Price Dynamics in Storage Models

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Storage in a Markov Economy

Bertrand Villeneuve
Université de Tours, CREST, FIME and CESifo

What is stored? Commodities and others

- Agricultural products

  Soy, coffee, cocoa, palm oil, sugar, wheat, corn, etc.

- Mineral products
  - Oil, natural gas
  - Metals

- Water
- Rights

Basic competitive storage logic

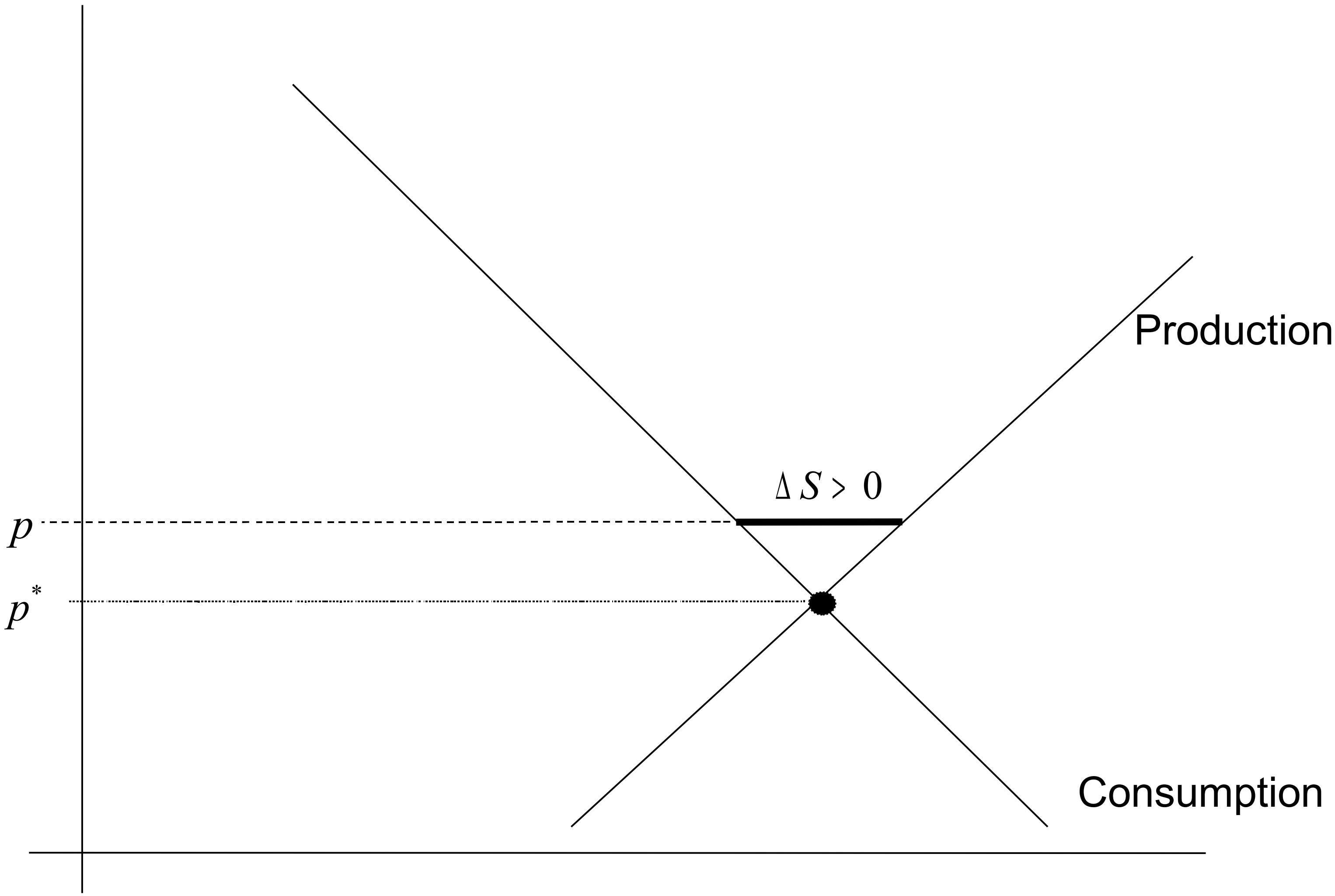
- $p_t > Ep_{t+1}$ implies no storage $S_t = 0$
- Storage $S_t > 0$ implies $p_t = E\,p_{t+1}$
- *Variations*

