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Modeling Power Generation Switch as a Real Switching Option: Nuclear Vs. Gas

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Abstract

Given the volatility of the prices of fossil fuels and of environmental constraints, the nuclear power plants can be the least expensive solution to satisfy the demand of electricity. In this paper, we present a dynamic modeling for an optimal operational planning by considering the possibility to switch from natural gas to nuclear power. A switching options approach is applied to address an optimal power generation strategy. We show that, when we exchange the current asset with another alternative, this turns out to be equivalent to bearing the cost of the alternative operational option to earn the current cost. The results of the analysis justify the economic worth of the decision to adopt nuclear energy generation. The switching option increases the profit of the power generation schedule. The benefit of this decision is explained in the presence of a no correlation condition and the switching costs.

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1 Introduction

For many countries, the nuclear power technology adoption is considered as an energy perspective with many technological and economic advantages. The nuclear power generation cost is relatively lower comparing to other fossil fuel technologies. However, the introduction of nuclear power technology requires the elaboration of an investment planning. Furthermore, it is crucial to think beyond the power plant building stage to elaborate a strategic generation planning process. Given the availability of fossil fuel technologies and the fluctuation of their generation costs, the exchanging electricity generation to nuclear power must be analyzed trough a dynamic valuation technique. This make the choice of energy technology a dynamic control process of the competitiveness of available generation assets. Decision maker must proceed with an optimal dynamic choice of the generation technologies, under uncertainties. The dynamic assessment of alternative generation technologies is developed in order to deal with the continuing changes in the energy market and technological development. This development can be made based on the dual variables such as Cost/Price of generating electricity.

In this paper, we discuss possible solution involving changes in the underlying technology under uncertain conditions using real options technique. This technique makes the producer able to model the operating flexibility trough the valuation of the switching option from one production alternative to another with satisfying its goal which is a higher profitability. Especially, we focus on the switching decision from natural gas to nuclear technology under uncertainty. Generally, the real switching options approach is applied when a planner faces complex decision making process with multiple numbers of customers or multiple production alternatives. Such real option was linked particularly to the Entry/Exit strategies (McDonald and Siegel (1986), Pindyck (1993), Dixit and pindyck (1994), Shackleton et al. (2004)). Moreover, the application of this kind of real options becomes very useful to solve the dynamic opportunity choice.

The literature on power investment valuation using real options shows many cases illustrating the value of flexibility of waiting as an option [Dixit and Pindyck (1994), Gollier et al (2005), Rothwell (2006)]. The irreversibility of investment makes the waiting decision appropriate with keeping the option to invest open. In this sense, the decision maker will wait for more favorable market conditions. However, the decision to wait may not satisfy the development of an analysis framework incorporating several existing opportunities. When a monopoly's firm must manage a growing and continuous power demand trough a diversity of production technologies, the company has no time to wait. In this case, it is appropriate to study the decision to continue or to switch to another generation strategy. It is the same problem when a decision maker faces the problem to switch from fossil fuel-based electricity production to nuclear power generation under industry regulation. In the case of pre-liberalization stage of electricity market, the dynamic control of variable generation costs of each technology plays a key role in order to formulate the decision making process (Roques et al, 2006).

In the previous paper, we attempt to formulate a dynamic switching decision from fossil fuel to nuclear power technology. Our starting point is to model the profit generated by power generation planning based on current natural gas generation strategy with considering the possibility to switch to nuclear power. We assume that the generation technologies (nuclear and gas) are available and ready to the operational state and the decision maker may adopt only one generation strategy. The exchanging technology problem is derived when the transition to nuclear power requires the investment of a switching cost. We model the value of the power generation planning, using the Margrabe (1978) approach. We demonstrate that the transition problem to nuclear power is equivalent to investing in the alternative energy option to receive a profit, which is the generation cost of the current mode minus the switching cost. In this way, we applied dynamic programming to evaluate the switching option with stochastic generation costs. Here, the decision maker, faces random generation costs, attempts to choose between the electricity generation based on the current strategy and the switching decision to nuclear power alternative. In this setting, we formulate the optimal generation strategy by calculating the threshold values of each generations cost in the presence of switching cost.

The basic intention of this paper is to study the impact of generation cost level of the two generation assets and the correlation between them on the value of the switching option in the presence of switching cost. This is measured by the changes on the thresholds curve. By using dynamic programming framework, the competitiveness of generation assets is linked to a dynamic choice which can increase the generation power planning value. The contribution of this paper is to take into account the switching cost to present a bi-dimensional mathematical model of the switching option valuation tanks to market entry model of Dixit and Pindyck (1994)[1] .

This paper is organized as follows. In the second section, we present a literature review which is concerned with the employment of the real switching option. Section 3 exposes the problem assumptions. Section 4 shows the dynamic formulation of a power generation planning. In section 5, we provide a solution using dynamic programming technique. In section 6, we

[1]Dixit and Pindyck (1994), pp. 267-271.

highlight the positive externality of the decision to switch from natural gas to nuclear power. Section 7 illustrates numerically the developed model of switching strategy.

2 Theoretical preliminaries: The switching option analysis

In this section, we review some literature closely related to our analysis. In particular, we discuss multiple complex processes for managing real assets with possibility to switch from a real asset to another, or from one investment to another alternative investment, using the real options theory.

Margrabe (1978), the progenitor of the switching option valuation literature, focuses on the switching from one asset to another. He develops a technique attempted to exchange one financial risky asset for another, based on the option valuation model of Black and Scholes (1973). His model addresses the way to evaluate a switching option with two stochastic underlying assets. Furthermore, the model states that the option value depends on the random path of each asset and the degree of correlation between them.

Stulz (1982) applies the theory of Myers (1977), the progenitor of real options theory, in which a real investment can be considered as a financial option. He analyses the example of an area with two alternative uses who can be two kinds of building. He demonstrates that the alternative uses of this land with a low degree of correlation between the two risky asset makes the land more valuable. The value added is created by the flexibility of the possibility of being able to switch between all of the multiple alternatives.

The real options literature is filled with many applications of these extended options pricing tools. Indeed, the usefulness of these applications consists of two insights. On the one hand, there is the valuation option to switch from one investment to another. On the other hand, there is the study of the interaction between multiple investment opportunities. Trigeorgis (1993) uses real options theory to study the flexibility given by multiple existing investment alternatives. He postulates that multiple operating options in a given project create value. He examines the size (large or small) of investment and the sign (negative or positive) of the interaction between them on real option valuation.

