Modelling the dependance between oil and gas future markets

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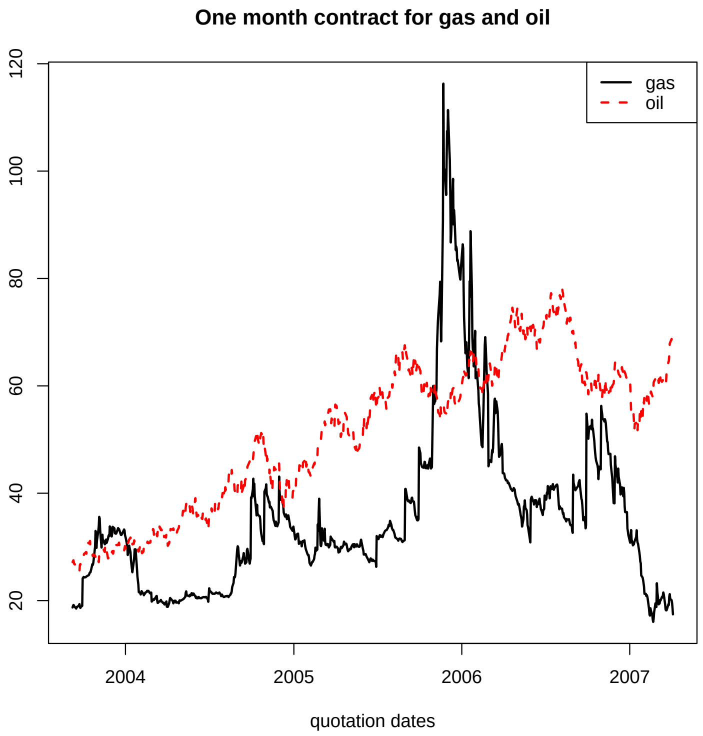
Work supported by .

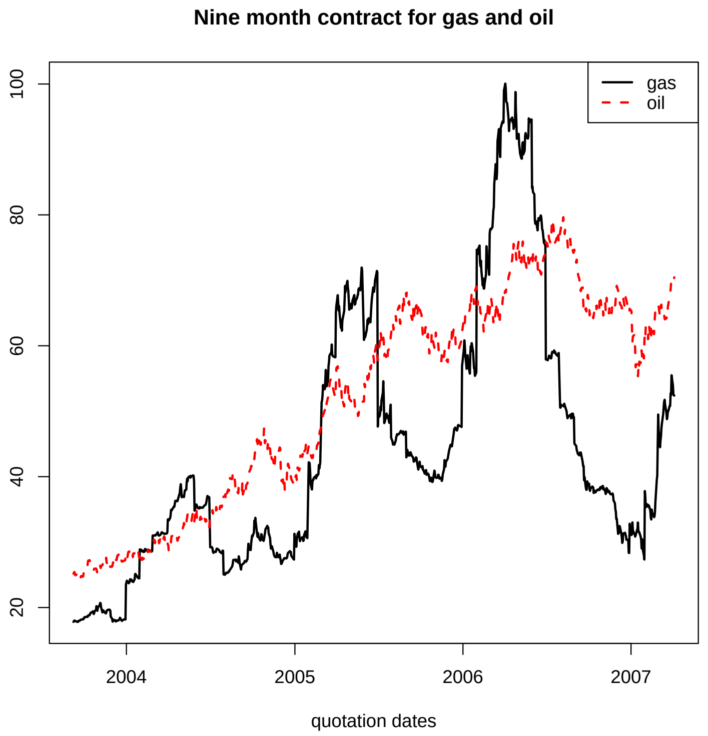
Motivation

- Need to model future prices of gas and oil simultaneously
  - to optimize energy portfolios
- Structural link between prices of both energies
  - Dependence explained with gas long term contracts (Indexed on oil and oil products prices)
  - Statistically shown

Data from ICE (InterContinentalExchange) Market

Quotations from September 8th, 2003 to April 5th, 2007.





Forward contracts delivering one unit of natural gas or crude oil in one month or nine months (p/th for natural gas and \$/bbl for crude oil).

Despite the seasonality of gas, natural gas and crude oil prices seem to have a common long term tendency.

