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A simple equilibrium model for a commodity market with spot and futures trades*

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Abstract

We propose a simple equilibrium model, where the physical and the derivative markets of the commodity interact. There are three types of agents: industrial processors, inventory holders and speculators. Only the two first of them operate in the physical market. All of them, however, may initiate a position in the paper market, for hedging and/or speculation purposes. We give the necessary and sufficient conditions on the fundamentals of this economy for a rational expectations equilibrium to exist and we show that it is unique. This is the first contribution of the paper. The model exhibits a surprising variety of behaviours at equilibrium; the second contribution is that we propose a unique generalized framework for the analysis of price relationships. The model indeed allows for the generalization of the normal backwardation theory and shows how it is connected to the storage theory. Meanwhile, it allows to study simultaneously the two main economic functions of derivative markets: hedging and price discovery. In its third contribution, the model illustrates what happens in a derivative market when speculation increases.

JEL Codes: D40; D81; D84; G13; Q00.

1 Introduction

In the field of commodity derivative markets, some questions are as old as the markets themselves, and they remain open today. Speculation is a good example: in his famous article about speculation and economic activity, Kaldor (1939) wrote: "Does speculation

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exert a price-stabilising influence, or the opposite? The most likely answer is that it is neither, or rather that it is both simultaneously". More than 70 years later, in June 2011, the report of the G20 (FAO et al. (2011)) states: "The debate on whether speculation stabilizes or destabilizes prices resumes with renewed interest and urgency during high price episodes. [...] More research is needed to clarify these questions and in so doing to assist regulators in their reflections about whether regulatory responses are needed and the nature and scale of those responses." Our simple (perhaps the simplest possible) model of commodity trading provides insights into this question. It also proposes a way to understand how these markets function and how the futures and spot prices are formed. Finally, it illustrates the interest of a derivative market in terms of the welfare of the agents.

In this model, the financial market interacts with the physical market. There are two periods, a single commodity, a numéraire and two markets: the spot market at times $t = 1$ and $t = 2$, and the futures market, which is open at $t = 1$ and settled at $t = 2$. The spot market is physical (there is a non negativity constraint on inventories), while the futures market is financial (shorting is allowed). There are three types of traders: inventory holders and industrial processors of the commodity, both of which operate on the two markets, and speculators who operate on the futures market only. All of them are utility maximizers and have mean-variance utility (this choice is discussed in the presentation of the model). There is also a background demand (or supply) attributed to spot traders, which helps clear the spot market. The sources of uncertainty are the amount of commodity produced and the demand of the spot traders at $t = 2$. Their realization is unknown at $t = 1$, but their law is common knowledge. Moreover, only the difference between these two quantities matters. All decisions are taken at $t = 1$ conditionally on expectations about $t = 2$.

Our main contributions are three: qualitative, quantitative and normative. They are the consequences of the tractability of the model.

Qualitatively, despite the presence of highly non linear equilibrium equations, we give necessary and sufficient conditions on the fundamentals of this economy for a rational expectations equilibrium to exist and we show that it is unique. Moreover, it has a very interesting economic interpretation. It indeed provides a unified framework for the theory of price relations in commodity futures markets, whereas in the literature this analysis is usually split into two strands: the storage theory and the normal backwardation theory (also named the hedging pressing theory after De Roon et al. (2000)). The first focuses on the cost of storage of the underlying asset, the second on the risk premium. Although they are complementary, to the best of our knowledge these two strands remained apart up to now.

With our model, we characterize four possible regimes in equilibrium, given the non-negativity constraints on physical positions and on prices. While each of these four regimes is simple to understand on economical grounds, we believe that our model is the first to allow them and to give explicit conditions on the fundamentals of the economy determining which one will actually prevail in equilibrium. In each of the regimes, we also give explicit formulas for the equilibrium prices. This enables us to characterize regimes in detail and to perform complete and novel comparative statics. For instance, as is done in the storage theory, we can explain why there is a contango (in such a case, the "current basis", defined as the difference between the futures price and the *current* spot price, is positive) or a backwardation (the current basis is negative) on the futures market. Towards this analysis, we give insights into the question of the informational content of the futures price and the price discovery function of futures markets. As done in the normal backwardation theory, we can also compare the futures price with the expected spot price and ask whether or not there is a bias in the futures price (we define the "expected basis" as the difference between the futures price and the *expected* spot price). The sign and the level of the bias depend directly on which regime prevails. In the third one, for instance, there is no bias whereas in the first regime, there are two sub-regimes, one where the futures price is higher than the expected spot price, and one where it is lower. Here, the model depicts the way futures markets are used to reallocate risk between operators, the price to pay for such a transfer, and thus provides insights into the main economic function of derivative markets: hedging [1].

On a quantitative point of view, our model allows for an important number of comparative statics. We show that when the number of speculators increases, for example because access to the futures market is relaxed, the volatility of spot prices at date 2 goes up. This effect may sound inefficient. Our interpretation is that speculation increases the informativeness of prices: volatility brings more efficiency. The mechanism is quite simple. As the number of speculators increases, the cost of hedging decreases and demand for futures grows along with physical positions. Smaller hedging costs make storers and processors amplify the differences in their positions in response to different pieces of information, implying that their market impact increases. This increases in turn the volatility of prices.

Normatively, we use our model to perform a welfare analysis. This question, again, is as old as derivatives markets. Newbery (2008) summarizes well the usual yet dual appreciation of the impact of derivatives markets on welfare. The author makes a difference between what he calls the "layman" and "the body of informed opinion". He explains that to the first, "the association of speculative activity with volatile markets is often taken as proof that speculators are the cause of the instability", whereas to the second, "volatility creates a demand for hedging or insurance". Our model also exhibits a dual conclusion about welfare, but it is differently stated. First, the model allows for a clear separation between the utility of speculation and that of hedging. Then, the analysis of the impact of an increasing number of speculators shows that, storers and processors, as far as their hedging activities are concerned, have opposite views on the desirability of speculators. The latter are worthless when the positions of storers and processors match exactly; but when one type of agents has needs higher than what the other type can supply, then the former wants more (the latter wants less) speculators because this reduces his costs of hedging. To the best of our knowledge, such an effect has never been investigated before.

Short literature review Of course, the questions we have raised have been investigated before. Contrary to what is done in this paper, the literature on commodity prices however separates the question of the links between the spot and the futures prices and that of the bias in the futures price. The latter has been investigated first by Keynes (1930) through the theory of normal backwardation whereas the former is usually associated to

[1] It is worth noticing that our model operates even without any risk-aversion at all: if we assume that all operators (or even a single one) are risk-neutral, then our model is still valid and gives the four regimes described earlier.

the theory of storage, initiated by Kaldor (1940), Brennan (1958) and Working (1949). The same is true for the equilibrium models developed so far.

An important number of equilibrium models of commodity prices focuses on the bias in the futures price and the risk transfer function of the derivative market. This is the case, for example, of Anderson and Danthine (1983a), Anderson and Danthine (1983b), Hirshleifer (1988), Hirshleifer (1989), Guesnerie and Rochet (1993), and Acharya et al. (2013). Anderson and Danthine (1983a) is an important source.

Compared with this work, our model is more simple (the producers are not directly modeled) and completely specified. This gives us the possibility to obtain explicit formulas for the equilibrium prices and to investigate further economics issues, like welfare for example. The models developed by Hirshleifer (1988) and Hirshleifer (1989) are also inspired by Anderson and Danthine (1983a). In these papers, Hirsleifer analyzes two points which are interesting for our model but that we did not take the time to develop: first, the simultaneous existence of futures and forward markets; second, the role of the spot traders. Hirshleifer (1989) also asks whether or not vertical integration and futures trading can be substitute means of diversifying risk. We focus instead, in the comparative statics, on the impact by type of agent, with a rich variety of cases.

Let us also mention that, contrary to Anderson and Danthine (1983b), Hirshleifer (1989) and Routledge et al. (2000), we do not undertake an inter-temporal analysis in the present version of the model. Anderson and Danthine (1983b) is the "inter-temporal" extension of Anderson and Danthine (1983a): they allow the futures position to be revised once within the cash market holding period. To obtain results while keeping tractable equations, the authors however must simplify their model so that only one category of hedger remains in the new version. When equilibrium analysis stands at the heart of all concerns (which is our case), this is a strong limitation. Routledge et al. (2000) give another interesting example of inter-temporal analysis. It is however not adapted to normative analysis.

Beyond the question of the risk premium, equilibrium models have also been used in order to examine the possible destabilizing effect of the presence of a futures market and to analyze welfare issues. This is the case of Guesnerie and Rochet (1993), Newbery (1987), and Baker and Routledge (2012). As the model proposed by Guesnerie and Rochet (1993) is devoted to the analysis of mental ("eductive") coordination strategies, it is more stripped down than ours. As in Newbery (1987), our explicit formulas for equilibrium prices allows for interesting comparisons depending on the presence or absence of a futures market. Finally, contrary to Baker and Routledge (2012), we are not interested in Pareto optimal risk allocations. We focus instead on utilities *per head*.

Another strand of the literature on equilibrium models focuses on the *current* spot price and the role of inventories in the behavior of commodity prices, as in Deaton and Laroque (1992), and in Chambers and Bailey (1996). In these models, however, there is no futures market; there is in fact a single type of representative agent, which prevents examining risk allocation and the political economy of structural change.

Apart from the specific behavior of prices, the non-negativity constraint on inventories raises another issue. Empirical facts indeed testify that there is more than a non-negativity constraint in commodity markets: the level of inventories never falls to zero, leaving thus unexploited some supposedly profitable arbitrage opportunities. The concept of a convenience yield associated with inventories, initially developed by Kaldor (1940) and Brennan (1958) is generally used to explain such a phenomenon, which has been regularly confirmed, on an empirical point of view, since Working (1949) [2]. In their model, Routledge et al. (2000) introduce a convenience yield in the form of an embedded timing option associated with physical stocks. Contrary to these authors, we do not take into account the presence of a convenience yield in our analysis. While this would probably constitute an interesting improvement of our work, it is hardly compatible with a two-period model.

Recent attempts to test equilibrium models must also be mentioned, as they are rare. While not totally operable in our context, the tests undertaken by Acharya et al. (2013) could be used as in fruitful source of inspiration for further developments of our model. As far the analysis of the risk premium is concerned, the empirical tests performed by Hamilton and Wu (2012) and Szymanowska et al. (2013), as well as the simulations proposed by Bessembinder and Lemmon (2002) are other possible directions.

2 The model

This is a two-period model. There is one commodity, a numéraire, and two markets: the spot market at times $t = 1$ and $t = 2$, and a futures market, which is open at $t = 1$ and such that contracts are settled at time $t = 2$. It is important to note that short positions are allowed on the futures market. When an agent sells (resp. buys) futures contracts, his position is short (resp. long), and the amount f he holds is negative (resp. positive). On the spot market, such positions are not allowed: you can't sell what you don't hold. In other words, the futures is a financial market, while the spot is a physical market.

There are three types of traders.

- *Industrial users*, or processors, who use the commodity to produce other goods which they sell to consumers. Because of the inertia of their own production process, and/or because all their production is sold forward, they decide at $t = 1$ how much to produce at $t = 2$. They cannot store the commodity, so they have to buy all of their input on the spot market at $t = 2$. They also trade on the futures market.
- *Inventory holders*, who have storage capacity, and who can use it to buy the commodity at $t = 1$ and release it at $t = 2$. They trade on the spot market at $t = 1$, where they buy, and at $t = 2$, where they sell. They also operate on the futures market.
- *Money managers*, or speculators, who use the commodity price as a source of risk, to make a profit on the basis of their positions in futures contracts. They do not trade on the spot market.

In addition, we think of these markets as operating in a partial equilibrium framework: in the background, there are other users of the commodity, and producers as well. These additional agents will be referred to as *spot traders*, and their global effect will be described by a demand function. At time $t = 1$, the demand is $\mu_1 - mP_1$, and it is $\tilde{\mu}_2 - m\tilde{P}_2$ at time $t = 2$. P_t is the spot price at time t and the demand can be either positive or negative; the sign $\sim$ indicates a random variable.

All decisions are taken at time $t = 1$, conditional on the information available for $t = 2$. The timing is as follows:

[2] For a recent and exhaustive study on this question, see for example Symeonidis et al. (2012)

- for $t = 1$, the commodity is in total supply ω_1, the spot market and the futures market open. On the spot market, there are spot traders and storers on the demand side, the price is P_1. On the futures markets, the processors, the storers and the speculators all initiate a position, and the price is P_F. Note that the storers have to decide simultaneously how much to buy on the spot market and what position to take on the futures market.

