.34}$$

where

$$V_{\underline{\omega}}(\omega, w_0) := \ln\left(\frac{p_1^e+p_1^x+W^{d_x}(\omega)}{p_1^x(1+\alpha_M)}\right)+\alpha_M\ln\left(\frac{\alpha_M(p_1^e+p_1^x+W^{d_x}(\omega))}{p_1^e(1+\alpha_M)}\right), \tag{2.35}$$

$V^{\varnothing}_{m,0}(\omega, w_0)$, $V^{\varnothing}_{M,M}(\omega, w_0)$ and $W^{d_x}(\omega)$ are defined respectively by (2.31), (2.33) and in Proposition (2.16).

Let's define the following utility difference:

$$\Delta V_{M,m}(\omega, w_0) = V_{\underline{\omega}}(\omega, w_0) - V^{\varnothing}_{M,m}(\omega, w_0), \tag{2.36}$$

This variation represents the difference in utility at $t = 1$ when the household does or does not receive fuel poverty insurance. Its analytical expression can be found in Appendix B.

2.5.3 Willingness to pay for fuel poverty insurance

The lowest income that the household is likely to earn in $t = 1$ is $\omega^m w_0$ where $\omega^m \geqslant 0$. If the household income in $t = 1$, $w_1 = \omega w_0$ is such that $\omega > \bar{\omega}$ then the insured household does not receive $\underline{e}$ and has therefore lost ΔV_0 (defined by (2.30)) compared to the situation without insurance. Therefore, the utility expectation generated by fuel poverty insurance ΔU defined by (2.28) is rewritten as

$$\Delta U = \Delta V_0 + \beta \left(\int_{\omega^m}^{\frac{p_1^x \underline{x}}{w_0}} \bar{\alpha}_m \ln(\underline{e} + 1) f(\omega)\, d\omega + \int_{\frac{p_1^x \underline{x}}{w_0}}^{\underline{\omega}(w_0)} \Delta V_{M,m}(\omega, w_0) f(\omega)\, d\omega \right). \tag{2.37}$$

The maximum price of insurance that the household accepts, denoted $\bar{p}$, is the one that cancels ΔU if it is less than $W_0^d(w_0)$. Therefore,

$$\bar{p} = \min \left\{ W_0^d, \left(1 - \exp\left(-\frac{\beta W}{1 + \alpha_M}\right)\right) (W_0^d(w_0) + p_0^e + p_0^x)) \right\} \tag{2.38}$$

where

$$W = \bar{\alpha}_m \ln(\underline{e} + 1) \int_{\omega^m}^{\frac{p_1^x \underline{x}}{w_0}} f(\omega)\, d\omega + \int_{\frac{p_1^x \underline{x}}{w_0}}^{\underline{\omega}(w_0)} \Delta V_{M,m}(\omega, w_0) f(\omega)\, d\omega. \tag{2.39}$$

For all energy insurance prices greater than $\bar{p}$, it is not optimal for the household to contract this insurance. Note that $\bar{p}_m \leqslant \bar{p} \leqslant W_0^d(w_0)$ where

$$\bar{p}_m = \min \left\{ W_0^d, \left(1 - (\underline{e} + 1) \exp\left(-\frac{\beta \bar{\alpha}_m}{1 + \alpha_M} \int_{\omega^m}^{\frac{p_1^x \underline{x}}{w_0}} f(\omega)\, d\omega\right)\right) (W_0^d(w_0) + p_0^e + p_0^x)) \right\}, \tag{2.40}$$

where α_m is defined by (2.20).

3 Simulations

3.1 Focus on the utility function

To quantify the three parameters ($\underline{x}$, $\underline{e}$, and α_M)[19] of the utility function defined by (2.9), we use the 2019 wave of France's Statistics on Resources and Living Conditions (2019SRCV). Note that in this database, data is collected for the current year and only income data is from the previous year. We have a sample of 11,737 households representative of the French population. We first restrict the sample to households whose standard of living is above the poverty line, threshold set by convention in France at 60% of the median standard of living of the population[20]. This leaves 10,176 households. Of these 10,176 non-poor households, we focus on the following two types of households characterized by their perception of their current financial situation[21].

- The first ones respond that they are struggling but are getting by, or that it is fair and that they should be careful. We therefore assume that these non-poor households have a financial capacity that only allows them to consume the essential level of composite good (including at least food and housing), $\underline{x}$ and the essential level of energy, $\underline{e}$. These households, for which $W_0^d(w_0)$ is zero, are in a strained financial. We call them "**strained**" households. 42% of households report being in this situation. This is more than the data from Statista[22] where 33% of households have just enough to make ends meet, but no more. If we remove the households that do not declare energy expenditure, or for which this expenditure is nil, or whose income per consumption units (CUs) is greater than 30,620 euros, then the number of households in this situation is 4,935, i.e. 37% of the sample.

- The latter respond to the perception of their financial situation with: "*we are fine*", "*we are rather comfortable*", "*we are really comfortable*". They represent 42% of the sample. These non-poor households that can consume more than $\underline{x}$ and $\underline{e}$, i.e. those whose income w_0 is such that $W_0^d(w_0) > 0$, are hereinafter called **"well-off"** households.

The first type of household is used to calibrate the parameters $\underline{x}$ and $\underline{e}$ while the second allows us to calibrate α_M.

[19] These three parameters and $\alpha_m(.)$ should be indexed by i because they depend on the household i, but in order not to make the notation more cumbersome in the previous section we have omitted this index.

[20] see INSEE: https://www.insee.fr/fr/statistiques/5759045

[21] Question code: NVACTB for perception by the household of its current financial situation.

[22] See https://fr.statista.com/statistiques/1070289/equilibre-budgetaire-francais/

3.1.1 Living standards for composite, excluding energy expenditure for housing, $\underline{x}$

Recall that this level $\underline{x}$ does not include energy expenditures for housing. To simplify our analysis, we assume that it depends only on the composition of the family, more precisely on the consumption units (CUs). Then we have the following assumption.

Assumption 3.1. *Let i and j be two households. If the consumption units of i and j are the same ($CUs_i = CUs_j$) then the minimum quantity of composite good for i and j are the same for both households, i.e. $\underline{x}_i = \underline{x}_j$.*

To determine $\underline{x}$ we look at "strained" household expenditure in 2019SRCV. For each of these 4,935 households, we consider that the value of the minimal level of the composite good per CUs excluding energy $p_0^x \underline{x}_i/\text{CUs}_i$ (for $i = 1, \cdots, 4935$) is equal to $(w_{0,i} - p_0^e e_i)/\text{CUs}_i$, where $w_{0,i}$ is the disposable income of household i, $p_0^e e_i$ is the energy expenditure reported by household i and consumption units of i, CUs_i. Thereafter, we normalize the price of $\underline{x}$, i.e. $p_{2019}^x = 1$. From Assumption 3.1, we take the median of $p_0^x \underline{x}_i/\text{CUs}_i$ as the value of the minimum level of the composite good by CUs. This value is equal to **1,644 euros per month in 2019**[23]. In appendix C.1 the distribution of $\underline{x}$ for the 4,935 households is given. $\underline{x}$ surely does not only depend on the size of the household. To refine the analysis, we should calculate $\underline{x}$ not only as a function of CUs but also, among other things, depending on the geographical location, to take into account, among other things, differences in cost of existing housing between regions/size of agglomeration and so on.

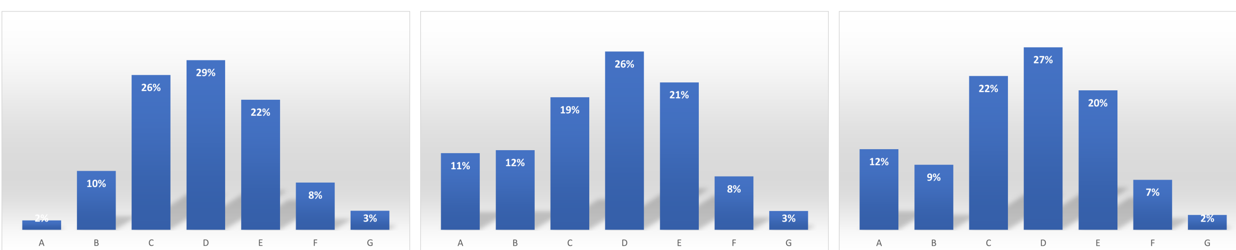
3.1.2 What decent level of energy for the household i?

Homes where the only energy used is electricity. As previously mentioned the decent level of energy $\underline{e}$ depends mainly on the characteristics of the dwelling, i.e. its surface, its energy efficiency as well as the fuel used for heating, for the production of domestic hot water, for cooking. In order not to multiply the cases to be analyzed, we are only interested in households that only use electricity as energy for their homes. To quantify the decent electricity level of a household i living in an all-electric dwelling $\underline{e}_i$, we are interested in the electricity expenditure per square meter of "strained" households living in this type of housing. By definition, the quantity of energy consumed by these households (i.e. 1,150 households out of 4,935 in the sample) is $\underline{e}$.

[23] Obviously, this value varies according to the years considered in order to take into account the variation of the price compared to 2019. By way of comparison, the sum of the food and private accommodation items for a household of two adults with two children is €1,419 (excluding heating) in 2014 according to ONPES [2014-2015].

Electricity at regulated prices. Since we know the (declared) energy expenditure of housing of the household and not the quantity, we must make assumptions about the price of the energy used. We assume that this household subscribes a regulated electricity price with peak/off-peak option, i.e. an *EDF's Blue peak/off-peak tariff*. The average price it pays depends on the one hand on the power it subscribes (according to the surface of the dwelling), and on the other hand on the quantity consumed at each period (peak and off-peak). All prices considered are inclusive of all taxes and can be found on the French government database[24]. Appendix C.2 provides information on the assumptions made concerning the subscribed power as well as the cost of the electricity contract and the peak/off-peak tariff. Given the energy expenditure declared by the household, we deduce the quantity (in kWh) of electricity it consumes from the electricity tariff taken according to the size of the accommodation.

Homes classified according to their energy performance. As the value of $\underline{e}$ depends not only on the surface of the dwelling, but also on its energy performance, we must get an estimate. To do this, we determine from the declared energy expenditure in which score of the Energy Certificate Performance (EPC) label[25] is situated the household's dwelling. Thus, we calculate the amount of primary energy per m^2 that the households consumes in a year (KWhpe/m^2/year) according to the conversion factor of final energy into primary energy for the electricity, i.e. the factor of 2.58 given in Annex 3.2 of the decree[26] of 8/2/2012. This amount is used to classify dwellings and Figure 2 gives, for different types of households, a representation of EPC scores based on energy expenditure reports of households living in all-electric dwellings.



All households "Strained" households "Well-off" households

Interpretation: The curves above represent the percentage in each EPC score of all households living in an all-electric dwelling (left figure); those of these households that declare themselves to be in a strained financial (center figure); and those who have no financial difficulties (right figure).

Figure 2: Percentage of all-electric dwellings in the different EPC scores by household type

[24]See "Historique des tarifs réglementés de vente d'électricité pour les consommateurs résidentiels" on https://www.data.gouv.fr/fr/datasets/

[25]This system, in French the Diagnosis of Energy Performance or DPE, provides information on the energy performance of a dwelling, by rating its energy consumption and its impact in terms of greenhouse gas emissions from A (best performance) to G (worst one). The EPC scores are in Appendix C.3.

[26]https://www.legifrance.gouv.fr/jorf/id/JORFTEXT000025509925/

Let us note that $\underline{e}$ can be defined or chosen by the household when subscribing the contract: not everyone is equally sensitive to the cold, for example.

A reference quantity of primary energy per m^2 for each EPC score for "well-off" households. First, for each EPC score j ($j \in \{$A, B, C, D, E, F, G$\}$), we calculate the average consumption of primary energy per m^2 and per year (kWhpe/m^2/year) of "strained" households denoted $\underline{\epsilon}_j$ (see Table 1). These averages are then used as a reference to determine the value of $\underline{e}$ of a household belonging to the "well-off" households type. Thus, if we consider a "well-off" household i living in a dwelling of EPC score j and a s m^2 surface area then it wishes a minimum energy level $\underline{e}_i$ equal to $2.58 \times s \times \min(\underline{\epsilon}_j; \epsilon_i)$ kWh/year where ϵ_i is the consumption of primary energy per m^2 per year of household i. Note that if $\underline{\epsilon}_j = \epsilon_i$ then the electricity consumption of i is equal to $\underline{e}_i$. Figure 3 shows frequency histogram of $\underline{e}$ for the "well-off" households (with $W^d_{2019,i} > 0$)[27].

