Is it sensible to invest in home energy renovation?

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ABSTRACT

To accelerate the energy renovation of buildings (housing and tertiary) necessary to achieve carbon neutrality by 2050, the French government has launched an energy renovation plan for buildings. This plan allocates funds across different subsidy and aid mechanisms for housing renovation. What is the impact of these investment subsidies for energy renovation on such investment and on the consumption and savings of a home-owning household that is not in fuel poverty? To answer this question, we develop a two-period, two-goods model that integrates "essential baskets", i.e., baskets of goods (with a "minimum energy" level and an "essential composite good"). We confirm that the investment subsidy for energy renovation is effective if it is targeted. However, this targeting should not be based solely on income. Indeed, we highlight non-monetary side effects as one of the key parameters of policy effectiveness. To illustrate our remarks, we estimate and calibrate the parameters of the model with data from French households that only use electricity as an energy source for their homes.

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1. INTRODUCTION

The urgent climate situation has made reducing emissions one of the greatest challenges we face. Although a change in consumption behaviour, so-called "*energy sobriety*" has emerged as a necessary joint effort on behalf of all, current state policies in France focus on how to tap into energy efficiency potential: increasing the fuel efficiency of material systems. The built environment is at the core of the emission-reduction strategy, as it accounts for 44% of total French energy consumption and nearly a quarter of the country's greenhouse gas (GHG) emissions. Increasing residential energy efficiency is critical, not only for decreasing emissions but also to reduce vulnerability to exogenous shocks and fuel poverty[1]. The thermal renovation of buildings is believed to be the most effective way to achieve this reduction in energy consumption and energy price dependence. It is therefore essential to study and understand the decision-making process for renovating a dwelling and the impact of renovation aid on this process. This is the main objective of our study.

Consequently, in 2015, the French government published a "road map" under the name *"Nation-wide Low Carbon Emissions Strategy"*[2]. Although dwellings are progressively being renewed, the pace is too slow to keep up with the objectives set for 2030, according to the High Council for Climate (HCC). Indeed, most renovations are only small gestures, which consist in renewing a specific unique part of a dwelling, follow specific subsidy opportunities, and only a few consist of global renovation work that addresses all characteristics of the dwelling. French governmental agencies such as ADEME[3] provide physical and statistical evidence that small gestures are far from enough to

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[1] The recent war in Ukraine has shown that consumers may have to bear high price volatility in unusual circumstances.

[2] From "Stratégie Nationale Bas Carbonne", SNBC.

[3] French Agency of Environment and Energy Management

achieve efficiency in housing, compared to a global renovation. For more information, the reader can refer to Appendix 5.1. Unfortunately, as stated by the French Observatory of Energy Renovation (ONRE) in 2021, *"in the absence of a clear definition of what exactly an energy renovation is, accounting for renovated units remains a delicate process"*. Indeed, although the global number of renovations increased tenfold between 2017 and 2022, only 10% of them were estimated to be global. Ultimately, among the 700,000 projects registered as renovations in 2022, only 2% were labeled as low-consumption buildings. What appears at first glance to be a major effort conceals a completely different reality: the number of leaky homes, that is to say housing units ranked in the two bottom categories of energy efficiency, increased from 4.8 million in 2018 to 5.2 million in 2022. This one-percentage-point increase brings the proportion of leaky homes to 17.4%. As a comparison, the 1.5 million most efficient dwellings (the two top ranks in efficiency) only represent 5% of the French housing stock. Despite a strategy that claims to be focused on energy-poor housing units, the current state of practice could increase the remaining energy efficiency potential: in addition to being empty gestures from the perspective of net efficiency gains, small gestures might represent sunk costs for households, especially poorer households, that deter them from engaging in further work. From this perspective, policies might encourage self selection towards the most "profitable" renovations for households, which may represent wasted potential that decrease overall welfare (Allcott and Greenstone (2017)).

Such under-investment by households in thermal renovation is a substantial public policy challenge. Obviously, current financial mechanisms and incentives are insufficient to realise from the energy savings potential that thermal renovation represent. Of course, the financial up-front cost is a major obstacle for people to conduct a total renovation. Several studies (HCC, 2020; ADEME, 2021; Rudinger and Gaspard, 2022) estimate the amount of a global renovation to exceed €50,000, which represents a large amount for most households. Even when accumulating all subsidies, the cost to bear seems too high: nearly half of households report financial reasons behind their discouragement. The explanation of the efficiency gap is often attributed to a set of behavioural market failures. In the case of energy efficiency investment, the literature highlights probable intangible costs, mostly due to a misperception of the ratio between the up-front cost and the energy operating cost (Houde and Myers (2019)). These costs that we regard as negative side effects play a major role in the decision not to invest in energy renovation. The issue of information is crucial because a number of *ex post* empirical studies show that providing information about benefits, emphasising increased comfort or decreased energy expenses, usually increases participation in programs and investment in energy-efficient appliances (Buckley (2020); Allcott and Rogers (2014); Bartiaux et al. (2006); Herring et al. (2007); Jacobsen (2015); Nair et al. (2010)). This literature considers small energy-efficient appliances, but negative perceived externalities might be emphasised in the case of renovation: households may have to search for trustworthy professionals, among which only a few are government certified to perform global and efficient renovations. Furthermore, households often cite the substantial disturbances engendered by considerable amount of work as a reason for discouragement, but few statements are made about the comfort gain and "warm glow" for the dwelling when a sufficient investment is made, which might cause the subsidies to be wrongly calibrated (Allcott and Greenstone (2017)). From this perspective, it seems that the current level of household knowledge about energy renovation means that the perceived negative side effects outweigh the perceived positive effects. The lack of information has multiple sources: it may come from people lacking experience with renovation (Miller et al. (2022); Allcott and Greenstone (2012); Gillingham and Palmer (2013); Howarth and Sanstad (1995)) or miscommunication about available subsidies; for example the average household tends to be ignorant of its eligibility.[4]

Our research contributes to the literature on *ex ante* theoretical models of investment decisions in the energy renovation of housing and energy efficient appliances by deploying a groundbreaking

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[4] A inquiry lead by OpinionWay showed that nearly 60% of subjects were unaware of any available subsidy.

utilitarian approach. Most of the literature focuses on either empirical exploitation of behavioural biases or the net present monetary value of energy efficient investment. We choose to capture the effect of selecting a renovation approach in lieu of another, that is, the tangible and intangible costs and benefits and their impact on household utility that we define later. Household decisions are based on the trade-off between their consumption, their investment and their savings as well as non-monetary positive and negative side effects. We show that the impact of an exogenous investment subsidy on the amount of renovation work completed, the household's savings and therefore the consumption of the two goods in both periods resides in non-trivial mechanisms. Based on our results, we confirm that energy retrofit investment assistance is "effective" (i.e., at a minimum leads to reduced energy consumption) if it is targeted. Targeting households is not obvious, as the effectiveness of the policy does not only depend on income. It also depends, among other things, on the composition of the household, its dwelling (surface, Energy Performance Certificate, etc.), the prices of the goods, and on the return on savings and the efficiency of the renovation of the dwelling. We cannot resolve the public debate on whether a subsidy should be distributed at a particular time during a renovation. We therefore emphasise that in a certain environment and with complete information, the decision-making of households regarding energy renovation provided by our utilitarian model is complex. It is obviously more complex under uncertainty and a lack of information. From this perspective, it is not surprising that both Risch (2020) and Blaise and Glachant (2019) find a lack of investment in thermal renovations of housing, with an average of approximately €4000 per year, compared to the €50,000 typically required. We choose to focus on owner-occupied housing to avoid any problem of information asymmetry between landlords and tenants, which might be another direction to explore. In terms of policy implications, this article presents different levers on which to act to improve incentives, whether it is the amount of subsidy, the timing of issuance, or changing people's perception. In Section 2, we present the main hypothesis and define and report the model's principal results. Then, we gather data mainly from two French databases[5] to conduct a numerical example in Section 3 that allows us to calibrate and compute all model parameters. Finally, Section 4 concludes by reporting policy implications.

2. THE MODEL

2.1 General assumptions and notation

We consider a two-period household decision-making model. In the first period ($t = 0$), a household makes a choice under budgetary constraints among consuming, saving or renovating its housing. Investing in energy renovation allows it to benefit from energy savings in the second period. During these two periods, the household also makes a decision concerning the consumption of two goods: a composite good X, and energy E. We denote $x_t \geq 0$ and $e_t \geq 0$ consumption values at period t, and denote p_t^x and p_t^e as their prices. Specifically, at each time $t \in \{0, 1\}$, the household has income w_t which allows it to consume an amount e_t of transformed energy for its housing and a quantity x_t of another composite good.[6] The subjective discount factor between the two periods is β $(0 < \beta \geq 1)$. Note that although the first period can be one year, this need not be the case for the second. Indeed, it is obvious that the household will benefit from its investment beyond one year. Therefore, the duration of the second period can be the life of the investment, which we define later in Section 2.2.2.

Sufficient level of each good: A household is not considered precarious if it consumes a minimum level of transformed energy[7] $\underline{e}$ as well as a minimum level of composite good $\underline{x}$. If the household consumes at least $\underline{x}$ but does not have the financial capacity to consume $\underline{e}$ (i.e., $w_t > p_t^x \underline{x}$

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[5] Statistics, Resources and Living Conditions.

[6] This composite accounts for every other good in the consumer basket.

[7] We use transformed energy. Therefore, the value of $\underline{e}$ depends not only on the household's dwelling (e.g., area, energy efficiency, location) but also on the household (composition, time spent in the dwelling, specific needs for vital reasons).

and $w_t < p_t^x \underline{x} + p_t^e \underline{e}$) it is considered fuel poor in t. Attaining at least the minimum levels of energy and composite goods means that the household lives in a decent situation.

Household preferences: We use certain assumptions adopted by Chaton and Guillerminet (2022) to specify the utility function of a household. Thus, "*the household prefers first to consume a certain quantity of food, then to afford housing (which thus determines $\underline{x}$), and finally to heat its housing. In other words, the household will never stop consuming the composite good, regardless of its income, and will not consume any energy good until the consumption of the composite good reaches the minimum level $\underline{x}$*". Therefore, the household utility function, U, satisfies

$$\forall \epsilon > 0,\ U\left(\underline{x},\ \frac{w_0 - p_0^x \underline{x}}{p_0^e}\right) > U\left(\underline{x} - \epsilon,\ \frac{w_0 - p_0^x \underline{x}}{p_0^e} + \epsilon\right). \tag{2.1}$$

Moreover, "*[a]s long as a poor household does not consume $\underline{e}$, its consumption of the composite good will not exceed $\underline{x}$.*" (Chaton and Guillerminet, 2022). Under these circumstances, this means that we consider the goods X and E to be substitutes.

Energy renovation and its impact: Let c ($c \geq 0$) be the expenditure in $t = 0$ of the household on the energy renovation of its home. Following the most notable studies by ADEME (2021), Jacobsen (2015) and Risch (2020), we assume that the renovation of a dwelling implies that the efficiency of the dwelling is improved. Any further case of quality uncertainty can be the object of further studies. Specifically, it is assumed that an investment in $t = 0$ of c euros in the renovation of a dwelling contributes to reducing by a fraction $\epsilon \times c$ the minimum (decent) energy level necessary to have minimum (decent) comfort. It is possible that ϵ is null. This is the case if the rebound effect cancels out the energy gain generated by the renovation. In practice, the literature often finds large rebound effects, as much from a microeconomic (Wei and Liu, 2017) as from a macroeconomic perspective (Stern, 2020). Hache et al. (2017) describes heterogeneity in household preferences and energy consumption that depends on socio-economic characteristics, as well as dwelling and regional characteristics. This heterogeneity is also observed for the rebound effect, i.e., ϵ depends on (i) the household's social and economic characteristics and (ii) the dwelling's location and efficiency. The assumption of the linearity of the reduction in the necessary energy needs following the investment may seem strong. This simplifying hypothesis, which makes it possible to subsequently obtain analytical solutions, is not necessarily reductive. Indeed, it can be seen as a linear approximation of the efficiency gains generated by the renovation. Therefore, to be closer to reality, these gains could be specified as a piecewise linear function of investment.

We assume that engagement in renovation creates moral inconveniences and benefits. For example, following a survey conducted by the ADEME[8], we learn that in addition to monetary costs, "moral" costs, as such transaction, information, and opportunity costs and discomfort, play a role in households' decisions[9], consistent with Allcott and Greenstone (2017). Households could of course gain utility from non-monetary results (temperature, self-satisfaction with an environmental friendly decision, etc.). Under these circumstances, we can assume that c represents a renovation strategy, or a renovation technology. The better this strategy or this technology is, the greater the efficiency at the end of the retrofit work. There also might be greater comfort and potential disturbances implied by the time and the size of the work. In summary, households can suffer from a disutility in $t = 1$ and have a bonus in utility other than from monetary gains in $t = 1$.