- for $t = 2$, the commodity is in total supply $\tilde{\omega}_2$, to which one has to add the inventory which the storers carry from $t = 1$, and the spot market opens. The processors and the spot traders are on the demand side, and the price is $\tilde{P}_2$. The futures contracts are then settled at that price, meaning that every contract brings a financial result of $\tilde{P}_2 - P_F$.

There are N_S speculators, N_P processors, N_I storage companies (I for inventories). We assume that all agents (except the spot traders) are risk averse inter-temporal utility maximizers. To take their decisions at time $t = 1$, they need to know the distribution of the spot price $\tilde{P}_2$ at $t = 2$. We will show that, under mean-variance specifications of the utilities, there is a unique price system $(P_1, P_F, \tilde{P}_2)$ such that all three markets clear.

Uncertainty is modeled by a probability space $(\Omega, \mathcal{A}, P)$. Both $\tilde{\omega}_2$, $\tilde{\mu}_2$ and $\tilde{P}_2$ are random variables on $(\Omega, \mathcal{A}, P)$. At time $t = 1$, their realizations are unknown, but their distributions are common knowledge.

Before we proceed, some clarifications are in order.

- Production of the commodity is inelastic: the quantities ω_1 and $\tilde{\omega}_2$ which reach the spot markets at times $t = 1$ and $t = 2$ are exogenous to the model. Traders know ω_1 and μ_1, and share the same priors as to $\tilde{\omega}_2$ and $\tilde{\mu}_2$.

- This said, a negative spot demand can be understood as extra spot supply: if for instance $P_1 > \mu_1/m$, then the spot price at time $t = 1$ is so high that additional means of production become profitable, and the global economy provides additional quantities to the spot market. The coefficient m is the elasticity of demand (or production) with respect to prices. The number μ_1 (demand when $P_1 = 0$) is the level at which the economy saturates: to induce spot traders to demand quantities larger than μ_1, one would have to pay them, that is, offer negative price $P_1 < 0$ for the commodity. The same remark applies to time $t = 2$.

- We separate the roles of the industrial user and the inventory holder, whereas in reality industrial users may also hold inventory. It will be apparent in the sequel that this separation need not be as strict, and that the model would accommodate agents of mixed type. In all cases, agents who trade on the physical markets would also trade on the financial market for two separate purposes: hedging their risk, and making additional profits. In the sequel, we will see how their positions reflect this dual purpose.

- Note also that the speculators would typically use their position on the futures market as part of a diversified portfolio; our model does not take this into account.

- We also suppose that there is a perfect convergence of the basis at the expiration of the futures contract. Thus, at time $t = 2$, the position on the futures markets is settled at the price $\tilde{P}_2$ then prevailing on the spot market.

- For the sake of simplicity, we set the risk-free interest rate to 0.

In what follows, as we examine an REE (rational expectation equilibrium), we look at two necessary conditions for such an equilibrium to appear: the maximization of the agent's utility, conditionally on their price expectations, and market clearing.

3 Optimal positions and market clearing

3.1 Profit maximization

All agents have mean-variance utilities. For all of them, a profit $\tilde{\pi}$ brings utility:

$$E[\tilde{\pi}] - \frac{1}{2}\alpha_i \mathrm{Var}[\tilde{\pi}] \qquad (1)$$

where α_i is the risk aversion parameter of a type i individual.

Beside their mathematical tractability, there are good economic reasons for using mean-variance utilities. They are not of von Neumann - Morgenstern type, i.e formula (1) cannot be put in the form $E\left[u\left(\tilde{X}\right)\right]$ for a suitable function u, so they are poorly suited to model the behaviour of individuals under uncertainty. However, they are well suited to describe the behaviour of firms operating under risk constraints. The capital asset-pricing model (CAPM) in finance, for instance, consists of maximizing $E\left[\tilde{R}\right]$ under the constraint $\mathrm{Var}\left[\tilde{R}\right] \leq \rho$, where $\tilde{R}$ is the return on the portfolio, which is equivalent to maximizing $E\left[\tilde{R}\right] - \lambda \mathrm{Var}\left[\tilde{R}\right]$, where λ is the Lagrange multiplier. In financial markets, as in commodities markets, agents are mostly firms, not individuals, and they have risk constraints imposed on them from inside (managers controlling traders) and from outside (regulators controlling the firm). This is what formula (1) is trying to capture. For the sake of simplicity, we have kept the variance as a measure of risk, but we expect that our results could be extended to more sophisticated ones (coherent risk measures), at the cost of some mathematical complications.

Speculator For the speculator, the profit resulting from a position in the futures market f_S is the r.v.:

$$\tilde{\pi}_S(f_S) = f_S\,(\tilde{P}_2 - P_F),$$

and the optimal position is:

$$f_S^\star = \frac{\mathrm{E}[\tilde{P}_2] - P_F}{\alpha_S \mathrm{Var}[\tilde{P}_2]}. \qquad (2)$$

This position is purely speculative. It depends mainly on the level and on the sign of the bias in the futures price. The speculator goes long whenever he thinks that the expected spot price is higher than the futures price. Otherwise he goes short. Finally, he is all the more inclined to take a position as his risk aversion and volatility of the underlying asset are low.

Storer The storer can hold any non-negative inventory. However, storage is costly: holding a quantity x between $t = 1$ and $t = 2$ costs $\frac{1}{2}Cx^2$. Parameters C (cost of storage) and α_I (risk aversion) characterize the storer. He has to decide how much inventory to buy at $t = 1$, if any, and what position to take in the futures market, if any.

If he buys $x \geq 0$ on the spot market at $t = 1$, resells it on the spot market at $t = 2$, and takes a position f_I on the futures market, the resulting profit is the r.v.:

$$\tilde{\pi}_I(x, f_I) = x\,(\tilde{P}_2 - P_1) + f_I\,(\tilde{P}_2 - P_F) - \frac{1}{2}Cx^2.$$

The optimal position on the physical market is:

$$x^\star = \frac{1}{C}\max\{P_F - P_1, 0\}. \tag{3}$$

The storer holds inventories if the futures price is higher than the current spot price. This position is the only one, in the model, that directly links the spot and the futures prices. This is consistent with the theory of storage and, more precisely, its analysis of contango and the informational role of futures prices.

The optimal position on the futures market is:

$$f_I^\star = \frac{\mathrm{E}[\tilde{P}_2] - P_F}{\alpha_I \mathrm{Var}[\tilde{P}_2]} - x^\star. \tag{4}$$

This position can be decomposed into two elements. First, a negative position $-x^*$, which simply hedges the physical position: the storer sells futures contracts in order to protect himself against a decrease in the spot price. Second, a speculative position, structurally identical to that of the speculator, which reflects the storer's risk aversion and his expectations about the relative level of the futures and the expected spot prices.

Processor The processor decides at time $t = 1$ how much input y to buy at $t = 2$, and which position f_P to take on the futures market. The revenue from sales at date $t = 2$ is $(y - \frac{\beta}{2}y^2)\,P$, where P is our convention for the forward price of the output, and the other factor reflects decreasing marginal revenue. Due to these forward sales of the production, this revenue is known at time $t = 1$. The resulting profit is the r.v.:

$$\tilde{\pi}_P(y, f_P) = \left(y - \frac{\beta}{2}y^2\right) P - y\tilde{P}_2 + f_P(\tilde{P}_2 - P_F).$$

An easy computation then gives his optimal decisions, namely:

$$y^\star = \frac{1}{\beta P}\max\{P - P_F, 0\}, \tag{5}$$

$$f_P^\star = \frac{\mathrm{E}[\tilde{P}_2] - P_F}{\alpha_P \mathrm{Var}[\tilde{P}_2]} + y^\star. \tag{6}$$

The futures market is also used by the processor to plan his production, all the more so if the price of his input P_F is below that of his output P. The position on the futures market, again, can be decomposed into two elements. First, a positive position y^*, which hedges the position on the physical market: the processor goes long on futures contracts in order to protect himself against an increase in the spot price. Then, a speculative position reflecting the processor's risk aversion and his expectations about the level of the expected basis.

Remarks on optimal positions In this framework, all agents have the possibility to undertake speculative operations. After having hedged 100 percent of their physical positions, they adjust this position according to their expectations. The separation of the physical and the futures decisions was derived by Danthine (1978). As shown by Anderson and Danthine (1983a), this property does not hold if the final good price is stochastic, unless a second futures market for the final good is introduced. As we shall see, this separation result is very convenient for equilibrium analysis. This is one of the reasons why we choose, for the processor, not to introduce uncertainty on the output price and/or on the quantities produced.

3.2 Market clearing

Although we assume that all individuals are identical in each category of agents, more subtle assumptions could be retained without much complication. For example, remark that if the storers had different technologies, say, storer i with $i = 1, \ldots, N_I$ had technology C_i, then, instead of $\frac{N_I}{C}\max\{P_F - P_1, 0\}$, total inventories would be $(\sum_i 1/C_i)\max\{P_F - P_1, 0\}$. In other words, storers are easily aggregated. In the following, when relevant, we shall use the index n_I representing a synthetic number of storage units, and per-unit inventories X^* defined by:

$$n_I = \begin{cases} N_I/C & \text{if storers are identical,} \\ \sum_i 1/C_i & \text{otherwise,} \end{cases}$$
$$X^\star = \max\{P_F - P_1, 0\}.$$

Similarly, if processors had different technologies, say, processor i with $i = 1, \ldots, N_P$ had technology β_i, then total input demand would be $\sum_i 1/(\beta_i P) \cdot \max\{P - P_F, 0\}$ instead of $\frac{N_P}{\beta P}\max\{P - P_F, 0\}$. Thus, when relevant, we shall use the index n_P representing a synthetic number of processing units, and per-unit demand Y^* defined by:

$$n_P = \begin{cases} \frac{N_P}{\beta P} & \text{if processors are identical,} \\ \frac{1}{P}\sum_i \frac{1}{\beta_i} & \text{otherwise,} \end{cases}$$
$$Y^\star = \max\{P - P_F, 0\}.$$

The spot market at time 1 On the supply side we have the harvest ω_1. On the other side we have the inventory $n_I X^\star$ bought by the storers, and the demand of the spot traders. Market clearing requires:

$$\omega_1 = n_I X^\star + \mu_1 - mP_1,$$

hence:

$$P_1 = \frac{1}{m}\left(\mu_1 - \omega_1 + n_I X^\star\right). \tag{7}$$

The spot market at time 2 We have, on the supply side, the harvest $\tilde{\omega}_2$, and the inventory $n_I X^\star$ sold by the storers; on the other side, the input $n_P Y^\star$ bought by the processors and the demand of the spot traders. The market clearing condition is:

$$\tilde{\omega}_2 + n_I X^\star = n_P Y^\star + \tilde{\mu}_2 - m\tilde{P}_2,$$

with $X^\star$ and $Y^\star$ as above. We get:

$$\tilde{P}_2 = \frac{1}{m}\left(\tilde{\mu}_2 - \tilde{\omega}_2 - n_I X^\star + n_P Y^\star\right). \tag{8}$$

The futures market Market clearing requires:

$$N_S f_S^\star + N_P f_P^\star + N_I f_I^\star = 0.$$

Replacing the $f_i^\star$ by their values, we get:

$$\mathrm{E}[\tilde{P}_2] - P_F = \frac{\mathrm{Var}[\tilde{P}_2]}{\frac{N_P}{\alpha_P} + \frac{N_I}{\alpha_I} + \frac{N_S}{\alpha_S}}\left(n_I X^\star - n_P Y^\star\right). \tag{9}$$

Remark that if, say, different storers had different risk aversions α_{Ij} (for $j = 1, \ldots, N_I$), then we would see $\sum_j 1/\alpha_{Ij}$ instead of N_I/α_I in equation (9). This is an illustration of a more general fact: we sum up the inverse of the risk aversions of all agents to represent the inverse of the overall (or market) risk aversion.

Equation (9) gives a formal expression for the bias in the futures price, which confirms the findings of Anderson and Danthine (1983a). It shows indeed that the bias depends primarily on fundamental economic structures (the characteristics of the storage and production functions, which are embedded in $X^\star$ and $Y^\star$) and the number of operators, secondarily on subjective parameters (the risk aversion of the operators), and thirdly on the volatility of the underlying asset. Note also that the sign of the bias depends only on the sign of $(n_I X^\star - n_P Y^\star)$. As the risk aversion of the operators only influences the speculative part of the futures position, it does not impact this sign. Finally, when $n_I X^\star = n_P Y^\star$, there is no bias in the futures price, and the risk transfer function is entirely undertaken by the hedgers, provided that their positions on the futures market are the exact opposite of each others. Thus the absence of bias is not exclusively the consequence of risk neutrality but may have other structural causes.