$$p_t + c = Ep_{t+1}$$

$$p_t = \frac{1}{1+r} Ep_{t+1}$$

$$p_t = \frac{1-d}{1+r} Ep_{t+1}$$

Excess supply function $\Delta_\sigma[p] = \Delta S$



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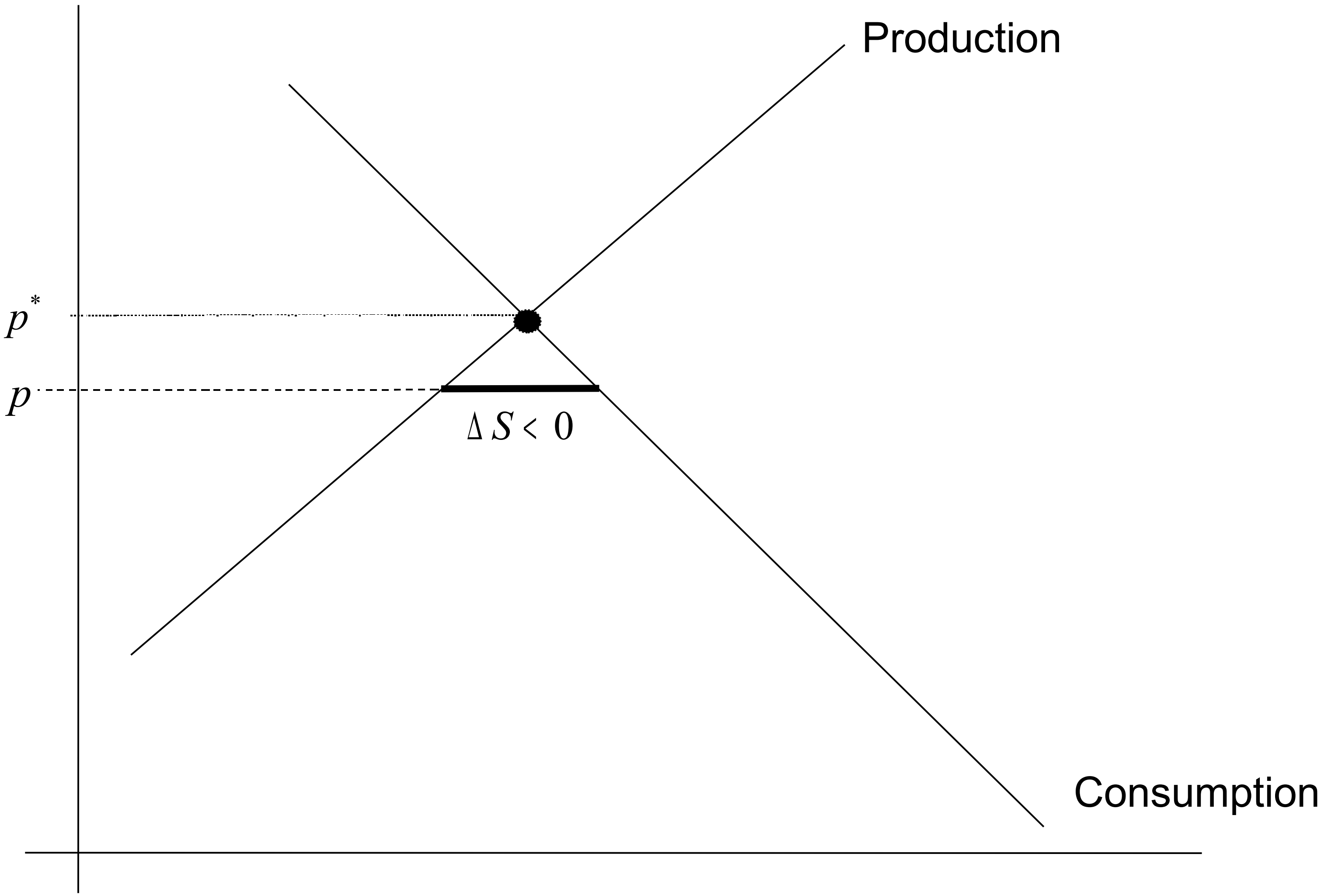
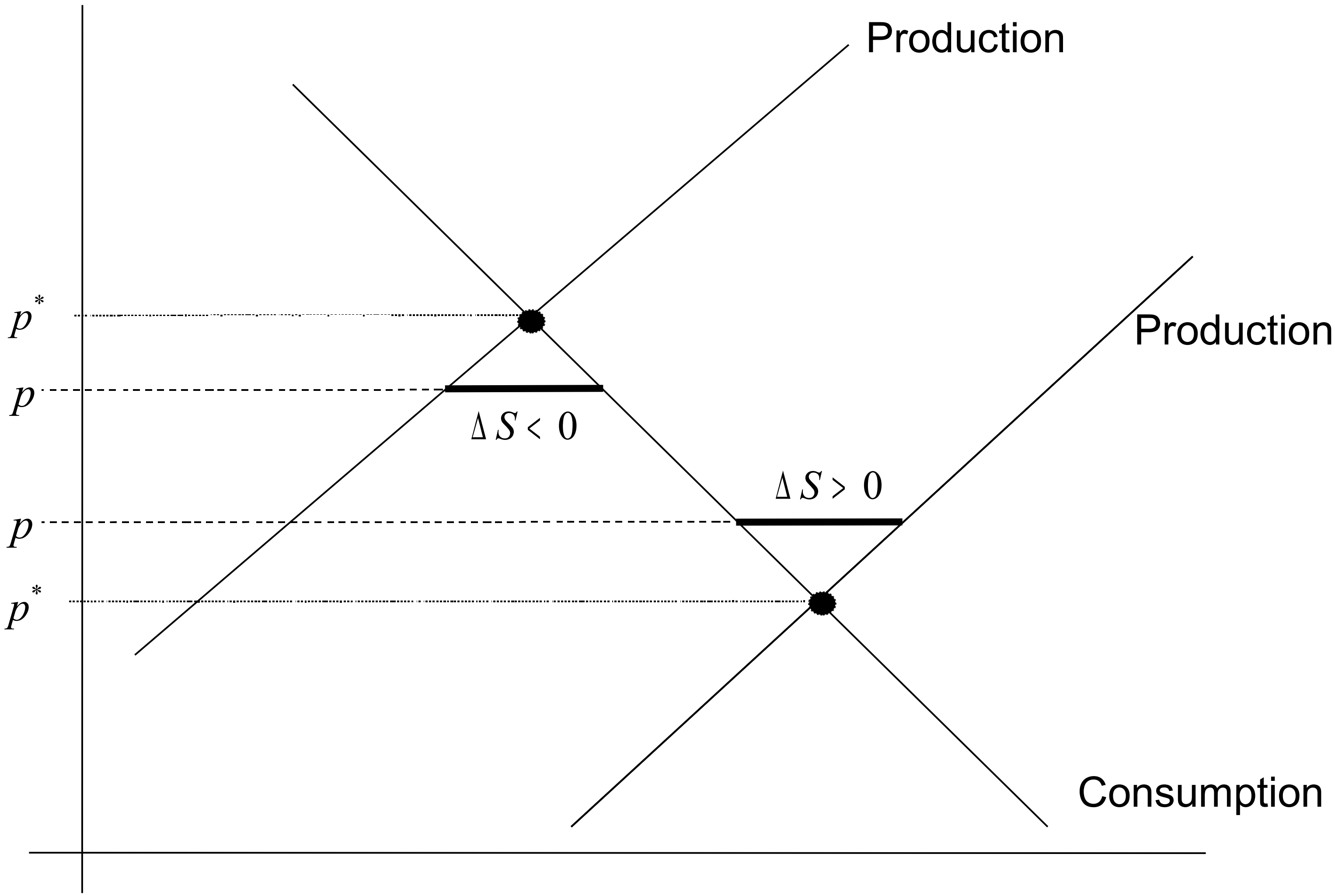


Random factor σ



The equilibrium

- Fundamental data $\Delta_{\sigma}[p]$ where σ follows some exogenous process
- Equilibrium S_0 and $p_t(S, \sigma, X)$
  - Conservation of matter $\Delta_{\sigma_t}[p_t(S_t, \sigma_t, X_t)] = S_{t+1} - S_t$
  - Rational expectations
  - No arbitrage = CRS storers
  - No negative storage constraint