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[8] A survey on the "Travaux de Rénovation Energétique des Maison Individuelles" (TREMI), that is, renovation work engaged in by individual household. The survey was conducted between 2014 and 2016 on French private and public housing stock.

[9] They can feel discouraged due to having workers in the home for during the period of renovation, be deterred from consciously searching for a quality professional, or consciously follow the work. Investing and emptying household finances could also represent a moral cost.

Incentives for the energy renovation of housing: Depending on the amount invested to renovate the house c and the standard of living, the household can benefit from financial subsidies. The objective of our study is to specify the subsidy that on the one hand encourages owner-occupied households to renovate their dwellings and on the other hand that this renovation allows a reduction in its minimum energy needs by a certain percentage γ. Of course, for poor and fuel poor households, the renovation is funded by another party before the start of the renovation work. As a result, precarious owner-occupied households that wish to renovate their housing receive subsidy $Z \times c$ at $t = 0$. Other households, which might not be eligible for such help, can obtain a tax credit (in France, the CITE, or *"Crédit d'Impôt Transition Energétique"*). They advance c in $t = 0$ and receive $Z \times c$ at the beginning of the second period ($t = 1$). Even if currently non-fuel-poor households, the population on which we focus later, do not receive the subsidy in $t = 0$ but in $t = 1$. In 2.2.6, we study the two possibilities to analyse the impact of the date of subsidy receipt.

2.2 Choice of non-fuel-poor households

We assume that in $t = 0$ the household is not in fuel poverty and believes that it will not be in $t = 1$, that is, during each period, it can consume at least $\underline{x}$ and $\underline{e}$. In the databases we use and describe further in Section 3, owner-occupiers who are not in fuel poverty represent more than 85.7% in 2017 and 87% in 2018[10]. Given our assumptions, the model follows a two-period timing.

- **First period** ($t = 0$)
  - For households that can afford a minimum of both goods, there is a trade-off in the first period among saving money to enjoy it later, investing in energy renovation of the housing to reduce its energy needs and therefore its energy expenditure, or consume more goods. Thus, the household chooses (x_0, e_0, s, c), that is, its consumption of composite goods and energy, and the remaining share that goes to savings, s, or renovation investment, c.
  - We assume the household correctly anticipates the return on savings between the two periods, r, as well as the impact of the renovation on future spending on energy, ϵ.
- **Second period** ($t = 1$)
  - The household decides its consumption of the two goods (x_1, e_1) and does not save or invest.
  - Following the renovation of the dwelling, its energy efficiency is increased so that the minimum (decent) energy consumption is now $\underline{e} - \epsilon c$.

The model is solved by backward induction.

Assumption 2.1. *We assume that the renovation subsidy Z is received by the household in the first period.*

The case where the household receives it in $t = 1$ is developed in 2.2.6.

To simplify the writing, we assume that the duration of the second period is also one year. However, as mentioned above, the benefit of an investment in energy renovation is not limited to one year. To account for the lifetime of this investment, the cost of the investment considered in $t = 0$ is annualised and discounted and equal to $\frac{c}{d_\beta}$ where d_β is the discounted life of the investment[11]. Given our assumptions, the household budget constraints are as follows:

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[10] This is a weighted mean, with the weights given in the database for proper representation. Amounts without representation weights and for all households are given in 5.2.1

[11] $d_\beta = \frac{1-(1+\beta)^{-T}}{\ln(1+\beta)}$ where T is the life of investment.

- First period

$$w_0 + \frac{Z}{d_\beta} c \geq p_0^x x_0 + p_0^e e_0 + \frac{c}{d_\beta} + s, \tag{2.2}$$

- Second period

$$w_1 + rs \geq p_1^x x_1 + p_1^e e_1. \tag{2.3}$$

Definition 2.2. *We define the disposable (or missing) income at period t of the household after (for) the purchase $\underline{x}$ and $\underline{e}$, that is,*

$$W_t^d = w_t^s - p_t^x \underline{x} - p_t^e \underline{e}_t, \tag{2.4}$$

where

$$w_t^s = \begin{cases} w_0 & \text{if } t = 0, \\ w_1 + rs & \text{if } t = 1, \end{cases} \tag{2.5}$$

and

$$\underline{e}_t = \begin{cases} \underline{e} & \text{if } t = 0, \\ \underline{e} - \epsilon c & \text{if } t = 1. \end{cases} \tag{2.6}$$

As we are interested in non-fuel-poor households[12], W_t^d is assumed to be greater than or equal to zero hereafter.

Assumption 2.3. *The renovation of dwellings does not go beyond energy autonomy, that is,* $\underline{e}_t \geq 0$.

With the aim of obtaining the most explicit results possible, we represent the preferences of the household towards the goods consumption, at time t, by the following separable utility function based on logarithmic utilities,

$$\mathrm{U}(x_t, e_t) = \ln(1 + x_t - \underline{x}) + \alpha \ln(1 + e_t - \underline{e}_t) + \delta \ln(1 + \mathbb{1}\{t = 0\}c), \tag{2.7}$$

where α parameterises the elasticity of substitution between energy E and the composite good X; $\delta = -\delta_d + \beta\delta_u$, $((\delta_d, \delta_u) \in \mathbb{R}_+^2)$ where $-\delta_d$ represents the inconvenience suffered during the work and δ_u represents the non-financial benefits in $t = 1$. These non-monetary positive side effects (δ_u) are discounted by a factor $\beta \in [0, 1]$. Parameter δ can be regarded as the difference between two side effects caused by the renovation work, one positive and one negative. In the following, we often distinguish whether one type of side effect outweighs the other, that is, play with the sign of δ. Since we focus on households that are not in fuel poverty, i.e., that consume at least $\underline{x}$ in each period and $\underline{e}_t$, the utility function (2.7) is defined.

2.2.1 Second period

The household maximises its second-period utility under its budget constraint that, according to Assumption 2.1, is

$$\max_{x_1, e_1} \ln(1 + x_1 - \underline{x}) + \alpha \ln(1 + e_1 - \underline{e} + \epsilon c), \tag{2.8}$$

$$\text{s.t. } w_1 + rs_0 \geq p_1^x x_1 + p_1^e e_1.$$

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[12] We assume that the investment made by the household in $t = 0$ does not cause it to fall into fuel poverty in $t = 0$.

As a result, the optimal values of a household's consumption in the second period (functions of savings and investment in energy renovation) are[13]

$$\begin{aligned} e_1(s,c) &:= \underline{e} + \frac{\alpha W_1^d - p_1^e \epsilon c}{p_1^e(1+\alpha)} + \frac{p_1^x \alpha - p_1^e}{p_1^e(1+\alpha)}, && (2.9) \\ x_1(s,c) &:= \underline{x} + \frac{W_1^d + p_1^e \epsilon c}{p_1^x(1+\alpha)} - \frac{p_1^x \alpha - p_1^e}{p_1^x(1+\alpha)}. && (2.10) \end{aligned}$$

As the household is not in fuel poverty, $e_1(s,c)$ (respectively $x_1(s,c)$) must be greater than or equal to $\underline{e}$ (respectively $\underline{x}$). This is verified if for s and c provided that the price of the energy good in period 1 (p_1^e) is included in

$$\left[\frac{p_1^x\alpha - W_1^d}{1+\epsilon c}, \frac{\alpha(W_1^d + p_1^x)}{1+\epsilon c}\right].$$

Obviously, the energy consumption in $t = 1$ (respectively the consumption of composite good in $t = 1$) is increasing linearly with $\underline{e}$ (respectively with $\underline{x}$).

2.2.2 First period

In the first period, the household maximises the following inter-temporal utility function:

$$\max_{x_0,e_0,c,s} \ln(1+x_0-\underline{x}) + \alpha\ln(1+e_0-\underline{e}) + \delta\ln(1+c) + \beta \mathrm{U}\left(x_1(s,c); e_1(s,c)\right), \qquad (2.11)$$

$$\text{s.t. } w_0 + \frac{Z}{d_\beta}c \geq p_0^x x_0 + p_0^e e_0 + \frac{c}{d_\beta} + s,$$

where e_1 and x_1 satisfy (2.9) and (2.10), respectively.

2.2.3 Optimal consumption and conditions on the subsidy (Z)

Let us define

$$\begin{aligned} \Omega_t &= w_t + p_t^x\left(1-\underline{x}\right) + p_t^e\left(1-\underline{e}\right), && (2.12) \\ \Omega &= r\Omega_0 + \Omega_1, && (2.13) \\ P^E &= p_1^e\epsilon - \frac{r(1-Z)}{d_\beta}, && (2.14) \\ \Gamma &= (1+\alpha)(1+\beta) + \delta. && (2.15) \end{aligned}$$

Ω_t is close to the household's remaining income at t, after it has consumed the minimum decent amount of both goods $\underline{x}$ and $\underline{e}$.[14] The quantity P^E is the difference between the marginal return engendered by energy savings $p_1^e\epsilon$ and the out-of-pocket expenses for the dwelling $\dfrac{r(1-Z)}{d_\beta}$.

Assumption 2.4. *We assume that Γ is greater than 0. In other words, the inconvenience suffered during the renovation (δ_d) is smaller than $1+\alpha+\beta(1+\alpha+\delta_u)$.*

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[13] Note that the last term of (2.9) and (2.10) are due to the number "1" present in the first two logarithms of the utility function (2.7).

[14] The difference from the remaining income is equal to $p_t^x + p_t^e$.

Lemma 2.5. *The optimal consumption values are given by*

$$e_0^* = \underline{e} - 1 + \alpha \left(\frac{\Omega - P^E}{r p_0^e \Gamma} \right), \tag{2.16}$$

$$x_0^* = \underline{x} - 1 + \frac{\Omega - P^E}{r p_0^x \Gamma}, \tag{2.17}$$

$$c^* = -\frac{1}{\Gamma} \left[(1+\alpha)(1+\beta) + \frac{\Omega \delta}{P^E} \right], \tag{2.18}$$

$$s^* = \Omega_0 - \frac{1-Z}{d_\beta} c^* + \frac{(1+\alpha)(P^E - \Omega)}{r\Gamma}, \tag{2.19}$$

$$e_1^* = \underline{e} - \epsilon c^* - 1 + \frac{\alpha\beta(\Omega - P^E)}{p_1^e \Gamma}, \tag{2.20}$$

$$x_1^* = \underline{x} - 1 + \frac{\beta}{p_1^x \Gamma} (\Omega - P^E). \tag{2.21}$$

The equilibrium values below have been determined for a non-fuel-poor household at $t = 0$, under the assumption that it will remain in this state and that its investment in renovation implies $c^* \geq 0$. In Appendix 5.2.2, we show that in equilibrium, the household is not fuel poor, i.e., $y \geq \underline{y}$ with $(y, \underline{y}) \in \left\{ (x_0^*, \underline{x}); (e_0^*, \underline{e}); (e_1^*, \underline{x}); (e_1^*, \underline{e} - \epsilon c) \right\}$ if subsidy Z is less than or equal to Z_y. On the other hand, we show that $c^* \geq 0$ is equivalent to $\underline{Z}_c(\delta) \leq Z \leq \overline{Z}_c(\delta)$ where

$$\underline{Z}_c(\delta) = \begin{cases} 1 - d\beta \times \frac{p_1^e \epsilon}{r} & \text{if } \delta < 0, \\ 1 - d\beta \left(\frac{p_1^e \epsilon}{r} + \frac{\delta\Omega}{r(1+\alpha)(1+\beta)} \right) & \text{if } \delta \geq 0, \end{cases} \tag{2.22}$$

$$\overline{Z}_c(\delta) = \begin{cases} 1 - d\beta \left(\frac{p_1^e \epsilon}{r} + \frac{\delta\Omega}{r(1+\alpha)(1+\beta)} \right) & \text{if } \delta < 0, \\ 1 - d\beta \times \frac{p_1^e \epsilon}{r} & \text{if } \delta \geq 0. \end{cases} \tag{2.23}$$

Note that $Z < 1 - d\beta \times \frac{p_1^e \epsilon}{r} \Leftrightarrow P^E < 0$. The conditions for $c^* > 0$ change with respect to the sign of δ, that is, whether individuals are convinced of the benefits of energy renovation or deterred from engaging in work for numerous reasons such as those cited in Appendix 5.1. Specifically, the value of δ, which represents the valuation of the entire renovation process for a given household, has a substantial influence on the equilibrium, specifically on policy sizing. If intangible benefits exceed intangible costs,[15] if households trust the process of renovation, if renovation is valued in their utility (without considering energy retrofits), that is, in the case where $\delta > 0$, the subsidy should at least be such that the remaining costs to bear $1 - Z$ do not exceed the financial benefit of renovation compared to that of savings $\frac{p_1^e \epsilon}{r}$, plus the net actualised side effect $d_\beta \times \frac{\delta\Omega}{r(1+\alpha)(1+\beta)}$. Both periods are covered. Conversely, when $\delta < 0$, the disutility exceeds the positive side effects (comfort, etc.). Consequently, the preference factor δ is critical in the decision process of the household and should be considered carefully when sizing policies. Ideally, subsidies should be associated with mechanisms that encourage households to reveal their type, which might be beyond the scope of traditional household characteristic analysis. Given the conditions on Z, which are expressed by setting $Z_{\text{nfp}} = \min \left\{ Z_y \right\}$, we can state the following proposition.