4 Existence and uniqueness of the equilibrium

The equations characterizing the equilibrium are the optimal choices on the physical market (equations (3) and (5)), the clearing of the spot market at dates 1 and 2 (equations (7) and (8)), as well as the clearing of the futures market (9):

$$\left\{\begin{array}{lcll} X^\star & = & \max\{P_F - P_1, 0\} & (3) \\ Y^\star & = & \max\{P - P_F, 0\} & (5) \\ P_1 & = & \frac{1}{m}\left(\mu_1 - \omega_1 + n_I X^\star\right) & (7) \\ \tilde{P}_2 & = & \frac{1}{m}\left(\tilde{\mu}_2 - \tilde{\omega}_2 - n_I X^\star + n_P Y^\star\right) & (8) \\ P_F & = & \mathrm{E}[\tilde{P}_2] + \frac{\mathrm{Var}[\tilde{P}_2]}{\frac{N_P}{\alpha_P} + \frac{N_I}{\alpha_I} + \frac{N_S}{\alpha_S}}\left(n_P Y^\star - n_I X^\star\right) & (9) \end{array}\right.$$

Let us also remind that the distribution of $\tilde{\mu}_2 - \tilde{\omega}_2$ is common knowledge. We introduce the following notations:

$$\xi_1 := \mu_1 - \omega_1,$$

$$\tilde{\xi}_2 := \tilde{\mu}_2 - \tilde{\omega}_2,$$
$$\xi_2 := \mathrm{E}[\tilde{\mu}_2 - \tilde{\omega}_2],$$
$$\rho := 1 + \frac{1}{m} \frac{\mathrm{Var}[\tilde{\mu}_2 - \tilde{\omega}_2]}{\frac{N_P}{\alpha_P} + \frac{N_I}{\alpha_I} + \frac{N_S}{\alpha_S}}$$

where m is the elasticity of demand.

From (8), we can derive useful moments:

$$\mathrm{E}[\tilde{P}_2] = \frac{1}{m}\left(\xi_2 - n_I X^\star + n_P Y^\star\right), \tag{8E}$$
$$\mathrm{Var}[\tilde{P}_2] = \frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2}. \tag{8V}$$

We assume $\mathrm{Var}[\tilde{\xi}_2] > 0$, so there is uncertainty on the future availability of the commodity. It is the only source of uncertainty in the model. Likewise, we assume (for the time being) that α_P, α_I and α_S all are non-zero numbers. These restrictions will be lifted later on.

4.1 Definitions

Definition 1. An *equilibrium* is a family $\left(X^\star, Y^\star, P_1, P_F, \tilde{P}_2\right)$ such that all prices are non-negative, processors, storers and speculators act as price-takers, and all markets clear.

Technically speaking, $\left(X^\star, Y^\star, P_1, P_F, \tilde{P}_2\right)$ is an equilibrium if equations (3), (5), (7), (8), and (9) are satisfied, with $X^* \geq 0$, $Y^* \geq 0$, $P_1 \geq 0$, $P_F \geq 0$ and $\tilde{P}_2(\omega) \geq 0$ for all $\omega \in \Omega$. Note that the latter condition depends on the realization of the random variable $\tilde{P}_2$, which can be observed only at $t = 2$, while the first four can be checked at time $t = 1$. This leads us to the following:

Definition 2. A *quasi-equilibrium* is a family $\left(X^\star, Y^\star, P_1, P_F, \tilde{P}_2\right)$ such that all prices except possibly $\tilde{P}_2$ are non-negative, processors, storers and speculators act as price-takers and all markets clear.

Technically speaking, a quasi-equilibrium is a family $\left(X^\star, Y^\star, P_1, P_F, \tilde{P}_2\right) \in \mathbb{R}_+^4 \times L^0(\Omega, \mathcal{A}, P)$ such that equations (3), (5), (7), (8) and (9) are satisfied.

We now give two existence and uniqueness results, the first one for quasi-equilibria and the second one for equilibria.

4.2 Quasi-equilibrium

Theorem 1. *There is a quasi-equilibrium if and only if* (ξ_1, ξ_2) *belongs to the region:*

$$\xi_2 \geq -\rho n_P P \qquad \text{if } \xi_1 \geq 0, \tag{10}$$
$$\xi_2 \geq -\rho n_P P - \left((m + \rho n_P)/n_I + \rho\right)\xi_1 \qquad \text{if } -n_I P \leq \xi_1 \leq 0, \tag{11}$$
$$\xi_2 \geq -(m/n_I + \rho)\xi_1 \qquad \text{if } \xi_1 \leq -n_I P, \tag{12}$$

and then it is unique.

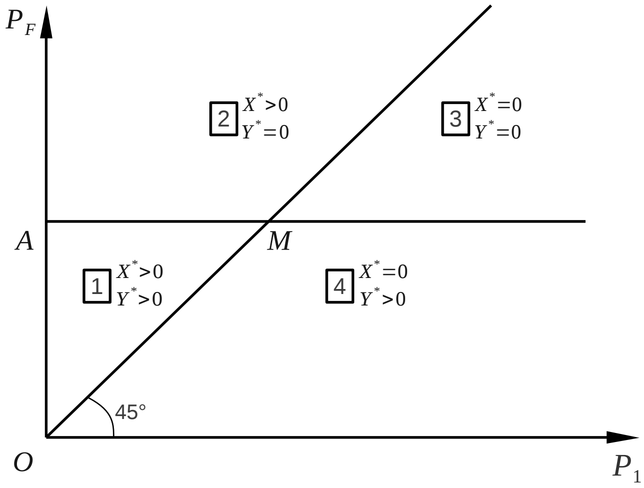

Figure 1: Phase diagram of physical and financial decisions in space (P_1, P_F).

Proof. To prove this theorem, we begin by substituting equation (8E) in equation (9). We get:

$$mP_F - \rho\left(n_P Y^\star - n_I X^\star\right) = \xi_2. \tag{13}$$

We now have two equations, (7) and (13) for P_1 and P_F. Replacing $X^\star$ and $Y^\star$ by their values, given by (3) and (5), we get a system of two nonlinear equations in two variables:

$$mP_1 - n_I \max\{P_F - P_1, 0\} = \xi_1, \tag{14}$$

$$mP_F + \rho\left(n_I \max\{P_F - P_1, 0\} - n_P \max\{P - P_F, 0\}\right) = \xi_2. \tag{15}$$

Remark that if we can solve this system with $P_1 > 0$ and $P_F > 0$, we get $\tilde{P}_2$ from (8). So the problem is reduced to solving (15) and (14). Consider the mapping $F : \mathbb{R}^2_+ \to \mathbb{R}^2$ defined by:

$$F(P_1, P_F) = \begin{pmatrix} mP_1 - n_I \max\{P_F - P_1, 0\} \\ mP_F + \rho\left(n_I \max\{P_F - P_1, 0\} - n_P \max\{P - P_F, 0\}\right) \end{pmatrix}.$$

In $\mathbb{R}^2_+$, take P_1 as the horizontal coordinate and P_F as the vertical one, as depicted by Figure 1. There are four regions, separated by the straight lines $P_F = P_1$ and $P_F = P$:

- Region 1, where $P_F > P_1$ and $P_F < P$. In this region, both $X^\star$ and $Y^\star$ are positive.
- Region 2, where $P_F > P_1$ and $P_F > P$. In this region, $X^\star > 0$ and $Y^\star = 0$.
- Region 3, where $P_F < P_1$ and $P_F > P$. In this region, $X^\star = 0$ and $Y^\star = 0$.
- Region 4, where $P_F < P_1$ and $P_F < P$. In this region, $X^\star = 0$ and $Y^\star > 0$

Moreover, in the regions where $X^\star > 0$, we have $X^\star = P_F - P_1$ and in the regions where $Y^\star > 0$, we have $Y^\star = P - P_F$. So, in each region, the mapping is linear, and it is obviously continuous across the boundaries. Denote by O the origin in $\mathbb{R}^2_+$, by A the point $P_1 = 0$, $P_F = P$, and by M the point $P_1 = P_F = P$ (so, for instance, region 1 is the triangle OAM). In region 1, we have:

$$F(P_1, P_F) = \begin{pmatrix} mP_1 - n_I\,(P_F - P_1) \\ mP_F + \rho\,(n_I\,(P_F - P_1) - n_P\,(P - P_F)) \end{pmatrix}.$$

The images $F(O)$, $F(A)$, and $F(M)$ are easily computed:

$$\begin{aligned} F(O) &= (0, -\rho n_P P), \\ F(A) &= P\,(-n_I, m + \rho n_I), \\ F(M) &= m\,P\,(1, 1). \end{aligned}$$

From this, one can find the images of all four regions (see Figure 2). The image of region 1 is the triangle $F(O)F(A)F(M)$. The image of region 2 is bounded by the segment $F(A)F(M)$ and by two infinite half-lines, one of which is the image of $\{P_1 = 0, P_F \geq P\}$, the other being the image of $\{P_1 = P_F, P_F \geq P\}$. In region 2, we have:

$$F(P_1, P_F) = \begin{pmatrix} mP_1 - n_I\,(P_F - P_1) \\ mP_F + \rho n_I\,(P_F - P_1) \end{pmatrix}.$$

The first half-line emanates from $F(A)$ and is carried by the vector $(-n_I,\, m + \rho n_I)$. The second half-lines emanates from $F(M)$ and is carried by the vector $(1, 1)$. Both of them (if extended in the negative direction) go through the origin. The image of region 4 is bounded by the segment $F(O)F(M)$ and by two infinite half-lines, one of which is the image of $\{P_F = 0\}$, the other being the image of $\{P_1 \geq P, P_F = P\}$. In region 4, we have:

$$F(P_1, P_F) = \begin{pmatrix} mP_1 \\ mP_F - \rho n_P\,(P - P_F) \end{pmatrix},$$

so the first half-line emanates from $F(O)$ and is horizontal, with vertical coordinate $-\rho n_P P$, and the second emanates from $F(M)$ and is horizontal. The image of region 3 is entirely contained in $\mathbb{R}^2_+$, where it is the remainder of the three images we described. To prove the theorem, we have to show that the system (15) and (14) has a unique solution. It can be rewritten as:

$$F(P_1, P_F) = \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix},$$

and it has a unique solution if and only if the right-hand side belongs to the image of F, which we have just described. This leads to the conclusion of the proof: based on the previous remark summarized in Figure 2, we easily find the expressions of the theorem. □

4.3 Equilibrium

To get an equilibrium instead of a quasi-equilibrium, we need the further condition, calculated last, $\tilde{P}_2 \geq 0$. By equation (8), this is equivalent to:

$$\inf\{\tilde{\mu}_2 - \tilde{\omega}_2\} \geq n_I X^\star - n_P Y^\star. \tag{16}$$

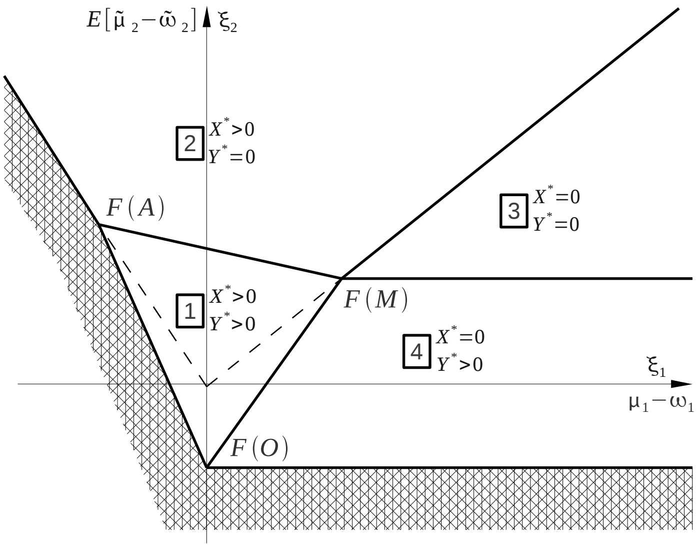

Figure 2: Phase diagram of physical and financial decisions.