Agenda

- Bibliography
  - Econometric models versus arbitrage free models
- Cointegration and Vectorial Error Correction Model
  - Cointegration
  - VECM
  - Cointegration and correlation
- The model
  - Dynamics under risk neutral and historical probability
  - Estimation/Calibration
- Some numerical applications
  - Simulation with and without cointegration
  - Spread option risk management

Econometric models

- Discrete time
- Interdependence between gas and oil with cointegration
- Useful for risk management purposes (Value at Risk)
- Can not be used for pricing or hedging issues (**models are not free of arbitrage**)
- Link between energy prices

References

- Asche, F., Osmunddsen, P., Sandssmark, M., 2006. The UK market for natural gas, oil and electricity: are the prices decoupled? The Energy Journal 27 (2), 27-40
- Bachmeier, L., Griffin, J., 2006. Testing for market integration: crude oil, coal, and natural gas. The Energy Journal 27 (2), 55-71.
- Panagiotidis, T., Rutledge, E., 2007. Oil and gas markets in the UK: evidence from a cointegrating approach. Energy Economics 29, 329-347.

Arbitrage free models

- In order to consider energy contracts and related pricing or hedging issues
  - Arbitrage free theory
  - Forward contracts are martingales under risk neutral probability
  - Usual factor models for spot and forward contracts

References

- Brooks, R., 2001. Value at risk applied to natural gas forward contracts. The University of Alabama, Economics, Finance and Legal Studies Working Paper Num 01-08-01.
- Clewlow, L., Strickland, C., 2000. Energy Derivatives: Pricing and Risk Management. Lacima Publications.
- Geman, H., 2005. Commodities and commodity derivatives. Modeling and Pricing for Agriculturals, Metals and Energy. Wiley Finance.

From Spot to Forward

$$F^c(t,T) = \mathbb{E}_{\mathbb{Q}}(S^c_T|\mathcal{F}_t) \qquad \text{and} \qquad F^g(t,T) = \mathbb{E}_{\mathbb{Q}}(S^g_T|\mathcal{F}_t).$$

Joint model for S^c and S^g for good dependence between energies;

- should lead to explicit representation of F^c and F^g;
- then, change $\mathbb{Q}$ to $\mathbb{P}$. What dynamics under $\mathbb{P}$?

From Forward to Spot

$$\frac{\mathrm{d}F^e(t,T)}{F^e(t,T)} = \sigma^e_T(t) \cdot dB_t$$

under $\mathbb{Q}$.

Arbitrage free models (Brooks)

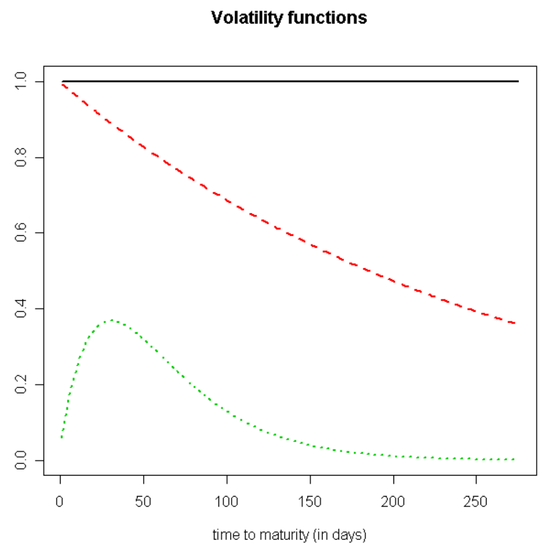
$$\frac{\mathrm{d}F^e(t,T)}{F^e(t,T)} = \sigma_1 \mathrm{d}W_t^1 + \sigma_2 e^{-\frac{T-t}{\tau_2^e}} \mathrm{d}W_t^2 + \sigma_3 \frac{T-t}{\tau_3^e} e^{-\frac{T-t}{\tau_3^e}} \mathrm{d}W_t^3$$

where

- $F(t,T)$ is the forward contract (of energy e) quoted in t and delivered in T,
- $W_t = \left(W_t^1, W_t^2, W_t^3\right)^*$ is a 3-dimensional correlated Brownians motions under risk neutral probabilities.