Proposition 2.6. *The equilibrium stated in Lemma 2.5 holds as long as subsidy Z satisfies*

$$\underline{Z}_c(\delta) \leq Z \leq \min \left\{ Z_{nfp}; \overline{Z}_c(\delta) \right\}, \tag{2.24}$$

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[15] Alternatively, if positive side effects exceed negative side effects.

where $\underline{Z}_c(\delta) \leq Z \leq \overline{Z}_c(\delta) \Longleftrightarrow c^* \geq 0$ *and* $Z \leq Z_{nfp} \Longleftrightarrow$ *the household is not in fuel poverty. The* Z_{nfp} *limits are specified in Appendix 5.2.2, and* $\underline{Z}_c(\delta)$ *(respectively* $\overline{Z}_c(\delta)$*) is defined by (2.22) (resp. (2.23)).*

Proposition 2.7. *The conditions* $\underline{Z}_c(\delta) \leq Z \leq \overline{Z}_c(\delta)$ *implies that Assumption 2.3 is verified.*

Proof. See Appendix 5.2.3. □

Unlike the investment amount, c^*, the savings, s^*, can be negative. Indeed, the household can borrow in period 0. This may be the case, for example, if it anticipates that its income in the second period is high enough to meet its needs for this period. Intuitively, if the return on savings is lower than the return on investment, household income must be high enough to allow for savings. However, because of the existence of psychological gain/loss due to the renovation (δ), this is more complex, as underlined by the following proposition, which provides the conditions for s^* to be positive.

Proposition 2.8. *There are two* δ *thresholds* ($\delta_1 < \delta_2$) *such that if*

1. $\delta \in [\delta_1; \delta_2]$ *and if*
   - $P^E > 0$, *i.e.,* $Z > 1 - d\beta \times \frac{p_1^e \epsilon}{r}$ *the household borrows;*
   - $P^E < 0$, *the household saves.*
2. $\delta \notin [\delta_1; \delta_2]$, *then there are two thresholds* ($Z_{1,s}(\delta) < Z_{2,s}(\delta)$) *such as the household saves if the subsidy* (Z) *satisfies*
   - $\max\left\{1 - d\beta \times \frac{p_1^e \epsilon}{r}; Z_{1,s}(\delta)\right\} < Z < Z_{2,s}(\delta)$,
   - *or* $Z < \min\left\{1 - d\beta \times \frac{p_1^e \epsilon}{r}; Z_{1,s}(\delta)\right\}$ *or*
   - *or* $Z_{1,s}(\delta) < Z < 1 - d\beta \times \frac{p_1^e \epsilon}{r}$.

   In other cases, the household borrows.

Proof. See Appendix 5.2.4 □

2.2.4 Sensitivity analysis

The impacts of the renovation subsidy on household equilibrium decisions are expressed below. In Section 3.2, we illustrate the impact of other parameters on this equilibrium for a median household.

Consumption variation: An increase in the investment subsidy, Z, reduces consumption in the first period and the consumption of the composite good in the second period.[16] Specifically,

$$\frac{\partial e_0^*}{\partial Z} = \frac{\alpha p_0^x}{p_0^e} \times \frac{\partial x_0^*}{\partial Z} = \frac{\alpha p_1^x}{r\beta p_0^e} \times \frac{\partial x_1^*}{\partial Z} = -\frac{\alpha}{d_\beta p_0^e \Gamma}. \tag{2.25}$$

Intuitively, the higher the subsidy is, the more the household will tend to invest in the first period; this should have a negative impact not only on its savings but also on its consumption in the first period (which decreases). Investment in energy efficiency reduces energy consumption in the second period (unless the rebound effect is significant). The impact of the subsidy on the consumption of the composite good is not obvious. Indeed, the reduction in energy consumption induced by the investment should allow the household to consume more of the composite good. This holds unless this investment is made at the expense of savings, and consequently the disposable income in the

---

[16] It should be noted that if Assumption 2.4 is not satisfied, the increase in the investment subsidy generates an increase in consumption.

second period decreases with the increase in the subsidy. This decrease in disposable income has the effect of reducing consumption in the second period. However, as we demonstrate below, the impacts of the subsidy on savings and investment are ambiguous. The same is true for energy consumption in the second period. Indeed,

$$\frac{\partial e_1^*}{\partial Z} = -\frac{r}{d_\beta \Gamma} \times \left(\frac{\alpha\beta}{p_1^e} + \frac{\Omega\delta\epsilon}{(P^E)^2} \right).$$

Therefore, if $\Gamma > 0$ and $\delta > \frac{\alpha\beta(P^E)^2}{p_1^e \Omega\epsilon}$, then e_1^* is decreasing with ζ on the following intervals $\left[0; \frac{r - d_\beta p_1^e \epsilon}{r}\right[$ and $\left]\frac{r - p_1^e \epsilon}{r}; 1\right]$.

Investment decision: When the amount of the subsidy increases, the amount invested in renovation increases when δ is positive; otherwise, it decreases or is constant. Indeed,

$$\frac{\partial c^*}{\partial Z} = \frac{r\delta\Omega}{\Gamma d_\beta (P^E)^2}.$$

Savings decision: Since

$$\frac{\partial s^*}{\partial Z} = -\frac{1}{d_\beta \Gamma} \times \left((1+\alpha)\beta + \frac{p_1^e \Omega\delta\epsilon}{(P^E)^2} \right),$$

s^* is an increasing (decreasing) function of subsidy Z if side effect δ is less than

$$\delta_s = -\frac{(1+\alpha)\beta(d_\beta p_1^e \epsilon - r(1-Z))^2}{d_\beta^2 p_1^e \epsilon\Omega}. \tag{2.26}$$

As δ_s is negative, when the household has positive side effects from renovating its housing, an increase (decrease) in the subsidy (Z) leads to an increase (decrease) in its savings.

Given the complexity of the impact of the investment subsidy for energy renovation on investment and savings, and especially the role of side effects δ, it is not easy for the public authorities to determine the aid needed to encourage households to renovate their homes.

2.2.5 Financial assistance in the first period for what energy efficiency?

Insofar as the objective of aid for the energy renovation of housing is to reduce the energy needs of households, it is important to determine the amount of subsidy necessary for a reduction of a fraction γ of $\underline{e}$ (namely an increase of a fraction γ of the intrinsic energy efficiency of the dwelling). This amount, $Z(\gamma)$, is a solution to $\underline{e} - \epsilon c_0^* = \underline{e}(1-\gamma)$. Then,

$$Z(\gamma) = 1 - \frac{d_\beta \epsilon}{r} \left(p_1^e + \frac{\delta\Omega}{\gamma \underline{e} \Gamma + (1+\alpha)(1+\beta)\epsilon} \right). \tag{2.27}$$

Once again, factor δ plays a highly important role in the policy design, as the variation in the subsidy with respect to different parameters such as the inter-temporal available income, Ω, or energy prices, as below:

$$
\begin{aligned}
\frac{\partial Z(\gamma)}{\partial \Omega} &= -\frac{d_\beta \epsilon \delta}{r\underline{e}\gamma\Gamma + r\,(1+\alpha)\,(1+\beta)} \;\propto\; -\delta,\\
\frac{\partial Z(\gamma)}{\partial p_0^e} &= -\frac{d_\beta(\underline{e}-1)\delta\epsilon}{\gamma\underline{e}\Gamma + (1+\alpha)(1+\beta)\epsilon} \;\propto\; \delta, \qquad\qquad (2.28)\\
\frac{\partial Z(\gamma)}{\partial p_1^e} &= \frac{p_1^e \epsilon}{r}\left(-1 + \frac{(\underline{e}-1)\delta}{\underline{e}\gamma\Gamma + (1+\alpha)(1+\beta)\epsilon}\right).
\end{aligned}
$$

Consequently, if $\delta > 0$ (respectively if $\delta < 0$), the needed subsidy to achieve increase energy efficiency by a fraction γ decreases (resp. increases) with the disposable income Ω and increases (resp. decreases) with the energy price in the first period p_0^e. For the energy price in the second period p_1^e, the subsidy increases when $\delta > -\dfrac{(1+\alpha)(1+\beta)(\underline{e}\gamma+\epsilon)}{1+\underline{e}(\gamma-1)}$.

However, the subsidy $Z(\gamma)$ that is necessary to generate a reduction in $\underline{e}$ by a fraction γ does not imply that the actual energy consumption in the second period drops by a fraction γ for this dwelling. For an increase in energy efficiency by a fraction γ, the necessary subsidy $Z(\gamma)$ has an impact on the energy consumption in period one given by:

$$
e_1^*(Z \leftarrow Z(\gamma)) = -1 + \underline{e}(1-\gamma) + \frac{\alpha\beta(\underline{e}\gamma+\epsilon)\Omega}{p_1^e\left(\underline{e}\gamma\Gamma + (1+\alpha)\,(1+\beta)\,\epsilon\right)}. \qquad (2.29)
$$

The expression above, to some extent, could be considered **an approximation of the direct rebound effect after the renovation**. The literature emphasise the difficulty of estimating rebound effects in energy savings investments and policies, as it is empirically difficult to develop a counterfactual of what the energy savings should have been. Moreover, the gap between the predicted energy savings and the actual energy savings might not be entirely due to the rebound effect but also be due to measurement and prediction errors that may derive from technical issues. In our model and by our definitions, it is not possible that a technical issue can alter the efficiency gain and the reduction in minimum intrinsic energy consumption.

2.2.6 Impact of the date of receipt of the subsidy

As noted in Section 2.1, in practice, there is a possibility that the household receives the subsidy before the work, that is, it does not have to advance cash, or after the work (generally in the form of a credit or a tax credit). There is a concern in policy questions on whether subsidising before the renovation could deter households from being proactive in monitoring the work. Additionally, Glachant (2022) suggests that households might less carefully study competition in the market for builders and have a lower propensity to sue them in the event of defects. Based on Lemma 2.5, we compute a new equilibrium in which the subsidy is received after the work, at $t = 1$. The period during which the household's budget is constrained is different, which thus affects the results of the model. The result is a new equilibrium that we express as a function of the old equilibrium in the following Lemma.

Lemma 2.9. *When the subsidy is received in the second period, that is, when Assumption 2.1 is no* *longer satisfied, we have:*

$$x^*_{0,t=1} = x^*_0 - \frac{(1-r)Z}{d_\beta p_0^x r \Gamma}, \tag{2.30}$$

$$e^*_{0,t=1} = e^*_0 - \frac{(1-r)\alpha Z}{d_\beta p_0^e r \Gamma}, \tag{2.31}$$

$$c^*_{t=1} = c^* - \frac{(1-r)\Omega\delta Z}{\Gamma P^E (r - d_\beta p_1^e \epsilon - Z)}, \tag{2.32}$$

$$x^*_{1,t=1} = x^*_1 + \frac{c^*_{t=1} Z}{d_\beta p_1^x (1+\alpha)}, \tag{2.33}$$

$$e^*_{1,t=1} = e^*_1 + \frac{c^*_{t=1} \alpha Z}{d_\beta p_1^e (1+\alpha)}. \tag{2.34}$$

From these expressions, it is easy to deduce that following this framework, we have the results below:

- **If the household has a positive valuation of energy renovation** ($\delta > 0$), it invests more when the subsidy is received in the first period if $P^E\left(r - d_\beta p_1^e \epsilon - Z\right) > 0$, that is, if the amount of subsidy Z is such that $Z \in]1 - \frac{d_\beta p_1^e \epsilon}{r}; r - d_\beta p_1^e \epsilon[$.
- **If the household has a negative valuation of energy renovation** ($\delta < 0$), it invests more when the subsidy is received in the first period if $P^E(r - d_\beta p_1^e \epsilon - Z) < 0$, that is, if $Z < 1 - \frac{d_\beta p_1^e \epsilon}{r}$ or $Z > r - d_\beta p_1^e \epsilon$.