Takezawa (2001) develops a theoretical model, based on a real options approach to define the optimal production policy. He models the optimal production strategy when the producer has the flexibility to switch from a production program to another depends on the prices variability between the commencement and the completion date of the final production time. Likewise, many pervious papers take into account the flexibility due to the production alternatives exchanging in manufacturing sector. Chen et al. (2002) study the manufacturing process and the flexible facilities when it is possible to substitute the current state with other multiple alternative strategies in the presence of switching costs. They highlight the impact of management flexibility on the investment valuation with production facilities.

Shackleton et al. (2004) use real options approach to elaborate an entry decision strategy by considering only two player firms. They study the rival relationship between two firms by assuming a monopolistic structure for the concerned market in the presence of an entry cost. They calculate the threshold times at whose each player can take an active position depends on his variable profitability. The optimal timing entry is solved using dynamic programming technique and Bellman equation with adopting the homogeneity assumption to reduce the problem optimization to one dimensional analysis. The main result of the analysis shows that the optimal entry timing, which is measured by the profitability threshold, increases in the opportunity cost and decreases in the correlation between profitability of rivals.

Many studies discuss the switching options analysis in the energy sector. This framework is applied to solve complex investment decisions, when there are multiple generation technologies to choose from, and to evaluate possible fuel source switching opportunities. Tseng et al. (2002) study the optimal operating rule in a thermal power plant. They show that the operating flexibility can be represented as an option to switch from the "on" to "off" state. Their model takes into account the physical operational constraints of a power plant, when starting and stopping the thermal power plant take time. They define these constraints as a nonzero startup/shutdown time. In this case, the optimal decision depends on four different states. The remaining decision can be made at any time, based on observation of variable parameters at the decision time. Using dynamic programming and the Monte Carlo Simulation (MCS), the authors solve the generation problem which is a function of fuel and power variable prices and demand uncertainty. They present a valuation model for short term power generation planning, which introduces some extensions of the financial option valuation theory.

Siclari and Giuseppe (2003) assume that the choice of power generation technologies, under uncertain conditions, must satisfy the goal of minimizing the operation cost. This problem can be solved by evaluating the switching fuel decision. They evaluate the option of a maximum spread between the energy prices and a minimum of costs using MCS, when the option depends on various energy market factors.

Vedenov et al (2006) study the optimal decision policy, for a fuel buyer, when there is the possibility to switch from gasoline to ethanol blends, based on the approach taken by Dixit and Pindyck (1994). They postulate that each type of fuel follows a Geometric Brownian motion (GBM) process. They solve the problem of the choice between a conventional fuel and renewable energy, using the Bellman equation. The authors analyze the sensitivity of the optimal switching thresholds that depends on horizon time, discount rate and the volatility degree of fuel alternatives.

3 Assumptions

The analysis studies the optimal power generation decision to switch from fossil fuel to nuclear technology that will provide a cheaper source of energy. This section exposes some assumptions about the economic and technical states that characterize the different parameters of the analysis: electricity prices and generation costs to model the power supplying technology choice. The problem arises if it is worthy to switch from the current generation strategy to an alternative electricity supplying technology. In this case, we consider two electricity generation strategies:

- The current power generation strategy: the electricity supply planning is based on natural gas power generation.

- The alternative power generation strategy: the electricity is generated by the nuclear power technology.

We aim to model the switching decision from natural gas to nuclear technology. We assume that a producer can adopt only one generation strategy (natural gas or nuclear) and the switching to nuclear power involves a switching cost.

The switching process problem is presented in the paper of Tseng and Barz (2002) with considering that the switching from online power generation technology to offline one takes time to be operational. This technological constraint will generates, certainly, other costs than the switching cost and the switching decision will take time to be operational. In this work, we omit this constraint to simplify the analysis. No time delay is considered between the choice of the alternative generation technology and the time at which the nuclear technology can be ready to the operational state. The realistic explanation, for such assumption, is argued by the possibility to import electricity. For example, a country which decides to switch from fossil fuel to nuclear power generation may import power within the context of regional or bilateral nuclear power programme. The technical constraint related to the operation delay of nuclear power plant will be bearing by the supplier country. Indeed, the power importer will be able to make the decision to switch without taking the technological constraints into account.

The main goal of the analysis is to present a dynamic model which allows to a decision maker (producer) to optimize the switching decision in uncertain economic environment. Here, we use real options approach to find the optimal solution which defines the switching decision from gas to nuclear. It's clear that finding a solution for the optimal power exchanging decision involve uncertainty. For this reason, we propose some underlying assumptions to model the uncertainties of different parameters involved in the decision making process.

Assumption 1. We assume that natural gas generation costs, C_G follows a GBM process:

$$\frac{dC_G}{C_G} = \alpha_G dt + \sigma_G dz_G \tag{1}$$

where α_G and σ_G are, respectively, the drift and the volatility of the natural gas generation cost and dz_G represents the increments of a Wiener process. The choice of the stochastic GBM process reflects the randomness of the generation cost and its non-negative value. Such assumption was noted by Näsäkkälä and Fleten (2005), and Siddiqui and Marnay (2007) based on the theory of Schwartz and Smith (2000) of the stochastic modeling of the evolution of natural gas prices.

Assumption 2. We consider that the nuclear generation cost, C_N represents the sum of the uranium fuel prices, the O&M cost and the investment cost. Suppose that C_N follows a GBM process:

$$\frac{dC_N}{C_N} = \alpha_N dt + \sigma_N dz_N \tag{2}$$

where α_N and σ_N are, respectively, the drift and the volatility of the nuclear power generation cost and dz_N represents the standard increment of the Wiener process. Such assumption is adopted by Takizawa and Suzuki (2002) to describe the variability of the electricity production cost of nuclear power plant (the generation cost includes both fuel and O&M cost components)[2].

The adoption of GBM process to model the generation cost of nuclear power plant assumes that this cost evolves in a dynamic way. This can be

[2] Takizawa and Suzuki (2002), Decision Support Systems, P. 451. When the production cost is C, the authors consider that C follows: $dC = C\sigma dz$.

justified theoretically through a number of observed facts, in energy markets, that described the relative volatility of the nuclear generation:

- The volatility of the uranium supply due to the uncertain fuel prices;
- The uncertainty of the fuel cycle and the volatility of the enrichment;
- The variability of the Operation and Maintenance (O&M) cost;
- The increase of investment cost (Pindyck, 1993).