Refereed as LSC model (*Level*, *Slope*, *Curvature*).

- Gaussian model.
- Estimation by PCA.
- Easy to simulate (related to Ornstein-Uhlenbeck processes).

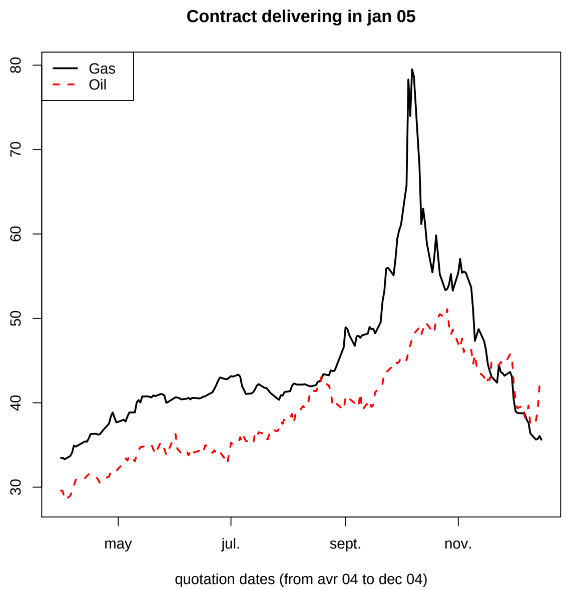


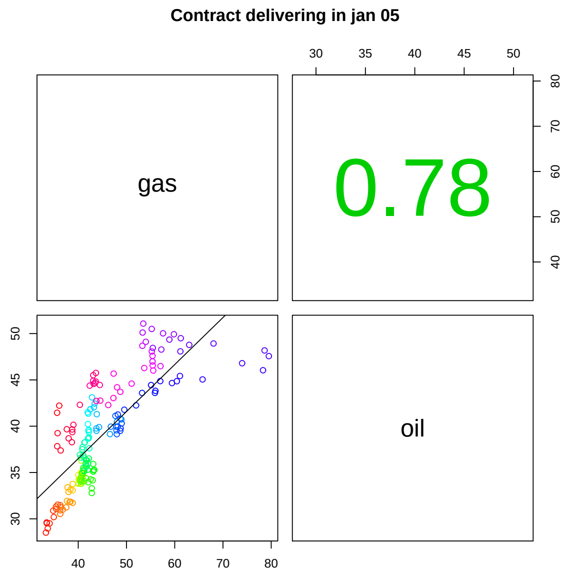
Dependence between gas and oil

⇝ Add <u>correlation</u> between Brownian motions of different energies.

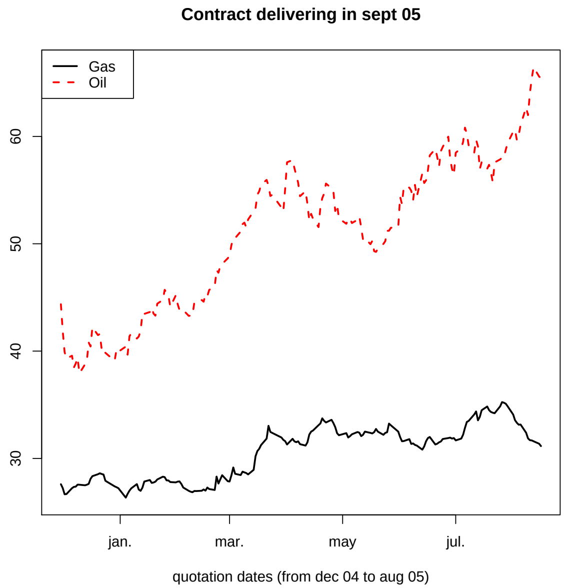
☹ It does not work!!