This amount to

$$P_F \leq \frac{\inf\{\tilde{\mu}_2 - \tilde{\omega}_2\} + n_I P_1 + n_P P}{n_I + n_P} \quad \text{in region 1,} \tag{17}$$

$$P_F \leq P_1 + \frac{\inf\{\tilde{\mu}_2 - \tilde{\omega}_2\}}{n_I} \quad \text{in region 2,} \tag{18}$$

$$0 \leq \inf\{\tilde{\mu}_2 - \tilde{\omega}_2\} \quad \text{in region 3,} \tag{19}$$

$$P_F \leq P + \frac{\inf\{\tilde{\mu}_2 - \tilde{\omega}_2\}}{n_P} \quad \text{in region 4.} \tag{20}$$

Theorem 2. *Let* (ξ_1, ξ_2) *belong to the region (10), (11), (12), so there exists a unique quasi-equilibrium. It is an equilibrium if and only if* $\tilde{\mu}_2 - \tilde{\omega}_2$ *satisfies an additional condition, namely:*

$$\inf\{\tilde{\mu}_2 - \tilde{\omega}_2\} \geq \frac{n_P(m+n_I)(\xi_2 - mP) + mn_I(\xi_2 - \xi_1)}{n_P(m+n_I)\rho + m(m+(1+\rho)n_I)} \text{ in region 1;}$$

$$\inf\{\tilde{\mu}_2 - \tilde{\omega}_2\} \geq \frac{n_I(\xi_2 - \xi_1)}{m + (1+\rho)n_I} \text{ in region 2;}$$

$$\inf\{\tilde{\mu}_2 - \tilde{\omega}_2\} \geq 0 \text{ in region 3;}$$

$$\inf\{\tilde{\mu}_2 - \tilde{\omega}_2\} \geq -\frac{mn_P(P - \frac{\xi_2}{m})}{m + \rho n_P} \text{ in region 4.}$$

Proof. The proof for region 1 comes from applying F on equation (17). For region 2, a direct application of F shows that equation (18) implies

$$\xi_2 - \xi_1 \leq \frac{m + (1+\rho)n_I}{n_I} \inf\{\tilde{\mu}_2 - \tilde{\omega}_2\},$$

which must be read directly as a restriction on $\inf\{\tilde{\mu}_2 - \tilde{\omega}_2\}$ given ξ_2. For region 3, the theorem is directly derived from equation (16), since $X^\star = 0$ and $Y^\star = 0$. For region 4, a direct application of F shows that equation (20) gives the condition. Note that $\inf\{\tilde{\mu}_2 - \tilde{\omega}_2\} \geq 0$ is a sufficient condition for an equilibrium to exist in region 4. □

Remark that the condition for region 1 is general in the following sense. Take $n_P = 0$, you get the condition for region 2; take $n_I = 0$, you get the condition for region 4; take now $n_I = n_P = 0$, you get the condition for region 3. This simple shortcut works for other analytical results. The explicit expressions of the prices, for each region, in given in Appendix.

5 Equilibrium analysis

In this section we analyze the equilibrium in two steps. Firstly, we examine the four regimes depicted in Figure 1. They correspond to very different types of decisions undertaken in the physical and the financial markets. Secondly, we turn to Figure 2 and enrich the discussion with the analysis of the net scarcity of the commodity, both immediate and expected.

5.1 Prices, physical and financial positions

A first general comment on Figure 1 is that in Regimes 1 and 2 where $X^\star > 0$, the futures market is in contango: $P_F > P_1$. Inventories are positive and they can be used for intertemporal arbitrages. In Regimes 3 and 4, there is no inventory ($X^\star = 0$) and the market is in backwardation: $P_F < P_1$. These configurations are fully consistent with the theory of storage. The other meaningful comparison concerns P_F and $E[\tilde{P}_2]$. From Equation (9), we know that $n_I X^\star - n_P Y^\star$ gives the sign and magnitude of $E[\tilde{P}_2] - P_F$, i.e. the way risk is transferred between the operators on the futures market.

The analysis of the four possible regimes, with a focus on Regime 1 (it is the only one where all operators are active and it gathers two important subcases), enables us to unfold the reasons for the classical conjecture: backwardation on the expected basis, i.e. $P_F < E[\tilde{P}_2]$. More interestingly, we show why the reverse inequality is also plausible, as mentioned by several empirical studies.[3]

The equation $n_I X^\star - n_P Y^\star = 0$ cuts Regime 1 into two parts, 1U and 1L. It passes through M as can be seen in Figure 3. This frontier can be rewritten as:

$$n_I(P_F - P_1) - n_P(P - P_F) = 0. \tag{21}$$

- Along the line Δ, there is no bias in the futures price, and the risk remains entirely in the hands of the hedgers (storers and producers have perfectly matching positions).
- Above Δ, $n_I X^\star > n_P Y^\star$ and $P_F < E[\tilde{P}_2]$. This concerns the upper part of Regime 1 (Regime 1U) and Regime 2.
- Below Δ, $n_I X^\star < n_P Y^\star$ and $P_F > E[\tilde{P}_2]$. This concerns the lower part of Regime 1 (Regime 1L) and Regime 4.

When $n_I X^\star > n_P Y^\star$, the net hedging position is short and speculators in long position are indispensable to the clearing of the futures market. In order to induce their participation,

[3] For extensive analyses of the bias in a large number of commodity markets, see for example Fama and French (1987), Kat and Oomen (2007) and Gorton et al. (2013).

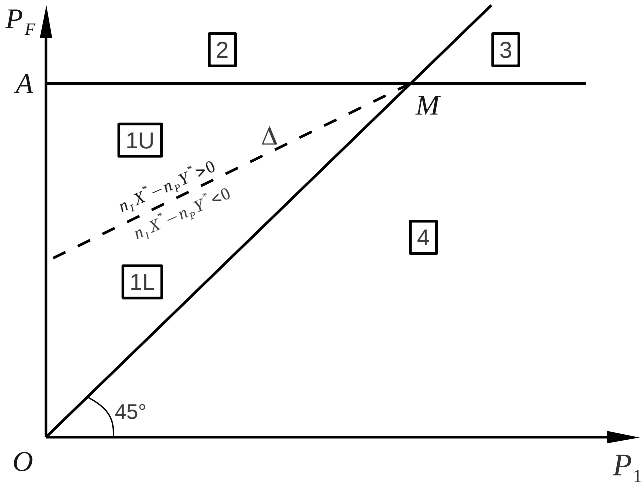

Figure 3: Phase diagram of physical and financial decisions in space (P_1, P_F) (zoom on Regime 1).

there must be a profitable bias between the futures price and the expected spot price: the bias $E[\tilde{P}_2] - P_F$ is positive. This backwardation on the expected basis corresponds to the situation depicted by Keynes (1930) as the normal backwardation theory. On the contrary, when $n_I X^\star < n_P Y^\star$, the net hedging position is necessarily long and the speculators must be short. The expected spot price must be lower than the futures price, and the bias $E[\tilde{P}_2] - P_F$ is negative.

Table 1 summarizes for each regime the relationships between the prices and the physical and financial positions. An attentive scrutiny of the table shows that the regimes are very contrasted.

For example, in Regime 2, we have simultaneously a contango on the current basis and a backwardation on the expected basis (or a positive bias). In short, $P_1 < P_F < E[\tilde{P}_2]$. In Regime 3, in the absence of hedging of any sort, the futures market is dormant, and this is no bias on the expected basis. Regime 4 is the opposite of Regime 2: the market is in backwardation and, as $X^\star = 0$, the net hedging position is long, the net speculative position is short and the bias is negative. In short, $P_1 > P_F > E[\tilde{P}_2]$.

5.2 Supply shocks

To exploit usefully Figure 2, one must bear in mind that the horizontal and vertical variables measure scarcity, not abundance: $\xi_1 = \mu_1 - \omega_1$ is the extent to which current production ω_1 fails short of the demand of spot traders, and $\xi_2 = \mathrm{E}[\tilde{\mu}_2 - \tilde{\omega}_2]$ is the (expected) extent to which future production falls short of the demand of spot traders.

Assume that no markets are open before ξ_1 is realized and assume that ξ_1 brings no news (or revision) about ξ_2. We can fix ξ_2, and see what happens on equilibrium variables, depending on ξ_1. To fix ideas suppose that the expected situation at date 2 is a moderate scarcity, situated at $\xi_2 = \overline{\xi}_2$. The level of $\overline{\xi}_2$ is common knowledge for the operators. Take it as drawn in Figure 4. In the case of a low ξ_1 (abundance in period 1), we are in Regime

1U	$P_1 < P_F$	$P_F < E[\tilde{P}_2]$	$P_F < P$
	$X^\star > 0$	$f_S > 0$	$Y^\star > 0$
Δ	$P_1 < P_F$	$P_F = E[\tilde{P}_2]$	$P_F < P$
	$X^\star > 0$	$f_S = 0$	$Y^\star > 0$
1L	$P_1 < P_F$	$P_F > E[\tilde{P}_2]$	$P_F < P$
	$X^\star > 0$	$f_S < 0$	$Y^\star > 0$
2	$P_1 < P_F$	$P_F < E[\tilde{P}_2]$	$P_F > P$
	$X^\star > 0$	$f_S > 0$	$Y^\star = 0$
3	$P_1 > P_F$	$P_F = E[\tilde{P}_2]$	$P_F > P$
	$X^\star = 0$	$f_S = 0$	$Y^\star = 0$
4	$P_1 > P_F$	$P_F > E[\tilde{P}_2]$	$P_F < P$
	$X^\star = 0$	$f_S < 0$	$Y^\star > 0$

Table 1: Relationships between prices, physical and financial positions.

1U. If ξ_1 is bigger, we are in Regime 1L, and if ξ_1 is even bigger, the equilibrium is in Regime 4.

The interpretation is straightforward. If period 1 experiences abundance (Regime 1U), there is massive storage (the current price is low and expected profits are attractive, since a future scarcity is expected). Storers need more hedging than processors, first because inventories are high, second because the expected release of stocks reduces the needs of the processors. Thus, there is a positive bias in the futures price and speculators have a buy position. For a less marked abundance (Regime 1L), storage is more limited. The hedging needs of the storers diminishes while those of the processors increase. So the net hedging position is long, the bias in the futures price becomes negative and the speculators have a sell position. If the commodity is even scarcer (Regime 4), there is no storage, only the processors are active and they hedge their positions.

The combination of the exogenous variables of the model (i.e. current or expected scarcity) with the activities on the physical market makes it possible to create a link between the storage and the normal backwardation theories. For example, it explains why, when there is a contango on the current basis in Regime 1, we can have either an expected backwardation or an expected contango.

6 The impact of speculation on prices, quantities and welfare

To examine the impact of speculators, we study the influence of an increase in their number. Such a phenomenon could be due for example, to the fact that the access to the futures market is relaxed. We first see how such a change impacts prices and quantities; then we perform a welfare analysis.

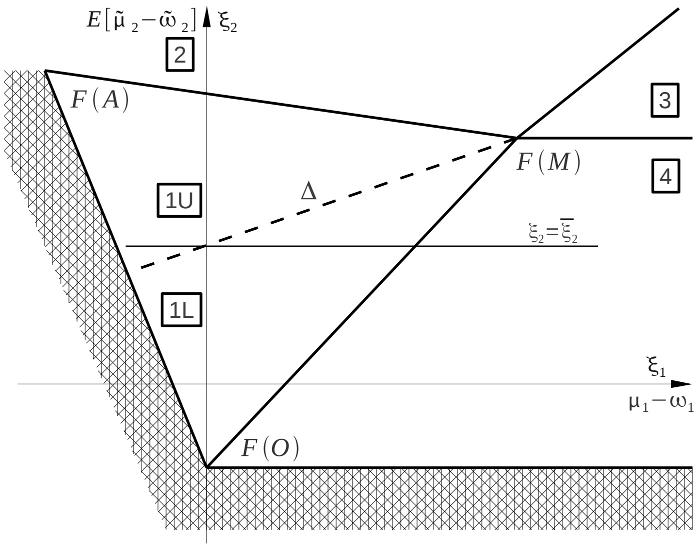

Figure 4: Phase diagram of physical and financial decisions in space (zoom on Regime 1).

6.1 Prices and quantities

It this subsection we show how a change in N_S influences the futures price and its variance, the level and variances of the quantities on the physical market, and the level and variances of spot prices. To do so, the best is to focus first on Regimes 2 and 4, in order to examine simple mechanisms, then discuss Regime 1, in which the previous effects are mixed in interesting ways. The price equations are drawn from the Appendix A.

Up to now, the analysis has assumed that ξ_1 is known when markets open. In this subsection, in order to perform a variance analysis, we consider prices too as random variables at a previous stage: we thus focus on $\text{Var}[\cdot]$ instead of $\text{Var}[\cdot|\ \xi_1]$, as was done implicitly up to now. Only unconditional variances are indeed affected by changes in the number of speculators. Thus both $\tilde{\xi}_1$ and $\tilde{\xi}_2$ are now random, with $\xi_i = \text{E}[\tilde{\xi}_i]$. Moreover, we assume that they are independent.