A	B	C	D	E	F	G
30.75	70.97	121.62	189.62	268.27	363.57	544.09

Table 1: Average primary energy consumption (kWh$_{\text{pe}}$/m^2/year) of "strained" household of each EPC score, i.e. $\underline{\epsilon}_j$

3.1.3 Determination of the household energy sobriety parameter, α_M

From equation (2.15), since by assumption $p^x_{2019} = 1$, we deduce for each "well-off" household i

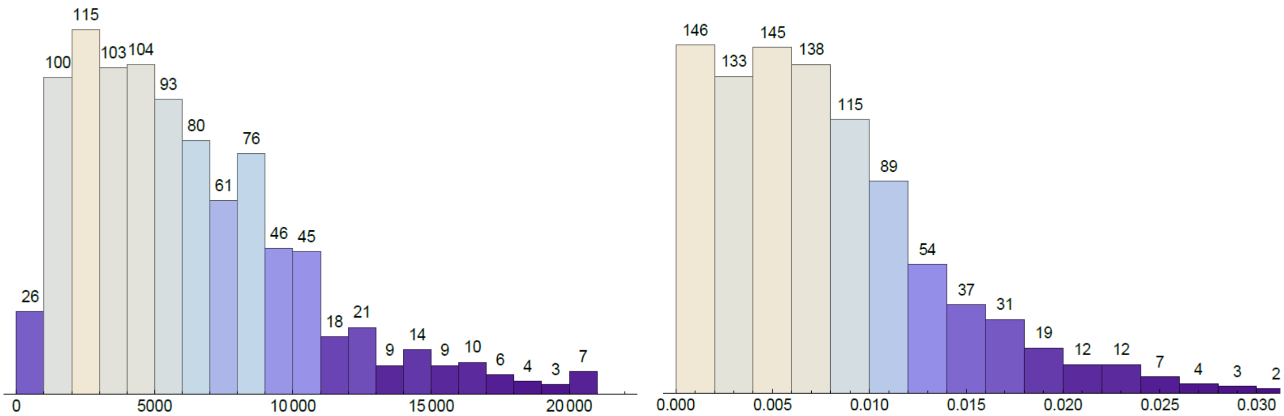
$$\alpha_{M_i} = \frac{(1 + e_i - \underline{e}_i)p^e_{2019,i}}{(\underline{e}_i - e_i)p^e_{2019,i} + 1 + W^d_{2019,i}(w_{2019,i})}, \tag{3.1}$$

where $p^e_{2019,i}$ is the average price of electricity in 2019 for i. This average price takes into account the subscription and consumption divided into 60% during peak hours and 40% during off-peak hours. Figure 3 shows frequency histogram of α_{M_i}.

3.1.4 Determination of the household energy restriction parameter, α_m

To estimate the value of α_m, we have to focus on households that report that they cannot do it without going into debt. But, we need to know the EPC score of their dwelling to assign them a value of $\underline{e}$. However, we do not know the real value of the EPC score of their dwelling

[27] Some households that declare themselves financially comfortable are nevertheless constrained (i.e. for these households, $W^d_{2019} \leqslant 0$). They can therefore no longer considered as "well-off" households. From now on, the number of "well-off" households in 2019SRCV is 954, i.e. 4.81% of all households (or 24.81% of households living in all-electric housing).



Histogram of $\underline{e}_i$ Histogram of α_{M_i}

Interpretation: The decent level of energy $\underline{e}$ for "well-off" households is between almost 265 and $30,058$ kWh per year, with an average level of $6,221$ kWh per year and a median at almost $5,291$ kWh per year. The distribution of $\underline{e}_i$ (left figure) is estimated by the gamma distribution with shape parameter of 2.135 and scale parameter of 2,949. The elasticity of substitution between composite good and energy α_M, which is a parameter of energy sobriety for these "well-off" households, is between almost 0% (0.0014%) and almost 10% (9.99%). The mean is 0.81% and the median is 0.67%

Figure 3: Distribution of "well-off" households according to their decent level of energy and their energy sobriety

and we cannot estimate it from their declared energy expenditure (as we did for the non-poor households). Indeed, these households restrict their consumption (their income is below $p^e_{2019}\underline{e} + \underline{x}$) and consequently their energy expenditure underestimates the EPC score of their dwelling and thus their needs. This estimate is not necessary for our study. The function α_m defined by (2.19) is determined as soon as $\underline{x}$, $\underline{e}$ and prices are known.

3.1.5 Illustration for a median household

In this subsection, we are interested in a representative household living in an all-electric dwelling that we call a median household and that we characterize[28] from 2019SRCV.

A median household. According to 2019SRCV, in 2019, the median number of people in a household is 2 (i.e. 1.5 CU) and the median size[29] of all-electric dwellings is 80 m^2. A medium insulated dwelling corresponds to a D-score dwelling rated according to the EPC label. The same is true for the calculated EPC score[30] for 80 m² all-electric housing in the SRCV database.

Definition 3.2. *The median household consists of two people living in a D-score all-electric dwelling of 80* m^2. *This household has an electricity supply contract at the EDF's Blue peak/off-*

[28] We ensured consistency with the data of INSEE and Selectra, which is a French company specializing in comparisons between electricity, gas and internet offers.

[29] The median size of all dwellings is 92 m^2.

[30] Assuming as above that the median household subscribes a regulated electricity price with peak/off-peak option, i.e. an *EDF's Blue peak/off-peak tariff*.

peak tariff for a power of 9kV: this household's consumption is divided into 40% in off-peak hours and 60% in peak hours.

According to 3.1.1, the value of the minimum level of composite good in 2019 for this median household is equal to $1,644 \times 1.5 = 2,466$ €/month. Following the same reasoning as in 3.1.2, we assume that the minimum energy level, $\underline{e}$, of the median household is equal to 4,744 kWh/year and that the electricity price for this household is equal to 17.6 c€/kWh. The median energy bill for 2-person households living in a dwelling of 80 m^2 without gas calculated with 2019SRCV is equal to €835. In the same year, there were three changes to the regulated electricity tariffs. The weighted average of the different elements that make up these tariffs[31], gives us a electricity price for this household equal to 17.6 c€/kWh. For this median household, we can deduce a consumption of 4,744 kWh/year, or $4,744 \times 2.58/80 \approx 153$ in primary energy per m^2 and per year (kWhpe/m^2/y). Of course, this amount depends on the weather conditions. The value of 153 kWhpe/m^2/y ranks the dwelling of this median household at the upper limit of the D score of the EPC label – it is a very good D score (at the limit of the C score) for the dwelling (see Table 5).

For this median household, α_M is assumed to be the median α_M of 2-person households living in an all-electric dwelling of 80 m^2, i.e. 0.65%. As $p^x_{2019} = 1$, the household is precarious when its annual income w_{2019} is below €28,890 and is in fuel poverty if $w_{2019} \leqslant 29,724.9$ euros. Figure 4 (respectively 5) shows the optimal consumption (respectively utility) without fuel poverty insurance of the median household according to its income. If the household paid for insurance in the previous year, then it only receives $\underline{e}$ if its income is such that $W^d < 0$ which is equivalent with our assumptions of an income lower than €29,724.9. In this case, the optimal consumption and utility are represented in Figure 6.

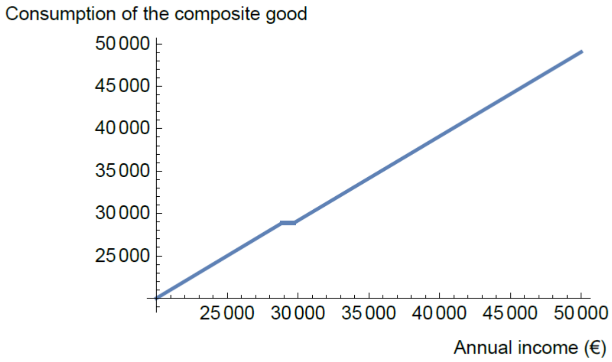
3.2 Income and its evolution

In the following we focus our analysis on French households that are likely to find interest in the energy insurance, i.e. the "well-off" households whose income w_0 verifies $W^d_0(w_0) \geqslant p^I$, where $W^d_0(w_0)$ is defined by (2.10). Therefore we exclude the following households[32].

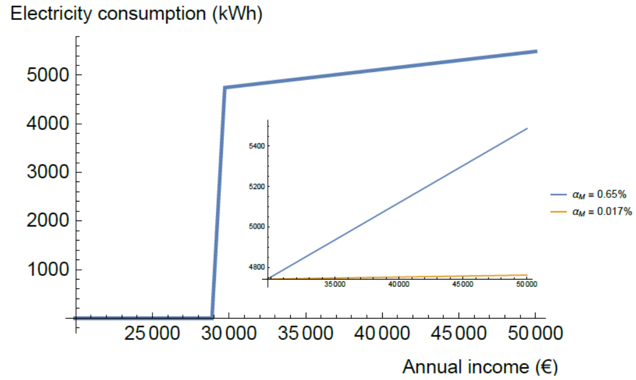
- First of all, this insurance is obviously not intended for households in extreme poverty, i.e. those whose income w_t is less than $p^x_t \underline{x}$. For these *precarious households*, the minimum service can be applied. Thus, a consumption-based system has been effective since April 1,

[31] i.e. a subscription for 9 kV equals to €120.73 and the price at peak (resp. off-peak) hours equals to 16.55 (resp. 12.82) cents euros per kWh.

[32] Note that we only detail the measures concerning energy for the households that we exclude, knowing that we must also take into account the social minima and other measures concerning housing as well as the threshold effects between these measures and the insurance system.



Optimal consumption of the composite good. Since, in 2019, the price of the composite good is equal to 1, the the y-axis also represents the value of the composite good in euros.



Optimal electricity consumption

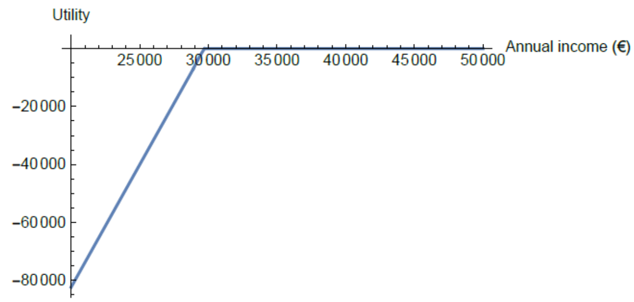
Interpretation: On the left (respectively on the right) the optimal quantity of composite good, x_{2019} (respectively of electricity, e_{2019}) is represented as a function of the annual income of the median household. Given that the price of the composite good is set equal to 1, on the income range $[0; 28,890]$, x_{2019} is on the first bisector while e_{2019} is zero. On the interval $[28,890; 29,724.9]$, x_{2019} is constant and equal to 28,890 while e_{2019} increases (e_{2019} is on the line of slope $1/p^e_{2019} = 1/0.176$). For annual incomes above €29724.9 both goods increase with income: x_{2019} is on the line with slope $\frac{1}{p^x_t(1+\alpha_M)} \approx 0.993$ and e_{2019} is on the line with slope $\frac{\alpha_M}{p^e_t(1+\alpha_M)} \approx 0.037$. Inside the figure on the right, another figure shows the impact of the sobriety parameter, α_M, on electricity consumption. More precisely, the blue (respectively orange) line corresponds to the energy consumption when α_M is equal to 0.65% (respectively 0.017% which is the minimal value of α_M for 2-person households living in an all-electric dwelling of 80 m^2). The slope of the orange line is approximately equal to 0.00097 (much smaller than the blue line). We have not represented the impact of α_M on x_{2019} because it is very small. Indeed, with $\alpha_M = 0.017\%$ the slope of the line on which x_{2019} is located is approximately equal to 0.999.

Figure 4: Optimal consumption of the median household

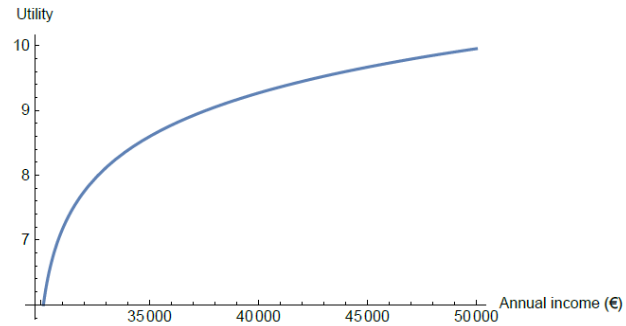
2022 in France. With this system, the supplier, EDF, will no longer cut off the electricity of its individual customers in the event of non-payment but will limit the guaranteed minimum power at 1 kVA (implicitly, these households consume the maximum power when they are present at home). This minimum service will be applied until the customer regularizes his situation by paying the remaining energy bills.

- *Fuel poors* households are those that limit their energy consumption (to a level below $\underline{e}$), i.e. those with an income w_t that do not allow them to purchase the minimum of two goods (i.e. $W^d_t(w_t) < 0$). These households, like the previous ones, should receive the energy voucher.
- Finally, households that we do not consider fuel poors but whose income w_0 is not enough to pay for the decent level of energy $\underline{e}$ plus the cost of insurance p_I, i.e. $W^d_0(w_0) < p^I$, should not buy insurance. Among these households are the "strained" households.

To calculate a household's willingness to pay for energy insurance, we need to know the prob-



Utility for $w_{2019} \in [0; 50,000]$



Utility for $w_{2019} \in [29,725; 50,000]$

Interpretation: The figure on the left shows the high disutility of not being able to consume the essential minimum of the two goods because of insufficient income. The extreme case of zero annual income generates an utility equal to $-\ln(1+\underline{x}) - \alpha_m \ln(1+\underline{e}) \approx -251,629$. If the household income allows it to consume $\underline{x}$ but not to consume electricity ($e_{2019} = 0$), i.e. $w_{2019} = 28,890$ euros, then the utility is -7,069.16. When the household income makes it possible to acquire more than $\underline{x}$ and $\underline{e}$, the utility is obviously positive (left-hand curve) and its variation generated by a variation of the income is much less pronounced than in the left-hand curve. Of course, the cases where incomes are very low are almost non-existent because of the possibility for households to receive aids.

Figure 5: Optimum utility for the median household

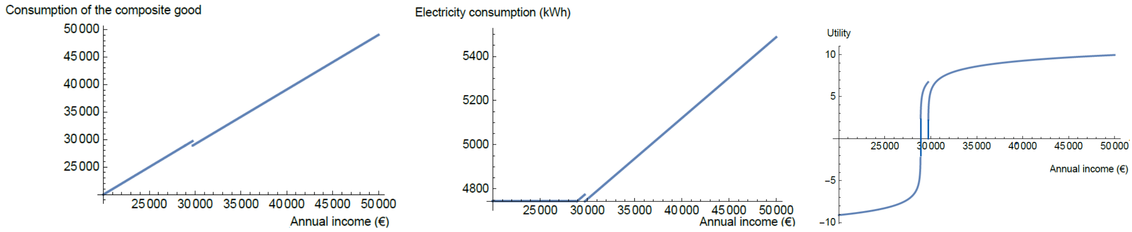
ability of an annual change in household income. This is necessary because among households that have purchased energy insurance in $t = 0$, only households whose income in $t = 1$ is such that $W_1^d < 0$ will be covered in $t = 1$. In order to have an analytical expression for the utility expectations defined in 2.5, we proceed as follows.