Once again, there is a condition on the amount of the subsidy that depends on household preferences towards energy renovation. From a policy perspective, this corroborates that household preferences and considering the intangible externalities of renovation work on household utility[17] (to use Allcott and Greenstone (2017) terms) may drive more efficient policies than simple characteristics such as revenue.

3. SIMULATIONS

3.1 Estimation of model parameters

Each household is characterised by a set of parameters that are recalled in Table 1. The values we determine in the following are the closest approximation to the parameters that we could find with available data.

Table 1: Calibrated Parameter and Initial Variable Values

Parameter	Brief Description
$\underline{x}$	Minimum level of the composite good for a decent standard of living
$\underline{e}$	Minimum level of energy for decent heating
α	Parametrises the elasticity of substitution between the two goods
δ	Valuation of side-effects in the utility function
Z	Subsidy received by the household
ϵ	Efficiency of the renovation technology (or strategy) that linearly decreases in $\underline{e}$

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[17] i.e., the side effects of renovation work.

3.1.1 Data and purpose

Information on energy renovation, especially that concern parameters of our utility model, is sparse in the literature. We use two databases, the French Statistics on income and living conditions survey (SRCV) and a survey conducted by ADEME called TREMI [18]. Because TREMI has data available only for 2017, we use the 2017 and 2018 waves of SRCV. Other information is available in other reports from CIRED[19] and France Stratégie[20].

The SRCV panel provides detailed information on the dwelling (e.g., area) and on the household (e.g., composition, income, energy expenditure, . . .). Thus, we use the 2017 and 2018 waves of this panel to determine the basket of goods $(\underline{x}, \underline{e})$ necessary for a household to live decently. Ultimately, this also allows us to compute the elasticity parameter (α in (2.7)) for each household. Table 2 reports the main statistics over the merged waves of SRCV for households with a sufficient completion rate. Among the obtained total of 11,068 households in 2017, we were able to identify 8,424 living in the same dwelling in 2018.

Table 2: Summary statistics – SRCV – full dataset

Variable	**Mean**	**Stand. Dev**	**Med**	**Max**
2017 Income	40,781	56,380.7	34,200	4,547,830
2018 Income	40,968	55,518.8	34,002	4,416,562
Consumption units	1.6	0.53	1.5	6.1
Surface	101.3	90.3	91	6,120
Energy expenditures	1,351.7	922.3	1,416	10,000

To determine the side effects parameter δ, we use TREMI. We refer to the literature to find information necessary for the other parameters.

3.1.2 Determination of the basket of goods for a decent living, $(\underline{x}, \underline{e})$

We first determine **the basket of decent goods excluding energy**, that is, $\underline{x}$. For this, we are interested in households that cannot consume more than $\underline{x}$ and $\underline{e}$. As in Chaton and Guillerminet (2022), we call these households "strained" households. We assume that these households are those in the SRCV survey that answer the following regarding their perception of their standard of living[21] "*It's just that you have to be careful*". These households (4,408 out of 11,068) are not in deep poverty but still have to manage their budget with care, so we assume that their budget constraint binds in that case. Of the 4,408 that reports themselves as precarious, we removed those with high purchasing power (on an income by decile criterion). Consequently, of the households in the 2017 sample that heat using electricity and are owner occupied, 31% are considered to be strained. For each of them, indexed by i, the value of $\underline{x}_i$, i.e., $p^x_t \underline{x}_i$ is equal to "Disposable income of i – Energy expenses for housing of i". We take the median of $p^x_{2017}\underline{x}_i/\text{CUs}_i$ as the value for the minimum level of the composite good per consumption unit (CU). This value is equal to €20,915, i.e., €1,742 a month. This is similar to and consistent with the view of the French Observatory of Poverty and Social Exclusion, which states that in 2015, €3,284 are required for a couple with a single child (1.8 CUs) to live decently, i.e., €1,824 per month per CU. Note that this decent standard of living of €1,824 per month, which includes energy expenditure, allows the family to go on holidays, give presents, and follow cultural and sporting activities. The value of $\underline{x}$ depends on the geographical location of the household (because of the

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[18] Energy Renovation Work for Individual Housing.

[19] International Center of Research for Environment and Development.

[20] An economic advising service for the French prime minister.

[21] Question code: NVACTB.

cost of living, which can differ from one city to another). However, to simplify our analysis, in the following we ignore this difference and assume that in 2017 the price of the composite good is equal to unity and that $\underline{x}$ per CU is equal €20,915 .

Minimum energy consumption, $\underline{e}$ is crucial in our model. In 2012 in France, the average energy expenditure for dwellings fitted with individual heating equipment was €1,620 (Commissariat Général au Développement Durable (2015)). The notion of minimum energy consumption highly depends on the dwelling's characteristics, its equipment, its insulation, its main energy source, etc. We can assume that the number of occupants is partly encapsulated by the area of the dwelling, so that the normal consumption of a dwelling of a given type with given characteristics can be framed between two limit values. We assume that the lower limit is determined by the minimum value of the EPC[22] rank of the dwelling (energy class), and it defines $\underline{e}$, which is as a result associated with a dwelling. As this minimum value is in primary energy consumption, the resulting $\underline{e}$ (in final energy) differs from one energy source to another. In other words, for the same quantity of energy consumed, the corresponding EPC calculated on the bill differs from one type of heating to another. For convenience, we limit our analysis to owner-occupiers of all-electric homes (i.e., 650 households of the 11,068 in the sample). In 2017, for electricity, 1 kWh of final energy is equivalent to 2.58 kWh of primary energy. Therefore, $\underline{e}$ per m^2 values associated with EPC are summarised in Table 3. Hereafter, we study the decision of households to contract energy work to change EPC class. As a result, none of the households studied live in a dwelling with an "A" EPC score in $t = 0$. Households seeking to renovate their housing have the objective of achieving an EPC A/B, specifically, that after renovation the $\underline{e}$ characterising their housing is equal to 19.77 kWh per m^2 (see Table 3)

Table 3: Minimum energy per m^2 according to the EPC rank for an all-electric dwelling

EPC rank	**B**	**C**	**D**	**E**	**F**	**G**
$\underline{e}$ / m^2	19.77	35.27	58.53	89.53	128.29	174.81

The SRCV survey does not communicate the EPC of the accommodation. This database provides information on energy expenditure and not on the quantities consumed. Thus, to determine the EPC on the bill of an all-electric household, it is necessary to make an assumption on the price of electricity. We assume that in 2017 it was €15.13/MWh (this corresponds to the regulated tariff of the energy supplier EDF for subscribed electrical power of 9kVA). Thus, in 2017, the quantity of primary energy per m^2, e^{pe}_{2017}, for each all-electric dwelling of non-precarious owner households is determined by

$$e^{pe}_{2017} = \frac{2.58}{p^{e}_{2017}} \times \frac{\text{Energy expenses}}{\text{Housing size}}.$$

This value is used to calculate the EPC on the invoice for these accommodations. For example, if $e^{pe}_{2017} = 200$ kWh$_{\text{PE}}$/m^2/year then the dwelling has a D rating according to the EPC label (see Table 10 in Appendix 5.3 for the upper limit of scores).

Using the SRCV data, we calculated the EPC on the bill of all-electric housing in which the owner households live who report that they do not restrict their consumption. Since we are considering households that report not restricting their consumption, none of their dwellings should have an estimated EPC score lower than the true score. However, it is possible that a household wastes electricity. In this case, the EPC score of the household's dwelling is higher than its real score, and therefore the minimum energy level attributed to the household is too high.

Figure 1 shows the distribution of EPCs for owners' all-electric primary residences in 2017. The D and E labels are the most frequent (51% of the data) and the share of high energy consuming housing is quite large (7%). Note that according to the French Ministry of Ecological Transition,

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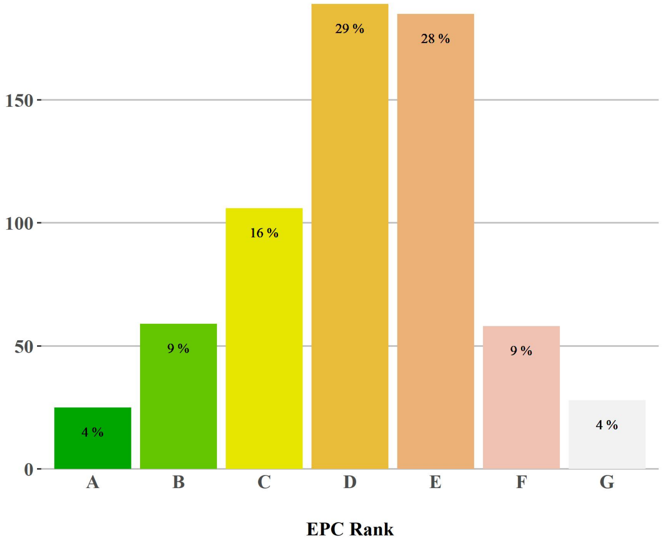
[22] See Appendix 5.3.

the most common EPC rank in the French housing stock is D, representing approximately 34% of dwellings[23]. This means that when taking a median household of class D, people tend to consume at least 151 $\text{kWh}_{\text{PE}}/\text{m}^2$/year.

Figure 1: Estimated EPC distribution in 2017 SRCV (all-electric housing for owner-occupiers)



Note: Each bar corresponds to the number of dwellings for each EPC rank in the 2017 SRCV datasets. Classes C and D are the most common. Moreover, consistent with the literature, A and B dwellings represent a smaller share than the two worst ranks F and G.

3.1.3 Investment-related parameters

Investment subsidy (Z): A report by France Stratégie (2021) estimates the average help received by French renovating households, depending on their income and whether they use all the available subsidies that are accessible. The state is able to cover, through different agencies, up to 60% of households' renovation costs for very modest households. A middle-income household can expect up to 30% of renovation work done to be covered. For the wealthiest households, subsidies do not exceed 16%. In the following, we assume that the household can obtain the full range of subsidies (in practice, additional subsidies can differ by region). Depending on a household's income decile, we assign it a value different from Z. These values in Table 4 directly come from France Stratégie (2021).

Table 4: Share of renovation covered by subsidies

Household	Income deciles	Coverage (Z)
Very Modest	4 to 5	0.60
Modest	6 to 7	0.45
Intermediate	8 to 9	0.30
Superior	10	0.16

Work efficiency (ϵ): We use the same method as in the example in Appendix 5.2.5 and assume that every household that conducts renovations wants to obtain a minimum level of

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[23] Data from the French Ministry of Ecological Transition.

energy $\underline{e} = 19.77$ kWh/m^2·year (see Table 3). Although it is likely that ϵ depends on the amount of money invested in the renovation, we do not have enough information and data to compute the real effectiveness of the work. Data on household consumption following previous detailed work could be used to find a good estimate of ϵ (including the rebound effect) relative to the amount of money invested. As every dwelling is characterised by an efficiency level, we use Initiative "Rénovons" (2017) studies on transition costs between EPC ranks as estimates for investment amounts necessary to reach the A/B score of the EPC. ADEME and CIRED propose different versions of the transition cost matrix. In our study, we show in Table 5 the values of ϵ for each transition cost (to rank B) and for each study.

Table 5: Transition costs to achieve EPC score B and corresponding ϵ

		Initial rank				
		C	**D**	**E**	**F**	**G**
ADEME	***Transition cost*** **(€/m^2)**	75	122	163	227	304
	ϵ	*0.53*	*0.81*	*1.1*	*1.23*	*1.31*
CIRED	***Transition cost*** **(€/m^2)**	290	480	620	710	750
	ϵ	*0.13*	*0.21*	*0.29*	*0.39*	*0.53*

Another study by France Stratégie (2021) reports mixed values, which can be found in Table 6. In the following, we only consider these values.

Table 6: Retained values for ϵ

Origin rank	**C**	**D**	**E**	**F**	**G**
Transition Cost **(€/m^2)**	93	169	232	287	351
ϵ	0.43	0.59	0.77	0.97	1.14

For example, any dwelling ranked E on the EPC scale is characterised by $\underline{e} = 231$ kWh$_{\text{PE}}$/m^2 · year. If an owner occupying a class E dwelling wants to improve to rank B, it must invest 232 €/m^2 in renovation. Under the hypothesis that the renovation is completely effective at improving dwelling's efficiency, the household is associated with $\epsilon = 0.772$kWh/€.

The estimation of parameter ϵ is key in every theoretical model that addresses energy renovation for households, as it represents the actual conversion of the renovation work into the energy efficiency of the dwelling. Moreover, good knowledge of ϵ could influence the household's decision mechanism. Although some studies have sought to obtain such information, there is still a lack of precision in the estimates. Furthermore, the estimation methods themselves and their sources remain unclear. Additionally, as can be seen from Tables 5 and 6, the transition costs from one EPC score to another vary considerably from one study to another. These differences do not seem to be solely due to the end dates of the studies (as there is no correspondence from one matrix of transition costs to another).