- Generally Markov (time is eliminated from *p*(.))
- No bubbles

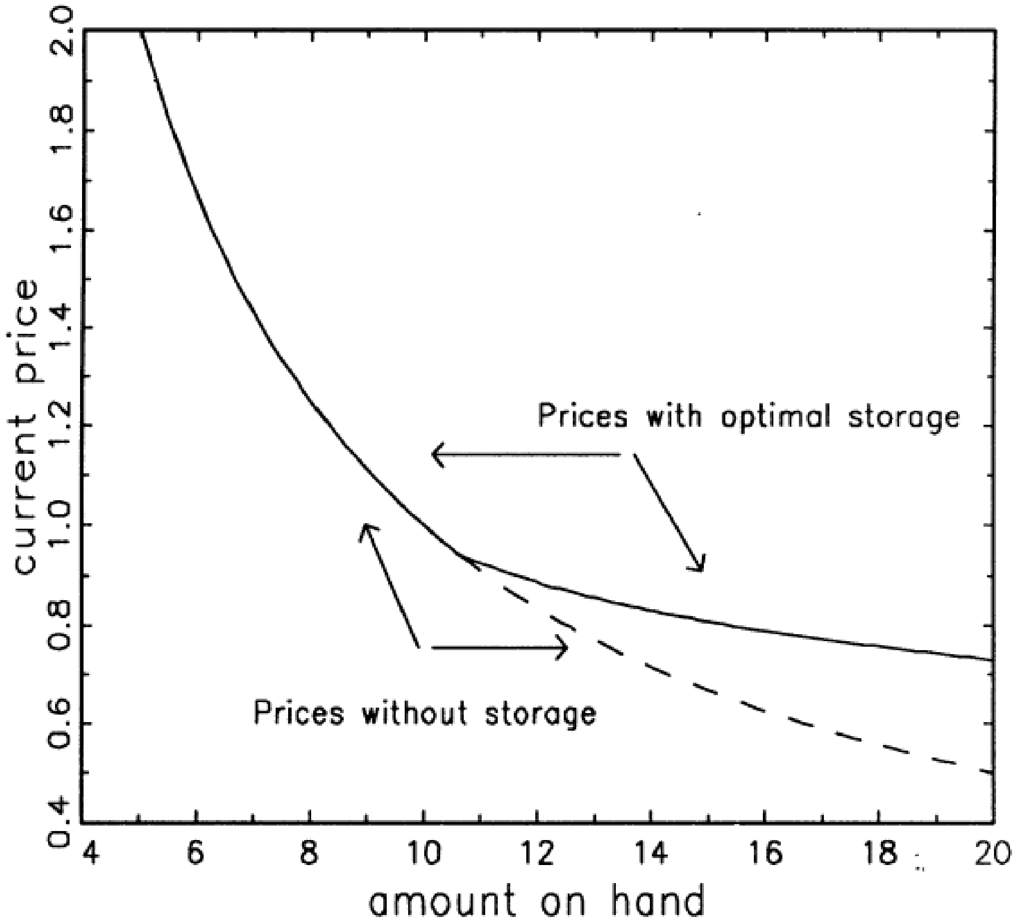


FIGURE 2

Prices with and without storage

From Deaton-Laroque RES92

Randomness source σ

- Some combination of fundamental consumption and production functions
- Typical cases
  - iid shocks, stock only state var.
  - correlated shocks
  - Tb explored: with some capital dynamics (goes further with endogeneization)

Statistics

- Stationary nonlinear process
- Pure theory
  - – price a function of former shocks
  - – iid case: future price a (random) fonction of current price
- If only prices are observed
  - – iid case: future price predicted with current only
    - Prices are serially, NL correlated
  - – AR shocks: 2 lagged prices
  - – etc.

Deaton-Laroque (RES96, JPE96, JAppE97)

- Correlation puzzle: yearly prices are highly correlated
- Does the basic storage model explain well this fact?
  - iid version: not quite
  - AR version: yes but
    - with high *exogenous* correlation
    - does not perform so much better than (linear) AR
- Ways out
  - Information on stocks
  - Capital dynamics
  - Spot and future prices

Routledge-Seppi-Spatt (JF00)

- Discrete time
- Finite set of possible shocks+Markov
- Focus on relationship between spot and future prices
  - Contango
  - Backwardation
- Convenience yield
  - Fully endogenized here (embedded option)
  - Alternative: not so convincing
- Samuelson effect

Natural Gas

- Results from

Creti (U Bocconi) and Villeneuve

- Limited diversification in Europe (Russia, Norway, Algeria)

- Precautionary Storage: supply disruptions
  - Exogenous
  - Discrete
  - Reversible

Our approach

- Dynamic model under perfect competition
  - Equilibrium = Optimum
  - Notion of target stock
  - Optimal stockpiling and drainage rules
  - Evaluation of “simple” suboptimal policies
  - “Statistical” properties of the equilibrium

Model

- Continuous time
- Exogenous random discrete state variable
  - Abundance A
  - Crisis C

- Endogenous continuous state variable S = Stocks, which depends on
  - Stockpiling rule
  - History