All of these observations can explain the volatility of the nuclear generation. This variability is measured by σ_N. The increase of investment cost is detected by the drift which is a measure of the annual growth of nuclear power generation cost. Choosing the GBM process for nuclear power generation states a variable production cost of the nuclear technology in the future.

Assumption 3. We retain that the electricity price is constant. This assumption can be accepted under the regulated electricity market condition with a monopolistic structure of electricity supplying industry. In many countries including Tunisia, Egypt, Thailand, Japan [the Japanese electricity industry; Takizawa and Suzuki (2004)] and many other developing countries the prices of electricity is fixed by the government (Nagayama, 2009). The only adjustment of electricity prices is made to accommodate those prices with the marginal generation cost. Even in this condition, the revision of the price level requires a governmental permission to keep a fixed tariff structure.

In this setting, the switching problem formulation is presented within the context of electricity industry regulation. The pre-liberalization context leads to a problem formulation whose solution depends on the level of generation costs level and its uncertainties.

3.1 Dynamic Analysis of The Power Generation Planning

Considering the possibility to adopt a nuclear power generation alternative, it is crucial to analyze dynamically the choice between natural gas (G) and nuclear (N) generation strategy. In this case, the choice must be made through an economic comparison between the current fossil fuel generation strategy and an alternative nuclear option. The decision maker, which can be the monopolistic electricity supplier, attempts to maximize his profit. From this view point, the choice of the electricity generation technology is formulated in term of two variables "electricity price"–"generation cost" analysis. In the electricity regulation context, the output prices are already certain and proposed by the monopolist electric company and approved by the authorities of regulation. This condition of fixed output prices, under regulation structure, makes the choice of generation strategy an analysis based, only, on the inputs prices.

The assessment of a Power Generation Planning (PGP) must satisfy the possibility to adopt the cheaper power generation technology. We assume that the two generation technologies (N and G) are ready at the time of switching decision t. The expected value of the PGP with taking into account the flexibility allowed by the switching decision from $G \rightarrow N$, at time t, is represented as:

$$V(\Pi_N, \Pi_G, t) = Max\left[Max\left(\Pi_N(t), \Pi_G(t)\right), 0\right] + \Pi_G(t) \tag{3}$$

with

- $\Pi_G(t) = Q_G(PE - C_G(t))$ represents the profit of the natural gas power generation strategy, at time t.
- $\Pi_N(t) = Q_N(PE - C_N(t))$ represents the profit of the nuclear power generation alternative, at time t.

PE is the fixed electricity price, Q_i (kWh) with $i = G, N$, represents the kWh electricity produced quantities by each power plant, which is defined as $Q_i = k_i \times f_i^a$. Here, k_i is the kWe power plant capacity and f_i^a is the power plant availability factor, with $0 < f_i^a \leq 100\%$. To simplify the study, we retain the same power plant capacity and availability factor for the two technologies (Roques et al, 2006)[3]. In this case, we have $Q_G = Q_N = Q$.

The difference between a dynamic assessment of PGP, $V(\Pi_N, \Pi_G, t)$, which takes in to account the possibility to adopt the alternative generation strategy and the static policy $\Pi_G(t)$ equals the added value of the switching flexibility, which is represented as a switching option:

$$V(\Pi_N, \Pi_G, t) - \Pi_G(t) = H\left(\Pi_N(t), \Pi_G(t), t\right). \tag{4}$$

We use the proposed technique of Margrabe (1978) to evaluate the flexibility added by the switching option from natural gas to nuclear power. This theory gives the pricing equation of the option to exchange one risky asset

[3]The authors consider that is possible to retain the same availability factor for nuclear power plant and natural gas turbine.

with another. This kind of switching option is implicit in some complex investment decisions or operation planning. In this case, the switching option measures the flexibility of a power generation scheme, when a rational decision maker has the choice between two operating technologies. The payoff of the switching option $H(\Pi_N(t), \Pi_G(t), t)$ when the decision maker takes the switching act is given by:

$$\begin{aligned} H(\Pi_N(t), \Pi_G(t), t) &= Max(\Pi_N - \Pi_G, 0, t) = Max(C_G - C_N, 0, t) \\ &= H(C_G, C_N, t). \end{aligned} \quad (5)$$

Using Margrabe (1978) techniques, we have a maximization problem in order to minimize the cost. The decision maker applies his switching option $H(C_G, C_N, t)$ to switch from the natural gas technology to the nuclear one. So this decision depends on the sign of the result $C_G - C_N$, at time t. Here, we can present the following remark.

Remark

The switching decision from natural gas to nuclear power must be taken in order to maximize the PGN profit. Anytime, the optimal decision is defined by the following conditions:

- *If $C_G - C_N > 0$, the producer takes the decision to switch;*
- *If $C_G - C_N \leq 0$, the producer decides to continue with natural gas generation.*

This reflects that an agent may pay to earn C_G following an immediate switching decision. In the real context, the owner decides to sell the real asset C_N (the alternative cost) to get the real asset C_G (the production cost of the current generation strategy). We introduce, later, the switching cost to the analysis. Equation (4) is represented

$$V(\Pi_N, \Pi_G, t) = Max(\Pi_N - \Pi_G, 0, t) + \Pi_G(t). \quad (6)$$

Mathematically, when the stochastic production costs affect the profitability of each power generation technology, the optimal generation rule takes two states:

$$V = \begin{cases} \Pi_G, & \text{if } \Pi_N \leq \Pi_G, \\ \Pi_N, & \text{if } \Pi_G \leq \Pi_N. \end{cases} \quad (7)$$

This situation satisfies the continuity of the electricity supply with taking into account the flexibility to switch which guarantees the highest profit to the supplier. In this way, we value the flexibility associated with the possibility of switching between two operating modes. The investment decision has two states: “continue” or “switch”.

4 Dynamic Programming solution

In the context of the optimization problem, the switching option valuation can be derived by starting the analysis at the end of the decision process, precisely at the maturity time, then we come back to find the present optimal decision. This section tries to develop a dynamic programming framework to solve the switching decision problem in the presence of the switching cost from natural gas to nuclear generation strategy. First, we discuss the exercising condition of the switching option. For this reason, we use the Bellman equation to solve the option valuation problem. Next, we examine the threshold curves that define the decision to switch or to continue in the presence of the switching cost. Finally, we present the optimal operational strategy, given in terms of generation cost thresholds.