Before we proceed, note that $\text{Var}[\tilde{\mu}_2 - \tilde{\omega}_2]$, α_P, α_I and α_S appear only through the single parameter:

$$\rho = 1 + \frac{1}{m}\frac{\text{Var}[\tilde{\mu}_2 - \tilde{\omega}_2]}{\frac{N_P}{\alpha_P} + \frac{N_I}{\alpha_I} + \frac{N_S}{\alpha_S}}.$$

Thus an increase in N_S leads to a decrease in ρ. In what follows, we will concentrate our attention on this latter parameter.

Regime 2 The equilibrium is such that:

$$\tilde{P}_1 = \frac{(m + n_I\rho)\frac{\tilde{\xi}_1}{m} + n_I\frac{\xi_2}{m}}{m + n_I(1+\rho)}\ ; \tilde{P}_F = \frac{n_I\rho\frac{\tilde{\xi}_1}{m} + (m + n_I)\frac{\xi_2}{m}}{m + n_I(1+\rho)}\ ; \tilde{P}_2 = \frac{\tilde{\xi}_2}{m} - \frac{n_I\left(\frac{\xi_2}{m} - \frac{\tilde{\xi}_1}{m}\right)}{m + n_I(1+\rho)}\ ;$$

$$\tilde{X}^\star = \frac{m\left(\frac{\xi_2}{m} - \frac{\tilde{\xi}_1}{m}\right)}{m + n_I(1+\rho)}\ ; \tilde{Y}^\star = 0.$$

The fact that $\tilde{P}_1$ and $\tilde{P}_F$ are weighted averages of $\tilde{\xi}_1/m$ and ξ_2/m, and that $\frac{\xi_2}{m} \geq \frac{\tilde{\xi}_1}{m}$ in Regime 2, determine the comparative statics.

An increase in the number of speculators has first of all a positive influence on the futures prices: in this regime, as $\mathrm{E}[\tilde{P}_2] > \tilde{P}_F$, there is an incentive to take long speculative positions. Under the pressure of a more intense speculative activity, the variance of the futures price diminishes. Remark, indeed, that:

$$\mathrm{Var}[\tilde{P}_F] = \left(\frac{n_I \rho}{m + n_I(1+\rho)}\right)^2 \frac{\mathrm{Var}[\tilde{\xi}_1]}{m^2}.$$

The increase of $\tilde{P}_F$ induces a rise in the stocks hold on the physical market, for two reasons: first, $\tilde{X}^\star$ is determined by the spread $(\tilde{P}_F - \tilde{P}_1)$, which raises. Second, the decrease of the spread $(\mathrm{E}[\tilde{P}_2] - \tilde{P}_F)$ diminishes the cost of hedging for the storer. Meanwhile, the variance of the inventories increases:

$$\mathrm{Var}[\tilde{X}^\star] = \left(\frac{-1}{m + n_I(1+\rho)}\right)^2 \mathrm{Var}[\tilde{\xi}_1].$$

This variance is indeed directly impacted by that of the shock on the first period. This impact is enhanced proportionally to the number of storers when N_S increases.

The spot price $\tilde{P}_1$ also rises; this results from a more intense storage activity. The variance decreases. On the one hand, this finding is intuitive because an increase of the stocks provides a cushion absorbing prices fluctuations. On the other hand, it is counterintuitive because the variance of the physical quantities rises. What is important however, is the combination of these two contradictory effects. Remind indeed that from equation 3, we have $\tilde{P}_1 = \tilde{P}_F - \tilde{X}^\star$. Thus:

$$\mathrm{Var}[\tilde{P}_1] = \mathrm{Var}[\tilde{P}_F] + \mathrm{Var}[\tilde{X}^\star] - 2\mathrm{Cov}[\tilde{P}_F, \tilde{X}^\star].$$

Conditionally to the fact that the negative covariance between the futures price and the stocks doesn't change (which is reasonable), the decrease in the variance of $\tilde{P}_1$ comes from the fact that the change in $\mathrm{Var}[\tilde{P}_F]$ dominates that of $\mathrm{Var}[\tilde{X}^\star]$.

Finally $\mathrm{E}[\tilde{P}_2]$ diminishes because the stocks built in period 1 are released in period 2. The same is true for $\tilde{P}_2$ but not for its variance, which increases:

$$\mathrm{Var}[\tilde{P}_2] = \frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2} + \left(\frac{n_I}{m + n_I(1+\rho)}\right)^2 \frac{\mathrm{Var}[\tilde{\xi}_1]}{m^2}.$$

This change in the variance is due only to what happened in period 1, which is transmitted into period 2 *via* the inventories. Relying on equation (8), which depicts the equilibrium on the spot market at date 2 and on an analysis similar to that proposed for $\mathrm{Var}[\tilde{P}_1]$, it is indeed easy to show that this increase in $\mathrm{Var}[\tilde{P}_2]$ is due to that of $\mathrm{Var}[\tilde{X}^\star]$.

Regime 4 The equilibrium is such that:

$$\tilde{P}_1 = \frac{\tilde{\xi}_1}{m}\,;\, \tilde{P}_F = \frac{m\frac{\xi_2}{m} + n_P \rho P}{m + n_P \rho}\,;\, \tilde{P}_2 = \frac{\tilde{\xi}_2}{m} + \frac{n_P\left(P - \frac{\xi_2}{m}\right)}{m + n_P\rho}\,;$$
$$\tilde{X}^\star = 0\,;\, \tilde{Y}^\star = \frac{m\left(P - \frac{\xi_2}{m}\right)}{m + n_P\rho}.$$

Again, $\tilde{P}_1$ and $\tilde{P}_F$ are weighted averages of $\tilde{\xi}_1/m$ and ξ_2/m. This, in addition to the fact that $\frac{\xi_2}{m} \leq P$ in Regime 4, determine the comparative statics.

An increase of N_S tends to reduce $\tilde{P}_F$ because the negative basis induces short speculative positions. Here however, there are no inventories, thus no link between $\tilde{P}_1$ and $\tilde{P}_F$ and/or between $\tilde{\xi}_1$ and the other variables. Consequently, there is no change in the spot price at date 1, and all variances (for the physical quantities, the two spot and the futures prices) remain the same. The decrease in $\tilde{P}_F$ induces a rise in the commitments of the processors for two reasons: first, $\tilde{Y}^\star$ is determined by $(P - \tilde{P}_F)$, which raises. Second, the decrease of $(\tilde{P}_F - \mathrm{E}[\tilde{P}_2])$ diminishes the cost of hedging. The spot price $\tilde{P}_2$ increases: this results from higher commitments by the processors.

Regime 1 The equilibrium is such that:

$$\tilde{P}_1 = \frac{(m + (n_I + n_P)\rho)\frac{\tilde{\xi}_1}{m} + n_I\frac{\xi_2}{m} + n_I n_P \rho m^{-1} P}{m + (n_I + n_P)\rho + n_I + n_I n_P \rho m^{-1}} \tag{22}$$

$$\tilde{P}_F = \frac{n_I\rho\frac{\tilde{\xi}_1}{m} + (m + n_I)\frac{\xi_2}{m} + (m + n_I) n_P \rho m^{-1} P}{n_I\rho + (m + n_I) + (m + n_I) n_P \rho m^{-1}} \tag{23}$$

$$\tilde{P}_2 = \frac{\tilde{\xi}_2}{m} + \frac{n_I\frac{\tilde{\xi}_1}{m} - ((m + n_I) n_P m^{-1} + n_I)\frac{\xi_2}{m} + (m + n_I) n_P m^{-1} P}{n_I\rho + (m + n_I) + (m + n_I) n_P \rho m^{-1}} \tag{24}$$

$$\tilde{X}^\star = \frac{-(m + n_P\rho)\frac{\tilde{\xi}_1}{m} + m\frac{\xi_2}{m} + n_P\rho P}{n_I\rho + (m + n_I) + (m + n_I) n_P \rho m^{-1}} \tag{25}$$

$$\tilde{Y}^\star = \frac{-n_I\rho\frac{\tilde{\xi}_1}{m} - (m + n_I)\frac{\xi_2}{m} + (m + (1+\rho) n_I) P}{n_I\rho + (m + n_I) + (m + n_I) n_P \rho m^{-1}}. \tag{26}$$

The two Sub Regimes 1U and 1L are separated by the line Δ, already encountered, defined by $n_I \tilde{X}^\star - n_P \tilde{Y}^\star = 0$, ie :

$$n_I \left(\xi_2 - \tilde{\xi}_1\right) + n_P (m + n_I) \left(\frac{\xi_2}{m} - P\right) = 0.$$

Taking into account the fact that prices and quantities have the form $\frac{A+B\rho}{C+D\rho}$, with positive numerators and denominators, and that such expressions are increasing with respect to ρ if $BC - DA \geq 0$, it is easy to show that 1U and 1L are the relevant sub-regimes, the former resembling Regime 2 and the latter Regime 4. This is true for the levels of $\tilde{P}_F$, $\tilde{X}^\star$, $\tilde{P}_1$, $\tilde{Y}^\star$, and $\tilde{P}_2$, which changes are summarized in table 2.

The position with regard to Δ is however not relevant for the variances, which evolve in the same way whatever the sub regime is concerned.

Under the pressure of a more intense speculative activity, the variance of the futures price diminishes:

$$\mathrm{Var}[\tilde{P}_F] = \left(\frac{n_I\rho}{m + (n_I + n_P)\rho + n_I + \frac{n_I n_P}{m}\rho}\right)^2 \frac{\mathrm{Var}[\tilde{\xi}_1]}{m^2}.$$

The variances the physical quantities diminish. As far as $\mathrm{Var}[\tilde{X}^*]$ is concerned, if we compare this result to that obtained in Regime 2, it looks like the presence of the processors has a stabilizing effect on the quantities stored: indeed, their commitment is known at date 1 by the storers. Moreover, the variance of $\tilde{P}_1$ diminishes, which can be seen as the result of the decrease in $\mathrm{Var}[\tilde{P}_F]$ and $\mathrm{Var}[\tilde{X}^*]$.

Finally, the variance of $\tilde{P}_2$ rises. This results can be explained by equation (8), which gives the following expression for this variance:

$$\mathrm{Var}[\tilde{P}_2] = \left(\frac{1}{m^2}\right)\left(\mathrm{Var}[\tilde{\xi}_2] + n_I^2\mathrm{Var}[\tilde{X}^\star] + n_P^2\mathrm{Var}[\tilde{Y}^\star] - 2n_I n_P \mathrm{Cov}[\tilde{X}^\star, \tilde{Y}^\star]\right) \tag{27}$$

The decrease in the variances of $\tilde{X}^\star$ and$\tilde{Y}^\star$ is more than compensated by the rise of their negative covariance. The latter is due to the increase in N_S, which amplifies the difference in the physical quantities hold by the operators.

	$\tilde{P}_F$	$\tilde{X}^\star$	$\tilde{Y}^\star$	$\tilde{P}_1$	$\tilde{P}_2$	$\mathrm{Var}[\tilde{\mathrm{P}}_\mathrm{F}]$	$\mathrm{Var}[\tilde{\mathrm{P}}_1]$	$\mathrm{Var}[\tilde{\mathrm{P}}_2]$
2	↗	↗		↗	↘	↘	↘	↗
1U	↗	↗	↘	↗	↘	↘	↘	↗
4	↘		↗	⟷	↗	⟷	⟷	⟷
1L	↘	↘	↗	↘	↗	↘	↘	↗

Table 2: Impact of speculators on prices and quantities

Speculation, prices and quantities in summary Table 2 shows that Regimes 2 and 4 are only subsets of Regimes 1U and 1L. Remark for example that $\tilde{P}_1$ decreases in Regime 1L whereas it is constant in Regime 4. This is due to the fact that the storers are active in Regime 1L but not in Regime 4, and underlines how important the stocks are for the functioning of a commodity market. Inventories indeed appear as the transmission channel for shocks in the space (between the paper and the physical markets) and in the time (between dates 1 and 2). For this is through inventories that a shock appearing in the paper market (i.e. the rise in N_S) impacts the level and variances of the physical quantities and the prices. This result is close to the analysis in Newbery (1987).

As far as the level of the different variables is concerned, our model shows that the impact of an increase in N_S depends, in the end, on which side of the hedging demand dominates. The physical quantities, for example, increase for the operators benefiting from lower hedging costs whereas they decrease for the others. This amplifies the difference in the positions of the operators and consequently their market impact.

The analysis of the variances is less straightforward. The most simple effect is the impact on $\mathrm{Var}[\tilde{\mathrm{P}}_\mathrm{F}]$, which always diminishes under the pressure of a more intense speculative activity (provided that there are stocks in the economy). As regards to the spot prices, a raise in N_S has a stabilizing effect at time 1 and a destabilizing one at time 2. The latter result however might be modified in a three-period model, where the quantities at time 2 would be influenced by the futures price of a contract expiring at time 3. It could also be changed if the price of the output, P, could be adjusted as an answer to a shock. Up to now, indeed, there is nothing in the model that could absorb a shock at time 2.