Random process

- A $\longrightarrow$ C : Bernoulli process

  Jump probability between *t* and *t*+*dt* = λ_C dt

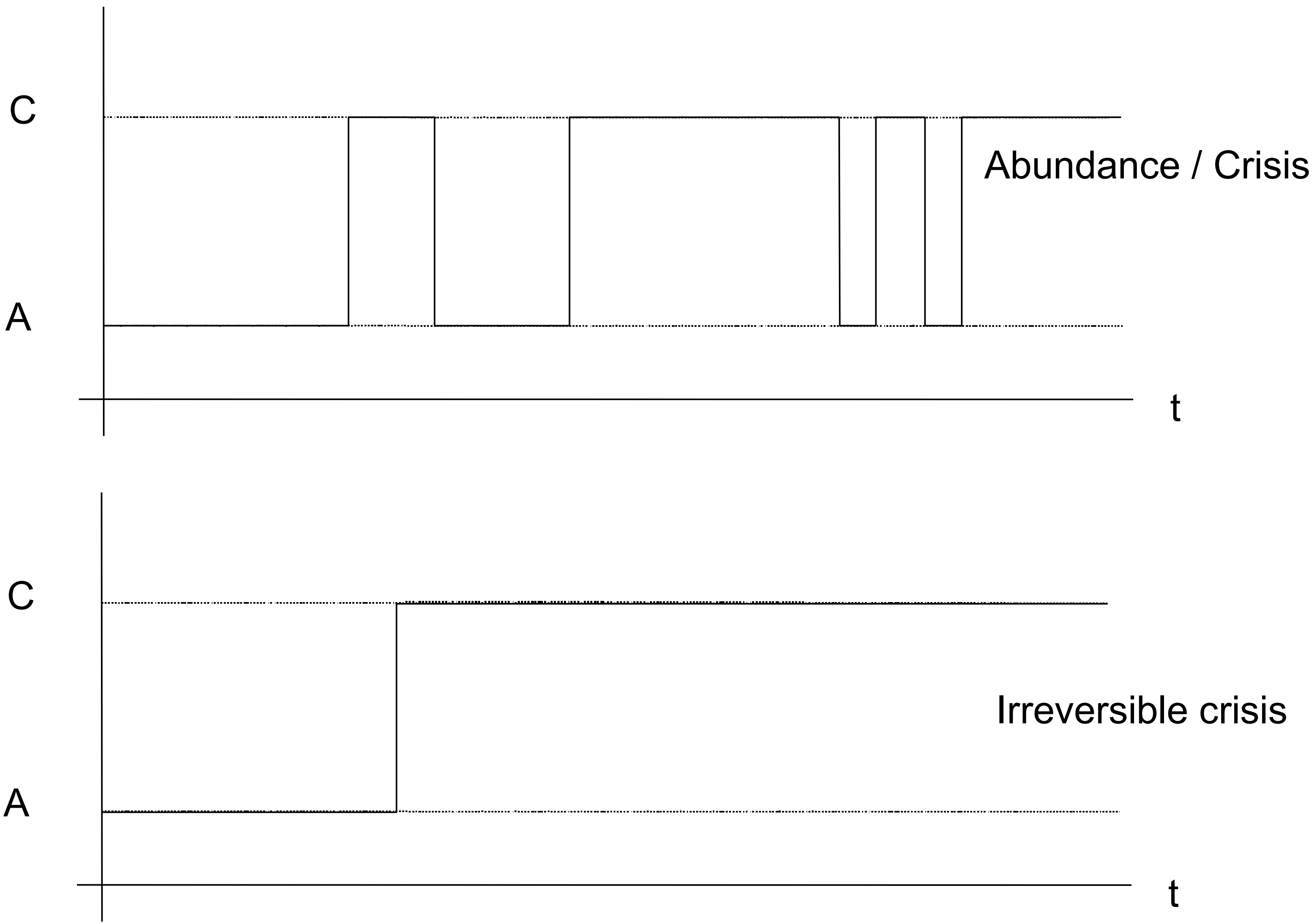
- C $\longrightarrow$ A : idem

  Jump probability between *t* and *t*+*dt* = λ_A dt

- Unconditional Prob. of state C : $\lambda_C / (\lambda_A + \lambda_C)$

- Unconditional Prob. of state A : $\lambda_A / (\lambda_A + \lambda_C)$

A possible history



Supply and demand

- *p* current price
  - Current final consumption and production only depend on *p* [strong assumption]
- Short term

  Supply = Demand

  meaning

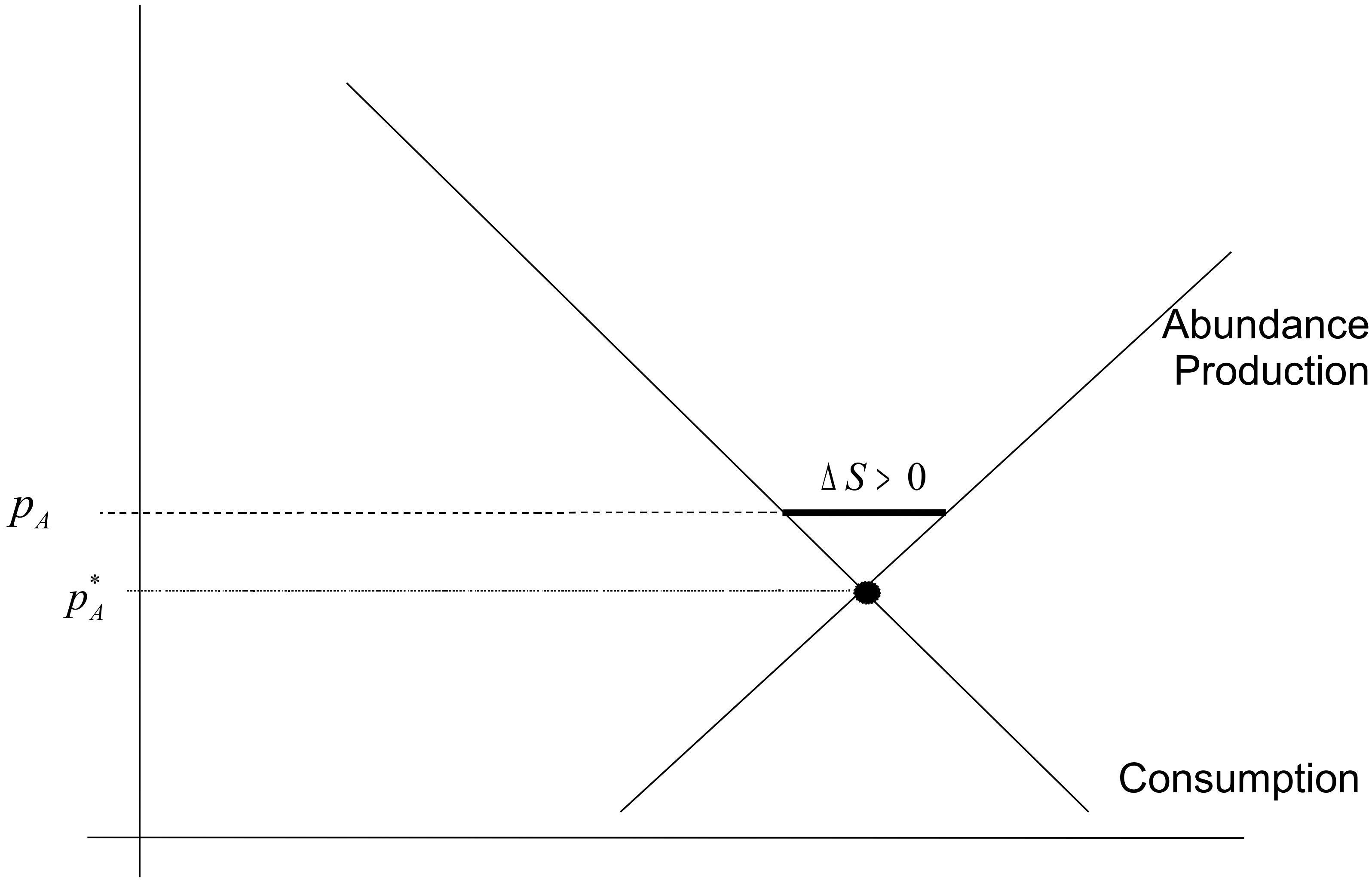
  Demand = Final consumption + Stockpiled commodities

  Supply = Production + Released commodities
- Can be summarized with Excess supply functions