4.1 The Bellman partial differential equation with no correlation condition

Proposition 1. *the decision maker decides to apply the switching option only if he observes, at time t, the following state:* $C_N(t) - C_G(t) > 0$.

We denote by V^{SW} the expected payoff of the switching option which is equal to

$$\begin{aligned} V^{SW} &= E\left[H\left(C_G, C_N, t\right)\right] \\ &= E\left[Q\int_t^T \left(C_G\left(\tau\right) - C_N\left(\tau\right)\right)e^{-r\tau}d\tau \ /C_G\left(0\right) = C_G; C_N\left(0\right) = C_N\right] \end{aligned} \quad (8)$$

We assume that the decision maker is a risk-neutral agent. We denote by r the risk free rate. Using the Bellman equation, the expected value of the switching option during the interval of time equals the expected rate of capital change:

$$rHdt = E\left(dH\right). \quad (9)$$

We applied Ito Lemma. The Bellman equation may be represented as the following partial differential equation (EDP)

$$\begin{aligned} rH &= \alpha_G C_G H_{C_G} + \alpha_N C_N H_{C_N} + H_t \\ &+ \frac{1}{2}\left(\sigma_G^2 C_G^2 H_{C_G C_G} + \sigma_N^2 C_N^2 H_{C_N C_N} + 2\rho_{C_G C_N} C_N C_N H_{C_G C_N}\right). \end{aligned} \quad (10)$$

with $\rho_{C_G C_N} = E\left(dz_G dz_N\right)$ is the correlation coefficient between natural gas and nuclear generation costs. In our study, we aim to test the hypothesis where there is no correlation between these two generation costs based on the World Nuclear Association (WNA)[4] For this reason, we retain the following assumption.

Assumption 4 (WNA; 2008) There is no correlation between the nuclear generation cost and the generation cost based on natural gas. Note that the natural gas generation is, significantly, affected by the risk of volatility of fossil fuel prices which can be captured by electricity prices. We retain the no impact of the risk of a fossil fuel generation on the cost of nuclear generation, at least in the medium term:

$$\rho_{C_G C_N} = 0. \tag{11}$$

We pay no attention to the dependence of H on time; this implies that the EDP becomes

$$\alpha_G C_G H_{C_G} + \alpha_N C_N H_{C_N} + \frac{1}{2}\left(\sigma_G^2 C_G^2 H_{C_G C_G} + \sigma_N^2 C_N^2 H_{C_N C_N}\right) - rH = 0. \tag{12}$$

To solve this EDP, we use boundary conditions to obtain the threshold values of C_G^* and C_N^*. Here, we introduce the switching option related to the exercising of the exchanging decision.

4.2 Boundary conditions in the presence of switching cost

To solve the EDP, we need to precise the boundary conditions. We started to solve the problem at $t = 0$, next we derived the solution at $t + dt$ presented as a subsequent period of the analysis. Fixing $t = 0$ simplifies the analysis. We have an initial boundary condition and two other kinds of boundary conditions that are the value matching and smooth pasting conditions [Dixit and Pindyck (1994)]:

- The function H will be increasing in C_G and decreasing in C_N. Consequently, if C_G goes to zero, the real switching option has no value because the decision makers may consider that is more worthwhile to retain the current natural gas generation which generates a negligible operating cost:

$$H\left(0, C_N\right) = 0. \tag{13}$$

[4] WNA report, Structuring Nuclear Projects for Success, 2008.

- **Value matching condition**. This condition matches the value of the unknown function $H(C_G, C_N)$ to a termination payoff $V^{SW}(C_G, C_N)$ minus the switching cost K^{SW} at the optimal thresholds that define the optimal exercising time. We consider that the switching cost K^{SW} may be equal to s, a constant part of C_G with $0 < s < 1$, then we have

$$K^{SW}(G \longrightarrow N) = sC_G. \tag{14}$$

We have already noted that the decision to exercise the switching option generates a switching cost. If the planner decides to adopt the nuclear generation in order to economize the natural gas generation cost. In this case, the planner must pay s part of his profit. Then the profit generated by the switching decision is lowered to $(1-s)C_G$. We have the following boundary condition

$$H(C_G^*, C_N^*) = V^{SW}(C_G^*, C_N^*) - K^{SW}. \tag{15}$$

Considering equation (14), the value matching condition becomes

$$H(C_G^*, C_N^*) = V^{SW}(C_G^*, C_N^*) - sC_G. \tag{16}$$

- **Smooth pasting condition**. This condition requires that the two functions meet tangentially at the optimal thresholds. In this case, the first derivate of the switching option H must equal its termination payoff V^{SW} at the threshold values of each costs (Dumas, 1991). We have two smooth pasting conditions

$$H_{C_G}(C_G^*, C_N^*) = V_{C_G}^{SW}(C_G^*, C_N^*), \tag{17}$$

$$H_{C_N}(C_G^*, C_N^*) = V_{C_N}^{SW}(C_G^*, C_N^*). \tag{18}$$

This condition becomes

$$H_{C_G}(C_G^*, C_N^*) = 1 - s, \tag{19}$$

$$H_{C_N}(C_G^*, C_N^*) = -1. \tag{20}$$

Using the smooth pasting condition, we conclude that the optimal value C_G^* depends on C_N^*. We can not solve the Bellman equation without knowing the value of the two generation costs (C_G^*, C_N^*). In this case, an analytical solution is not available and the numerical methods are difficult to implement in a two dimensional problem. However, by adopting the homogeneity assumption we can make one dimensional problem (see Dixit and Pindyck; 1994[5]).

[5] The valuation of the option to invest under cost and price uncertainty, pp. 207-211.

4.3 Solving Bellman EDP under homogeneity assumption

Since we have formulated the boundary condition with introducing the switching cost in the two dimensional problem, we retain the homogeneity assumption to reduce our problem to one dimensional analysis.