Lifetime of investment (T): We consider an average investment lifetime of 20 years, which corresponds to the raw return duration of investments in ADEME (2021).

3.1.4 Parameters capturing household preferences

Discount factor (β): According to Howarth and Sanstad (1995), the discount rate is generally high for energy investment and energy efficiency retrofits. This view is corroborated by Ruderman et al. (1987). Both argue that under-investment, which is commonly observed in the housing stock, may be due to market inefficiencies, such as a lack of intelligible information about the costs and benefits of investing in energy efficiency or the lack of financial capacity for the majority of households.

Ansar and Sparks (2009) provide a literature review of discount rates computed for different types of appliances, essentially refrigerators, air conditioners, thermal shells (devices used for refrigerators) and water heater type. The range of discount factors varies depending on the study and the data, from a lower average bound at 0.29 for a room air conditioner, up to 3 for a refrigerator.

Our study focuses on thermal renovation without specifying the amount of work. However, according to our quantitative analysis in 2.2.6, $\beta = 0.85$ seems to be a reasonable choice. Although the amount to invest might be high to have a good return on investment, the benefit engendered has high expected profit. As a consequence, the discount rate weight is high, and a lack of investment is due to other sources of inefficiency.

Interest rate (r): The interest rate is chosen to be consistent with people's savings. We suppose that savings do not refer to any particular financial process. Thus, we take the basic rate for most well-known French financial saving mechanism, the "Livret A", that is 3% from February 2023 to the end of July 2023. Consequently, we have $r = 1.03$. Given the past variations in this rate,[24] a sensitivity analysis of the equilibrium quantities at this rate is conducted in 3.2.

Side effects δ: As mentioned above, δ in the utility function (2.7) is computed based on the TREMI database. The literature estimating individuals' preference parameters is relatively large: among others, Arrondel et al. (2004) present some methods based on surveys, and Frischknecht et al. (2014) proposes a logit approach. However, the TREMI database does not allow us to compute any preference parameters following those traditional approaches. Indeed, the survey relies on whether households engaged in renovation work, so a logit choice model on the renovation decision would not highlight any preference mechanism, as for instance questions such as *"Why did you choose to engage work"* are asked of people that effectively engaged work, and questions such as *"Why did you not engage work"* are asked to people who did not engage work. Moreover, these questions do not truly represent a preference score, that is, the formulation does not follow the usual time-oriented template as in Arrondel et al. (2004), for instance *"Would you prefer 100€ now or x € in a year"*. Instead, the TREMI database is more a report on experiences that lists people's justification for having chosen to renovate (among other characteristics).

As we saw in 2.2, $\delta = -\delta_d + \beta\delta_u$, with δ_d representing the inconvenience suffered during the work and δ_u representing satisfaction in $t = 1$. We perform the estimation as follows:

- For the estimation of preferences in $t = 1$, we used the question **"Q53 - Have you already noticed a reduction in your energy costs as a result of the work?"**. We made sure to only consider work that was actually done. The options are ranked as follows: *"Yes, sensibly"*, *"Yes, a bit"*, *"No, not really"*, and *"No, not at all"*. We operationalised δ_u as the proportion of people answering *"yes"*, with any level of satisfaction. Note that there is another question about the improvement of comfort in the dwelling, **"Q52 Overall, did the work carried out enable you to improve the thermal comfort of your home?"**. We also created a score that is essentially the sum of Q52 and Q53, so that any form of improvement is taken into account. For Q53 only, we obtain $\delta_u = 0.3$. For both questions, we obtain $\delta_u = 0.225$. To better take into account psychological benefits, we only consider the second value of δ_u.
- The estimation of δ_d is much simpler, as we take people who, among other answers, selected *"The complexity or the difficulties of these first works discouraged me (research of the craftsmen, follow-up of the building site...)"* to the question **"Q51 What are the main reasons why you are not planning to do this work in the immediate future?"**, which is a multiple choice question that is detailed in Appendix 5.4.2. We find $\delta_d = 0.06$.

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[24] The highest value (8.5%) was reached in October 1981, and the lowest (0.5%) was observed in February 2020.

Thus, with $\beta = 0.85$, the value for δ is equal to $-0.06 + 0.85 \times 0.225 \approx 0.13$. However, it may differ from household to household. Therefore, we consider this value as a reference value and then perform a sensitivity analysis of the decision variables with respect to δ.

Energy-related preferences (α): From (2.16), we have

$$\alpha = \frac{rp_0^e\,(1+\beta+\delta)\,\left(e_0^* - \underline{e} + 1\right)}{\Omega - P^E - rp_0^e\,(1+\beta)\left(e_0^* - \underline{e} + 1\right)}. \tag{3.1}$$

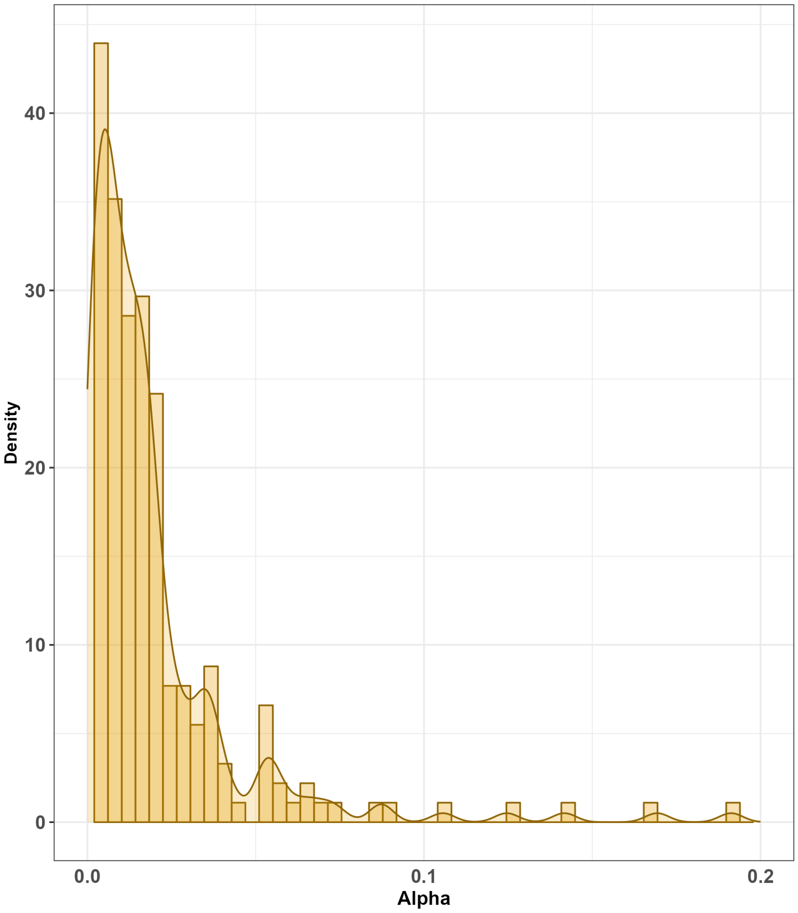
This parameter α is specific to the household and can be calculated from its income (in P^E defined by (2.14)), its energy expenditure ($p_0^e e_0^*$) reported in SRCV and its expenditure to have a decent level of the composite good ($p_0^x \underline{x}_i$) calculated in 3.1.2. As we are interested in the renovation decision to improve the energy efficiency of housing, we consider owner households that do not move between the two periods (2017 and 2018). We assume that households want to invest and have a direct return in 2018. Their goal is, as mentioned above, to reach rank A/B of the EPC score.

Each household is characterised in period 0 (i.e., in 2017) by a value of Z (according to its income); a value of $\underline{e}$ (according to its EPC); and an ϵ (according to its initial EPC and the EPC it wishes to reach). The value e_0^* for a non-fuel-poor owner-occupied household living in an all-electric dwelling is given by its declared energy expenditure in 2017 (for SRCV) divided by the price of electricity in 2017, p_{2017}^e. As already mentioned, $p_{2017}^e = 15.13$€/MWh, $p_{2017}^x = 1$. Thus, the price of the composite good is normalised for the first year, and we assume that inflation of 1.8% is applied to the price for 2018. For the price of electricity in 2018, as for 2017, we refer to the regulated energy supplier EDF tariff for a subscribed electrical power of 9kVA, i.e., $p_{2018}^e = 15.61$€/MWh. We have all the information to calculate the α of each household defined by (3.1). Figure 2 shows the distribution of α for non-fuel-poor owner-occupied households living in an all-electric dwelling.

Varying parameter T does not significantly influence preferences for energy consumption, so we choose to keep $T = 20$. However, we find evidence that the actualisation parameter β influences the distribution. In particular, the higher future preferences are, the wider the α density, meaning that more households are susceptible to have higher a higher elasticity of substitution between energy and the composite good. Although parameter β is a mean that might in reality vary across individuals (following, for example, their socio-economic characteristics), it is still consistent with the fact that households valuing the future and the return on their investment might be those valuing the share of energy in their consumption.

In Appendix 5.4.1, we study different characteristics of parameters $\underline{e}$ and α following the socio-economic characteristics of households. Note that $mean(\underline{e})$ and $median(\underline{e})$ may increase with household income because higher household income often implies a higher value for total consumption units.

Figure 2: Plot of α - density



3.1.5 Heterogeneity analysis

As we suggested in the section above, parameters may vary across individuals and households. In this section, we assume that preferences for the future (β) are constant across individuals, and we seek heterogeneity in the elasticity of substitution between the consumption of energy and composite goods (characterised by the parameter α of the utility function (2.7)).

To identify potential heterogeneity, we plotted the boxplots of α as a function of income decile (see Figure 3). The income deciles can be found in Appendix 5.4.1. We consider standard of living (i.e., income per CU[25]) deciles to characterise household wealth. Note that we take only deciles 4 to 10 to exclude fuel-poor households.

As α parameterises the elasticity of substitution between energy E and the composite good X, a higher value of α means greater substitutability between the two goods in the model, specifically more credit for energy consumption in households' utility level. In Figure 3, the poorer households are, the higher α. Very modest households might suffer from a weaker balance between disposable income and the magnitude of subsidies. The richest households do not receive as much subsidy as others, but they have more investment power (and simply a higher disposable income), which could

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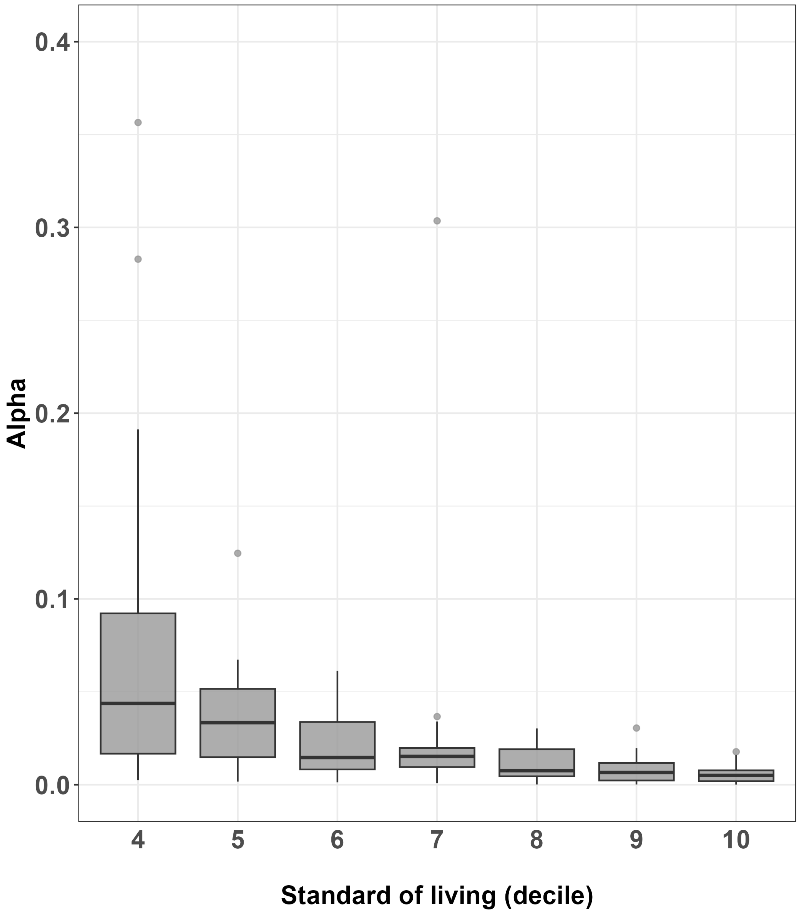
[25] Recall that one person is equal to one unit of consumption, CU, two people in one dwelling equals 1.5 units, adding a child increases the total by 0.3, etc.

explain their ease and low alpha. Overall, the results regarding the α distribution are quite intuitive. Households that are just above their budget constraint being binding might have more satisfaction from a one additional degree of heating than for those further from their constraint. These are the households that are closer to fuel poverty.