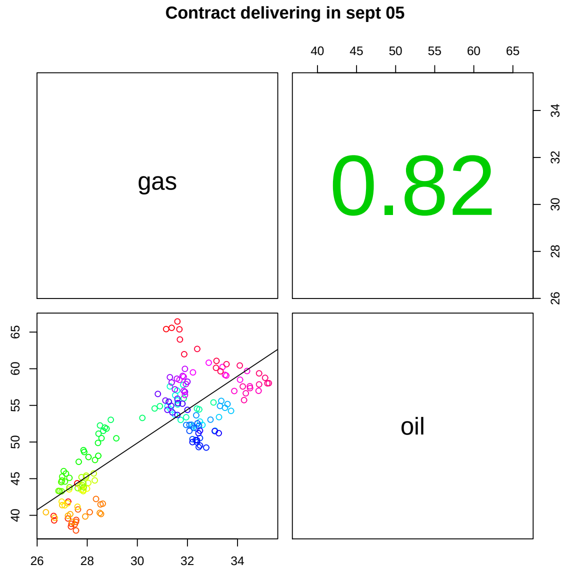
Visual proof



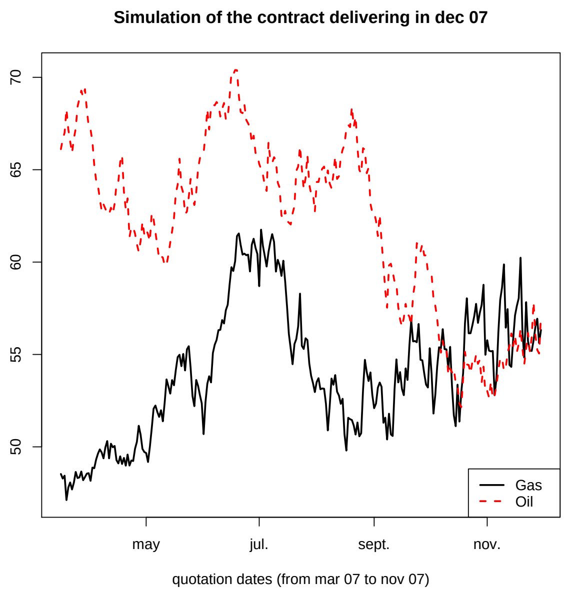


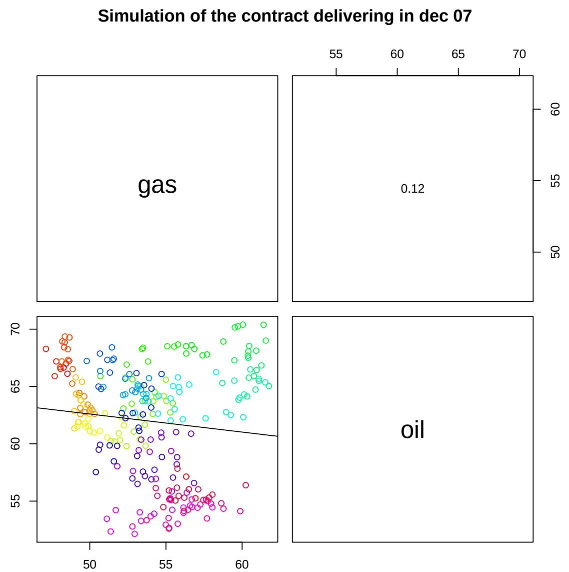
Contracts delivering in January 2005 (p/th for natural gas and \$/bbl for crude oil).





Contracts delivering in September 2005 (p/th for natural gas and $/bbl for crude oil).





Simulation for the contract delivering in December 2007 using **Brooks' model.**

Summary of Factor models

- Good adjustement of volatility (for each energy)
- Poor link between different energy prices

Strong Dependence between returns does not imply strong dependence between prices.

Our objective

- Continuous time model in gas and oil future markets
- Using cointegration
  - Tool coming from econometric models
  - Good way to model the link between gas and oil
- To be used for pricing and risk management issues (under risk-neutral probabilities and historical probabilities).

S. Ohana's PhD dissertation

- Dynamics in discrete time for the energy e:

$$\frac{\Delta F^e(t,T)}{F^e(t,T)} = e^{-k_e(T-t)}\Delta X_t^e + \Delta Y_t^e$$

- The vector $(\Delta X_t^g, \Delta Y_t^g, \Delta X_t^c, \Delta Y_t^c)$ is a sum of
  - A drift
  - A constant
  - A term related to the present
  - A term related to the past
  - A noise

Reference

- Ohana, S., 2006. Deux contributions en gestion des risques de matières premières. Ph.D. thesis, CEREG, Université de Dauphine, Paris, France.