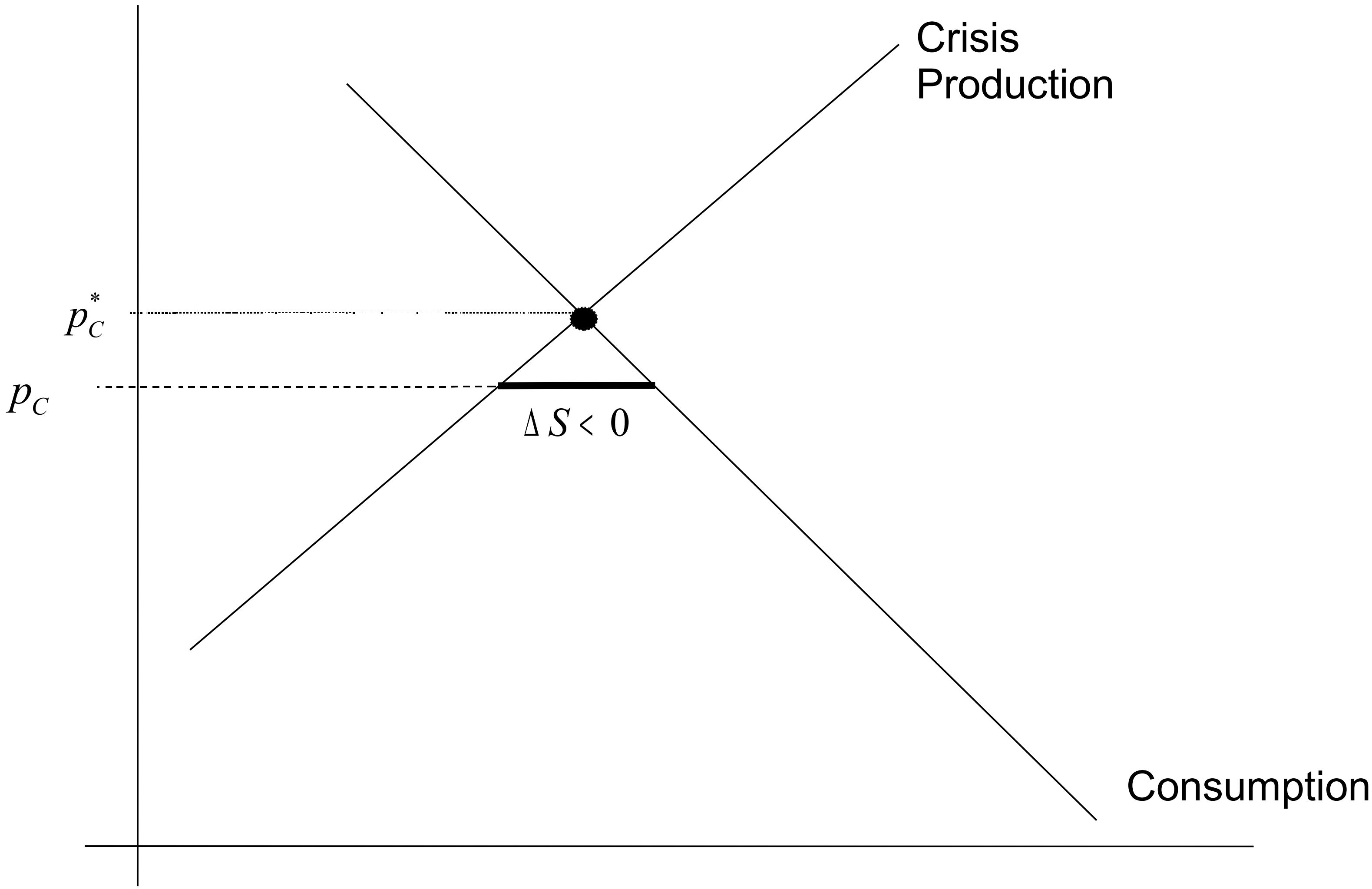
  $\Delta_C[p]$ = Stock variation if C and price *p*

  $\Delta_A[p]$ = Stock variation if A and price *p*

Excess supply: Abundance



Excess supply: Crisis



Equilibrium

- Conservation of matter
- No negative stocks
- Price taking behavior
- Competitive stockholding
- Rational expectations

Markovian equilibrium

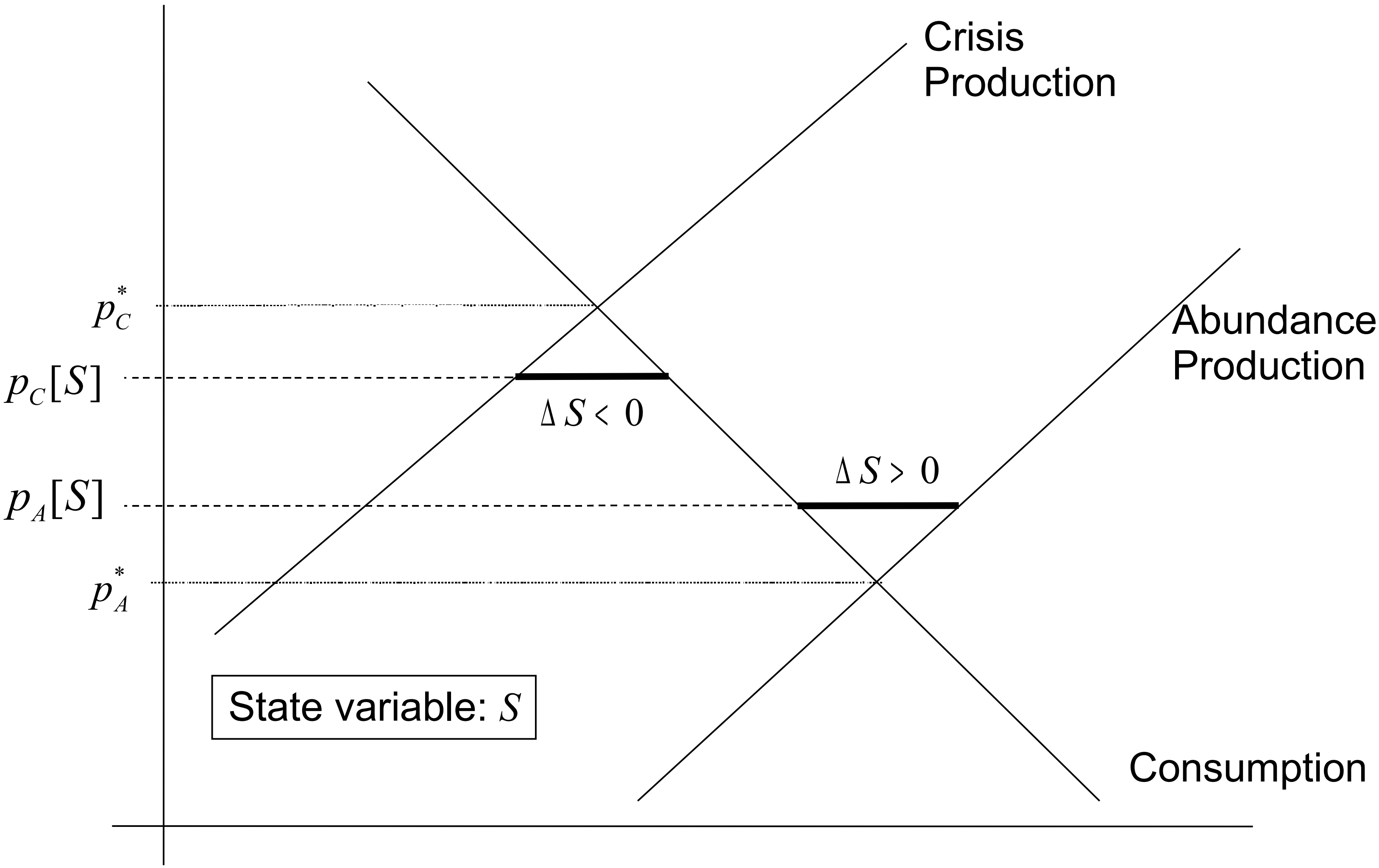
- What matters is *S* and A/C (not time per se)
- Equilibrium is a pair of functions

$$\mathbf{p_C[\mathit{S}] \text{ and } p_A[\mathit{S}]}$$

- Causes stock dynamics

$$\Delta_A[p_A[S]] \text{ and } \Delta_C[p_C[S]]$$

Supply shock



Differential equations

- Risk neutral storers bet on expected gains and losses

$$p_A[S] + c\,dt = (1 - r\,dt)\left[(1 - \lambda_C dt)\, p_A[S + dS] + (\lambda_C dt)\, p_C[S + dS]\right]$$

$$with$$

$$dS = \Delta_A[p_A]\,dt$$

+ similar equation for crisis price.
We eliminate time and we get:

$$\begin{cases} \Delta_A[p_A].\dfrac{dp_A}{dS} = (r + \lambda_C)\,p_A - \lambda_C p_C + c \\ \Delta_C[p_C].\dfrac{dp_C}{dS} = (r + \lambda_A)\,p_C - \lambda_A p_C + c \end{cases}$$