Figure 3: Boxplots of α across standard of living deciles in the 2017 SRCV dataset, (with $\beta = 0.8$)



Note: The boxplots are extracted from 2017 and 2018 SRCV data. The first three deciles of living standards correspond to households in fuel poverty, which are excluded. Details for the quantiles can be found in 5.4.1. The median and mean of α decrease with the income bracket. Note that for quantiles 7 and 9, the decline is not as steep as expected, mainly due to the mean surface values for these quantiles.

3.2 Simulation

Although in Section 2, we focused on the impact of the renovation subsidy on the equilibrium and therefore on the decision to renovate, it is nevertheless interesting to study the impact of the other parameters of the model on the equilibrium.[26] For simplicity, we assume that certain parameters have values common to non-fuel-poor owner-occupier households (see Table 7).

Although it is possible to illustrate the behaviour of all owner households that are not in fuel poverty, we conduct the exercise here for the median household.

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[26] The analytical expressions for these impacts are given in Appendix 5.4.3.

Table 7: Parameters characterizing the median household

Parameter	$\mathbf{p^x_{2017}}$ (€)	$\mathbf{p^x_{2018}}$ (€)	$\mathbf{p^e_{2017}}$ (€/MWh)	$\mathbf{p^e_{2018}}$ (€/MWh)	**r**	δ	β	**T** (years)
Value	1	1.018	15.13	15.61	1.03	0.13125	0.85	20

Definition 3.1. *The median household consists of an owner-occupied household composed of two adults (CU=1.5) living in a D-score all-electric dwelling of 88 m^2.*

From the definition of the median household and the determination of the level of composite good excluding energy necessary to live decently (equal to 20,915 €/CUs, cf. 3.1.2), we deduce that for this median household, $\underline{x} = 31,372$. The household lives in a D-score all-electric dwelling of 88 m^2, and the minimum energy level associated with it is therefore equal to $151 \times 88/2.58 = 5,150$ kWh. We assume that its standard of living (respectively α) is the median of the standard of living (α) of owner-occupied households whose CU is equal to 1.5 and living in a dwelling with an area between 85 m^2 and 90 m^2 and having an EPC score of D. Thus, this standard of living is equal in 2017 (2018) to 34,909 (39,491) euros and $\alpha = 0.017$.

Given the complexity of estimating ϵ, due among other things to the heterogeneity of the renovation work to be done to move from a given EPC class to another, we take an arbitrary ϵ ($\epsilon = 0.21$) and perform here an analysis of sensitivity of the decision variables to this ϵ. Note that this heterogeneity explains the existence of very different cost matrices to switch from one EPC to another (see tables 5 and 6). The sensitivity analysis below is not only done on ϵ but also on the parameters specific to the median household and summarised in the Table 8.

Table 8: Parameters characterizing the median household

Parameter	α (€)	$\mathbf{w_{2017}}$ (€)	$\mathbf{w_{2017}}$ (kWh)	$\underline{\mathbf{e}}$ (€)	$\mathbf{p^x_{2017}\underline{x}}$ (€)	$\mathbf{p^x_{2018}\underline{x}}$	**Z**
Value	0.017	52,363	59,236	5,150	31,372	33,882	0.3

The optimal household expenditures calculated from our assumptions are summarised in the Table 9. The energy expenditure corresponds to a consumption of 7,714 kWh in 2017 and 5,748 kWh in 2018. The investment reduced the energy level for a decent standard of living by 1,577 kWh ($\epsilon \times c^*$). With s^* being negative, the household borrowed to carry out the energy renovation work.

Table 9: Household expenses (current euros)

Parameter	$\mathbf{p^x_{2017}x_{2017}}$	$\mathbf{p^x_{2018}x_{2018}}$	$\mathbf{p^e_{2017}e_{2017}}$	$\mathbf{p^e_{2018}e_{2018}}$	**s**	**c**
Value	54,195	51,918	1,167	897	-6,233	7,510

Impact of future income, w_1: This household, which perfectly anticipates its future income and second-period prices, must have a second-period income at least equal to €32,633 to consume the basket of goods for a decent life (i.e., $\underline{x}$ and $\underline{e} - \epsilon \times c^*$)[27]. Since by hypothesis Γ defined by (2.15) is positive, the consumption of the composite good during the two periods and the consumption of energy in the first period increase when household income increases. Specifically, given the values of the parameters, $\frac{\partial x_0^*}{\partial w_1} \approx 0.482$; $\frac{\partial x_1^*}{\partial w_1} \approx 0.415$; $\frac{\partial e_0^*}{\partial w_1} \approx 0.0542$. On the other hand, the variation in energy consumption in the second period, as well as savings and investments in renovation following an increase in income, depend among other things on δ and the P^E. Under our assumptions $P^E \approx -0.411$ and $\delta \approx 0.131$; as a result, energy consumption in the second period and savings increase when income increases. Specifically, $\frac{\partial e_1^*}{\partial w_1} \approx 0.0126$ and $\frac{\partial c^*}{\partial w_1} \approx 0.1587$. On the other hand, savings decrease

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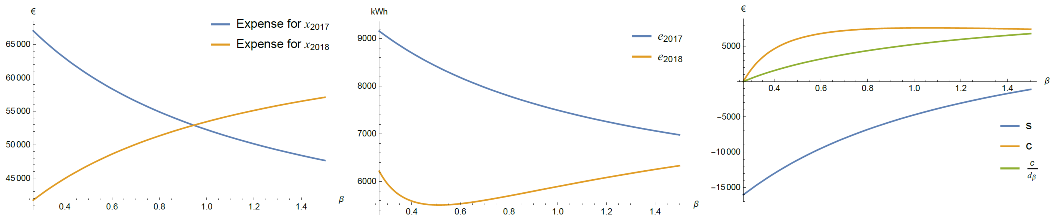
[27] If its income is less than €32,633, the household is in energy poverty.

when income increases ($\frac{\partial s^*}{\partial w_1} \approx -0.5589$). The household does not borrow or save when its income is approximately €48,084. It is possible to determine the second-period income at which the household borrows to finance its entire investment, i.e., the income in $t = 1$ that solves equation $-s^* = c^*(1 - Z)$. In the case of our median household, this income is equal to €57,056.

Impact of discount rate, β: Figure 4 shows the value of the optimal decision variables as a function of the discount rate (β) when the discount rate varies between 0.267 and 1.5. If $\beta < 0.267$, the household does not invest. It is clear that consumption in the first period (the second period) decreases (increases) when β increases. The higher the discount rate is, the higher the savings and the lower the investment. Savings are positive only if β is greater than 1.715. As long as β is lower than 1.989, energy consumption in 2017 will be higher than in 2018. For this value of β, compared to the reference situation $\beta = 0.85$, the household invests less and above all saves instead of borrowing.

Figure 4: Optimal values for the median household as a function of the discount rate



Expenditure for the composite good | **Optimal electricity consumption** | **Optimal savings and investment**

Note: The curves represent for two years (2017-2018), according to the discount rate, the optimal expenditure in composite goods (left figure); the optimal quantity of electricity (centre figure); and the optimal savings and investments (right figure).

Impact of discount rate, ϵ: While the impact of ϵ on first-period consumption is small, that on second-period energy consumption is obviously more significant. Given our assumptions, if ϵ is greater than 0.587, the household's residence will be energy self-sufficient. Since $\delta > 0$ and $\Gamma > 0$, the higher ϵ is, the higher the investment (c^*). Given the values of the parameters, the higher ϵ is, the greater the borrowing ($-s^*$). The higher ϵ is, the lower the consumption of the composite good during the two periods and the lower the consumption of the energy good in the first period. The decrease in the consumption of the energy good in the second period following an increase in ϵ depends on the value of the parameters. We assess this with the retained values. Note that even if investment does not improve the energy efficiency of the dwelling ($\epsilon = 0$), the household invests (€6922) because of the existence of a psychological gain from the investment (δ). **The subsidy should obviously be a function of ϵ.**

Impact of δ_u and δ_d: The impacts of positive side effects (δ_u) and those of the disadvantage suffered during the renovation work (δ_d) are obviously opposed. If $P^E < \Omega$ (a condition highly probable and satisfied in our simulations), the consumption of the composite good during the two periods and the consumption of the energy good in the first period decrease (increase) when δ_u (δ_d) increases. If $0 < P^E < \Omega$ (a condition satisfied in our simulations), we have the following:

1. the consumption of the energy good in the second period decreases (increases) when δ_u (δ_d) increases;

2. An increase in δ_u (δ_d) decreases (increases) the interest of the household in energy renovation of its dwelling. Therefore, the higher δ_u (δ_d) is, the higher (lower) the investment. On the other hand, it will lead to an increase (decrease) in borrowing.

4. CONCLUSION AND POLICY IMPLICATIONS

We are interested in the behaviour of households in terms of energy renovation of their housing. We emphasise the intricacy of analysing the impact of financial subsidies on renovation decisions that pose a significant challenge for public policies in effectively adjusting multiple parameters. Thus, it is unsurprising that public policies in France have encountered difficulties in effective implementation. These public policies, are intended to improve the energy efficiency of buildings, are not having the desired effect: household investment in renovation is too low. Faced with these "failures", subsidies have been multiplied and modified. In this article, we develop an *ex ante* two-good, two-period utility model that allows us to disentangle different parameters' effects on an optimal subsidy for people to invest in energy renovation. We notably take into account potential substitutability between energy and composite goods, and we assume that every investment implies negative and positive side effects, that is, losses and benefits other than financial (reduction in energy expenditures). These parameters are of great interest, so we decided to include their effect in the utility function.

Our results offer a positive outlook for policy makers and provide a road map for further studies on the subject. We provide insights into households' behaviour based on their characteristics, specifically on their preferences. The side effects that an investment engenders, of all kinds,[28] are critical in the investment decision. We show that if negative side effects outweigh the positive side effects in the utility function of a household, the scaling of a subsidy policy (which we consider to be exogenous) changes substantially. From the perspective of a state decision-maker, it could be an incentive for the implementation of a disclosure mechanism destined to reveal, ideally, each household's type, or at least launch experimental studies to expand our knowledge of these features that remain insufficiently understood. Additionally, according to our results, attention should be paid to the date of reception of the subsidy, as we show in Section 2.2.6. Additionally, we find income heterogeneity in the substitution effect between our two consumption goods, energy and a composite good, which conveys the idea that energy poverty should be a focus in other studies, especially regarding the actual price situation and potential future instabilities on the international energy market, with energy renovation (if well done) being an effective way for households to become insulated from exogenous crises.

Other policies could influence the value of factor δ. By alleviating informational issues, households could adopt a more positive view of energy renovation. The aim, according to both Rudinger and Gaspard (2022) and ADEME (2021), is to better inform households on both the availability of subsidies and the potential positive side effects that they imply. Ideally, the remaining up-front cost to pay would be covered by a loan whose reimbursement would be equal to or lower than the reduction in energy expenditures. The household would have no monetary loss or benefit during the repayment of the loan compared to its initial situation except for a wide set of positive side effects (comfort, embellishment, etc.). Unfortunately, although ADEME (2021) gives some scenario examples, an inquiry led by OpinionWay shows that it represents 40% of respondents, knowing that approximately 60% were unable to even mention one subsidy dedicated to energy renovation. With average costs that often exceed €50,000, communication policies that may affect δ are of first-order importance. The current use of subsidies is apparently inefficient: according to Rudinger and Gaspard (2022), 86% of work supported by *MaPrimeRenov'* (a subsidy available for all dwellings, with the amount depending on income) were small gestures, against 12% that included two or more items, and 3% that included three items or more.

Overall, there seems to be a double issue with the necessity of increasing the pace of renovations on the housing stock and their quality. As ADEME (2021) shows, technical and physical

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[28] Financial constraints, inconvenience suffered during the work, the time and dedication needed to find a trustworthy builder, the comfort level the improvement, the visual embellishment, household beliefs that the amount invested will have an impact *a priori*, etc.

limitations prevent efficiency from being achieved, and actual subsidies do not seem to be effective. This corroborates the broader literature about energy efficiency investment in the residential sector, as in Allcott and Greenstone (2017), that finds evidence that efficiency programs decrease overall welfare by $ 0.18, mainly because the of the design and calibration of policies, which fail to account sufficiently for intangible effects. A wider problem is self selection into subsidies that create low social values. Technically, this could be the case in France with the over-selection of subsidies that allow one or two renovation gestures. Considering how to size policies around household preferences, and implement mechanisms that reveal household type, may help better understand household behaviour towards renovation and subsidy policies, and increase the number of efficient renovations.