First, we find a simple functional form in order to fit the distribution g of data $\omega_{i,t} = \frac{w_{i,t+1}}{w_{i,t}}$ (where $w_{i,t+1}$ is the income of household i in $t+1$, and $w_{i,t}$ is the income for the previous year). Then, we modify the support of the distribution so that it equals the interval $[\min\{\omega_{i,t}\}, \max\{\omega_{i,t}\}]$ and we approximate the new distribution f_t by several Padé approximants on its support.

Thus, using the 2018 and 2019 waves of SRCV[33], we have $t = 2018$, $\omega^m = \min\{\omega_{i,2018}\} = 0.015$, $\omega^M = \max\{\omega_{i,2018}\} = 54.45$ and the median is 1.01. Figure 7 shows the frequency histogram of $\omega_{i,2018}$. We find that the distribution f_{2018} of $\omega_{i,2018}$ is a Student's t-distribution with a location parameter of 1.01176, a scale parameter of 0.1272, and 1.499 degrees of freedom (see Figure 8). Note that the probability that a household has an income at most equal to that of the previous year is 46%. The probability for a household to have only 80% (or 70%) of its income from the previous year is 12% (resp. 7%).

We consider five Padé approximants ($a = 1, 2, 3, 4, 5$) to this Student's t-distribution on the support $\left[\omega^m, \omega^M\right]$ about the point ω_a, with a numerator of order 10 and a denominator of order

[33] The number of observations is 8,368.



Interpretation: On the left (respectively in the center) the optimal amount of composite good, x_{2019} (respectively electricity, e_{2019}) of the median insured household is represented. The figure on the right represents the corresponding utility. Compared to the case without insurance, the difference in the quantity of composite good consumed is in the income range $[28,890; 29,724.9[$. The quantity is higher because some of the money that would have been spent on electricity without insurance is available to increase consumption of the composite good. The quantity of electricity consumed on the income range $[0; 28,890]$ is equal to $\underline{e}$ whereas it was null without insurance. On the interval $[28,890; 29,724.9[$ it is greater than $\underline{e}$ whereas it was less without insurance. The discontinuity of optimal quantities when $w = 29,724.9$ results in a discontinuity of the utility function. Insofar as on the interval $[28,890; 29,724.9[$ the quantities consumed are greater than those consumed when $w = 29,724.9$, the utility on this interval is higher than that obtained when $w = 29,724.9$.

Figure 6: Optimal consumption and utility of the median insured household

0, i.e.

$$f_{2018,a}(\omega) \approx \sum_{k=0}^{10} \gamma_{k,a}(\omega - \omega_a)^k. \tag{3.2}$$

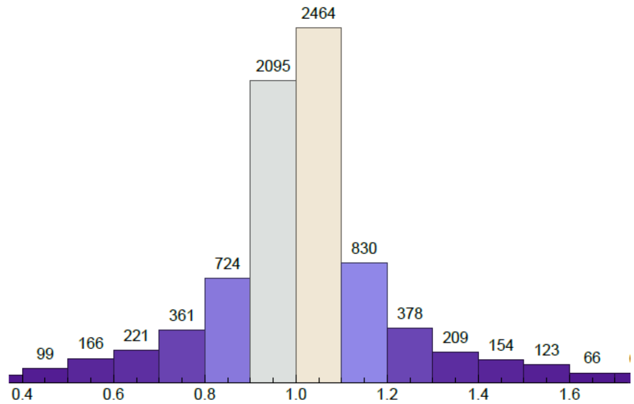
Let $\omega_1 = \omega^m = 0.015$, $\omega_2 = 0.65$, $\omega_3 = 0.85$, $\omega_4 = 1$ and $\omega_5 = 1.1$ (see Table 2 for the value of $\gamma_{k,a}$). We approximate f_{2018} on the interval $[\omega^m, 1.063]$ by

$$h_{2018}(\omega) = f_{2018,a}(\omega) \text{ if } \omega_{a-1,a} \leqslant \omega \leqslant \omega_{a,a+1} \text{ for all a}, \tag{3.3}$$

where $\omega_{0,1} = \omega_1$, $\omega_{5,6} = \bar{\omega}$ and $\omega_{a,a+1}$ ($a = 1, \ldots 5$) is the real solution on the interval $[\omega_a, \omega_{a+1}]$ of the equation $f_{2018,a}(\omega) = f_{2018,a+1}(\omega)$. With the data used and the selected ω_a, we obtain $\omega_{1,2} = 0.474$, $\omega_{2,3} = 0.775$, $\omega_{3,4} = 0.943$, $\omega_{4,5} = 1.035$ and $\bar{\omega} = 1.063$, (see Figure 8 for the representation of f_{2018} and its approximation h_{2018} for $\omega \in [0.015, 0.9]$). Note that for all $\omega > 0$, we have $h_{2018}(\omega) > f_{2018}(\omega)$. The percentage of error is between 1.6 and 1.74 (see Figure 8).

To get a better estimate and approximation of the distribution g of $\omega_{i,t}$, the methodology explained above is also performed for some income deciles[34]. We are not interested in the first two income deciles that do not have the financial capacity to buy insurance (no "well-off" household belongs to these deciles). Consequently, the distribution of $\omega_{i,2018}$ and its Padé approximation are made for income deciles 3 to 10 (See C.3.1).

[34]For the upper limits of the disposable income brackets (decile, in euros per year) according to 2018SRCV, see Table 6 in Appendix C.3.1.



Interpretation: The histogram of $\omega_i = \frac{W_{i,2019}}{W_{i,2018}}$ (the change in income) shown above is truncated on the right (for readability reasons). 6,113 households out of 8,368 have an ω_i value between 0.8 and 1.2.

Figure 7: Frequency histogram of observed w_i for $t = 2018$

k	$a = 1$	$a = 2$	$a = 3$	$a = 4$	$a = 5$
0	0.026	0.268	1.092	2.705	1.924
1	0.062	1.563	8.755	3.259	-13.24
2	0.106	6.039	36.115	135.01	6.98
3	0.154	18.757	48.465	-296.76	440.3
4	0.203	49.153	475.20	5919	2821
5	0.251	107.7	4257.7	19367	1198
6	0.296	180.013	18244	243319	88460
7	0.336	125.704	30761	1.09×10^6	544492
8	0.370	-628.224	180836	9.50×10^6	159349
9	0.397	-3866.44	1.84×10^6	5.67×10^7	1.7×10^7
10	0.414	-14230.8	8.31×10^6	-3.5×10^8	-1.01×10^8

Table 2: Values of the parameters $\gamma_{k,a}$ of the Pade approximation of the distribution of ω_{2018}

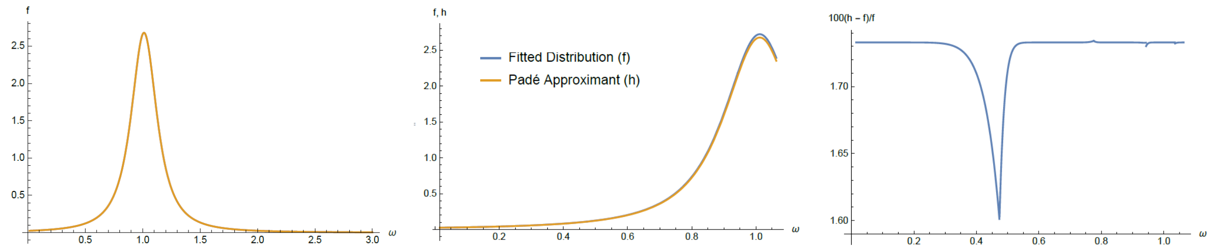
3.3 Assessing the willingness to pay for insurance

We determine the willingness to pay, in December 2021, of "well-off" households[35] to insure against a loss of purchasing power in 2022, due to a possible loss of income and/or an increase in electricity prices.

3.3.1 Electricity prices

Consequently, our interest is in EDF's Blue peak/off-peak tariff in December 2021 and for the year 2022. By subscribing to a tariff, the household pays an average price which decreases according to the quantities purchased due to the fixed part of this tariff. However, the household taking out the insurance must be able to buy at least a decent level of energy, $\underline{e}$. Indeed we

[35] 942 "well-off" households in 2019SRCV.



f_{2018} on $[0.015; 3]$ f_{2018} and h_{2018} on $[0.015; 1.063]$ $100(h - f)/f$ on $[0.015; 1.063]$

Interpretation: The curve on the left presents the distribution function of $\omega_{i,2018} = \frac{W_{i,2019}}{W_{i,2018}}$ truncated on the right (on the interval $[0.015; 3]$). In the center, this same distribution on $[0.015; 1.063]$ (in blue) and its approximation by Padé approximants (in orange) are shown. The figure on the right is the percent error, i.e. for all $\omega \in [0.015; 1.063]$, the representation of the function defined by $100\frac{h(\omega)-f(\omega)}{f(\omega)}$.

Figure 8: Fitted Distribution of $\omega_{i,2018}$, its approximation and the percent error

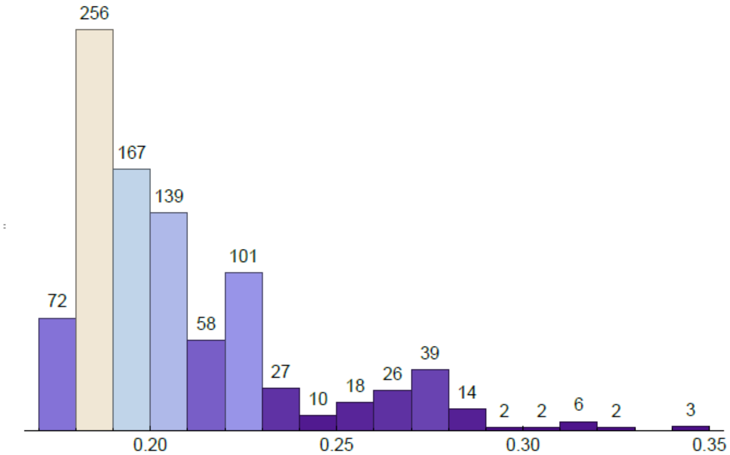
are interested in households that can afford insurance, i.e. whose income must be greater than $W^d_{2021} + p^I$, where W^d_{2021} involves consumption of $\underline{e}$. Consequently, the average price of electricity will be at most equal to that corresponding to $\underline{e}$. So, we focus on the price that corresponds to $\underline{e}$. In December 2021, the subscription for 9 kVA was €183.68; for 12 kVA was €221.5; for 15 kVA was €258.07; for 18 kVA was €292.35; and the price at peak (resp. off-peak) hours equals to 18.21 (resp. 11.93) cents euros per kWh.[36] Figure 9 shows the distribution of the average electricity price in 2021 for the "well-off" households that would be likely to purchase the insurance.

Illustration: Thus, for the median household, the decent level of energy $\underline{e}$ is equal to 4, 744kWh/y, split 40% in off-peak hours and 60% in peak hours. The annual energy bill amounts to €921, i.e. an average price is 19.41 cents euro per kWh.

To cope with the rising energy prices, and as mentioned in the introduction, French Prime Minister Jean Castex has announced, at the end of September 2021, an energy tariff shield for 2022 which was voted in *the 2022 Finance Act*. The government is thus fixed, by decree, a cap of 4% for the whole of 2022 on the increase in regulated electricity tariffs (whether offered to residential or professional consumers).[37] Indeed, the Government has refused to implement the tariff proposal of the French Energy Regulatory Agency (CRE). In its deliberation of January 18, 2022, CRE has calculated an average change in the regulated tariffs, on February 1, 2022, by 35% for all consumers. This proposal reflects the increase in electricity supply costs in a context of increases in wholesale electricity prices, induced by gas and CO_2 prices as well as by the reduced availability of the French nuclear power plants. The rise capped by the government

[36] Source: https://www.data.gouv.fr/fr/datasets/historique-des-tarifs-reglementes-de-vente-delectricite-pour-les-consommateurs-residentiels/

[37] See https://www.cre.fr/Actualites/proposition-d-evolution-des-tarifs-reglementes-de-vente-d-electricite-au-1er-fevrier-2022



Statistics	€/MWh
min	17.22
max	34.57
mean	20.85
median	19.88

Interpretation: The histogram represents the average electricity prices (in December 2021) from the EDF's Blue peak/off-peak tariff for "well-off" households consuming their minimum energy level, $\underline{e}$.

Figure 9: Electricity prices in December 2021 for "well-off" households

has taken place in February 2022. It is therefore the only one scheduled for 2022, so we assume that in 2022 the price will be that of February[38]. As a reminder, in February 2022, CRE did not rule out a possible catch-up from 2023. Even if it will not be in 2023, we must still think about the fact that this catch-up will take place in the future.

Illustration: For the median household, the subscription is then €182.29 and the price at peak (resp. off-peak) is 18.41 (resp. 14.70) cents euro per kWh. The median household pays now a bill of €943.78 for the decent energy level, i.e. an average price of 19.89 cents euro per kWh. The bill for the decent energy level has increased by €22.77 for an average price increase of 2.47%. This is less than the advertised 4%, but as a reminder, this 4% is calculated as an average of all consumers. Note that the price excluding subscription for the median household as well as all households at EDF's Blue peak/off-peak tariff increased between December 2021 and August 2022 by 3.13%. If prices had increased by 35% as proposed by CRE, then the average price of the median household would have reached 26.21 cents euro per kWh for the decent energy level. It pays an invoice of €1,243.