This version of the model illustrates the fact that financial markets may "destabilize" the underlying markets, though the term is inappropriate since it only refers to a statistical property. Of course a higher price volatility doesn't mean a lower welfare, quite the contrary: more volatility means that prices are more effective/informative signals. The impact of markets on prices volatilities is often a naïve aspect of welfare analysis. We will go further on this point in the next subsection.

A last word must be said about the quantities hold on the paper market. The analysis is rather simple so it does not deserve an extensive demonstration. Note that, whatever the evolutions encountered on the physical market, the increased speculation leads to a more important proportion of the hedging positions, compared to that of the the speculative positions.

Note finally that Appendix C completes this analysis with comparisons based on another scenario where the futures market is closed.

6.2 Indirect utilities

In the two following subsections, we will return, for the sake of simplicity, to an analysis where ξ_1 is known when markets open. We shall express the indirect utilities of the various agents in equilibrium, and compute their sensitivities with respect to the parameters. We proceed in two steps. First, we compute the indirect utilities of the agents in equilibrium, as functions of equilibrium prices P_1 and P_F. Second, we compute the elasticities of P_1 and P_F to deduce the elasticities of the indirect utilities. We restrict ourselves to the richer case, *i.e.* Regime 1, where all agents are active. Recall that then we have:

$$P_F < P \text{ and } P_1 < P_F; \tag{28}$$

$$\tilde{P}_2 = \frac{1}{m}\left(\tilde{\mu}_2 - \tilde{\omega}_2 - n_I\left(P_F - P_1\right) + n_P\left(P - P_F\right)\right); \tag{29}$$

$$mP_1 - n_I\left(P_F - P_1\right) = \xi_1; \tag{30}$$

$$mP_F + \rho\left(n_I\left(P_F - P_1\right) - n_P\left(P - P_F\right)\right) = \xi_2. \tag{31}$$

As above, we shall set $\xi_2 := \mathrm{E}[\tilde{\mu}_2 - \tilde{\omega}_2]$ and $\xi_1 := \mu_1 - \omega_1$.

The indirect utility of the speculators is given by:

$$U_S = f_S^*(\mathrm{E}[\tilde{P}_2] - P_F) - \frac{1}{2}\alpha_S f_S^{*2}\mathrm{Var}[\tilde{P}_2],$$

where we have to substitute the value of f_S^*, which leads to:

$$U_S = \frac{\left(\mathrm{E}[\tilde{P}_2] - P_F\right)^2}{2\alpha_S \mathrm{Var}[\tilde{P}_2]}. \tag{32}$$

Let us now turn to the storers. Their indirect utility is given by:

$$U_I = (x^* + f_I^*)\mathrm{E}[\tilde{P}_2] - x^*P_1 - f_I^*P_F - \frac{1}{2}Cx^{*2} - \frac{1}{2}\alpha_I(x + f_I^*)^2\mathrm{Var}[\tilde{P}_2],$$

where we substitute the values of f_I^*, x^* and y^*:

$$U_I = \frac{\left(\mathrm{E}[\tilde{P}_2] - P_F\right)^2}{2\alpha_I \mathrm{Var}[\tilde{P}_2]} + \frac{(P_F - P_1)^2}{2C}. \tag{33}$$

For the processors we have, in a similar fashion:

$$U_P = \frac{\left(\mathrm{E}[\tilde{P}_2] - P_F\right)^2}{2\alpha_P \mathrm{Var}[\tilde{P}_2]} + \frac{(P - P_F)^2}{2\beta P}. \tag{34}$$

We thus obtain, for all categories of agents, a clear separation between two additive components of the indirect utilities. The first is associated with the level of the expected basis and is clearly linked with speculation. The second is associated with the level of the current basis or the futures prices and is linked with the hedged activity on the physical market. We shall name U_{Si} this first component for the category of agent i, and U_{Hi} the second one.

Quite intuitively, for all operators, U_{Si} is all the more important as the futures market is biased, whatever the sign of the bias; it decreases with respect to risk aversion and to the variance of the expected spot price. U_{Hi} changes with the category of agent under consideration. For the storers, it is positively correlated to the current basis and diminishes with storage costs. For the processors, it rises with the margin on the processing activity and decreases with the production costs.

We will now particularize formulas (32), (33) and (34) to the case when the markets are in equilibrium. In that case, $\tilde{P}_2$ becomes a function of (P_1, P_F), and the formulas become (after replacing the n_i by their values in terms of the N_i):

$$U_S = \frac{\mathrm{Var}[\tilde{\xi}_2]}{2m^2\alpha_S \left(\sum \frac{N_i}{\alpha_i}\right)^2} \left(\frac{N_I}{C}(P_F - P_1) - \frac{N_P}{\beta P}(P - P_F)\right)^2; \tag{35}$$

$$U_I = \frac{\mathrm{Var}[\tilde{\xi}_2]}{2m^2\alpha_I \left(\sum \frac{N_i}{\alpha_i}\right)^2} \left(\frac{N_I}{C}(P_F - P_1) - \frac{N_P}{\beta P}(P - P_F)\right)^2 + \frac{(P_F - P_1)^2}{2C}; \tag{36}$$

$$U_P = \frac{\mathrm{Var}[\tilde{\xi}_2]}{2m^2\alpha_P \left(\sum \frac{N_i}{\alpha_i}\right)^2} \left(\frac{N_I}{C}(P_F - P_1) - \frac{N_P}{\beta P}(P - P_F)\right)^2 + \frac{(P - P_F)^2}{2\beta P}. \tag{37}$$

Note for future use that these are indirect utilities *per head*: for instance, there are N_I storers, they are all identical, and U_I is the indirect utility of each one of them. This will enable us to do a welfare analysis in the next subsection.

6.3 The impact of speculators on the welfare of others

Formulas (35), (36) and (37) give us the indirect utilities of the agents at equilibrium in terms of the equilibrium prices P_1 and P_F. These can in turn be expressed in terms of the fundamentals of the economy, namely ξ_1 and $\tilde{\xi}_2$ (see Appendix A): substituting formulas (43), (44) and (45), we get new expressions, which can be differentiated to give the sensitivities of the indirect utilities with respect to the parameters in the model.

However, it is better to work directly with formulas (35), (36) and (37). We will then need the sensitivities of P_1 and P_F with respect to the varying parameter, but these can be derived from the system (30)-(31) by the implicit function theorem. To see how it is done, let us compute the sensitivities with respect to N_S, the number of speculators. In other words, we will investigate whether an increase in the number of speculators increases or decreases the welfare of speculators, of inventory holders, and of industry processors.

Sensitivities of prices We first compute the sensitivities $\frac{dP_1}{dN_S}$ and $\frac{dP_F}{dN_S}$. We get them by differentiating (30)-(31):

$$
\begin{aligned}
m\frac{dP_1}{dN_S} - n_I\left(\frac{dP_F}{dN_S} - \frac{dP_1}{dN_S}\right) &= 0, \\
m\frac{dP_F}{dN_S} + \rho\left(n_I\left(\frac{dP_F}{dN_S} - \frac{dP_1}{dN_S}\right) + n_P\frac{dP_F}{dN_S}\right) &= -\frac{d\rho}{dN_S}\left(n_I\left(P_F - P_1\right) - n_P\left(P - P_F\right)\right),
\end{aligned}
$$

which yields:

$$
\frac{dP_F}{dN_S} = \left(\frac{m}{n_I} + 1\right)\frac{dP_1}{dN_S}, \tag{38}
$$

$$
\begin{aligned}
\frac{dP_1}{dN_S} &= -\frac{d\rho}{dN_S}\frac{n_I\left(P_F - P_1\right) - n_P\left(P - P_F\right)}{\left(\frac{m}{n_I} + 1\right)\left(m + \rho n_P\right) + \rho m} \\
&= \frac{1}{m\alpha_S}\frac{\mathrm{Var}[\tilde{\xi}_2]}{\left(\sum\frac{N_i}{\alpha_i}\right)^2}\frac{n_I\left(P_F - P_1\right) - n_P\left(P - P_F\right)}{\left(\frac{m}{n_I} + 1\right)\left(m + \rho n_P\right) + \rho m}.
\end{aligned} \tag{39}
$$

Sensitivity of U_S Differentiating formula (35) yields:

$$
\begin{aligned}
\frac{dU_S}{dN_S} =& \frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2\alpha_S\left(\sum\frac{N_i}{\alpha_i}\right)^2}\left(n_I(P_F - P_1) - n_P(P - P_F)\right)\left(m + n_P\left(1 + \frac{m}{n_I}\right)\right)\frac{dP_1}{dN_S} \\
&- \frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2\alpha_S^2\left(\sum\frac{N_i}{\alpha_i}\right)^3}\left(n_I(P_F - P_1) - n_P(P - P_F)\right)^2 \\
=& -\frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2\alpha_S^2\left(\sum\frac{N_i}{\alpha_i}\right)^3}\left(1 - \frac{\mathrm{Var}[\tilde{\xi}_2]}{m\sum\frac{N_i}{\alpha_i}}\frac{\left(m + n_P\left(1 + \frac{m}{n_I}\right)\right)}{\left(\frac{m}{n_I} + 1\right)(m + \rho n_P) + \rho m}\right) \\
&\times \left(n_I(P_F - P_1) - n_P(P - P_F)\right)^2. \\
=& -\frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2\alpha_S^2\left(\sum\frac{N_i}{\alpha_i}\right)^3}\frac{m^2 + n_I n_P + m(2n_I + n_P)}{m^2 + mn_I + \rho(n_I n_P + m(n_I + n_P))} \\
&\times \left(n_I(P_F - P_1) - n_P(P - P_F)\right)^2.
\end{aligned} \tag{40}
$$

The sign of $\frac{dU_S}{dN_S}$ is constant in region 1: it is negative. Adding speculators decreases the remuneration associated to risk bearing.

Sensitivity of U_I Differentiating formula (36) yields:

$$
\begin{aligned}
\frac{dU_I}{dN_S} =& -\frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2\alpha_S\alpha_I\left(\sum\frac{N_i}{\alpha_i}\right)^3}\frac{m^2 + n_I n_P + m(2n_I + n_P)}{m^2 + mn_I + \rho(n_I n_P + m(n_I + n_P))} \\
&\times \left(n_I(P_F - P_1) - n_P(P - P_F)\right)^2 + \frac{P_F - P_1}{C}\left(\frac{dP_F}{dN_S} - \frac{dP_1}{dN_S}\right)
\end{aligned}
$$

$$= -\frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2\alpha_S\alpha_I\left(\sum\frac{N_i}{\alpha_i}\right)^3}\frac{m^2+n_In_P+m(2n_I+n_P)}{m^2+mn_I+\rho(n_In_P+m(n_I+n_P))}$$
$$\times\left(n_I(P_F-P_1)-n_P(P-P_F)\right)^2$$
$$+\frac{P_F-P_1}{C}\frac{1}{n_I\alpha_S}\frac{\mathrm{Var}[\tilde{\xi}_2]}{\left(\sum\frac{N_i}{\alpha_i}\right)^2}\frac{n_I(P_F-P_1)-n_P(P-P_F)}{\left(\frac{m}{n_I}+1\right)(m+\rho n_P)+\rho m}. \tag{41}$$

As mentioned before, the utility due to speculative activities decreases when N_S increases. As far as the utility of hedging is concerned, the effect depends on the sign of: $n_I(P_F-P_1)-n_P(P-P_F)$. Remind that this line separates Region 1 into two subregions. In Region $1U$, the utility of hedging increases for the storers, because they need more hedging than processors. The opposite conclusion arises in Region $1L$.

As far as the total utility is concerned, we will not pursue the calculations further, noting simply that $n_I(P_F-P_1)-n_P(P-P_F)$ factors, so that the result is of the form:

$$\frac{dU_I}{dN_S}=A(n_I(P_F-P_1)-n_P(P-P_F))(K_1(P_F-P_1)+K_2(P-P_F)),$$

for suitable constants A, K_1, and K_2. This means that the sign changes across

- the line Δ, already encountered, defined by $n_I\left(P_F-P_1\right)+n_P\left(P-P_F\right)=0$;
- the line D, defined by the equation $K_1(P_F-P_1)+K_2(P-P_F)=0$.