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5. APPENDIX

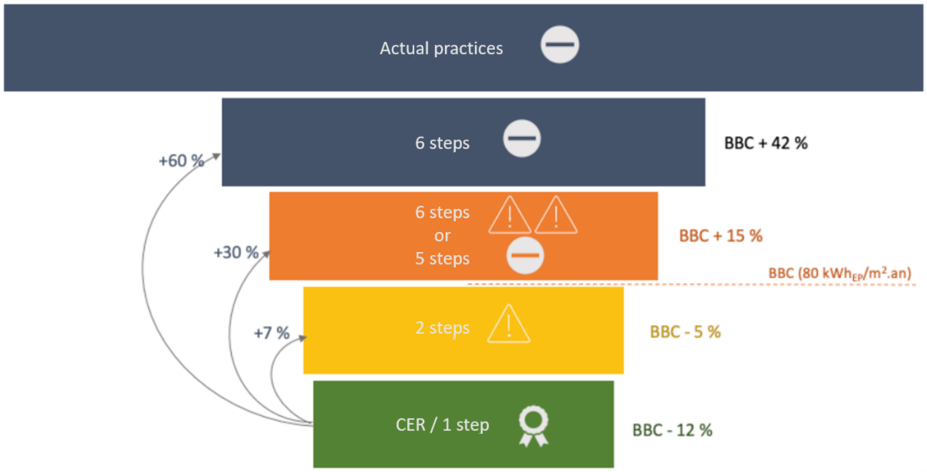
5.1 Housing renovation in France

The French Agency for the Environment and Energy Management (ADEME) provides several technical and economic characteristics of what could constitute a good, global and efficient energy renovation. A good and simple definition of thermal renovation would be "the act of sealing every heat leakage of a housing unit". Of course, the accumulation small gestures on top of one another is far from being enough to seal a whole dwelling. In a 2021 report (ADEME (2021)), the agency defines a highly technical and practical roadmap for global renovation by steps. Even when speaking about global renovation, the work must be highly meticulous to be efficient, mainly for very practical reasons, which they attribute to handling junctions. If the renovation occurs in too many steps, or at least if the steps are not organised, the builders might be unable to properly treat physical junctions between elements of the dwelling, and it might create thermal breaks: for example, walls and roof insulation should be done in a single step to ensure that their is no split between the two insulation installs.

Although this appears to be a detail, ADEME provides statistical evidence that step management is critical in the resulting energy efficiency level of a home. They divide renovation into six main items and junctions: wall insulation, roof insulation, floor insulation, venting, joinery, heating, and domestic hot water. They also encourage a renovation to be done in a maximum of three steps. To provide evidence for this necessity, they gather data on different energy renovations, which they classify following different renovation trajectories.

In Figure 1, there is clear evidence that for the same items renovated, thermal breaks have to be treated seriously to achieve a good energy efficiency level: a dwelling renovated with respect to the six items in six steps consumes 60% more than a dwelling renovated with respect to the six items in one step. Additionally, it is clear that more than three steps does not provide enough efficiency, that is, in the objective of being classified as a low energy consumption building (BBC). The state of actual practices is of course far from being efficient.

Figure 1: Primary energy consumption from renovated dwellings in the French housing stock built before 1982



Note: Each rectangle represents a renovation trajectory, with its size being proportional to the energy consumption at the end of the renovation work, relative to the low consumption building label level (BBC). Each arrow represents the differential consumption between different rectangles. For example, the arrow going from the bottom rectangle to the top rectangle is to be read “Dwellings that were renovated in one step for six items consume 60% less energy than dwellings that renovated the six items in six different steps”. CER (in the green rectangle) stands for complete and efficient renovation, a term introduced by the ADEME.

Source: Data and graph provided by ADEME, translated by authors.

5.2 Supplement to Section 2

5.2.1 Fuel poverty: quantitative definitions

In France, the main indicator for defining fuel poverty is the energy effort rate, which should not exceed 8% for the three lowest deciles of income per consumption unit[29] (EER_3D_08). According to this indicator, a household is in fuel poverty if the two following conditions are met: the ratio (energy expenditures)/(income of the household) > 8%, and (income of the household)/(consumption units) < 3rd decile of income. The indicator based on the EER_3D_08 is estimated annually by the Commissioner General for Sustainable Development using the Prometheus micro simulation tool. The estimate for 2017 (2018) is that 11.9% (12.6%) of households in mainland France were in fuel poverty. With another microsimulation model using the same database as Prometheus, namely the 2013 National Housing Survey, Chaton and Gouraud (2020) estimate that the percentage of households in fuel poverty for these two years was approximately 13%. We do not use the 2013 National Housing Survey, but taking into account all households in the SRCV database, we find a proportion of 13.3% of fuel poor for 2017 and 12.2% for 2018, using representation weights. Without weighting, the results change by less than 0.2 percentage points, so we consider it to be representative.

5.2.2 Equilibrium conditions to avoid fuel poverty

The solutions (2.16)–(2.21) are optimal for non-fuel-poor households. This imposes the following conditions on the subsidy received (according to Assumption 2.1) at $t = 0$. Thus, from equations (2.16), (2.17), (2.20) and (2.21) we deduce:

$$\left\{\begin{array}{ll} e_0^* \geq & \underline{e}, \\ x_0^* \geq & \underline{x}, \\ e_1^* \geq & \underline{e} - \epsilon c^*, \\ x_1^* \geq & \underline{e}, \end{array}\right. \quad \Leftrightarrow \left\{\begin{array}{ll} Z & \leq Z_{e_0}, \\ Z & \leq Z_{x_0}, \\ Z & \leq Z_{e_1}, \\ Z & \leq Z_{x_1}, \end{array}\right. \tag{5.1}$$

were

$$Z_{e_0} = 1 + d_\beta \left(\frac{\Omega - p_1^e \epsilon}{r} - \frac{p_0^e \Gamma}{\alpha} \right), \tag{5.2}$$

$$Z_{e_1} = 1 + d_\beta \left(\frac{\Omega - p_1^e \epsilon}{r} - \frac{p_1^e \Gamma}{r\alpha\beta} \right), \tag{5.3}$$

$$Z_{x_0} = 1 + d_\beta \left(\frac{\Omega - p_1^e \epsilon}{r} - p_0^x \Gamma \right), \tag{5.4}$$

$$Z_{x_1} = 1 + d_\beta \left(\frac{\Omega - p_1^e \epsilon}{r} - \frac{p_1^x \Gamma}{r\beta} \right). \tag{5.5}$$

5.2.3 Proof of Proposition 2.7

$\underline{Z}_c(\delta)$ (resp. $\overline{Z}_c(\delta)$) defined by (2.22) (resp. (2.23)) can be rewritten as

$$\underline{Z}_c(\delta) = \left\{\begin{array}{ll} Z_1 & \text{if } \delta < 0, \\ Z_2 & \text{if } \delta \geq 0, \end{array}\right. \tag{5.6}$$

$$\overline{Z}_c(\delta) = \left\{\begin{array}{ll} Z_2 & \text{if } \delta < 0, \\ Z_1 & \text{if } \delta \geq 0, \end{array}\right. \tag{5.7}$$

where $Z_1 = 1 - d\beta \times \frac{p_1^e \epsilon}{r}$ and $Z_2 = 1 - d\beta \left(\frac{p_1^e \epsilon}{r} + \frac{\delta\Omega}{r(1+\alpha)(1+\beta)} \right)$. According (2.18), Assumption 2.3 that is to say $\underline{e} - \epsilon \times c^* \geq 0$ is verified if

$$\left\{\begin{array}{ll} Z_1 < Z < Z_c & \text{if } \delta < 0, \\ Z_c < Z < Z_1 & \text{if } \delta \geq 0, \end{array}\right. \tag{5.8}$$

where

$$Z_c = 1 - \frac{d\beta\epsilon}{r} \times \left(p_1^e + \frac{\delta\Omega}{\underline{e}\Gamma + (1+\alpha)(1+\beta)\epsilon} \right). \tag{5.9}$$

But

$$Z_c - Z_2 = \frac{d\beta\underline{e}\Gamma\delta\Omega}{r(1+\alpha)(1+\beta)(\underline{e}\Gamma + (1+\alpha)(1+\beta)\epsilon)}.$$

As a result $Z_c > Z_2$ if $\delta > 0$ and $Z_c < Z_2$ if $\delta < 0$. By consequence if $\underline{Z}_c(\delta) \leq Z \leq \overline{Z}_c(\delta)$ (5.8) holds.

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[29] (In France, this represents the poorest 30% of people)

5.2.4 Proof of proposition 2.8

s^* defined by (2.19) can be rewritten as

$$s^* = \frac{F(Z)}{(d_\beta)^2 \Gamma r P^E}$$

where F a quadratic function of the subsidy Z. More precisely

$$F(Z) = AZ^2 + BZ + C,$$

where

$$\begin{array}{rcl} A & = & -r^2(1+\alpha)\beta < 0, \\ B & = & r(2r(1+\alpha]\beta + d_\beta(p_1^e(1+\alpha)(1-\beta)\epsilon - (1+\alpha+\delta)\Omega + r\Gamma\Omega_0)), \\ C & = & (1+\alpha)(d_\beta p_1^e \epsilon - r)(r\beta + d_\beta(p_1^e\epsilon - \Omega)) + d_\beta r(\delta\Omega + \Gamma(d_\beta p_1^e \epsilon - r)\Omega) \end{array}$$

The discriminant (Δ) of the polynomial $F(Z)$ is a quadratic function of δ more precisely,

$$\begin{array}{rcl} \Delta & = & \Omega_1^2\delta^2 + 2(1+\alpha)(\Omega_1(\Omega_1 - r\beta\Omega_0) + p_1^e\epsilon(r(1+\beta)\Omega_0 - (1-\beta)\Omega))\delta \\ & + & (1+\alpha)^2(p_1^e(1+\beta)\epsilon - \Omega_1 + r\beta\Omega_0)^2. \end{array}$$

The polynomial Δ vanishes at δ_1 and δ_1 where

$$\delta_1 = \frac{-(1+\alpha)(\Omega_1(\Omega_1 - p_1^e\epsilon - r\beta\Omega_0) + p_1^e\epsilon\beta(\Omega + r\Omega_0)) - 2\sqrt{p_1^e(1+\alpha)^2\beta\epsilon\Omega(\Omega_1 - p_1^e\epsilon)(\Omega_1 - r\beta\Omega_0)}}{\Omega_1^2},$$

$$\delta_2 = \frac{-(1+\alpha)(\Omega_1(\Omega_1 - p_1^e\epsilon - r\beta\Omega_0) + p_1^e\epsilon\beta(\Omega + r\Omega_0)) + 2\sqrt{p_1^e(1+\alpha)^2\beta\epsilon\Omega(\Omega_1 - p_1^e\epsilon)(\Omega_1 - r\beta\Omega_0)}}{\Omega_1^2}.$$

1. If δ is in the interval $[\delta_1; \delta_2]$ then the discriminant of $F(Z)$ (i.e. Δ) is negative or zero and therefore whatever Z, $F(Z) \geq 0$. Hence in this case, the sign of s^* is that of $-P^E$. So if P^E is positive (respectively negative) the household borrows (respectively saves).
2. If $\delta \notin [\delta_1; \delta_2]$ then $\Delta > 0$ and the equation $F(Z) = 0$ has the following two solutions

   $$\begin{array}{rcl} Z_{1,s}(\delta) & = & 1 - \dfrac{d\beta}{2r(1+\alpha)\beta} \times ((1+\alpha+\delta)\Omega - pe1(1+\alpha)(1-\beta)\epsilon - r\Gamma\Omega_0 + \sqrt{\Delta}), \\ Z_{2,s}(\delta) & = & 1 - \dfrac{d\beta}{2r(1+\alpha)\beta} \times ((1+\alpha+\delta)\Omega - pe1(1+\alpha)(1-\beta)\epsilon - r\Gamma\Omega_0 - \sqrt{\Delta}). \end{array}$$

   As $A < 0$, if $Z \in]Z_{1,s}(\delta) < Z_{2,s}(\delta)[$ then $F(Z) > 0$ and consequently s^* has the sign of P^E otherwise that of $-P^E$. But P^E is positive if $Z > 1 - d\beta \times \frac{p_1^e\epsilon}{r}$. Hence, we can state point 2 of Proposition 2.8.

5.2.5 Theoretical effectiveness: a quick numerical example

We wonder about the efficiency and profitability of investing in an energy-efficient house. In fact, we previously supposed that the reduction in minimal consumption needed by a given dwelling is linear and characterised in the utility function as $\epsilon \times c_0$; thus, the higher the investment is, the lower the minimum needed energy consumption in the second period. Our model does not highlight a simple mechanism behind the renovation decision or the amount invested. Consumers (households) trade off among savings s, consumption of both goods e and x, and investment c. However, a crucial parameter in our reasoning is the comparison between the marginal return on investment and the marginal return on savings. Whether one is greater than the other obviously implies different conclusions regarding household decisions.