A nonlinear system +
pC[0]=p*C
pA[S*]=p*A
S* is the target stock

- No explicit solution
  - non linear system
  - Bounded Value Problem
  - boundary conditions at singular points
  - target stock is implicit

- Propositions
  - existence
  - uniqueness
  - algorithm for resolution
  - comparative statics

Parameters

Table 1: Parameter values				
Financial and physical costs	$r = .1$	$c = .1$		
Linear excess supply	$\beta_C = 1$	$p_C^* = 5$	$\beta_C = 5$	$p_A^* = 1$
Rates of jumps	$\lambda_C = 1$	$\lambda_A = 1$		

- Stocks variations

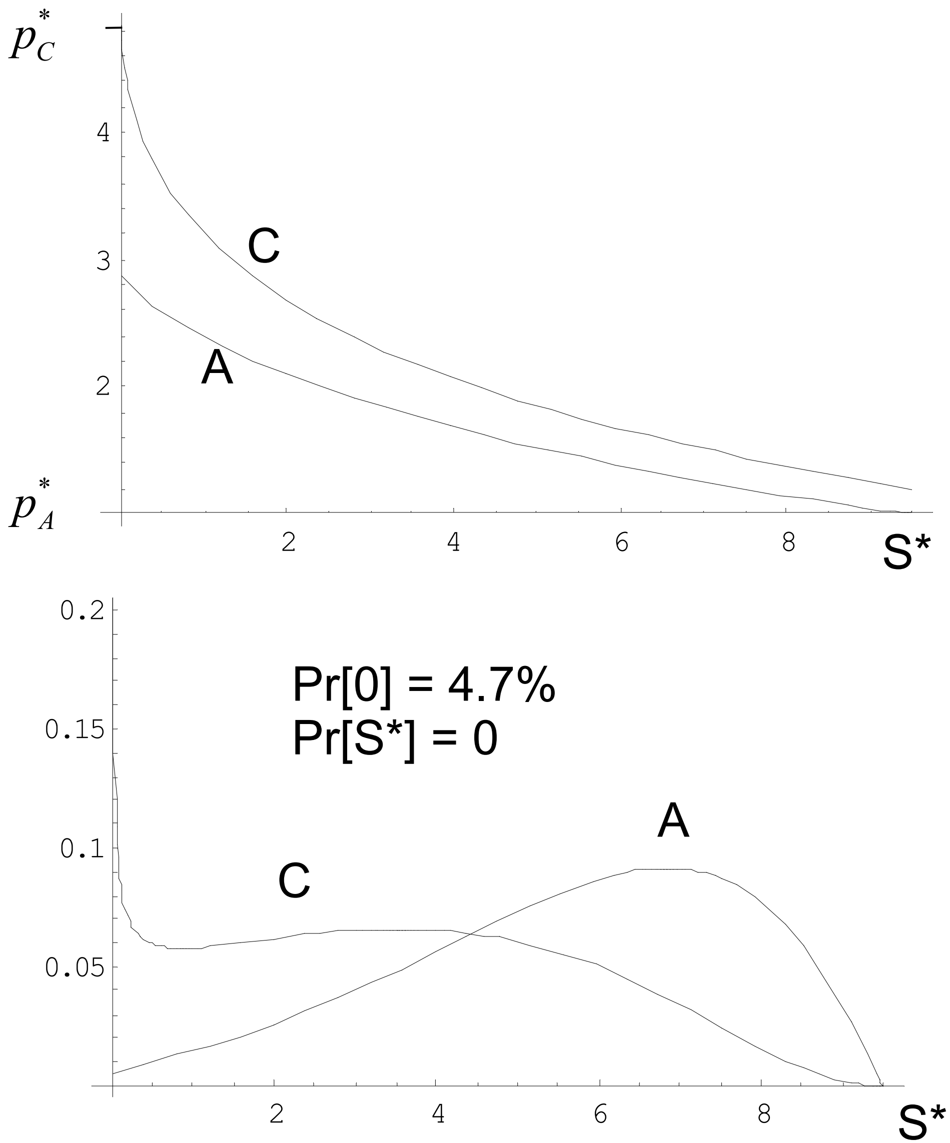
  $\Delta_C[p] = p - 5 < 0$

  $\Delta_A[p] = 5(p - 1) > 0$

=> Price between 1 and 5

=> $S^* = 9.5$

Equilibrium



Prices as
a function of the stock

State densities as
a function of the stock

Solving the equations

- Change of variables: integrate LHS in ODE

$$p_A(S) \rightarrow \; x_A = \int \Delta_A[p_A(S)].\frac{dp_A(S)}{dS}, \quad x_A(S^*) = 0$$

$$p_C(S) \rightarrow \; x_C = \int \Delta_C[p_C(S)].\frac{dp_C(S)}{dS}, \quad x_C(0) = 0$$

- System with additively separable variables

$$\begin{cases} \dfrac{dx_A}{dS} = a_{11}.g_A(x_A) + a_{12}.g_C(x_C) + a_{13} \\ \dfrac{dx_C}{dS} = a_{21}.g_A(x_A) + a_{22}.g_C(x_C) + a_{23} \end{cases}$$

Phase diagram
(*S* the underlying variable)