For "well-off" households, the variation in the average price of electricity between December 2021 and August 2022 is between 0.93% and 2.95% with a mean (respectively median) of 2.33% (respectively 2.44%). Thereafter, it is assumed that the price of electricity in $t = 0$ (respectively $t = 1$) for the "well-off" household i, noted $p_{2021,i}$ (resp. $p_{2022,i}$) is equal to the average price calculated from the blue tariff EDF's Blue peak/off-peak in December 2021 (resp. August 2022) that it pays to obtain its minimum energy level $\underline{e}_i$. In order to see the impact of the tariff shield,

[38] Bourgeois and Lafrogne-Joussier [2022] work from these two variations envisaged: 4% with the tariff shield and 35% without.

we also study the case where the energy price in $t = 1$ is equal to $p^e_{35\%,i} = p^e_{2021,i}(1 + 0.35) = 0.25893$. This case is characterised in the following by the index "35%".

Observation: all things being equal, with the tariff shield only 0.32% of "well-off" households fell into fuel poverty compared to 0.74% without the shield (i.e. with an increase in electricity prices of 35%).

3.3.2 Prices of the composite good

We have normalized the price of the composite good in 2019, i.e. $p^x_{2019} = 1$. To follow the evolution of inflation, we refer to the variations of the INSEE Consumer Price Index[39]. For 2020 and 2021, we have taken into account the annual weights of the energy good in the reference basket on which INSEE bases its calculations of Consumer Price Inflation. We then withdraw 50%[40] of the energy from this reference basket, assuming that it is energy for housing, i.e. the good e. The rest is therefore energy for transport. For 2022, we are directly removing energy from the housing item in the INSEE reference baskets calculated whether or not taking into account the tariff shield. We therefore obtain $p^x_{2020} = 1.007$ in 2020, $p^x_{2021} = 1.02$ in 2021. In 2022, with the tariff shield $p^x_{2022} = 1.068$ and without the tariff shield $p^x_{35\%} = 1.084$. As mentioned by Bourgeois and Lafrogne-Joussier [2022], the rise in energy prices has a double effect on inflation, one direct, the other transiting through the productive system. In a direct way, the rise in the price of good e weighs directly on the purchasing power of households. In a direct way, the rise in energy prices weighs directly on the purchasing power of households: these are the prices of the transport item in the composite good or the energy good e. Higher energy prices also increase the costs of firm, which pass through, at least in part, these increases to their own selling prices. These sales may be to households or to other firms, but in the latter case, cost increases continue to be transmitted down the value chain and end up affecting the purchasing power of households as well.

Observation: According to our hypotheses, at identical income, between 2019 and 2021, 3.05% of "well-off" households living in an all-electric dwelling fell into fuel poverty. Between 2021 and the second quarter of 2022, despite the tariff shield, this percentage is 5.64%. Without the shield this percentage would be much higher as it would be equal to 8.46%. Assuming that in 2022 there are 29 million French households and that 8.05% of them are "well-off" households living in all-electric dwellings having subscribed to a supply contract at the regulated tariff (EDF's Blue peak/off-peak tariff), we estimate that the cost of the tariff shield for these households is approximately 1.66 billion euros (€1.66bn), of which €0.43bn is due to the price

[39]See for 2020 and 2021 https://www.insee.fr/fr/statistiques/6036866, and for 2022 https://www.insee.fr/fr/statistiques/6523439 and Bourgeois and Lafrogne-Joussier [2022], https://www.insee.fr/fr/statistiques/6524161.

[40]We set this share according to the INSEE baskets and the baskets detailed by ONPES [2014-2015].

effect on quantities. Insofar as these households in 2021 were not all at the regulated electricity price, 1.66 billion is an overestimate of the cost borne by the supplier of the tariff shield applied in 2022 to these "well-off" households. The cost of 1.66 billion euros is therefore included in the cost announced for 2022 of the tariff shield of 10.5 billion euros to cap the increase in electricity tariffs at 4%. In total, the tariff shield on energy prices has amounted to 24 billion euros since its deployment in the fall of 2021 to cushion the shock of inflation (announcement of September 1, 2022 by the Ministry of the Economy and Finance). Moreover this amount of 1.66 billion euros should be a lower bound than the cost of the tariff shield announced for 2023. Indeed, on the one hand, while the announcements about the expected increase in the price of electricity seem to estimate it at 100%, the government would like to limit this increase to 10% or even 20%[41] and on the other hand, the even partial withdrawal of suppliers from the residential market (such as Iberdrola[42]). These announcements lead households that were not at the regulated electricity price to subscribe to new contracts at this regulated price.

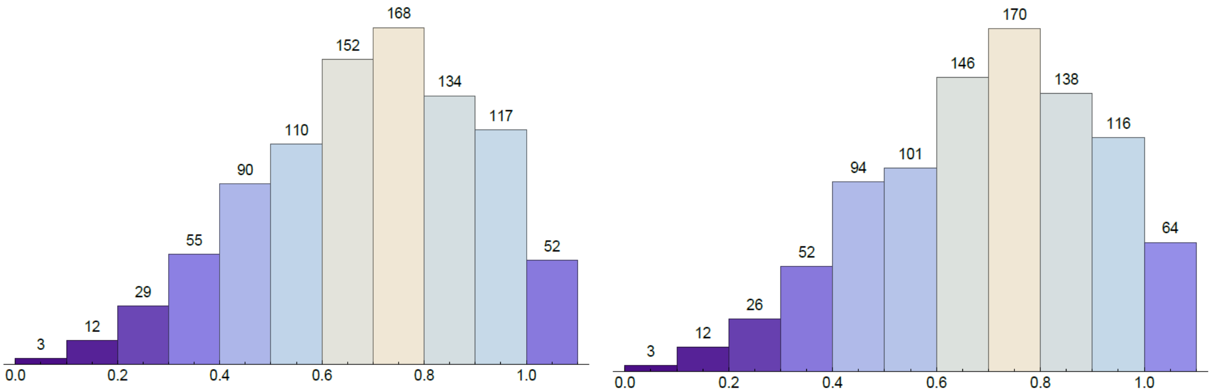
3.3.3 The insurance trigger, $\underline{\omega}$

As mentioned in Assumption 2.14, the insurance trigger must then be at least equal to $\underline{\omega}(w_{2021})$ (defined by 2.24) so that the household that purchased the insurance can consume a minimum of $\underline{e}$ kWh/year in all cases in 2022. Thereby considering this trigger, households hedge against the risk of loss of income but also against the price variations of energy and composite good, i.e. against the risks of loss of purchasing power. This insurance, whose trigger threshold is indexed to the price of energy, allows households that are not energy poor at $t = 0$ to avoid falling into this state even during positive energy price shocks such as those observed since the beginning of the war in Ukraine.

Observation: Assuming that "well-off" household incomes have not changed between 2018 and 2021, the distribution of insurance trigger levels with and without the tariff shield on the price of electricity is represented in Figure 10. Some statistics are also provided in this Figure. A Weibull distribution fits the distribution of $\underline{\omega}_i$: a Weibull distribution with shape parameter 8.17 (resp. 4.31), scale parameter 1.67 (resp. 1.01) and location parameter -0.882 (resp -0.198) for the case with (resp. without) tariff shield.

[41] On September 3, 2022, the Deputy to the Budget Minister, Gabriel Attal, confirmed that the price shield to contain the rise in energy prices would be maintained in 2023. These figures of a price increase capped at 0% or even 20% still make the subject of arbitration under the 2023 Finance Act, arbitration to be rendered in the coming days.

[42] Some of the French customers (less than 10,000 customers are concerned, i.e. 2% of customers in France) of the Spanish electricity supplier Iberdrola have been invited to "supply themselves elsewhere" to benefit from the regulated energy sale tariffs, and thus avoiding seeing "double or triple" the prices during an automatic renewal of the contract, confirmed Iberdrola on August 21 to the French Press Agency (AFP).



Distribution of insurance trigger levels with tariff shield ($\omega_{2022,i}$)

Distribution of insurance trigger levels without tariff shield ($\omega_{35\%,i}$)

Interpretation: The histograms represent the distribution of the triggering thresholds of the insurance against fuel poverty insurance for "well-off" households with the energy price shield (left figure) and without the shield (right figure). Recall that if the income in $t = 1$ is less than this threshold multiplied by the income in $t = 0$ then the household has fallen into fuel poverty in $t = 1$. With tariff shield (respectively without), $\min(\underline{\omega}_i) = 0.427\%$ (resp. 0.441%), $\max(\underline{\omega}_i) = 104.668\%$ (resp. 107.94%), $\text{mean}(\underline{\omega}_i) = 0.687$ (resp. 0.695), $\text{median}(\underline{\omega}_i) = 0.704$ (resp. 0.712).

Figure 10: Distribution of the insurance trigger

3.3.4 Expected cost to the insurer

For each "well-off" household i, the insurer knows $\underline{e}_i$ (which in practice could be declared by the household), but it does not know its cost. By making price scenarios (we consider two scenarios: the prices resulting from the tariff shield, and those without), it can estimate the insurance trigger $\underline{\omega}_i$ of each of these households. It can also estimate the laws that income variations ω_i follow and then calculate the expectation of the quantities necessary to guarantee that the households having subscribed to the insurance do not fall into fuel poverty. As an example, we calculated these expectations (with and without a tariff shield), for households living in all-electric dwellings[43] assuming that they had all subscribed to EDF's Blue peak/off-peak tariff. Assuming that in 2022 the number of French households is 29 million, the quantity of electricity that the insurer must guarantee so that these all-electric households do not fall into fuel poverty (in forecast) is about 1.517 TWh in which there is a tariff shield and 2.061 TWh without. Consequently, if the price at which the insurer is sourcing is p €/MWh, the expected cost of the fuel poverty insurance for these "well-off" households amounts with the tariff shield (resp. without shield) to $1.516 \times p$ (resp. $2.061 \times p$) million euros. As soon as the sourcing price is lower than 803 €/MWh the insurance is less expensive than the shield and this is all the more true as the insurer receives the contribution of the households for the insurance calculated below.

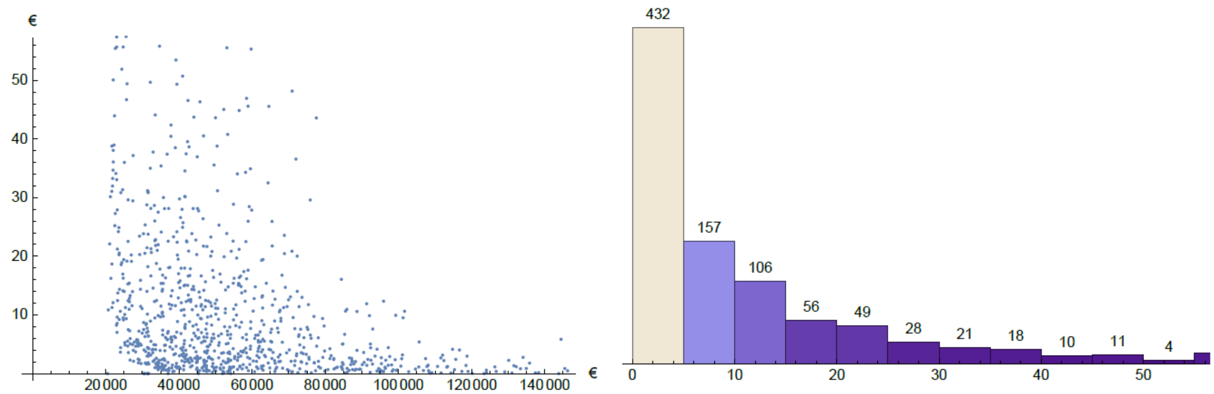
[43] Using 2019SRCV and considering households that would not have fallen into fuel poverty in 2021, this corresponds to 8.04% of all households.

3.3.5 The maximum price for fuel poverty coverage

The calculation of the maximum price that the "well-off" household i would be willing to pay for fuel poverty coverage using (2.38) requires some modification for the following reasons. If the household becomes more precarious, it will benefit from various forms of aid (minimum integration income, housing allowances, energy voucher). From an analytical point of view, these aids will result in a change in household income on the one hand, and of the utility function for low incomes ($w_t \leqslant p_t^x \underline{x}$) on the other hand, and more precisely of the parameter $\bar{\alpha}_m$ in (2.21). To the extent that the aid for consumption of the composite good should be the same with or without insurance, it will not impact the household's willingness to pay for fuel poverty coverage. This is not the case for energy-related assistance. Indeed, the insured household that has fallen into fuel poverty has no reason to receive an energy voucher, nor to benefit from the minimum service (1kVa). The utility function defined by (2.21) did not take these aids into account. However, the energy subsidies that must be taken into account in calculating the willingness to pay for insurance translate analytically into a reduction of $\underline{e}$ in the utility functions $U_{m,0}(.)$ and $U_{M,m}(.)$ defined in (2.21). The impact of a minimal energy service is studied in Appendix C.4. Note that $\underline{e}$ is found in $\bar{\alpha}_m$ and $\alpha_m(w_t)$. It should be noted that taking these aids into account has no impact on the calculations made previously, except for the representation of the utility of the median household (on the income interval $[0; 29, 725]$).