Cointegration and Vectorial Error Correction Model (VECM)

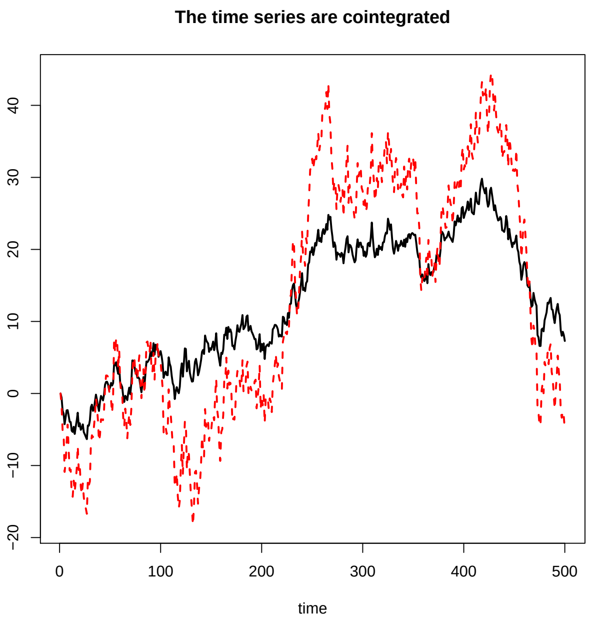
- **Definition:** A time series vector $\{y_t = (y_t^1, \cdots, y_t^n) : t \in \mathbb{N}\}$ is said to be cointegrated if
  - Each series (y_t^i) is integrated with an order of integration equal to 1;
  - Some linear combination of the series $\alpha \cdot y_t$ is stationary.
- The series $\alpha \cdot y_t$ is usually called the long term equilibrium.
- The Phillips-Ouliaris' test allows us to put the cointegration forward.

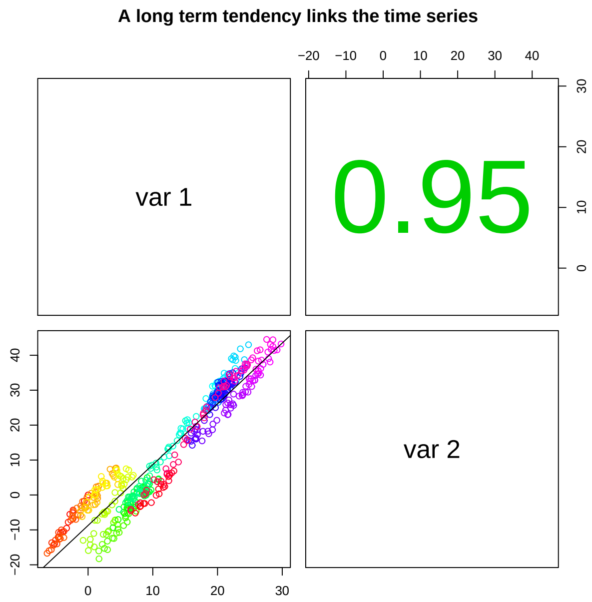
Example based on Hamilton (Time series Analysis) (1/2)

$$\begin{cases} \Delta y_t^1 = \Delta W_t^1, \\ y_t^2 = 2y_t^1 + \Delta W_{t-1}^2, \\ y_0^1 = y_0^2 = 0. \end{cases}$$

Take for $(\Delta W_t^1, \Delta W_t^2)$ uncorrelated white noise process.

Then (y_t^1, y_t^2) are cointegrated.