Assumption 5. Shackleton et al. (2004) like other papers assume the natural homogeneity of this kind of problem. For this reason, we assume that the function $H(C_G, C_N)$ is homogeneous in (C_G, C_N) of degree one. Assuming homogeneity, we consider that the real option function is one degree homogeneous in . This assumption is adopted by Dixit and Pindyck (1994) to solve a two dimensional problem. The function can be rewritten as

$$H(C_G, C_N) = C_N J\left(\frac{C_G}{C_N}\right) = C_N J(\theta). \tag{21}$$

Equivalently, we have

$$J(\theta) = \frac{1}{C_N} H(C_G, C_N). \tag{22}$$

We assume that, when a decision maker have the possibility to choose between two generation alternatives, then the choice is made in term of the ratio of the relative level of the generation costs. By considering at least two operating technologies to supply electricity, the generation strategy is planned beyond the stand alone cost method and takes into account the diversity of the production technologies. In this sense, assumption 5 reduces the problem to one dimensional analysis and the optimal solution depends only on the ratio

$$\theta = \frac{C_G}{C_N}. \tag{23}$$

With the non-correlation condition, the EDP becomes

$$\frac{1}{2}\left(\sigma_G^2 + \sigma_N^2\right)\theta^2 J_{\theta\theta} + (\alpha_G - \alpha_N)\,\theta J_\theta - (\alpha_N - r)\,J = 0. \tag{24}$$

The general solution is given by

$$J(\theta) = D\theta^{\mu_1} + F\theta^{\mu_2}. \tag{25}$$

Now, we need to obtain a solution during the interval $[t, t + dt]$ with adjusting our problem to the time, in the context of risk neutral decision. We set $t = 0$ and $t + dt = T$ the time to take the decision to switch, with $0 \leq t \leq T$. In the context of risk neutrality (with r is the risk-free rate), we discount the cash flow of each project by multiplying the natural gas and nuclear power generation cost respectively by the discounting coefficient $e^{-\delta_G T}$ and $e^{-\delta_N T}$ where $\delta_G = r - \alpha_G$ and $\delta_N = r - \alpha_N$ (see appendix 8.1). The value matching represented by the equation (16) is rewritten as

$$\begin{aligned} J(\theta^*) &= \frac{1}{C_N^*} H(C_G^*, C_N^*) \\ &= \frac{(1-s)\left(1-e^{-\delta_G T}\right)\theta^*}{\delta_G} - \frac{\left(1-e^{-\delta_N T}\right)}{\delta_N}. \end{aligned} \tag{26}$$

The smooth pasting conditions are reduced to one condition given by

$$J_\theta(\theta^*) = \frac{(1-s)\left(1-e^{-\delta_G T}\right)}{\delta_G}. \tag{27}$$

By solving the EDP, the optimal ratio is given by[6]

$$\theta^* = \frac{C_G^*}{C_N^*} = \frac{\mu_1}{\mu_1 - 1} \frac{\left(1-e^{-\delta_N T}\right)\delta_G}{(1-s)\left(1-e^{-\delta_G T}\right)\delta_N}. \tag{28}$$

The root μ_1 presented by Vedenov et al (2006) :

$$\mu_1 = \frac{1}{2} - \frac{\alpha_G - \alpha_N}{\sigma^2} + \sqrt{\left(\frac{\alpha_G - \alpha_N}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(r-\alpha_N)}{\sigma^2}} \tag{29}$$

with

$$\sigma^2 = \sigma_G^2 - 2\rho\sigma_G\sigma_N + \sigma_N^2. \tag{30}$$

If we assume that $\rho = 0$ then $\sigma^2 = \sigma_G^2 + \sigma_N^2$. In fact, each time we attempt to keep the correlation coefficient in the presentation of each equation involving ρ. This is useful when we study the impact of no correlation between generating technologies. In this case, we can show the two areas when the planner have the choice between the decision to switch to the nuclear power or to continue with the current power generation strategy. The regions are separated by the threshold curve. Dixit and Pindyck (1994) define the threshold curve as a "free boundary" that represents the entire trigger level. We can define the trigger level in this context as the critical level of the natural gas operating cost C_G^*. We can, then, represent the thresholds curve as $C_G^* = \theta^* C_N$. This function can be represented as a linear curve separating the decision to switch ($C_G < \theta^* C_N$) or to continue ($C_G \geq \theta^* C_N$) in the presence of a

[6]See the proof in Appendix 2.

switching cost. After defining the two states of decision making process, we return now to the real switching option value:

$$H\left(C_G, C_N\right) = \begin{cases} D\prime\left(\mu_1, C_N\right) C_N^{\mu_1} & \text{if } C_G < \theta^* C_N, \\ Q\left(\left(1-s\right) C_G \frac{1-e^{-\delta_G T}}{\delta_G} - C_N \frac{1-e^{-\delta_N T}}{\delta_N}\right) & \text{if } C_G < \theta^* C_N. \end{cases} \tag{31}$$

$D\prime\left(\mu_1, C_N\right) C_N^{\mu_1}$ corresponds to the option to continue with the current power generation strategy that given by general solution (equation (25)) and $D\prime$ is determined by substituting (28) in the smooth pasting condition (equation (38)). Here, we can state the following proposition defining the value of the option to continue with the current generation strategy.

Proposition 2. *The monopolistic supplier will not switch from natural gas to the nuclear power if the switching option and he will not invest a switching cost K^{SW} if $C_G < \theta^* C_N$. The value of the continuing option in this case equals $H = D' C_G^{\mu_1}$, with*

$$D' = Q\left(\frac{\left(\mu_1 - 1\right)\delta_N}{\left(1-e^{-\delta_N T}\right) C_N}\right)^{\mu_1 - 1}\left(\frac{\left(1-s\right)\left(1-e^{-\delta_G T}\right)}{\mu_1 \delta_G}\right)^{\mu_1}. \tag{32}$$

We can compare the option value to the value given by (6) to find the optimal value of a power generation plan in the presence of a switching option. We conclude that the presence of multiple technological choices in the power generation planning scheme creates value that corresponds to the option to continue even when the decision maker does not switch to a different energy production technology.

5 Illustration

In order to analyze the effect of the switching cost and the impact of no correlation between production costs, our illustration proposes two operating power plants with small capacity of 300 MW each. Table 1 shows the economic and technological parameters of each generation technology adopted in this section.

	Nuclear power	Natural gas Turbine
Production cost ($/kWh)	$3\ 10^{-3}$	$5\ 10^{-3}$
Standard derivation	5%	12%
Correlation	0	0
Capacity (MW)	300	300
Production factor	90%	90%
Risk free rate (r)	8%	8%

Table 1: Economic and technological Parameters. Sources: Adopted by the authors

As it is mentioned above, the generation planning will analyzed at the time, T, of the switching decision. This allows the calculation of the thresholds defining the switching from natural gas to nuclear. We retain the length of the analysis $T = 10$ years that corresponds to the possibility to exchange natural gas by nuclear generation. We assume that the risk-free rate adopted equals to 8%. We have already proposed that the two generation technologies will be available to the production, at the same time, with the same capacity, the same yearly operating hours (8760 hours per year) and the same production factor, which equals to 90%[7]. To illustrate the impact of the switching cost on the operation decision we started with $s = 10\%$. To begin the analysis with an initial case, we synthesize the numerical assumptions about all the parameter values in Table 2

In Table 2, we assume that the expected value of the variable at any time in the future is equal to its current value, so the predicted mean drift of nuclear and gas technologies are $\alpha_G = \alpha_N = 0\%$.First, we start with this case. Next, we try to evaluate the differing impacts of each parameter on the operation decision and switching option value. The sensitivity analysis can help make sense of our model.