Linear excess supply functions

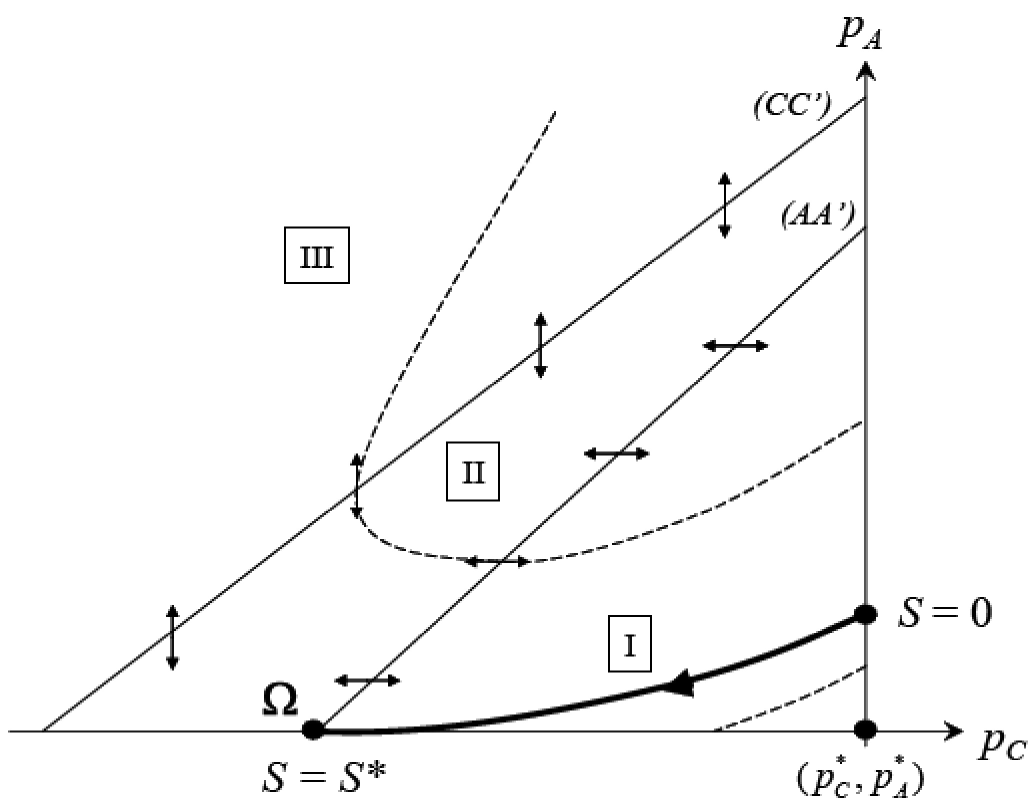
$$\Delta_A[p] = \beta_A\left(p - p_A^*\right)$$

$$\Delta_C[p] = \beta_C\left(p - p_C^*\right)$$

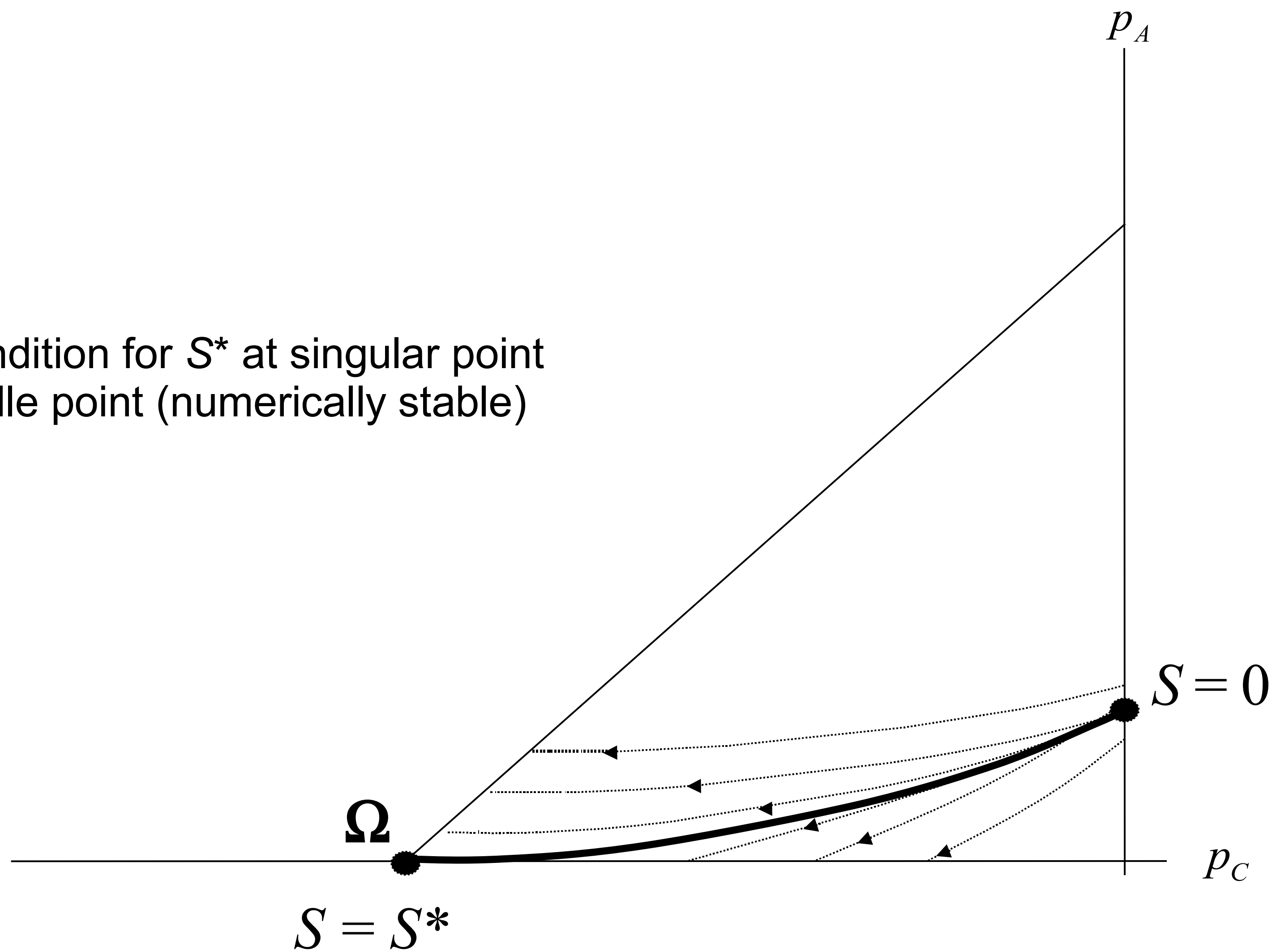
$$p_C^* > p_A^*$$

Most qualitative results are valid
for more general excess supply functions

Phase diagram



Limit condition for S^* at singular point
But saddle point (numerically stable)



- S^* increases
  - If crisis probability increases
  - If abundance probability decreases
  - If costs decrease (c or r)
  - Effects of $\Delta_A[.]$ and $\Delta_C[.]$ ambiguous
    - Level effects (e.g. depth of crisis)
    - Slope effects (e.g. flexibility during crisis)