Given current energy prices, it seems that investing in a future reduction in the amount of money spent on energy expenses is viable. A quick quantitative estimation can allow us to roughly estimate the benefits of investing in energy efficient devices. We use simple calculations based on the Energy Performance Certificate (EPC) scale[30] provided by the French's Code for Construction and Housing[31] and previous renovation cost estimates by Giraudet et al. (2018). They provide estimates of transition costs between all ranks on the EPC scale.

For instance, passing from rank E to a rank D (the most common in the dataset we use in Section 3) should, according to Giraudet et al. (2018), cost a mean of 232 €/m^2. Then, the minimum energy consumption would decrease from 231 $\text{kWh}_{\text{PE}}/\text{m}^2 \cdot year$ to 51 $\text{kWh}_{\text{PE}}/\text{m}^2 \cdot year$.

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[30] See Appendix 5.3.

[31] Article R126-15.

Let us give an average annual price of 15.3 €/MWh. Going from rank E to rank B, assuming that the dwelling heats itself using electricity only (i.e., a coefficient of conversion of final energy into primary energy (PE) equal to 2.58), would mean a reduction of $\frac{(231-51)\times 0.153}{2.58} = 10.7\ €/m^2$. For the average French dwelling, which according to INSEE occupies a space of 90.9 m^2, this represents approximately €900 in savings per year for an investment of €20,880, that is 5% profitability over a year.

Of course, in this quick quantitative study, we assume that dwelling consumes the strictly needed minimum. However, in a real situation, there could be several biases that would attenuate the profitability of an energy efficiency investment. For instance, we do not take into account people's consumption habits or any kind of rebound effect such for in Jevons (2007).

Furthermore, we there is no guarantee that the inconvenience costs that could be engendered, monetary or otherwise, are included in δ_d. For instance, a full renovation could cause people to rent a temporary dwelling or to find a place to stay (family, friend, etc.) while the work progresses.

Overall, the proportion of renovations that are completed are estimated by Giraudet et al. (2018) to only represent 6% of the renovation panel. The most common "renovations" are in fact small gestures that sometimes allow dwellings to be upgraded on the EPC scale (changing windows only, for example), which may allow households to increase their housing value or comply with rental laws, that tend to be more restrictive for low-ranked dwellings. According to Rudinger and Gaspard (2022), full renovations are essential for an efficient policy in terms of energy efficiency.

5.3 EPC Standards

The labels and ranks are strictly defined by French law. Sellers have no option but to hew to the regulation. Below is the standard new version of the label, which must include the original panel with energy consumption (in $kWh_{PE}/m^2 \cdot year$), coupled with a panel representing the equivalent in greenhouse gas emissions.

Figure 2: New standard for a EPC label

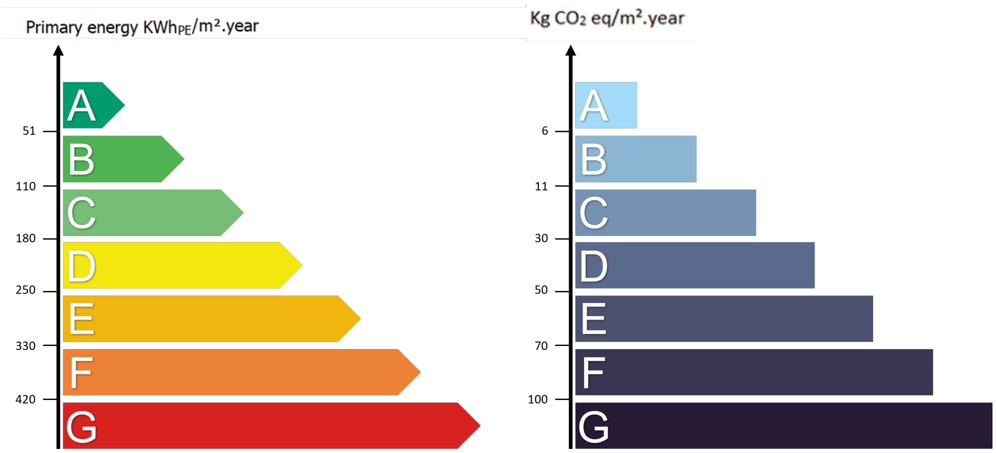

Table 10: Thresholds for primary energy consumption ($kWh_{PE}/m^2/year$)

Rank	A	B	C	D	E	F	G
$\underline{e}$	50	90	150	230	330	450	-

5.4 Supplement to Section 3

5.4.1 Characteristics of certain parameters

Table 11 gives the upper income limit for the first 9 standard of living deciles for the year 2017 for all households in the SRCV database.

Table 11: Standard of living deciles in the SRCV 2017 database in euros

Decile	10%	20%	30%	40%	50%	60%	70%	80%	90%
Standard of living (€)	12,389	15,483	17,970	20,146	22,239	24,690	27,928	32,734	41,837

Table 12 presents for 2017 the median of the main characteristics of owner households living in all-electric homes that impact their electricity consumption, i.e., α (which parameterises the elasticity of substitution between electricity and the composite good but can also be considered as an energy sobriety parameter), the minimum electricity level ($\underline{e}$) and surface area of the home.

Table 12: Main characteristics of parameters related to energy (with $\beta = 0.8$)

Household	**Standard of living** *(deciles)*	**med**(α)	**mean**(α)	**med**($\underline{e}$) *(kWh)*	**mean**($\underline{e}$) *(KWh)*	**med**(**Surface**) (m^2)
Very Modest	4 to 5	0.044	0.079	5,552	5,513	96
Modest	6 to 7	0.019	0.027	4,145	4,380	90
Intermediate	8 to 9	0.013	0.017	5,349	6,383	100
Superior	10	0.005	0.007	5,930	6,600	98

In Table 13, for 2017 and 2018, the medians of the composite good in value, of the minimum level of electricity, as well as of the surface area of the dwellings are shown. The increase in the cost of the composite good is explained by inflation. While the median surface area of owners' dwellings (living in their dwellings) has increased slightly, that of the minimum level of electricity has fallen sharply. This may be due to an improvement in housing efficiency between these two dates but especially the mild temperatures of 2018. Indeed, 2018 was listed as the hottest year since the turn of the 20th century. Therefore, it is necessary that the minimum energy level be corrected for the climate.

Table 13: Median value of certain characteristics of non-precarious owner households

Parameter	**Value**	**Data**
$med(p_0^x \underline{x}_0)$	€20,915	SRCV 2017
$med(p_1^x \underline{x}_1)$	€21,082	SRCV 2018
$med(\underline{e}_0)$	4,941 kWh	SRCV 2017
$med(\underline{e}_1)$	3,837 kWh	SRCV 2018
$med(\text{Surface}_0)$	84 m^2	SRCV 2017
$med(\text{Surface}_1)$	88 m^2	SRCV 2018

5.4.2 Questions used for the estimation of energy savings preferences

All questions had an answer of the type "did not answer", if the respondent was unable to answer. For example, people might not have experienced a change in their expenses because they are too close to the work they did.

- **"Q51 - What are the main reasons why you are not planning to do this work in the immediate future?"**

  1. I do not want to engage work (disturbing, lack of time, other projects...)
  2. My financial situation is too weak for me to engage work
  3. I am not sure of the economies realized justify the investment
  4. The complexity of the work discouraged me (search for craftsman, work follow-up)
  5. I do not plan to stay long in this dwelling
  6. The decision is independent from my will

- **"Q52 - Overall, did the work carried out enable you to improve the thermal comfort of your home?"**

  1. Yes, sensibly
  2. Yes, a bit
  3. No, not really
  4. No, not at all

- **"Q53 - Have you already noticed a reduction in your energy costs as a result of the work?"**

  1. Yes, sensibly
  2. Yes, a bit
  3. No, not really
  4. No, not at all

5.4.3 Impact of some parameters on the equilibrium

Impact of future income, w_1:

$$\begin{aligned}
\frac{\partial x_0^*}{\partial w_1} &= \frac{1}{p_0^x r\Gamma} > 0, && (5.10)\\
\frac{\partial x_1^*}{\partial w_1} &= \frac{\beta}{p_1^x\Gamma} > 0, && (5.11)\\
\frac{\partial e_0^*}{\partial w_1} &= \frac{\alpha}{p_0^e r\Gamma} > 0, && (5.12)\\
\frac{\partial e_1^*}{\partial w_1} &= \frac{p_1^e\delta\epsilon + \alpha\beta P^E}{p_1^e\Gamma P^E}, && (5.13)\\
\frac{\partial c^*}{\partial w_1} &= -\frac{\delta}{\Gamma P^E}, && (5.14)\\
\frac{\partial s^*}{\partial w_1} &= -\frac{1}{r}\left(\frac{1+\alpha}{r} - \frac{\delta(1-Z)}{d_\beta P^E}\right). && (5.15)
\end{aligned}$$

Impact of discount, ϵ:

$$\begin{aligned}
\frac{\partial c^*}{\partial \epsilon} &= \frac{d_\beta^2 p_1^e\delta\Omega}{\Gamma(d_\beta p_1^e\epsilon + r(-1+Z))^2}, && (5.16)\\
\frac{\partial s^*}{\partial \epsilon} &= \frac{(1+\alpha)(d_\beta p_1^e\epsilon - r(1-Z))^2 - d_\beta r\delta(1-Z)\Omega}{\Gamma(d_\beta p_1^e\epsilon - r(1-Z))^2}. && (5.17)
\end{aligned}$$

The sign of this derivative is that of $(1+\alpha)(d_\beta p_1^e\epsilon - r(1-Z))^2 - d_\beta r\delta(1-Z)\Omega$ which is negative for $\epsilon \in [0; 336.3]$.

$$\frac{\partial x_1^*}{\partial \epsilon} = \frac{p_1^x}{p_0^x r\beta} \times \frac{\partial x_1^*}{\partial \epsilon} = \frac{p_0^e}{p_0^x\alpha} \times \frac{\partial e_0^*}{\partial \epsilon} = -\frac{p_1^e}{p_0^x r\Gamma} < 0. \tag{5.18}$$

$$\frac{\partial e_1^*}{\partial \epsilon} = \frac{(1+\alpha+\beta)(d_\beta p_1^e\epsilon + r(-1+Z))^2 - d_\beta r\delta(1-Z)\Omega}{\Gamma(d_\beta p_1^e\epsilon + r(-1+Z))^2}. \tag{5.19}$$

The sign of this derivative is that of $(1+\alpha\beta)(d_\beta p_1^e\epsilon - r(1-Z))^2 - d_\beta r\delta(1-Z)\Omega$ which is negative for $\epsilon \in [0; 249[$.

Impact of δ_u and δ_d: We have

$$\begin{aligned}
&\frac{\partial x_0^*}{\partial \delta_u} = \frac{p_1^x}{p_0^x r\beta} \times \frac{\partial x_1^*}{\partial \delta_u} = \frac{p_0^e}{p_0^x\alpha} \times \frac{\partial e_0^*}{\partial \delta_u} = -\frac{p_1^e P^E}{p_1^e(1+\alpha+\beta+\alpha\beta-\delta_d+\beta\delta_u)^2 P^E} \times \frac{\partial x_1^*}{\partial \delta_u}\\
= \quad &-\beta\frac{\partial x_0^*}{\partial \delta_d} = -\frac{p_1^x}{p_0^x r} \times \frac{\partial x_1^*}{\partial \delta_d} = -\frac{\beta p_0^e}{p_0^x\alpha} \times \frac{\partial e_0^*}{\partial \delta_d} = -\frac{\beta p_1^e P^E}{p_1^e(1+\alpha+\beta+\alpha\beta-\delta_d+\beta\delta_u)^2 P^E} \times \frac{\partial x_1^*}{\partial \delta_d}.
\end{aligned}$$

$$\begin{aligned}
\frac{\partial c^*}{\partial \delta_u} &= -\beta\frac{\partial c^*}{\partial \delta_d} = \frac{(1+\alpha)\beta(1+\beta)(P^E-\Omega)}{(1+\alpha+\beta+\alpha\beta-\delta_d+\beta\delta_u)^2 P^E},\\
\frac{\partial s^*}{\partial \delta_u} &= -\beta\frac{\partial s^*}{\partial \delta_d} = \frac{(1+\alpha)\beta(r(1+\beta)(-1+Z) - d_\beta P^E)(P^E-\Omega)}{d_\beta r(1+\alpha+\beta+\alpha\beta-\delta_d+\beta\delta_u)^2 P^E}.
\end{aligned}$$