Both Δ and D go through the point M where $P_1=P_F=P$. If $K_2/K_1<0$, the line D enters region 1, if $K_2/K_1>0$, it does not. So, if $K_2/K_1<0$, region 1 is divided in three subregions by the lines D and Δ, and the sign changes when one crosses from one to the other. If $K_2/K_1>0$, region 1 is divided in two subregions by the line Δ, and the sign changes across Δ. In all cases, the response of inventory holders to an increase in the number of speculators will depend on the equilibrium.

Sensitivity of U_P Differentiating formula (37) yields:

$$\frac{dU_P}{dN_S}= -\frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2\alpha_S\alpha_P\left(\sum\frac{N_i}{\alpha_i}\right)^3}\frac{m^2+n_In_P+m(2n_I+n_P)}{m^2+mn_I+\rho(n_In_P+m(n_I+n_P))}$$
$$\times\left(n_I(P_F-P_1)-n_P(P-P_F)\right)^2+\frac{P_F-P}{\beta P}\frac{dP_F}{dN_S}$$
$$= -\frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2\alpha_S\alpha_P\left(\sum\frac{N_i}{\alpha_i}\right)^3}\frac{m^2+n_In_P+m(2n_I+n_P)}{m^2+mn_I+\rho(n_In_P+m(n_I+n_P))}$$
$$\times\left(n_I(P_F-P_1)-n_P(P-P_F)\right)^2$$
$$+\frac{P_F-P}{\beta P}\left(\frac{m}{n_I}+1\right)\frac{1}{m\alpha_S}\frac{\mathrm{Var}[\tilde{\xi}_2]}{\left(\sum\frac{N_i}{\alpha_i}\right)^2}\frac{n_I(P_F-P_1)-n_P(P-P_F)}{\left(\frac{m}{n_I}+1\right)(m+\rho n_P)+\rho m}. \tag{42}$$

Again, the utility due to speculation decreases and that linked with hedging depends on the sign of : $n_I(P_F - P_1) - n_P(P - P_F)$. In Region $1U$, the utility of hedging decreases for the processors, and it increases in Region $1L$.

We will not pursue the calculations further, noting simply that $n_I\,(P_F - P_1) - n_P\,(P - P_F)$ factors again, so that:

$$\frac{dU_P}{dN_S} = A^*(n_I(P_F - P_1) - n_P(P - P_F))(K_1^*(P_F - P_1) + K_2^*(P - P_F))$$

As in the preceding case, there will be a line D^* (different from D), which enters region 1 if $K_1^*/K_2^* < 0$ and does not if $K_1^*/K_2^* > 0$. In the first case, region 1 is divided into three subregions by D and Δ^*, in the second it is divided into two subregions by Δ, and the sign of $\frac{dU_I}{dN_P}$ changes when one crosses the frontiers.

Speculation and welfare in summary Remind that all agents are speculators in their ways. This activity gives the sign of the first term of the derivative of welfare with respect to N_S (the speculation term): if the speculators lose from being more, then all agents lose as far as only speculation is concerned. This said, remark that the second term in the derivative of welfare concerns only the storers and the processors (the hedging term). They go in opposite direction in Regime 1: in subcase 1U, if the number of speculators increases, the hedging term is positive for storers and negative for processors. It is the other way around in subcase 1L.

In terms of political economy (in the sense that economic interests may determine political positions), we can simplify the message as follows. Note that in the neighborhood of Δ, the speculation term is of second order with respect to the hedging term. Therefore, the interests of storers and processors are systematically opposed. Storers are in favor of (processors are against) an increase in the number of speculators if they demand more futures (in absolute value) than processors can offer (subcase 1U). The opposite positions are taken if processors are demanding more futures in absolute value (subcase 1L).

7 Conclusion

Our model, although extremely simple (perhaps the simplest possible), allows the interaction between a physical spot market and a financial futures market and exhibits a surprising variety of behaviors. In equilibrium, there may be a contango or a backwardation, the futures price may be higher or lower than the expected spot price, inventory holders may or may not hold inventory, industrial processors may or may not sell forward, adding speculators may increase or decrease the indirect hedging utilities of inventory holders and of industrial processors. All depends, in a way we determine, on market fundamentals and the realization of shocks in the physical market. This rich variety of behaviors can be found in commodities markets as they go, and we have not found in the literature a model which encompasses them all. While filing this gap, and to the best of our knowledge, the present paper offers for the first time since 1930 a unified framework for the analysis of price relationships in commodity futures markets.

Of course, our model is too simple to capture some important effects; for instance, we would like to understand the so-called convenience yield, which is usually explained as the option value of holding stock. This cannot be understood within a two-period model. For this reason, and also because we want to take into account possible differences in the investment horizons of the operators, developing an inter temporal approach is the next step. It would be interesting to see how the conclusions of the two-period model fare in a multi-period or even in a infinite-horizon models.

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A Prices: explicit expressions

Note that $\xi_1 := \mu_1 - \omega_1$, $\tilde{\xi}_2 := \tilde{\mu}_2 - \tilde{\omega}_2$, $\xi_2 := \mathrm{E}[\tilde{\mu}_2 - \tilde{\omega}_2]$, $n_I := N_I/C$ and $n_P := \frac{N_P}{\beta P}$.

(ξ_1, ξ_2) determine the regime, and the final expressions of equilibrium prices are as follows.

Regime 1 :

$$P_1 = \frac{(m+(n_I+n_P)\rho)\frac{\xi_1}{m} + n_I\frac{\xi_2}{m} + n_I n_P \rho m^{-1} P}{m + (n_I+n_P)\rho + n_I + n_I n_P \rho m^{-1}}, \quad (43)$$

$$P_F = \frac{n_I\rho\frac{\xi_1}{m} + (m+n_I)\frac{\xi_2}{m} + (m+n_I)n_P\rho m^{-1}P}{n_I\rho + (m+n_I) + (m+n_I)n_P\rho m^{-1}}, \quad (44)$$

$$\tilde{P}_2 = \frac{\tilde{\xi}_2}{m} + \frac{n_I\frac{\xi_1}{m} - ((m+n_I)n_P m^{-1} + n_I)\frac{\xi_2}{m} + (m+n_I)n_P m^{-1} P}{n_I\rho + (m+n_I) + (m+n_I)n_P\rho m^{-1}}, \quad (45)$$

$$X^\star = \frac{-(m+n_P\rho)\frac{\xi_1}{m} + m\frac{\xi_2}{m} + n_P\rho P}{n_I\rho + (m+n_I) + (m+n_I)n_P\rho m^{-1}}, \quad (46)$$

$$Y^\star = \frac{-n_I\rho\frac{\xi_1}{m} - (m+n_I)\frac{\xi_2}{m} + (m+(1+\rho)n_I)P}{n_I\rho + (m+n_I) + (m+n_I)n_P\rho m^{-1}}. \quad (47)$$

Remark that all denominators are equal. They are written in different ways only to show that P_1 and P_F are convex combinations of $\frac{\xi_1}{m}$, $\frac{\xi_2}{m}$ and P.

Note that starting from Regime 1, setting n_I or n_P to 0 in the expressions to get the prices for any other region works perfectly. For example, the prices for Regime 2 can be directly retrieved by posing $n_P = 0$ in equations (43)-(47).

Regime 2 :

$$P_1 = \frac{(m+n_I\rho)\frac{\xi_1}{m} + n_I\frac{\xi_2}{m}}{m+n_I(1+\rho)} \; ; \; P_F = \frac{n_I\rho\frac{\xi_1}{m} + (m+n_I)\frac{\xi_2}{m}}{m+n_I(1+\rho)} \; ; \; \tilde{P}_2 = \frac{\tilde{\xi}_2}{m} + \frac{n_I\left(\frac{\xi_1}{m} - \frac{\xi_2}{m}\right)}{m+n_I(1+\rho)} \; ;$$
$$X^\star = \frac{-m\left(\frac{\xi_1}{m} - \frac{\xi_2}{m}\right)}{m+n_I(1+\rho)} \; ; \; Y^\star = 0.$$

Regime 3 :

$$P_1 = \frac{\xi_1}{m} \; ; \; P_F = \frac{\xi_2}{m} \; ; \; \tilde{P}_2 = \frac{\tilde{\xi}_2}{m} \; ; \; X^\star = 0 \; ; \; Y^\star = 0.$$

Regime 4 :

$$P_1 = \frac{\xi_1}{m} \; ; \; P_F = \frac{m\frac{\xi_2}{m} + n_P\rho P}{m+n_P\rho} \; ; \; \tilde{P}_2 = \frac{\tilde{\xi}_2}{m} + \frac{n_P\left(P - \frac{\xi_2}{m}\right)}{m+n_P\rho} \; ;$$
$$X^\star = 0 \; ; \; Y^\star = \frac{m\left(P - \frac{\xi_2}{m}\right)}{m+n_P\rho}.$$

B Comparative statics when risk or risk aversion increase

This appendix depicts what happens with the model when risk or risk aversion increase.

Let us say that the distribution of (ξ_1, ξ_2) is given. Remind that, beside $\mu_1 - \omega_1$ and $E[\tilde{\mu}_2 - \tilde{\omega}_2]$, the model parameters are $\mathrm{Var}[\tilde{\mu}_2 - \tilde{\omega}_2]$, the forward price P at which the processor can sell his product, the numbers N_P, N_I and N_S, the risk aversion parameters α_P, α_I and α_S, the production coefficient β, the cost of storage C and the elasticity of spot traders' demand m.

Before we proceed with the comparative statics, remind also that $\mathrm{Var}[\tilde{\mu}_2 - \tilde{\omega}_2]$ and α_P, α_I and α_S appear only through the single parameter:

$$\rho = 1 + \frac{1}{m} \frac{\mathrm{Var}[\tilde{\mu}_2 - \tilde{\omega}_2]}{\frac{N_P}{\alpha_P} + \frac{N_I}{\alpha_I} + \frac{N_S}{\alpha_S}}.$$

We will thus once again concentrate our attention on this parameter.

Of particular interest is the case $\rho = 1$. This happens when one of the following conditions is verified:

- $\mathrm{Var}[\tilde{\mu}_2 - \tilde{\omega}_2] = 0$ (the future is deterministic and known);
- one category of agents (or more) is risk neutral (remind that even in this case, the four market configurations remain);
- at least one sector is extremely competitive: $N_i = +\infty$ for some i;
- m, the elasticity of the demand to the price, is zero.

In all other cases, ρ is higher than 1. In particular, it increases with the uncertainty on the future availability of the commodity and with risk aversion.

In the case where $\rho = 1$, we have:

$$\begin{aligned} F(O) &= (0, -n_P P), \\ F(A) &= P\,(-n_I, m + n_I), \\ F(M) &= mP\,(1, 1). \end{aligned}$$

and the upper boundary of Regime 3 has a slope equal to 1, so that Regime 3 has the minimum possible size. This means that the conditions for active physical markets are as favorable as possible when there is either no risk or no risk aversion.

Figure 5 illustrates what happens as ρ increases from 1 to $+\infty$ following a change in $\mathrm{Var}[\tilde{\mu}_2 - \tilde{\omega}_2]$ or of one of the α_i. The point $F(M)$ remains fixed, while $F(O)$ and $F(A)$ move vertically, the first one downwards and the second one upwards. While the size of Regime 1 is clearly enlarged, the effects are ambiguous for Regime 4. Regime 3 also enlarges, as the slope of the half-line emanating from $F(M)$ increases towards the vertical. Regime 2 is the only one to be unambiguously reduced.

As Regime 3 enlarges when risk or risk aversions increase, the overall activity in the physical market diminishes. Meanwhile Regime 1, in which all markets and all operators are active, becomes more likely.

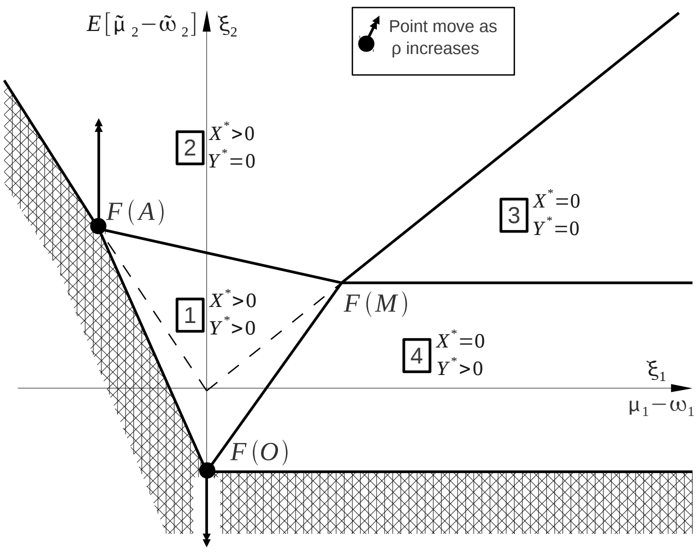

Figure 5: Phase diagram when there is an increase in ρ.