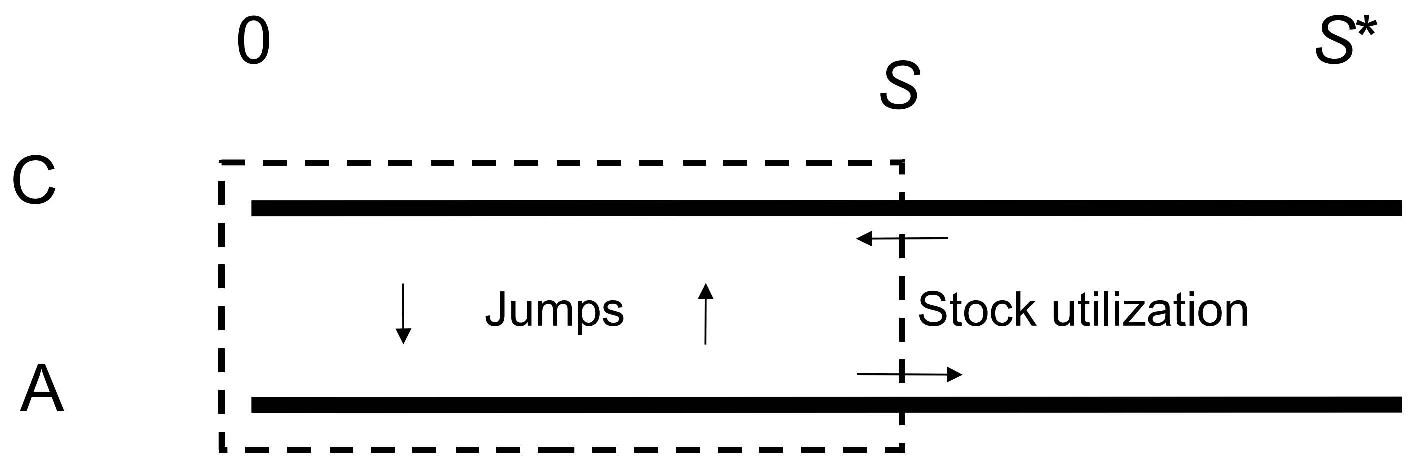
States distribution

- Densities for states A or C
- Probabilities of $S = 0$ and $S = S^*$

- Density and probability dynamics

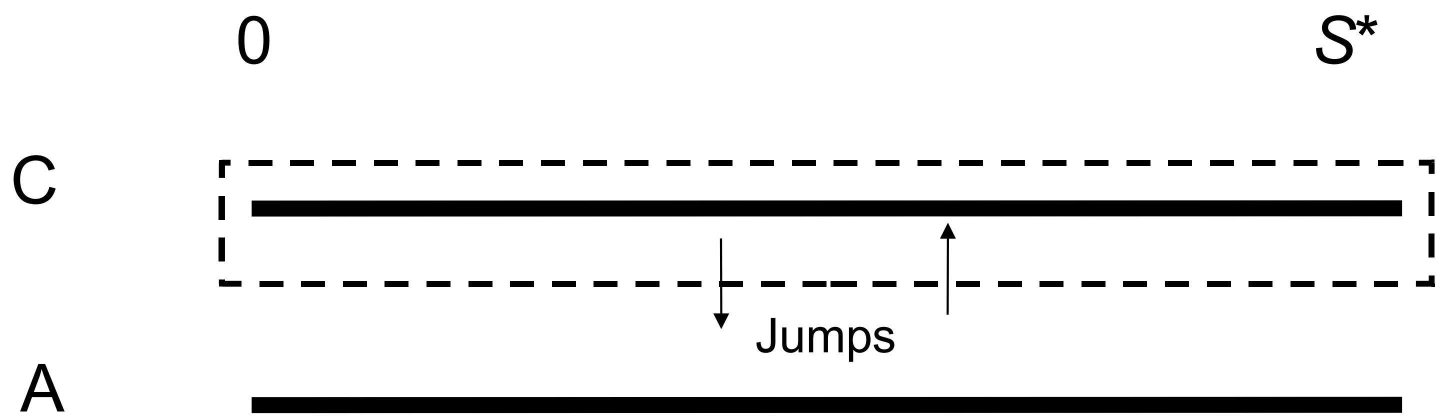
  Evolution between t and $t+dt$
- Stationary distribution

States distribution



$$\frac{d\,\Pr[S \in [0,S]]}{dt} = f_C(S,t).\Delta_C[p_C(S)] - f_A(S,t).\Delta_A[p_A(S)]$$

States distribution

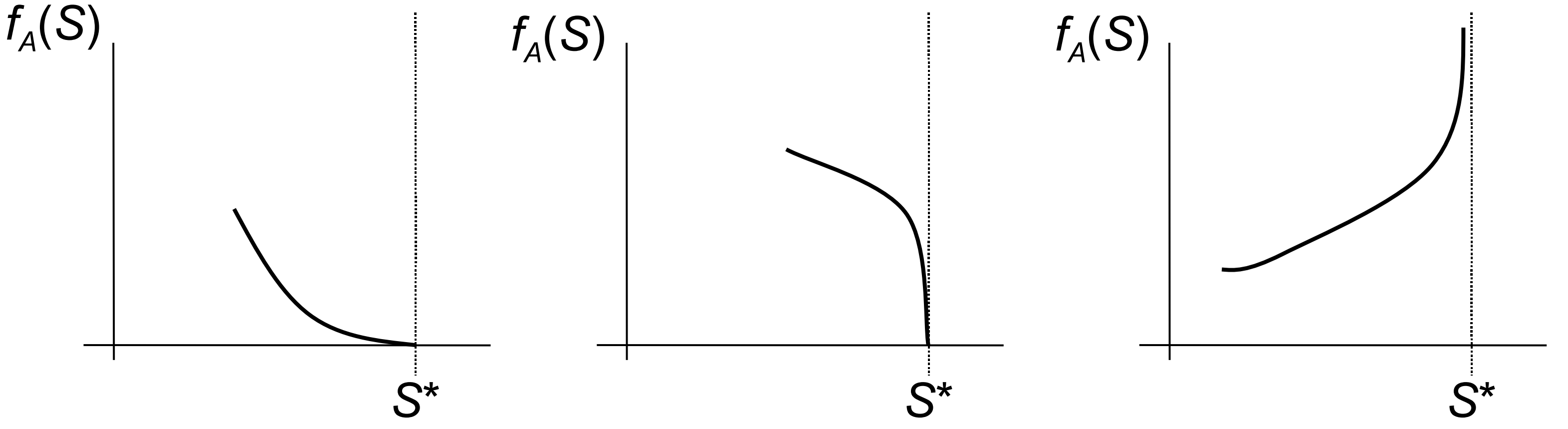


$$\frac{d\,\Pr[C]}{dt} = -\lambda_A \Pr[C] + \lambda_C \Pr[A]$$

- Gives ODE satisfied by densities

=> stationary distribution

- Explicit conditions on the shape of distribution around S = 0 and S = S^*



Extensions

- Injection and release costs
  - Nodal prices theory: different prices inside and outside the storage
  - Prices follow similar laws of motion
- Limited storage capacity
  - Changes boundary condition (upper limit on *S*)
  - Capacity constraint binds in finite time
  - Rents associated with scarcity of storage capacity (effect on value of storage capacity)
- Nonlinear excess demand

Extensions

- Price stabilization, e.g. constant price

$$p^* : \lambda_A \Delta_A[p^*] + \lambda_C \Delta_C[p^*]$$

  - If > 0 : stocks converge to infinity and price stabilized with Prob 1
  - If < 0 : stabilization fails (S = 0 with positive probability)
  - If = 0 : stocks follow a symmetric random walk

- Expropriation risk
- Market power on storage services