Estimation/calibration of the utility parameters for precarious households will be the subject of another paper. We approach the willingness to pay of "well-off" households for fuel poverty coverage in two ways. The first will not be through a totally utilitarian but also monetary approach by simply assuming that for household i, p_i^I is equal to $\tilde{p}_i = \text{Prob}(\omega_i < \underline{\omega}_i(w_{0,2021,i}))\mathbb{E}(p_{2022}^e) \times \underline{e}_i$. We know $\underline{e}$ as well as $\text{Prob}(\omega_i < \underline{\omega}_i(w_{0,2021,i}))$ and the expected price of electricity (regulated tariff). So we can calculate the consent of each i for the coverage. Obviously, the household i must be able to pay $\tilde{p}_i$. Therefore, its willingness to pay for the fuel poverty insurance is equal to $\min(W_{2021,i}^d, \tilde{p}_i)$ where $W_{2021,i}^d$ is defined by (2.10). Without tariff shield, this monthly consent (in €) is represented as a function of the household's income in Figure 11. The mean (respectively the median) is 13.23 (respectively 5.93) euros per month. As intuition suggests, this willingness to pay for insurance is low for high income. The dispersion is relatively important because for similar incomes the minimal level of energy demanded $\underline{e}_i$ varies, as it strongly depends on the characteristics of the housing. Finally, remember that the two main French energy suppliers EDF and ENGIE provide payment insurance to households that meet criteria such as a job loss, job stopping (total temporary disability), hospitalization, invalidity (total and irreversible loss of autonomy) and accidental death. ENGIE's insurance (Assurance Facture) costs 5 euros per month, allowing reimbursements of up to 5,000 euros for a maximum of one year (833 euros for a hospitalization). EDF's insurance (Assurénergie) costs range from 2 euros up to 8 euros per month. The reimbursement amounts range from 25 euros to 200 euros per month.

If all these households living in all-electric dwellings pay this premium, it would bring in (in expectation) 370 million euros for the insurer. Given the expected amount of electricity to be procured by the insurer, the insurer's expected profit will be positive if and only if it procures the electricity at a price below 179€/MWh!



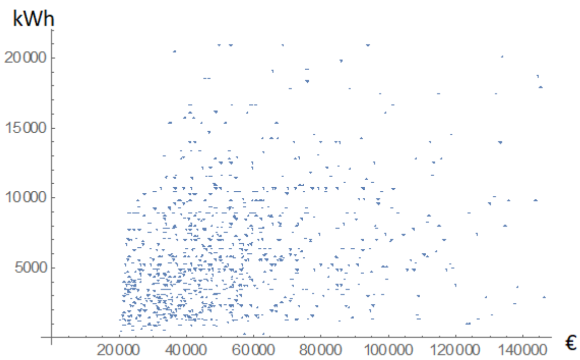
Household insurance prices by income Insurance premium distribution

Interpretation: The figure on the left represents the couple (income, insurance premium) of "well-off" households. The figure on the right gives the distribution of these premiums.

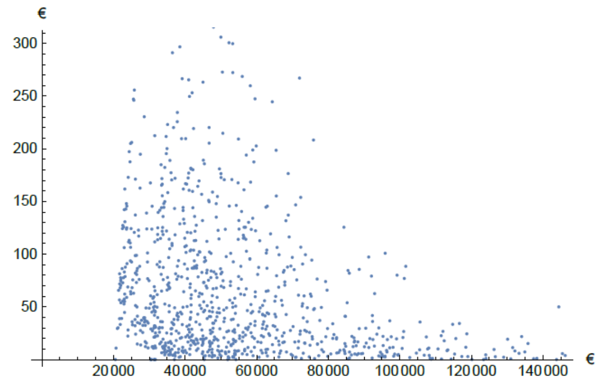
Figure 11: Household insurance prices

The second approach is based on equation (2.38). The assessment of willingness to pay for insurance will therefore be higher than with the previous method. In order to take into account the energy consumption subsidies that may be received by precarious households that would not have purchased the insurance we reduce their $\underline{e}$ as follows. If the power to which the household has subscribed is equal to ξ kVA, then the aid it will receive will be equal to $\frac{\underline{e}}{\xi}$. So for this household, $\underline{e}$ is now equal to $\frac{(1-\xi)\underline{e}}{\xi}$. This is a "quick" approximation of the amounts of energy and non-monetary (but dedicated) subsidies that the precarious household might receive. The 12 shows on the one hand the decent levels of energy according to the income, and on the other hand the willingness to pay the insurance depending on the income of "well-off" households. It illustrates our previous statement that willingness to pay for this energy insurance certainly depends on income but mainly on $\underline{e}$. Nevertheless, for the same $\underline{e}$, the lower the income, the higher the willingness to pay for insurance will be (as illustrated by the study of the median household below)

Finally, we examine the median household. We assume that it belongs to the 5th income decile (according to the SRCV2018 database, its annual income $w_{2018} \in [26,163; 34,002]$). Under our assumptions,



$\underline{e}$ according to income



Willingness to pay for insurance according to income

Interpretation: On the left, household income on the abscissa and their decent level of energy on the ordinate. On the right, the income of these same households (on the abscissa) and their willingness to pay (on the ordinate).

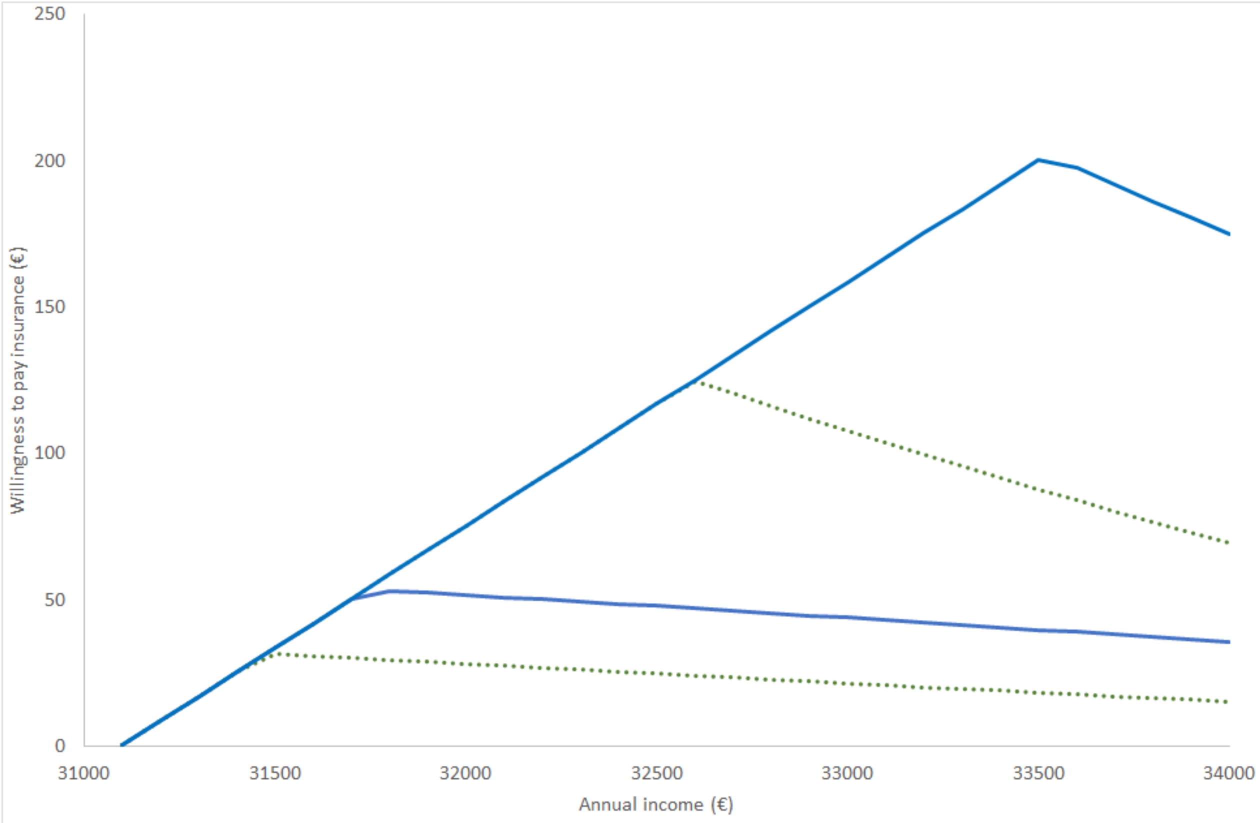
Figure 12: "Well-off" household insurance prices

- in 2021, the median household is precarious if its annual income is less than €30,168. It is in fuel poverty for any income below €31,089;
- in 2022,
  - with the energy price shield, for any annual income lower than €32,953 it is in precariousness. It is in fuel poverty if its annual income is less than €33,897;
  - without tariff shield and a 35% increase in electricity prices then if the household has an annual income below €33,434 it is precarious. If its income is lower than €34,678 it is in fuel poverty.

In Figure 13, the willingness to pay for insurance for the median household is shown. It obviously decreases with income as soon as the household has the financial capacity to pay the insurance. The amounts are high. This is explained by a high probability that the household falls into energy poverty (between $[0.34; 0.65]$ in the case where electricity prices increased by 35%) as well as a high probability that it will become precarious.

4 Conclusion and Policy implications

The proposed energy insurance system preserves the purchasing power of households that are not in financial difficulty but could become so, in particular due to the situation of rising energy prices since 2021 but also because of loss of income. It is a differentiated risk-sharing instrument, based on a utility function that takes into account two essential levels, energy and



Interpretation: The curves above represent the willingness to pay for fuel poverty insurance of the median household (not precarious in 2021) with the tariff shield (in green) and without the shield (in blue). For these two cases, there are two curves. The lower is the willingness to pay calculated via the first method (monetary) and the higher is the maximum price that the household is likely to pay (determined via utility differences more precisely from the equation (2.38). We see that if the household has an annual income of €33,500 it is ready to spend all its disposable income $W_0^d(w_0)$ on insurance.

Figure 13: Willingness to pay for fuel poverty insurance

a composite good (including at least food and housing). These essential levels are therefore set individually for each household. The level of these basic necessities depends, for example, on the composition of the family. It could also be a function of the geographical location of the household, the professional activity of the individuals in the household and integrate the cost of necessary travel. The minimum energy level depends on how the dwelling is heated, on its insulation or on the sensitivity of the household to cold. This level can also be chosen and declared by the household.

If not all "well-off" households take up the insurance, it is likely that the insurer will make a loss, especially if it has to pay more than the premium charged to households. This would have been the case in 2022 if it had purchased on the wholesale electricity market. Therefore, few private insurers can be expected to take up the challenge. Of course, as we have repeatedly pointed out, energy insurance already exists. But their guarantee is limited, e.g. 200 euros for "*Assurénergie*" and with stricter conditions. They do not prevent the subscribing households from falling into fuel poverty. Therefore, the fuel poverty coverage suggested in this paper should be at least compulsory for all "well-off" households. The only compulsory insurances are Civil Liability Insurance (since every French citizen is legally responsible for damage caused unintentionally to others), Health Insurance and Unemployement Insurance (in this case, all workers pay for this insurance, even civil servants when their probability of being unemployed is almost nil). There is still legal work to be done on this point.

More generally, an extension of this model amounts to questioning the governance of this "social energy insurance system". As we have indicated, the suppliers and the regulator, CRE, anticipate the increase in regulated tariffs from one year to the next. Should we think about it in the form of a public service mission, or modify the tariff structures? Of course, this reflection must take precarious households into account.

We show[44] that if the insurer sourcing price is below a certain threshold[45] then the insurance system is less costly for the society than the tariff shield. This energy price cap would be postponed to 2023, in order to "*continue to protect French households and their purchasing power*" as announced in September 2022, the Deputy to the Budget Minister, Gabriel Attal. However, the shield limits but does not prevent households from falling into fuel poverty unlike the insurance. Since the insurer may be loss-making, the mechanism to be put in place should be linked to public-private risk sharing. The public authorities could top up and guarantee the insurance fund. A compensatory public insurance system remains possible. This is the case, for example, in France with crop insurance: the State tops up the fund to limit insurance premiums for farmers. The State could also play a role in the insurer's sourcing price.

This differentiated policy proposal is based on the minimum subsistence levels for each household. We approximated them from the SRCV data and compared the results with those of ONPES [2014-2015]. But this study is old. Yet we stress the importance of having up-to-date data on individual households. Indeed in a society where sobriety will become more important, and for a fair transition, it seems to us really important to determine the the basket of essential goods and its cost.

Similarly, to determine the decent energy level, we relied on a EPC score obtained on bills. It could be refined thanks to the entry into force of the EPC label reform since January 1, 2022, after a transitional phase of 6 months, which makes it possible to establish energy efficiency and environmental performance scores based on building audits. In addition, dissemination of this information to households is crucial. These two scores are presented on the same label for housing. Real estate agencies have to indicate an estimate of the amount of the gas and electricity bills on the advertisements of real estate. ADEME has also made this data established

[44]admittedly only for all-electric housing, but the method can be generalized.
[45]803 €/MWh in our simulation.

by real estate diagnosticians available in OPEN DATA since August 3, 2022. It publishes this information in different formats to make it accessible to different audiences. We refer on this subject to the work of the HERMES project[46]. Moreover while EPC scores only realized at the time of the real estate transaction, the CESE[47] recommends in its notice of November 22, 2022 concerning "renovating for more sustainable buildings", to "make it compulsory to carry out a standardized audit for each building and each dwelling, including condominiums, within 5 years. This audit would be fully covered for the most modest households".

Among the possible extensions of the model, we can cite the inclusion of energy vouchers, the transport item in the subsistence minimum of households or the modification of household behavior. The problems of moral hazard (modifications of the behaviours of the household after the signature of the contract) and adverse selection (which may be due to asymmetric information as to the distribution of income or utility function parameters). If in the latter case, Alasseur et al. [2022] show the effectiveness of in-kind insurance is highlighted by a comparison with income insurance, the results nevertheless underline the need to regulate this insurance market. Theses questions of regulation and public policies have to been question in regards of risk sharing between private and public regarding energy efficiency in dwellings.

References

S. Awaworyi Churchill and R. Smyth. Energy poverty and health: Panel data evidence from australia. *Energy Economics*, 97:105219, 2021.