[7] In this study, some of the technical characteristics were omitted due to the complexity of the analysis.

	Nuclear power plant	Natural gas Turbine
Q (kWh)	270 000	270000
Operating cost ($/kWh)	$3\ 10^{-3}$	$5\ 10^{-3}$
σ	5%	15%
α	0%	0%
δ	8%	8%
s	10%	10%
ρ	0	0

Table 2: The Initial Case - Sources: authors

5.1 The Thresholds Curve

The thresholds curve defines two investment decisions. In our cases, the two regions illustrate two operation decisions: whether to continue or to switch.

5.1.1 The Correlation effects, ρ

The impact correlation coefficient is determined as the impact of the uncertainty on the switching option (Shackleton et al, 2004). The uncertainty is given by the variance indicated in (30). As we have shown, the change of the correlation coefficient affects the optimal ratio θ^* and the investment decision.

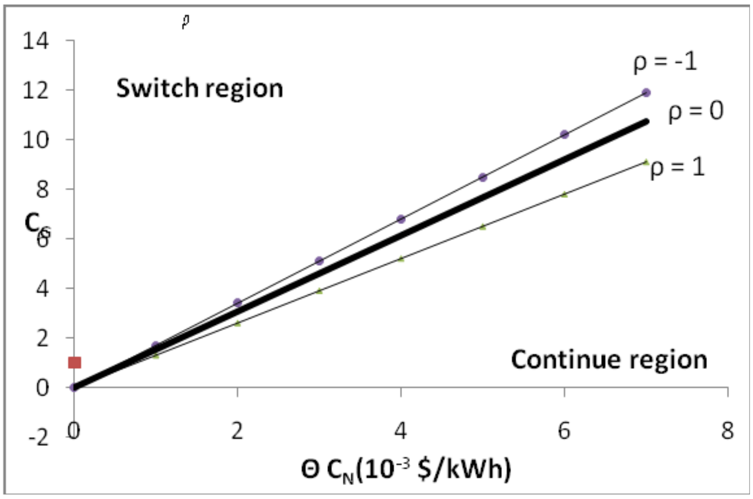

Figure 1: The impact of correlation on the investment decision

Figure 1 shows three cases. The curve in the middle represents the reference example with $\rho = 0$, when we have two assets (nuclear and gas generation) where there is no correlation between the generation costs, in the presence of a fixed switching cost. We can see that when $\rho = 1$, for example the case where there is a correlation between oil and gas technologies, this fact increases likelihood of switching option. This is logical because if ρ increases, the variance σ^2 decreases, and the optimal ratio θ^* decreases. The results are similar to those found by Shackleton et al (2004). The authors show that the increase of correlation incites the market change. In this way, when the uncertainty decreases the decision to continue becomes optimal. This result is economically sound, when the decision maker can choose the power generation technology with lower costs when there is a high correlation between the two alternatives.

In contrast, when the correlation is negative for the two power generating technologies, the uncertainty caused by a high σ^2 will require the non-exercise the switching option. We conclude that uncertainty has a positive impact on the switching option. When the correlation increases between two types of power generation technologies, the switching decision is worthwhile. Generally, this type of technology generates power with a higher power production cost. If we combined the no correlation with a lower generation cost for the nuclear energy, we can conclude that the switching decision may be considered to be optimal compared to the fossil fuel generation strategy.

5.1.2 The switching cost effects s

Now, we analyze the sensitivity of a switching decision relatively to the change in the switching cost. We considered the switching cost as a percentage of the natural gas operating cost: when it is viewed as the cash flow which is generated by switching decision.

Figure 2 shows the sensitivity of the threshold value of the natural gas generation depending on the switching cost. As we have shown, when the switching cost increases, the decision to switch is less likely and the current strategy is considered to be a worthwhile decision. This result is economically logical because the profit generated by the switch from gas to nuclear is lower. Therefore, we can conclude that a high switching cost makes the optimal ratio higher. This can have a negative impact on the switching option value.

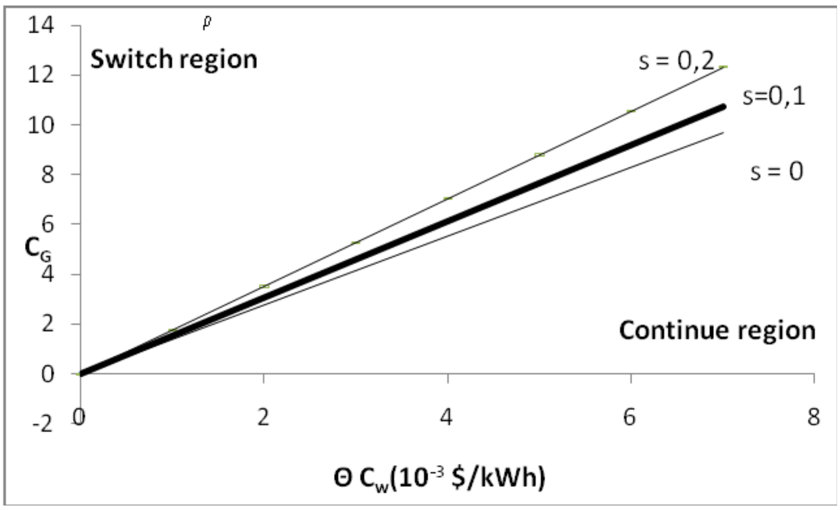

Figure 2: The impact of the switching cost on the investment decision

The main finding of the analysis is that the choice of generation technology is affected by the switching cost and the correlation level when a decision maker has two or many production mode opportunities. In the context of pre-liberalization of electric industry, the adoption of the competitiveness is based on the cost level. The adoption of the alternative technology decreases with the switching cost. Furthermore, the competitive choice increases with the correlation coefficient related to the uncertainty level.