C Comparison with the no-futures scenario (NF)

This appendix is devoted to the case where there is no futures market (scenario NF): speculators are inactive and there remains three kinds of operators: storers, processors, and spot traders.

Three questions are examined: the impact on the physical positions hold by the operators, the existence (and unicity) of the equilibrium, and the impact on prices and volatilities.

We economize on calculations by applying, on the previous results, the variables and parameters transposition described by Table 3. The validity of this method is formally established in the proof of Theorem 3.

We retain the following notation for the exogenous parameter representing the spot price volatility at date 2: $V_2 := \mathrm{Var}[\tilde{P}_2] = \frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2}$.

Basic scenario:	P_F	ρ	n_I	n_P
NF scenario:	$\mathrm{E}[\tilde{P}_2]$	1	$\underline{n}_I = n_I \times \frac{C}{C+\alpha_I V_2}$	$\underline{n}_P = n_P \times \frac{\beta}{\beta+\alpha_P \frac{V_2}{P}}$

Table 3: Variables and parameters transposition, from the basic to the NF scenario.

Optimal positions

The optimal position of the storer becomes:

$$\underline{x}^\star = \frac{1}{C + \alpha_I V_2} \max\{\mathrm{E}[\tilde{P}_2] - P_1, 0\}. \tag{48}$$

The storer holds inventory if the expected price is higher than the current spot price. As regards to the processor, his activity depends on the fact that the forward price of the output is higher than the expected spot price of the commodity:

$$\underline{y}^\star = \frac{1}{\beta P + \alpha_P V_2} \max\{P - \mathrm{E}[\tilde{P}_2], 0\}. \tag{49}$$

When there is no futures market, uncertainty on the future spot price determines the decisions undertaken in the physical market. In this scenario, the latter necessarily have a speculative aspect.

Except when one category of operators (or more) is risk neutral, the quantities are lower in the NF scenario. Thus, by allowing risk transfer, the paper market has a positive impact on the physical quantities.

Equilibrium

Theorem 3 (Existence conditions)**.** *With the notations $\xi_1 := \mu_1 - \omega_1$, $\xi_2 := \mathrm{E}[\tilde{\mu}_2 - \tilde{\omega}_2]$ and $V_2 := \mathrm{Var}[\tilde{\mu}_2 - \tilde{\omega}_2]/m^2$, existence conditions on ξ_1, ξ_2 and V_2 are stricter in scenario NF than in the basic case.*

The four regimes in the NF scenario are included in those of the basic scenario. Regime 1 diminishes. Regime 2 gains on the basic Regime 1 and it is cut on its left border. Regime 3 doesn't change. Regime 4 gains on the basic Regime 1 and it is cut on its bottom border.

See Figure 6.

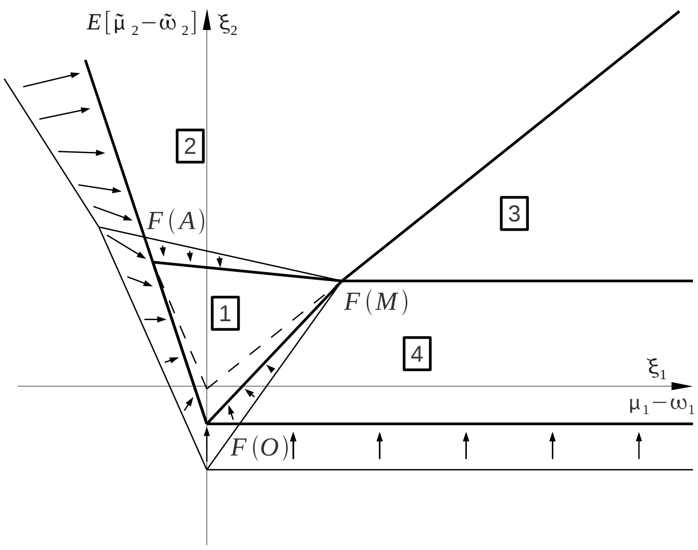

Figure 6: Changes on the four regimes when there is no futures market.

Proof. To prove this theorem, we begin by taking the equation depicting the expected equilibrium at date 2 (8):

$$\mathrm{E}[\tilde{P}_2] = \frac{1}{m}\left(\mathrm{E}[\tilde{\mu}_2 - \tilde{\omega}_2] - N_I y^\star + N_P y^\star\right).$$

We get the equation:

$$m\mathrm{E}[\tilde{P}_2] + N_I x^\star - N_P Y^\star = \mathrm{E}[\tilde{\mu}_2 - \tilde{\omega}_2]. \tag{50}$$

Hence $\mathrm{Var}[\tilde{P}_2] = V_2$, a constant. Consider the mapping $F_{NF} : \mathbb{R}_+^2 \to \mathbb{R}^2$ defined by:

$$F_{NF}(P_1, \mathrm{E}[\tilde{P}_2]) = \begin{pmatrix} mP_1 - \frac{n_I C}{C+\alpha_I V_2} \max\{\mathrm{E}[\tilde{P}_2] - P_1, 0\} \\ m\mathrm{E}[\tilde{P}_2] + \frac{n_I C}{C+\alpha_I V_2} \max\{\mathrm{E}[\tilde{P}_2] - P_1, 0\} - \frac{n_P P}{P\beta+\alpha_P V_2} \max\{P - \mathrm{E}[\tilde{P}_2], 0\} \end{pmatrix}.$$

Formally, the analysis is identical to the one done in the basic case. We see now the validity of Table 3. The images $F_{NF}(O)$, $F_{NF}(A)$, and $F_{NF}(M)$ are easily computed:

$$\begin{aligned} F_{NF}(O) &= (0, -\underline{n}_P P), \\ F_{NF}(A) &= P(-\underline{n}_I, m + \underline{n}_I), \\ F_{NF}(M) &= mP(1,1). \end{aligned}$$

From this, one can find the images of all four regions. In other words, the image of O moves towards the origin. The image of M doesn't move. The image of A goes to the south-east (this is directly visible in the expressions above), in a way that is characterized in detail to prove the theorem. The properties concerning O and M are obvious. Concerning the image of A, we need to check to facts: (1) $OF_{NF}(A)$ is steeper (in absolute value) than $OF(A)$; (2) $MF_{NF}(A)$ is flatter (in absolute value) than $MF(A)$. (1) We compare $\rho + m/n_I$ with $1 + m/\underline{n}_I$. The former can be rearranged as

$$\begin{aligned} 1 + m\frac{V_2}{\frac{N_P}{\alpha_P} + \frac{N_I}{\alpha_I} + \frac{N_S}{\alpha_S}} + \frac{m}{n_I} &= 1 + m\frac{C + \frac{V_2}{\frac{1}{\alpha_I} + \frac{1}{N_I}\left(\frac{N_P}{\alpha_P} + \frac{N_S}{\alpha_S}\right)}}{n_I C} \\ = 1 + m\frac{C + \frac{\alpha_I V_2}{1 + \frac{\alpha_I}{N_I}\left(\frac{N_P}{\alpha_P} + \frac{N_S}{\alpha_S}\right)}}{n_I C} &= 1 + \frac{m}{\underline{n}_I}\frac{C + \frac{\alpha_I V_2}{1 + \frac{\alpha_I}{N_I}\left(\frac{N_P}{\alpha_P} + \frac{N_S}{\alpha_S}\right)}}{C + \alpha_I V_2} \end{aligned}$$

which is clearly smaller than the latter. (2) We have to compare the two vectors:

$$\begin{aligned} F(M) - F(A) &= P(m + n_I, -\rho n_I), \\ F_{NF}(M) - F_{NF}(A) &= P(m + \underline{n}_I, -\underline{n}_I) \end{aligned}$$

Clearly, the ratios of coordinates are such that the latter is flatter than the former because $\rho > 1$ and $\alpha_I V_2 > 0$. □

Prices and volatility. The absence of a futures market also impacts price levels and volatilities. For instance equations (7) and (8) suggest that lower values for $\underline{X}^\star$ and $\underline{Y}^\star$ lead to lower levels of the spot price at date 1, and also, possibly, at date 2.

As was done in the beginning of section 6, in order to analyze the variances, we will consider ξ_1 as random.

Prices and quantities in the NF scenario are the following (they can be retrieved directly, or with Table 3 and the equations of Appendix A):

$$\tilde{P}_1 = \frac{(m + (\underline{n}_I + \underline{n}_P))\frac{\tilde{\xi}_1}{m} + \underline{n}_I \frac{\xi_2}{m} + \underline{n}_I \underline{n}_P m^{-1} P}{m + (\underline{n}_I + \underline{n}_P) + \underline{n}_I + \underline{n}_I \underline{n}_P m^{-1}}, \tag{51}$$

$$\mathrm{E}[\tilde{P}_2] = \frac{\underline{n}_I \frac{\tilde{\xi}_1}{m} + (m+\underline{n}_I)\frac{\xi_2}{m} + (m+\underline{n}_I)\underline{n}_P m^{-1} P}{\underline{n}_I + (m+\underline{n}_I) + (m+\underline{n}_I)\underline{n}_P m^{-1}}, \tag{52}$$

$$\tilde{P}_2 = \frac{\tilde{\xi}_2}{m} + \frac{\underline{n}_I \frac{\tilde{\xi}_1}{m} - ((m+\underline{n}_I)\underline{n}_P m^{-1} + \underline{n}_I)\frac{\xi_2}{m} + (m+\underline{n}_I)\underline{n}_P m^{-1} P}{\underline{n}_I + (m+\underline{n}_I) + (m+\underline{n}_I)\underline{n}_P m^{-1}}, \tag{53}$$

$$\tilde{X}^\star = \frac{-(m+\underline{n}_P)\frac{\tilde{\xi}_1}{m} + m\frac{\xi_2}{m} + \underline{n}_P P}{\underline{n}_I + (m+\underline{n}_I) + (m+\underline{n}_I)\underline{n}_P m^{-1}}, \tag{54}$$

$$\tilde{Y}^\star = \frac{-\underline{n}_I \frac{\tilde{\xi}_1}{m} - (m+\underline{n}_I)\frac{\xi_2}{m} + (m+2\underline{n}_I)P}{\underline{n}_I + (m+\underline{n}_I) + (m+\underline{n}_I)\underline{n}_P m^{-1}}. \tag{55}$$

Let thus compare the variance of $\tilde{P}_1$ in Regime 1 in the two scenarios, i.e.:

$$\mathrm{Var}[\tilde{P}_1] = \left(\frac{m + (n_I + n_P)\rho}{m + (n_I + n_P)\rho + n_I + \frac{n_I n_P}{m}\rho}\right)^2 \frac{\mathrm{Var}[\tilde{\xi}_1]}{m^2}.$$

with :

$$\mathrm{Var}\left[\underline{\tilde{P}}_1\right] = \left(\frac{m + \underline{n}_I + \underline{n}_P}{2\underline{n}_I + m + \underline{n}_P + \underline{n}_I \underline{n}_P m^{-1}}\right)^2 \frac{\mathrm{Var}\left[\tilde{\xi}_1\right]}{m^2}.$$

As $\rho > 1$, $n_I > \underline{n}_I$ and $n_P > \underline{n}_P$, the variance of $\tilde{P}_1$ is reduced by the presence of the futures market. This is true, however, unless the n_i become very different of the $\underline{n}_i$. Assume for example that the α_i are large (or any other reason making global risk aversion substantial). Then, the $\underline{n}_i$ would be very small, clearly making the variance smaller in the NF scenario.

The same conclusion can be reached through the comparison of the variances of $\tilde{P}_2$:

$$\mathrm{Var}[\tilde{P}_2] = \left(\frac{n_I}{(n_I + n_P + \frac{1}{m} n_I n_P)\rho + (m + n_I)}\right)^2 \frac{\mathrm{Var}[\xi_1]}{m^2} + \frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2}.$$

and :

$$\mathrm{Var}\left[\underline{\tilde{P}}_2\right] = \left(\frac{\underline{n}_I}{2\underline{n}_I + m + \underline{n}_P + \underline{n}_I \underline{n}_P m^{-1}}\right)^2 \frac{\mathrm{Var}\left[\tilde{\xi}_1\right]}{m^2} + \frac{\mathrm{Var}[\tilde{\xi}_2]}{m^2}.$$

To understand this higher variance, remark that the storer could react quite differently to different $\tilde{\xi}_1$ under different market organizations. If the absence of futures frightens the storers, so that they hardly store, their impact on the market in the second period would be negligible. On the contrary, if speculators were not very risk averse, they would accentuate the dependency of actual inventories on the current ξ_1. These more reactive actions will transport the volatility of the first period to the second one.

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