C. Alasseur, C. Corinne, and E. Hubert. Optimal contracts under adverse selection for staple goods such as energy: Effectiveness of in-kind insurance. *Energy Economics*, 106:105785, 2022.

A. Bourgeois and R. Lafrogne-Joussier. La flambée des prix de l'énergie : un effet sur l'inflation réduit de moitié par le « bouclier tarifaire ». *Insee Analyses*, 75, 2022.

C. Chaton and A. Gouraud. Simulation of fuel poverty in France. *Energy Policy*, 140:111434, 2020.

C. Chaton and E. Lacroix. Does France have a fuel poverty trap? *Energy Policy*, 113:258–268, 2018.

[46]cf. https://www.chaireeconomieduclimat.org/collaboration/hermes-heterogeneite-a-la-renovation-des-menages-simulations/

[47]The Economic, Social and Environmental Council (CESE) is the third constitutional assembly of the French Republic. It advises the Government and Parliament, and participates in the development and evaluation of public policies in its fields of competence.

F. Delmas and J. Guillaneuf. En 2018, les inégalités de niveau de vie augmentent. *Insee Première*, (1813), 2020.

Eurofund. *Inadequate housing in Europe: Costs and consequences.* Publications Office of the European Union, Luxembourg, 2016.

D. Kahneman and A. Tversky. Prospect theory: analysis of decision under risk. *Econometrica*, 47(2):263–291, 1979.

S. Kahouli. An economic approach to the study of the relationship between housing hazards and health: The case of residential fuel poverty in France. *Energy Economics*, 85:104592, 2020.

E. Lacroix and C. Chaton. Fuel poverty as a major determinant of perceived health: The case of France. *Public health*, 129(5):517–524, 2015.

ONPES. *Budgets de référence : une méthode d'évaluation des besoins pour une participation effective à la vie sociale.* 2014-2015.

A. Rodriguez-Alvarez, L. Orea, and T. Jamasb. Fuel poverty and well-being: a consumer theory and stochastic frontier approach. *Energy Policy*, 131:22–32, 2019.

M. Roys, M. Davidson, S. Nicol, D. Ormandy, and P. Ambrose. *The real cost of poor housing.* BRE FB 23. Bracknell, IHS BRE Press, 2010.

H. Schlesinger and B. Zhuang. Purchase insurance for future risk: A two-period model for insurance and saving/borrowing. unpublished, 2014.

H. Schlesinger and B. Zhuang. Whoops! It Happened Again. Demand for Insurance that Covers Multiple Risks. SSRN Scholarly Paper ID 2765962, Social Science Research Network, Rochester, NY, 2019.

A. Sharam. What the gas and electricity arrears of private low-income tenants can tell us about financial stress. *Journal of Economic and Social Policy*, 11(2), 2007.

B. Villeneuve. La précarité socialement construite : quelques remarques sur les effets indésirables de politiques fiscales et sociales bien intentionnées. *Revue Passages/ADAPES*, April, 2012.

Appendices

A Supplement to Section 2

A.1 Indifference curves

A.1.1 Expressions

If $\bar{U}$ is a given utility level, then the household indifference curves are defined by

$$x = \begin{cases} -1 + \exp(\bar{U} + \alpha_m \ln(1 - e_t + \underline{e})) + \underline{x} & \text{if} \quad x_t > \underline{x} \quad \text{and} \quad 0 \leqslant e_t \leqslant \underline{e}, \\ -1 + \exp(\bar{U} - \alpha_M \ln(1 + e_t - \underline{e})) + \underline{x} & \text{if} \quad x_t > \underline{x} \quad \text{and} \quad e_t > \underline{e}. \end{cases} \tag{A.1}$$

A.1.2 Convexity or concavity?

According to A.1 we have

$$\frac{d^2x}{de^2} = \begin{cases} -\alpha_m(1-\alpha_m)\exp(\bar{U})(1 - e_t + \underline{e})^{-2+\alpha_m} & \text{if} \quad x_t > \underline{x} \quad \text{and} \quad 0 \leqslant e_t \leqslant \underline{e}, \\ \alpha_M(1+\alpha_M)\exp(\bar{U})(1 + e_t - \underline{e})^{-2-\alpha_M} & \text{if} \quad x_t > \underline{x} \quad \text{and} \quad e_t > \underline{e}. \end{cases} \tag{A.2}$$

Consequently, since α_M is defined in $\mathbb{R}_+^*$, the indifference curves associated to α_M are convex. In order for the curves associated to the case $(M, 0)$ to be convex we need $\alpha_m > 1$.

A.2 Proofs of propositions in Section 2

A.2.1 Proof of Proposition 2.7

The following conditions must be verified: $x_t^{(M,m)}(w_t) \geqslant \underline{x}$; $e_t^{(M,m)}(w_t) < \underline{e}$.
$x_t^{(M,m)}(w_t) \geqslant \underline{x} \Leftrightarrow \frac{-p_t^x\alpha_m + p_t^e - W_t^d(w_t)}{p_t^x(\alpha_m - 1)} \geqslant 0$. But, by hypothesis α_m is greater than one. Consequently $x_t^{(M,m)}(w_t) \geqslant \underline{x}$ is rewritten as $-p_t^x\alpha_m + p_t^e - W_t^d(w_t) \geqslant 0 \Leftrightarrow W_t^d(w_t) \leqslant -p_t^x\alpha_m + p_t^e$.
$e_t^{(M,m)}(w_t) < \underline{e} \Leftrightarrow \frac{p_t^x\alpha_m - p_t^e + \alpha_m W_t^d(w_t)}{p_t^e(\alpha_m - 1)} < 0 \Leftrightarrow \alpha_m W_t^d(w_t) < -p_t^x\alpha_m + p_t^e \Leftrightarrow W_t^d(w_t) < \frac{p_t^e}{\alpha_m} - p_t^x$.
So to be in the state (M, m) it is necessary that $W_t^d(w_t) < \min\left(p_t^e - p_t^x\alpha_m; \frac{p_t^e}{\alpha_m} - p_t^x\right) \Leftrightarrow w_t - p_t^e\underline{e} - p_t^x\underline{x} < \min\left(p_t^e - p_t^x\alpha_m; \frac{p_t^e}{\alpha_m} - p_t^x\right) \Leftrightarrow w_t < W_t^m$, where $W_t^m = \min\left(p_t^e(\underline{e}+1) + p_t^x(\underline{x} - \alpha_m); p_t^e(\underline{e} + \frac{1}{\alpha_m} + p_t^x(\underline{x} - 1)\right)$.

A.2.2 Proof of Proposition 2.8

The following conditions must be verified: $x_t^{(M,M)}(w_t) \geqslant \underline{x}$; $e_t^{(M,M)}(w_t) \geqslant \underline{e}$.
$x_t^{(M,M)}(w_t) \geqslant \underline{x} \Leftrightarrow \frac{-p_t^x\alpha_M + p_t^e + W_t^d(w_t)}{p_t^x(\alpha_M + 1)} \geqslant 0$. But $\alpha_M \in]0; 1[$, then $x_t^{(M,M)}(w_t) \geqslant \underline{x} \Leftrightarrow -p_t^x\alpha_M + p_t^e + W_t^d(w_t) \geqslant 0 \Leftrightarrow p_t^e + W_t^d(w_t) \geqslant p_t^x\alpha_M \Leftrightarrow \alpha_M \leqslant \frac{p_t^e + W_t^d(w_t)}{p_t^x}$.
$e_t^{(M,M)}(w_t) \geqslant \underline{e} \Leftrightarrow \frac{p_t^x\alpha_M - p_t^e + \alpha_M W_t^d(w_t)}{p_t^e(\alpha_M + 1)} \geqslant 0 \Leftrightarrow p_t^x\alpha_M - p_t^e + \alpha_M W_t^d(w_t) \geqslant 0 \Leftrightarrow \alpha_M \geqslant \frac{p_t^e}{p_t^x + W_t^d}$.

A.2.3 Proof of Proposition 2.10

$p_t^e(\underline{e}+1)+p_t^x(\underline{x}-\alpha_m) < p_t^e(\underline{e}+\frac{1}{\alpha_m}+p_t^x(\underline{x}-1) \Leftrightarrow p_t^e\alpha_m+p_t^x\alpha_m-p_t^e-p_t^x\alpha_m^2 < 0 \Leftrightarrow (-1+\alpha_m)(-p_t^x\alpha_m+p_e^x) < 0$. The last inequality is true by hypothesis. Indeed, according to Assumption 2.9 $p_e^x < p_t^x$ and to Assumption 2.4 $\alpha_m > 1$. Therefore $W_t^m = p_t^e(\underline{e}+1) + p_t^x(\underline{x}-\alpha_m)$.

$p_t^x\underline{x} < p_t^e(\underline{e}+1) + p_t^x(\underline{x}-\alpha_m) \Leftrightarrow p_t^x\alpha_m < p_t^e(\underline{e}+1) \Leftrightarrow \alpha_m < \frac{p_t^e(\underline{e}+1)}{p_t^x}$.

$p_t^e(\underline{e}+\frac{1}{\alpha_m}) + p_t^x(\underline{x}-1) > p_t^e\underline{e} + p_t^x\underline{x} \Leftrightarrow \frac{p_t^e}{\alpha_m} > p_t^x \Leftrightarrow p_t^e > p_t^x\alpha_m$ (impossible according to 2.9). Then $p_t^e(\underline{e}+\frac{1}{\alpha_m}) + p_t^x(\underline{x}-1) < p_t^e\underline{e} + p_t^x\underline{x}$.

A.3 Optimal consumption without fuel poverty insurance when $\alpha_m > \frac{p_t^e(\underline{e}+1)}{p_t^x}$

If $\alpha_m > \frac{p_t^e(\underline{e}+1)}{p_t^x}$ then $p_t^x\underline{x} > p_t^e(\underline{e}+1) + p_t^x(\underline{x}-\alpha_m)$ and therefore the state (M,m) will never exist. Therefore, the optimal consumption is

$$x_t^{\varnothing}(w_t) := \begin{cases} x_t^{(m,0)} & \text{if } w_t < p_t^e\underline{e} + p_t^x\underline{x}, \\ x_t^{(M,M)} & \text{if } w_t \geqslant p_t^e\underline{e} + p_t^x\underline{x}, \end{cases} \tag{A.3}$$

$$e_t^{\varnothing}(w_t) := \begin{cases} 0 & \text{if } w_t < p_t^e\underline{e} + p_t^x\underline{x}, \\ e_t^{(M,M)} & \text{if } w_t \geqslant p_t^e\underline{e} + p_t^x\underline{x}, \end{cases} \tag{A.4}$$

where $x^{(m,0)}$, $x^{(M,M)}$ and $e^{(M,M)}$ are defined respectively by (2.11), (2.14) and (2.15).

A.4 Variations of optimal quantities, $x_t^{(M,M)}$ and $e_t^{(M,M)}$

The derivatives of the optimal quantities with respect to income, prices and α_M when the household is not in financial difficulty are the following:

$$\begin{aligned} \frac{\partial x_t^{(M,M)}}{\partial w_t} &= \frac{1}{p_t^x(1+\alpha_M)} > 0, & \frac{\partial e_t^{(M,M)}}{\partial w_t} &= \frac{\alpha_M}{p_t^e(1+\alpha_M)} > 0, \\ \frac{\partial x_t^{(M,M)}}{\partial p_t^x} &= -\frac{w_t + p_t^e(1-\underline{e})}{(p_t^x)^2(1+\alpha_M)} < 0, & \frac{\partial e_t^{(M,M)}}{\partial p_t^x} &= \frac{\alpha_M(1-\underline{x})}{p_t^e(1+\alpha_M)} < 0, \\ \frac{\partial x_t^{(M,M)}}{\partial p_t^e} &= \frac{1-\underline{e}}{p_t^x(1+\alpha_M)} < 0, & \frac{\partial e_t^{(M,M)}}{\partial p_t^e} &= -\frac{\alpha_M(w_t + p_t^x(1-\underline{x}))}{(p_t^e)^2(1+\alpha_M)} < 0, \\ \frac{\partial x_t^{(M,M)}}{\partial \alpha_M} &= -\frac{p_t^e}{p_t^x}\frac{\partial e_t^{(M,M)}}{\partial \alpha_M}, \end{aligned}$$

where

$$\frac{\partial e_t^{(M,M)}}{\partial \alpha_M} = \frac{w_t + p_t^x(1-\underline{x}) + p_t^e(1-\underline{e})}{p_t^e(1+\alpha_M)^2} > 0.$$

B Difference in utilities in $t = 1$

The analytical expression of the utility difference $\Delta V_{M,m}(\omega, w_0)$ defined in 2.5.2 are expressed in this section.

$$\begin{aligned}\Delta V_{M,m}(\omega, w_0) &= \alpha_M \ln\left(\frac{\alpha_M}{p_1^e}\right) - \ln(p_1^x) + \frac{(p_1^e - W_1^d)}{p_1^x}\ln\left(\frac{p_1^e - W_1^d}{p_1^e}\right) \\ &+ (1+\alpha_M)\ln\left(\frac{\omega w_0 + p_1^e + p_1^x(1-\underline{x})}{1+\alpha_M}\right). \end{aligned} \tag{B.1}$$

C Supplement to Section 3

C.1 Some statistics on the value of $\underline{x}_i/\text{CUs}_i$

In Table 3 are the minimum, maximum, mean and median values of $\underline{x}_i$ per year.

min	8436
max	30,563.3
mean	19,724.5
Median	19,257.2

Table 3: Statistics on the value of $\underline{x}_i/\text{CUs}_i$ (in €)

Figure 14 is the frequency histogram of $\underline{x}_i/\text{CUs}_i$' values for the 4,935 observations. The distribution