5.2 The switching option value

In this subsection, we evaluate the value of the flexibility which is measured by the value of the switching option $H(C_G, C_N)$ to give the optimal power generation schedule. The value of the switching option is presented by (31). We keep the same parameters adopted in the initial cases and we analyze the impact of the nuclear production cost C_N changes on the value of the flexibility and the threshold value of natural gas production cost.

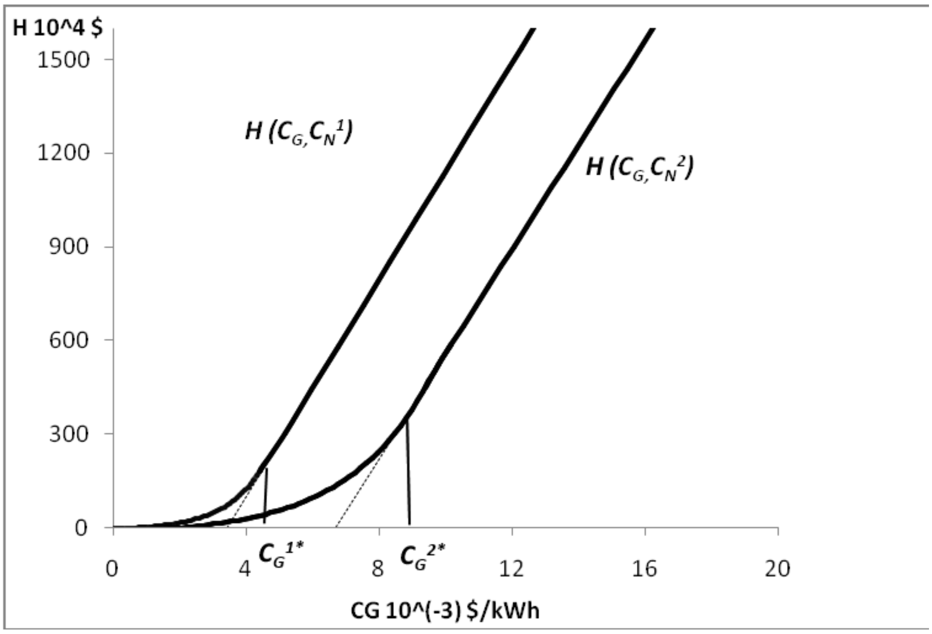

Figure 3: Value of Switching Option

Figure 3 plots the switching option as a function of natural gas and nuclear power production costs. The dense line illustrates the switching option H. As it is showed, the investment value when the planner decides to continue in the current generation strategy meets with the value of switching V^{sw} (the interrupted line) at the optimal threshold C_G^*. If $C_G \geq \theta^* C_N$, the planner exercises the switching option. In the other side, if $C_G < \theta^* C_N$ the planner prefers to retain the current fossil fuel strategy. Here, the option is out of the money and can be considered as a call option. We consider the left side of the curve, the optimal threshold is equal to 4.6\$/MWh (MegaWatt per hour) with optimal ratio $\theta^* = 1.55$. We examine that even if the gas operating cost is less than the critical value the value of the flexibility can reach $1,800,000$\$ cumulated during the length time of the analysis. Table 3 analyzes the value of flexibility added to the static valuation in term of the variation of cost level.

C_N =3 10^{-3} \$/kWh		C_N = 6 10^{-3} \$/kWh	
C_G 10^{-3} \$/kWh	Option Value 10^4 \$	C_G 10^{-3} \$/kWh	Option Value 10^4 \$
2	10,389	2	1,693
3	45,035	3	7,339
4	127,496	4	20,778
4,4	179,981	4,4	29,332
4,6	**211,377**	4,6	34,449
6	445,986	6	90,071
7	613,230	7	157,309
8	780,475	8	254,992
9	947,720	**9,5**	**474,787**
10	1114,965	10	571,582

Table 3: The impact of increasing in CN on the option value

If C_N increases, the threshold value of C_G becomes higher. Then, if $C_N = 6\$/\text{MWh}$ the threshold value equals $9.5\$/\text{MWh}$. An increase in nuclear generation cost reduces the switching possibility. This is shown by the fact that if C_N becomes higher the value of the switching option becomes lower. For the same thresholds value of C_G the planner receives less value when he decides to switch. In the case of increasing of C_N it is better to continue with keeping the switching option. The switching act is much more valuable when the timing is right.

6 Concluding remarks

This paper discusses the optimal power generation planning based on the real switching option valuation. A decision maker, which faces the choice of generation technology in the presence of uncertain generation costs, will look into the possibility to switch to the cheaper generation technology. In this context, we present a dynamic modeling for an optimal operational planning with considering the possibility to switch from natural gas to nuclear power. We consider that only one generation technology can be adapted to supplying electricity. Using the Margrabe (1978) and Dixit and Pindyck (1994) techniques, we show that when we exchange the current asset with another alternative asset, this is equivalent to bearing the cost of the alternative operational option to earn the current cost. This analysis considers that the switching act is costly.

In this paper, we apply the valuation model to the situation where a monopolistic producer may choose between fossil fuel technology and nuclear power to generate electricity, under electricity market regulation. We use dynamic programming solution based on Bellman differential equation, under some mathematical assumptions, which depends on the ratio of the power generation costs. We formulate the switching generation decision that's defined by the generation cost threshold. This allows us to be able to plot the free boundary curve. The adoption of new energy technology is sensitive to the switching cost and the uncertainty of generation costs. The uncertainty impact is shown trough the analysis of the impact of correlation changes on the threshold curve when a decision maker has two available production mode opportunities. The opportunity cost related to the switching decision is integrated within the specification of the boundary conditions to proceed with a dynamic programming solution. This cost affects negatively the switching decision to alternative technology. When the switching cost increases, it becomes optimal to keep the current generation strategy.

The analysis of the optimal operational planning shows when the nuclear power generation cost becomes higher, then the producer will continue with the current operational strategy. In this case, the threshold value calculated based on the level of natural gas generation cost becomes higher. In our setting, we assume that the decision to switch from fossil fuel to nuclear energy is characterized by the no correlation condition between generation costs. This condition combined with a low nuclear generation cost, will be able to make the real switching option valuable. This fact can be seen as motivating factor to adopting nuclear energy.