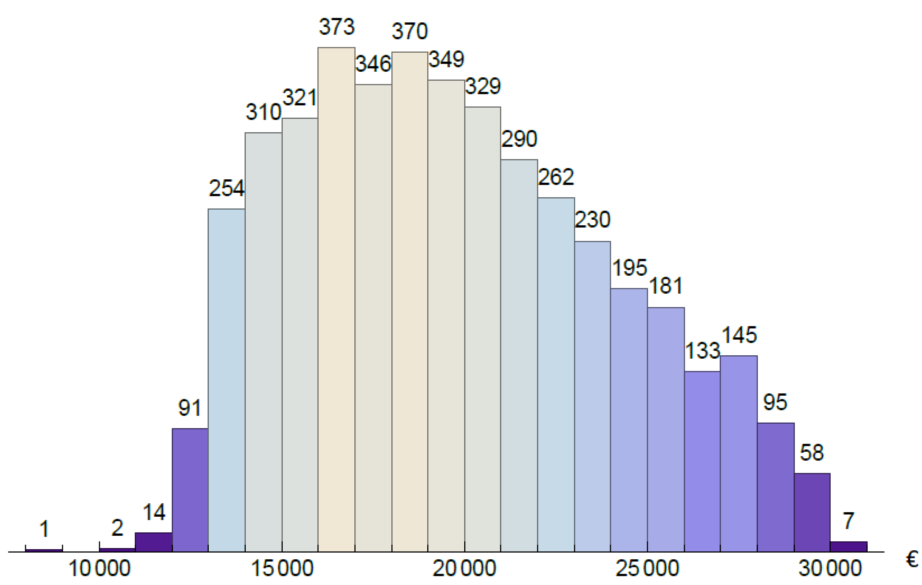


Figure 14: Frequency histogram of observed $p_{2019}^x\underline{x}_i/\text{CUs}_i$

of $\underline{x}_i/\text{CUs}_i$' is a mixture distribution whose the cumulative distribution function (CDF) is given as a sum of the CDFs of Gamma and LogNormal distributions: more precisely, Gamma distribution with mean

43.72 and scale parameter 385.945 and LogNormal distribution with mean 10.071 and scale parameter 0.125 each with weights of 0.627 and 0.373.

C.2 Electricity prices

The electrical power subscribed by the household depends on its consumption profile. We assume that the power is 6kVA for any dwelling less than 80m²; 9kVA for dwellings with an area between 80m² and 100m²; 12kVA for dwellings from 100m² to 160m² and 15 kVA for those with an area greater than 160m². All-electric households have an electricity supply contract at the EDF's Blue peak/off-peak tariff with the following shares: 40% off-peak and 60% in peak hours. Thus, for 2019, the prices are summarized in Table 4.

Power subscribed	Fixed part	Variable part
6 kVA	245	0.15
9 kVA	121	0.15
12 kVA	140	0.15
15 kVA	158	0.15

Table 4: Fixed part (subscription) in € and variable part in c€/kWh of the electricity tariff according to the subscribed power

C.3 The French EPC label, "Energy Performance Diagnostic, DPE"

The French EPC label provides information on the energy consumption and greenhouse gas emissions of a dwelling and estimates the amount of the annual energy bill of the household that resides in this dwelling. More precisely, concerning energy consumption, it is the consumption in primary energy, expressed in kWhep, per m^2 and per year, for heating, domestic hot water production and cooling. The primary energy takes into account both the final energy, that is to say the energy used by a house (gas, electricity, heating oil, etc) plus all the energy needed to extract, transport, store and produce them. Until January 1, 2021[48] for electricity, in France, 1 kWh corresponds to 2.58 kWhep. This coefficient varies according to the electrical energy mix and network losses. Consequently, this coefficient varies from country to country. Depending on the estimated level of energy consumption, a label score is assigned from A (the most efficient) to G (the most mediocre). Table 5 shows thresholds for primary energy consumption for all label scores. The greenhouse gas emissions score was published separately until July 1, 2021. That is why we only look at the primary energy consumption in our study: we do not have any information about greenhouse gas emissions. Thus a dwelling whose primary energy consumption of the household living in it is between 150 $kWh_{pe}/m^2/year$ and 230 $kWh_{pe}/m^2/year$ is assigned D-score.

[48]date of entry into force of the environmental regulation for buildings, under which the coefficient is set at 2.3.

A	B	C	D	E	F	G
50	90	150	230	330	450	–

Table 5: Thresholds for primary energy consumption ($kWh_{pe}/m^2/year$)

Figure 15 shows the distribution of label scores (calculated from the energy expenditure declarations) of households living in all-electric housing and having no financial difficulties according 2019 SRCV. For comparison, see statistics from the French Ministry of Ecological Transition[49].

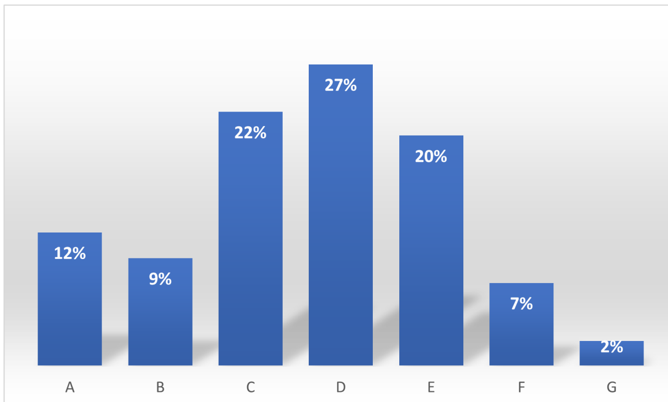


Figure 15: Distribution of EPC scores for households living in all-electric housing and having no financial difficulties

C.3.1 The probability of income loss by income decile

In this appendix, the distribution function of the annual change in household income is estimated and approximated for households belonging to income deciles 3 to 10. The methodology used is the same as that presented in 3.2.

Table 6 gives the upper limit of the disposable income bracket (decile, in euros per year) according to 2018SRCV.

Less than D1	D1 to D2	D2 to D3	D3 to D4	D4 to D5
16,046	20,409	24,407	29,163	34,002
D5 to D6	D6 to D7	D7 to D8	D8 to D9	
39,449	46,084	54,382	69,304.5	

Table 6: Upper income bracket limit (decile, in euros per year)

The distribution g of ω_i for each decile estimated from the 2018 and 2019 waves of SRCV is as follows:

[49]See https://www.statistiques.developpement-durable.gouv.fr/le-parc-de-logements-par-classe-de-consommation-energetique

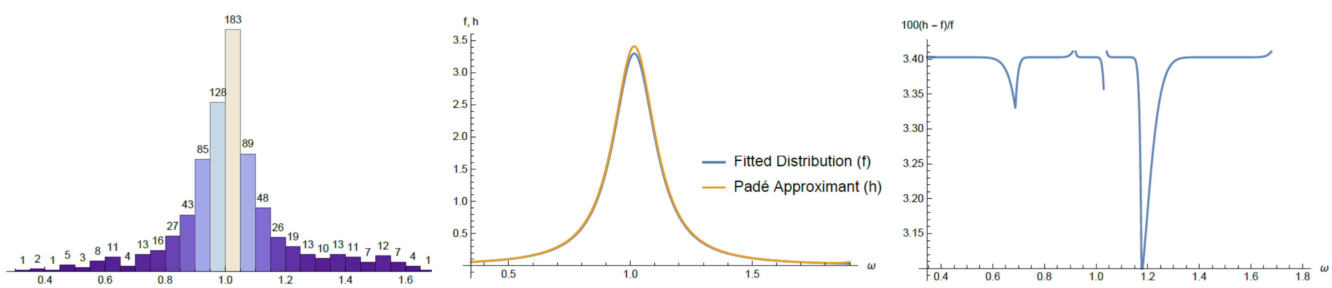
- For the fourth income decile, a mixture distribution, the cumulative distribution function (CDF) of which is given as a sum of the CDFs of Laplace and Cauchy distributions: more precisely, Laplace distribution with mean 1.028 and scale parameter 0.144 and Cauchy distribution with location parameter 1.92 and scale parameter 0.326 with weights of 0.925 and 0.0745.

- For the other deciles: a Student's t-distribution with location parameter μ, scale parameter σ and ν degrees of freedom. The values of μ, σ and ν for each decile are given in Table 7.

Decile	μ	σ	ν
3	1.01586	0.100581	1.26029
5	0.9927378	0.128261	1.80626
6	1.02839	0.113914	2.20967
7	1.00746	0.101331	1.51296
8	0.999744	0.118945	2.21006
9	0.984121	0.147164	2.39091
10	0.984121	0.147164	2.39091

Table 7: Values of the parameters of the Student's t-distributions

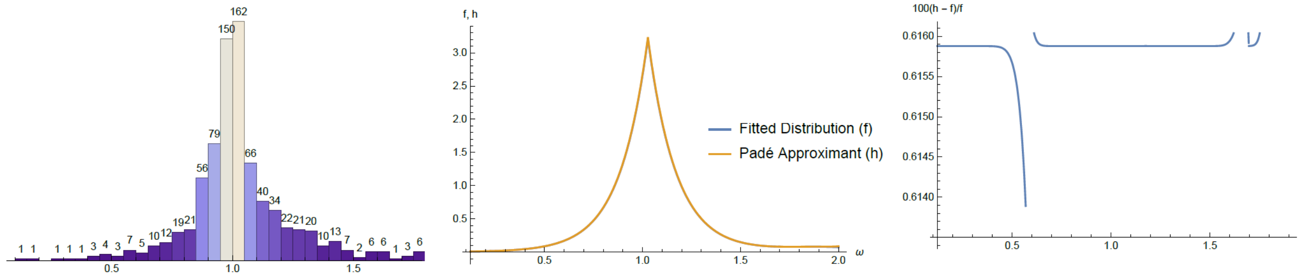
In order not to burden this appendix, the ten tables of the values of the parameters $\gamma_{k,a}$ of the Pade approximations of the distribution of ω_{2018} are not presented, only the graphic representations of the errors are drawn (see Figures 16–23)



Frequency histogram of observed w_i $\quad$ f_{2018} and h_{2018} on $[0.346, 1.5]$. $\quad$ $100(h-f)/f$ on $[0.346, 1.5]$.

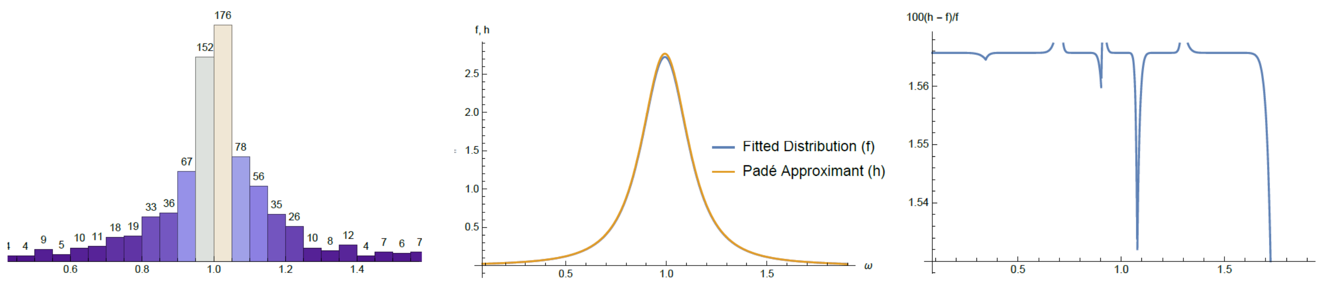
Interpretation: The curve on the left presents the frequency histogram of $\omega_{i,2018} = \frac{W_{i,2019}}{W_{i,2018}}$ observed for the 3rd income decile of 2018. In the center, the distribution function of $\omega_{i,2018} = \frac{W_{i,2019}}{W_{i,2018}}$ on $[\omega^m, 1.5]$ (in blue) and its approximation by Padé approximants (in orange) are shown ($\omega^m = 0.346$). The figure in the right shows the percent error, i.e. for all $\omega \in [\omega^m, 1.5]$ the representation of the function defined by $100\frac{h(\omega)-f(\omega)}{f(\omega)}$.

Figure 16: Change in income for households in the 3rd income decile in $t = 2018$



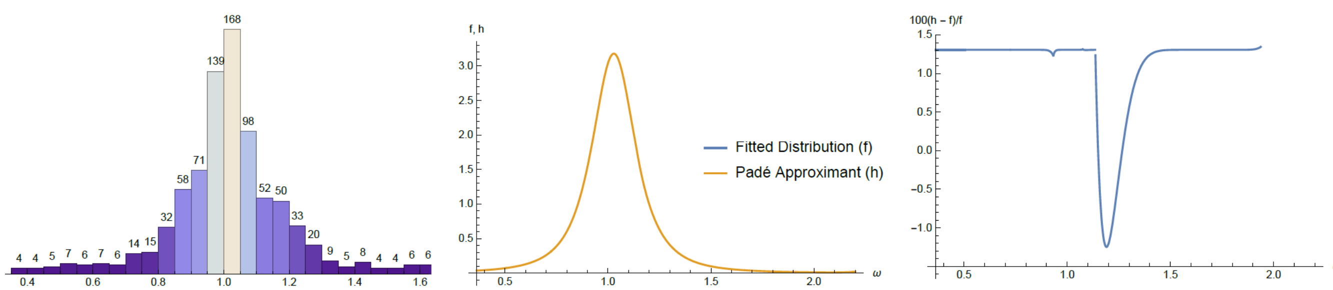
Frequency histogram of observed w_i — f_{2018} and h_{2018} on $[0.122, 2]$. — $100(h-f)/f$ on $[0.122, 2]$.