Even if the switching decision is costly, the benefit of adopting nuclear energy can be seen. This can be explained by the compensatory effect of the positive externality (no CO2 emission) of nuclear power as a clean energy. Therefore, we can conclude that the importance of investing in this technology, in terms of value of flexibility is economically optimal, even in the presence of switching costs. The proposed tool provides a new approach to evaluate the flexibility added by the possibility to switch in the presence of the switching cost. Our methodology, applied to the power generation planning, determines the optimal switching scheme between two generation assets that may be no correlated. In this way, we show in a dynamic way the economic worth of nuclear power plant investment. The methodology can be extended in two ways: (i) the numerical analysis of the positive externality of nuclear power; (ii) the introducing of the external cost of the nuclear power, like radioactive emission, in our model.

7 References

Black F. and M. Scholes, (1973). "The Pricing of Options and Corporate Liabilities", *Journal of Political Economy*, Vol. 81, pp: 637-654.

Brennan, M.J and E.S Schwartz, (1985). "Evaluating Natural Resource Investments", *Journal of Busines*s, Vol. 58, pp: 135-157.

Chen A., J. Conover and J. Kensinger (2002). "Evaluating complex flexible processes as multiple switching options"; November 31TH Meeting of The Euro Working Group on Financial Modeling.

Dixit A., and R. Pindyck, (1994). "Investment under Uncertainty", Princeton University Press.

Dumas B., (1991). "Super contact and related optimality conditions", *Journal of Economic Dynamics and Control*, Vol. 15, pp: 675-685.

Gollier C., D. Proult, F. Thais, and G. Walgenwitz, (2005) 'Choice of nuclear power investments under price uncertainty: valuing modularity', *Energy Economics*, Vol. 27, No. 4, pp: 667–685.

Margrabe W., (1978). "The Value of an Option to Exchange One Asset for Another", *Journal of Finance* Vol. 33, pp: 177-186.

Myers S., (1977). "Determinants of corporate borrowing", *Journal of Financial Economics* , Vol. 5, pp: 147-175.

Nagayama H., (2009). "Electric power sector reform liberalization models and electric power prices in developing countries: An empirical analysis using international panel data", *Energy Economics*, Vol. 31, pp: 463–472.

Näsäkkälä E., and S.-E. Fleten, (2005), "Flexibility and Technology Choice in Gas-fired Power Plant Investments", *Review of Financial Economics*, Vol. 14, pp: 371-393.

Pindyck R., (1993). “Investments of uncertain cost”, *Journal of Financial Economics*, Vol. 34, pp: 53-76.

Roques F. A., W. J. Nuttall and D. M. Newbery (2006). “Using Probabilistic Analysis to Value Power Generation Investments under Uncertainty”, Working Paper, CWPE 0650 and EPRG 065.

Rothwell G., (2006). “A real options approach to evaluation new nuclear power plants”, *The Energy journal*, Vol. 27, No. 1, pp:37–54.

Schwartz and Smith, (2000). “Short term-variations and long-term dynamics in commodity prices.” *Management Science*, Vol. 46, pp. 893–911

Shackleton M. B., E. T. Andrianos and R. Wojakowski, (2004). “Strategic entry and market leadership in a two-player real options game”, *Journal of Banking & Finance*, Vol.28, pp:179–201.

Siclari M., and C. Giuseppe (2003). “Beyond the spark spread option-fuel switching”. Energy Risk International 2003 Year Book.

Siddiqui A., and C. Marnay, (2007). "Distributed Generation Investment by a Microgrid under Uncertainty", Research Report No. 284, Department of Statistical Science, University College London.

Stulz, R. (1982). “Options on the Minimum or the Maximum of Two Risky Assets.” J*ournal of Financial Economics*, Vol. 10, pp: 161-185.

Takizawa S., A. Suzuki (2004), Analysis of the decision to invest for constructing a nuclear power plant under regulation of electricity price. Decision Support Systems, Vol. 37, pp: 449- 456.

Takezawa N. (2001). “The Option to Switch Scheduling Priority in Manufacturing”, 5th Annual Real Options Conference.

Tseng C. L. and G. Barz, (2002). "Short-term Generation Asset Valuation: a Real Options Approach," *Operations Research*, Vol. 50, pp. 297-310.

Vedenov V., J. Duffield, and M. Wetzstein, (2006). "Entry of Alternative Fuels in a Volatile U.S. Gasoline Market", *Journal of Agricultural and Resource Economics*, Vol. 3, pp: 1-13.

8 Appendices

8.1 Appendix 1

When the cash flow generated by project is discounted at the free rate r, we have the following generalization of Dixit and Pindyck (1994):

$$E\left(C_{G,t}\right) = C_G e^{\alpha_G t}. \tag{33}$$

The discounted return during the interval $[0, T]$:

$$E\left(\int_0^T C_{G,t} e^{-rt} dt\right) = \int_0^T C_G e^{(\alpha_G - r)t} dt = C_G \frac{e^{\delta_G T} - 1}{\delta_G}. \tag{34}$$

We denote by $\delta_G = r - \alpha_G$ and $\delta_N = r - \alpha_N$.

8.2 Appendix 2

The optimal decision of the one dimensional problem:

$$J\left(\theta^*\right) = \frac{(1-s)\left(1 - e^{-\delta_G T}\right)\theta^*}{\delta_G} - \frac{\left(1 - e^{-\delta_N T}\right)}{\delta_N}. \tag{35}$$

Applying the Dixit and Pindyck (1994), the general solution of the EDP (equation (24)) is written as equation (25). Let μ_1 is the positive root of the following equation

$$\frac{1}{2}\sigma^2 (\mu - 1)\mu + (\delta_N - \delta_N)\mu - \delta_N = 0. \tag{36}$$

With the condition $J(0) = 0$, we consider that the negative F coefficient of $J(\theta)$ equals 0. By equalizing the general solution (equation (25)) to the optimal decision conditions (value matching and smooth pasting conditions) as

$$D\theta^{*\mu_1} = \frac{(1-s)\left(1 - e^{-\delta_G T}\right)\theta^*}{\delta_G} - \frac{\left(1 - e^{-\delta_N T}\right)}{\delta_N}, \tag{37}$$

$$\mu_1 D\theta^{*\mu_1 - 1} = \frac{(1-s)\left(1 - e^{-\delta_G T}\right)}{\delta_G} \tag{38}$$

with

$$D = (\mu_1 - 1)^{\mu_1 - 1}\left(\frac{(1-s)\left(1 - e^{-\delta_G T}\right)}{\mu_1 \delta_G}\right)^{\mu_1}\left(\frac{\delta_N}{1 - e^{-\delta_N T}}\right)^{\mu_1 - 1}. \tag{39}$$

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