Figure 17: Change in income for households in the 4th income decile in $t = 2018$



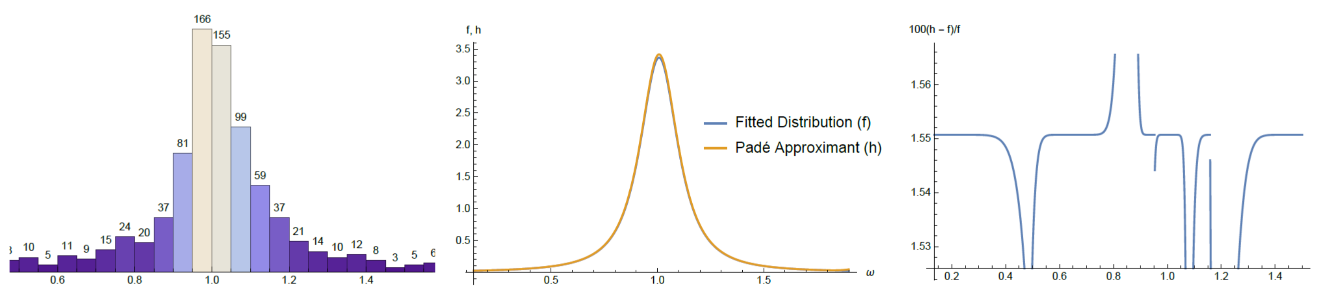
Frequency histogram of observed w_i — f_{2018} and h_{2018} on $[0.085, 1.9]$. — $100(h-f)/f$ on $[0.854, 1.9]$.

Figure 18: Change in income for households in the 5th income decile in $t = 2018$



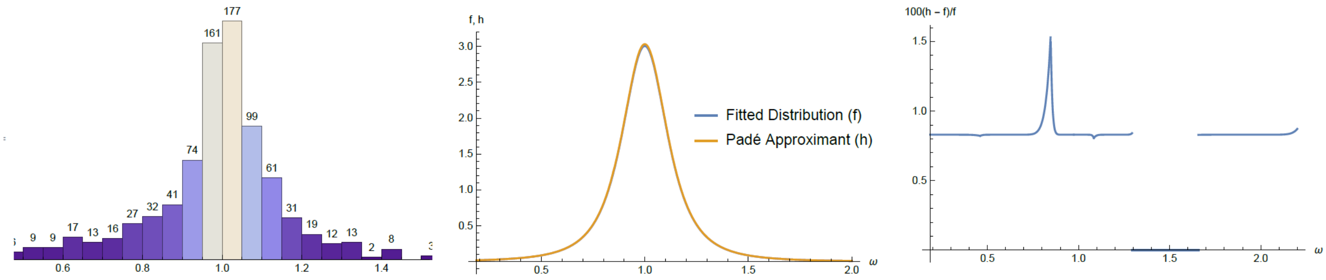
Frequency histogram of observed w_i — f_{2018} and h_{2018} on $[0.3628, 2.2]$. — $100(h-f)/f$ on $[0.3628, 2.2]$.

Figure 19: Change in income for households in the 6th income decile in $t = 2018$



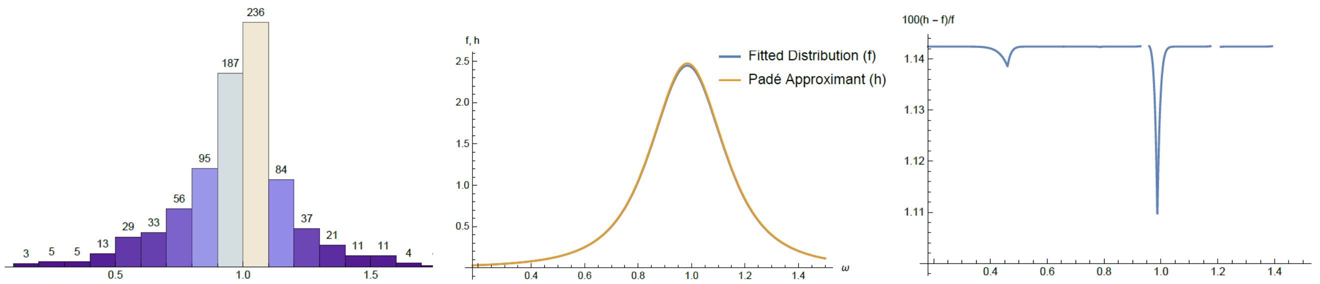
Frequency histogram of observed w_i — f_{2018} and h_{2018} on $[0.136, 1.9]$. — $100(h-f)/f$ on $[0.136, 1.9]$.

Figure 20: Change in income for households in the 7th income decile in $t = 2018$



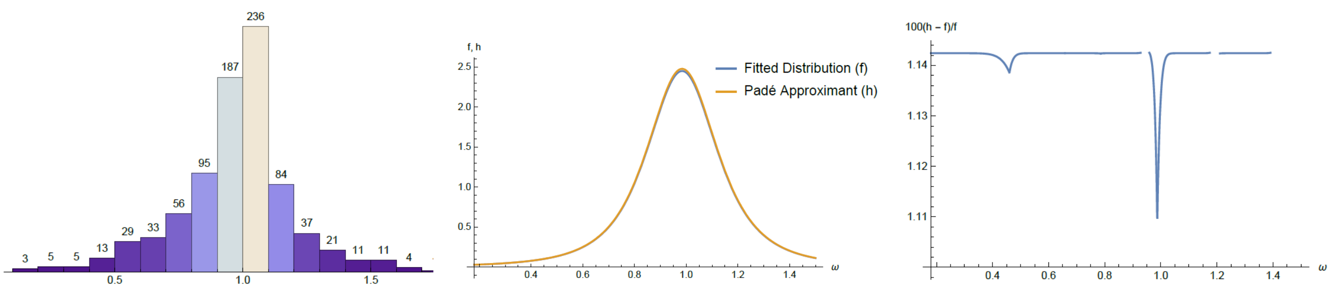
Frequency histogram of observed w_i f_{2018} and h_{2018} on $[0.186, 2]$. $100(h - f)/f$ on $[0.186, 2]$.

Figure 21: Change in income for households in the 8th income decile in $t = 2018$



Frequency histogram of observed w_i f_{2018} and h_{2018} on $[0.180, 1.5]$. $100(h - f)/f$ on $[0.180, 1.5]$.

Figure 22: Change in income for households in the 9th income decile in $t = 2018$



Frequency histogram of observed w_i f_{2018} and h_{2018} on $[0.180, 1.5]$. $100(h - f)/f$ on $[0.180, 1.5]$.

Figure 23: Change in income for households in the 10th income decile in $t = 2018$

C.4 Impact of 1kVa minimum service

To study the impact of the minimum service of 1kVa on the willingness to pay for insurance, it is necessary to transform this power into annual energy consumed. With this power, it is not possible to heat properly with electricity (according to EDF[50] only between 45 minutes and 1 hour in winter). The use of this minimum service depends on the equipment that the household has and its quality/efficiency. For example, if he has a 150W (Watts) refrigerator, he consumes at least an annual power of 1,314kWh (150W × 24 hours × 365 days /1000). At least, because you have to add lighting. Suppose that at any

[50] https://www.edf.fr/groupe-edf/espaces-dedies/l-energie-de-a-a-z/tout-sur-l-energie/l-electricite-au-quotidien/que-peut-on-faire-avec-1-kwh

given moment, he does not light more than one room in his house, that the power of the light bulb he uses is 10W, and that the average daily lighting time is 8 hours. The annual consumption of this lighting is therefore 29.2kWh (10W × 8h × 365days/1000). The household will be able to prepare their meals without unplugging their fridge while remaining on if the power of their hob is less than or equal to 840 W. Suppose that their hob is 840 W and that cooking meals takes an hour on average per day. Therefore, this generates a consumption of 306.6kWh (840 × 365/1000). If the household has a 250W television that it watches on average 5 hours a day, then its electricity consumption will be increased by 456.25kWh (250 × 5 × 365/1000). So this household with 1kVa consumes 2,106.05kWh per year.

Let us note $\underline{\underline{e}}$ the consumption associated with the 1 kVa. The existence of the minimal service implies that the state $(m, 0)$ no longer exists. It is replaced by the state $(m, \underline{\underline{e}})$. We suppose that as soon as the household can consume $\underline{x}$ i.e. $\omega w_0 \geqslant p_1^x \underline{x}$, it will receive for free in energy only the quantity $\kappa(\omega, w_0) = \min\left(\underline{\underline{e}} - \frac{\omega w_0 - p_1^x \underline{x}}{p_1^e}, 0\right)$. Then, if it has the financial capacity to pay $\underline{\underline{e}}$ i.e. $\omega w_0 \geqslant p_t^e \underline{\underline{e}} + p_1^x \underline{x}$ it does not receive any more free energy. This leads us to consider a new case : $(M, \underline{\underline{e}})$. Then the utility of the household without insurance defined in 2.5.2 is rewritten as follows. In

- the state $(m, \underline{\underline{e}})$, i.e. $\omega w_0 < p_1^x \underline{x}$

$$V^{\varnothing}_{m,\underline{\underline{e}}}(\omega, w_0) = -\ln\left(1 + \underline{x} - \frac{\omega w_0}{p_1^x}\right) - \frac{p_1^e}{p_1^x}(1 + \underline{e} - \underline{\underline{e}}) \ln(\underline{e} - \underline{\underline{e}} + 1). \tag{C.1}$$

- the state $(M, \underline{\underline{e}})$, i.e. $p_1^x \underline{x} \leqslant \omega w_0 \leqslant p_t^e \underline{\underline{e}} + p_1^x \underline{x}$

$$\begin{aligned} V^{\varnothing}_{M,\underline{\underline{e}}}(\omega, w_0) &= -\frac{p_1^e(1 - \kappa(\omega, w_0)) - W_1^d(\omega)}{p_1^x} \ln\left(\frac{p_1^e(1 - \kappa(\omega, w_0)) - W_1^d(\omega)}{p_1^e}\right), \\ &= \frac{p_1^e}{p_1^x}(1 + \underline{e} - \underline{\underline{e}}) \ln(\underline{e} - \underline{\underline{e}} + 1). \end{aligned} \tag{C.2}$$

- the state (M, m^+), i.e. $p_t^e \underline{\underline{e}} + p_1^x \underline{x} < \omega w_0 \leqslant p_t^e \underline{e} + p_1^x \underline{x}$, $V^{\varnothing}_{M,m}(\omega, w_0)$ defined by (2.32).

- the state (M, M), i.e. $\omega w_0 \geqslant p_t^e \underline{e} + p_1^x \underline{x}$, $V^{\varnothing}_{M,M}(\omega, w_0)$ defined by (2.33).

Let's define the following utility difference:

$$\Delta V_{M,\underline{\underline{e}}}(\omega, w_0) = V_{\underline{\omega}}(\omega, w_0) - V^{\varnothing}_{M,\underline{\underline{e}}}(\omega, w_0) \tag{C.3}$$

where $V_{\underline{\omega}}(\omega, w_0)$ is defined by (2.35). The analytical expression of this difference is

$$\begin{aligned} \Delta V_{M,\underline{\underline{e}}}(\omega, w_0) &= \alpha_M \ln\left(\frac{\alpha_M}{p_1^e}\right) \frac{p_1^e}{p_1^x}(1 + \underline{e} - \underline{\underline{e}}) + \ln(\underline{e} - \underline{\underline{e}} + 1) \\ &+ (1 + \alpha_M) \ln\left(\frac{\omega w_0 + p_1^e + p_1^x(1 - \underline{x})}{1 + \alpha_M}\right). \end{aligned} \tag{C.4}$$

Therefore, the utility expectation generated by fuel poverty insurance when a minimum energy service exists is

$$\begin{aligned}\Delta U &= \Delta V_0 + \beta \left(\int_{\omega^m}^{\frac{p_1^x \underline{x}}{w_0}} \frac{p_1^e}{p_1^x}(1 + \underline{e} - \underline{\underline{e}}) \ln(\underline{e} - \underline{\underline{e}} + 1) d\omega \right) \\ &+ \beta \left(\int_{\frac{p_1^x \underline{x}}{w_0}}^{\frac{p_1^x \underline{x} + p_1^e \underline{\underline{e}}}{w_0}} \Delta V_{M,\underline{\underline{e}}}(\omega, w_0) f(\omega)\, d\omega + \int_{\frac{p_1^x \underline{x} + p_1^e \underline{\underline{e}}}{w_0}}^{\underline{\omega}(w_0)} \Delta V_{M,m}(\omega, w_0) f(\omega)\, d\omega \right), \qquad \text{(C.5)}\end{aligned}$$

where ΔV_0 is defined by (2.30). Therefore, with the minimum energy service the maximum price of insurance that the household accepts is

$$\overline{p}_{\underline{\underline{e}}} = \left(1 - \exp\left(-\frac{\beta W_{\underline{\underline{e}}}}{1 + \alpha_M} \right) \right) \left(W_0^d(w_0) + p_0^e + p_0^x \right) \qquad \text{(C.6)}$$

where

$$\begin{aligned}W_{\underline{\underline{e}}} &= \int_{\omega^m}^{\frac{p_1^x \underline{x}}{w_0}} \frac{p_1^e}{p_1^x}(1 + \underline{e} - \underline{\underline{e}}) \ln(\underline{e} - \underline{\underline{e}} + 1) d\omega + \int_{\frac{p_1^x \underline{x}}{w_0}}^{\frac{p_1^x \underline{x} + p_1^e \underline{\underline{e}}}{w_0}} \Delta V_{M,\underline{\underline{e}}}(\omega, w_0) f(\omega)\, d\omega \\ &+ \int_{\frac{p_1^x \underline{x} + p_1^e \underline{\underline{e}}}{w_0}}^{\underline{\omega}(w_0)} \Delta V_{M,m}(\omega, w_0) f(\omega)\, d\omega. \qquad \text{(C.7)}\end{aligned}$$