Example based on Hamilton (Time series Analysis) (2/2)

$$\begin{cases} \Delta y_t^1 = \Delta W_t^1, \\ y_t^2 = 2y_t^1 + \Delta W_{t-1}^2, \\ y_0^1 = y_0^2 = 0. \end{cases}$$

The model can be rewritten:

$$\begin{cases} \Delta y_t^1 = \Delta W_t^1, \\ \Delta y_t^2 = 2y_t^1 - y_t^2 + 2\Delta W_t^1 + \Delta W_t^2. \end{cases}$$

or under a vectorial form

$$\Delta y_t = \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{\text{mean reverting speed}} \underbrace{\begin{pmatrix} 2 & -1 \end{pmatrix} y_t}_{\text{long term equilibrium}} + \underbrace{\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}}_{\text{short term correlation}} \Delta W_t.$$

Vectorial Error Correction Model

- General form (without lag)

$$\Delta y_t = \Pi y_t \Delta_t + \Sigma \Delta W_t.$$

- Continuous time form:

$$\mathrm{d}y_t = \Pi y_t \mathrm{d}t + \Sigma \mathrm{d}W_t.$$

It is in the class of non stationnary Ornstein-Uhlenbeck processes.

The matrix Π has not a full rank (the rank is related to the number of long term relations).

Cointegration $\neq$ correlation

⚠ If we take W^1 and W^2 two correlated Brownian motions (say $\rho = 0.7$), then although well correlated, the paths are not cointegrated (the Phillips-Ouliaris test is negative).

Our model - Under $\mathbb{Q}$

We start from a model under the risk-neutral probability $\mathbb{Q}$ $\rightsquigarrow$ **the model is arbitrage free.**

$$\begin{cases} \frac{\mathrm{d}F^g(t,T)}{F^g(t,T)} = \sigma^g(T-t)\mathrm{d}X_t, \\ \frac{\mathrm{d}F^c(t,T)}{F^c(t,T)} = \sigma^c(T-t)\mathrm{d}X_t, \\ \mathrm{d}X_t = \Sigma \mathrm{d}B_t, \end{cases}$$

where we set

- N^e for the factors' number for the energy e;
- $B_t = (B_t^1, \ldots, B_t^{N^g+N^c})^*$ for independant $\mathbb{Q}$-Brownian motions;
- Σ for a non-degenerate $(N^g + N^c) \times (N^g + N^c)$ matrix (to correlate returns);
- $(\sigma_i^e(T-t))_i$ for the set of normalized volatility functions for the energy e;
- $\sigma^g(T-t)$ and $\sigma^c(T-t)$ for the volatilities of each energy as $(N^g + N^c)$-dimensional row vectors defined by

$$\sigma^g(.) = (\sigma_1^g, \ldots, \sigma_{N^g}^g, 0, \ldots, 0)(.), \qquad \sigma^c(.) = (0, \ldots, 0, \sigma_1^c, \ldots, \sigma_{N^c}^c)(.).$$

Our model - Under $\mathbb{P}$

The passage from the neutral risk world to the historical one is made by modeling the market of risk $(\lambda_t)_t$:

$$B_t = W_t + \int_0^t \lambda_s \mathrm{d}s,$$

$$\lambda_{\mathbf{t}} = \mathbf{\Sigma}^{-1}[\mathbf{\Pi X_t} + \eta_{\mathbf{t}}].$$

Hence, under $\mathbb{P}$, X has a VECM dynamics

$$\mathrm{d}\mathbf{X_t} = \mathbf{\Pi X_t}\mathrm{d}\mathbf{t} + \mathbf{\Sigma}\mathrm{d}\mathbf{W_t} + \eta_{\mathbf{t}}\mathrm{d}\mathbf{t}.$$

The motions driving oil and gas are cointegrated.

Model's parameters:

- The functions σ^g and σ^c for the volatilities.
- The matrix Σ for the short-term correlation.
- The matrix Π driving the long-term relation.
- The time dependent function $(\eta_t)_t$?

Estimation in 5 steps

1. Principal Components analysis (PCA) on the returns (they are computed by $\frac{F^e(t+1 \text{ day}, T) - F^e(t,T)}{F^e(t,T)}$).
2. Estimation of the parameters τ_i^e $(i = 1, 2)$ with a nonlinear regression between $1, e^{-\frac{T-t}{\tau_1^e}}, \frac{T-t}{\tau_2^e} e^{-\frac{T-t}{\tau_2^e}}$ and the motions X deduced from the PCA.
3. Reconstruction of the differences $\mathrm{d}X_t^e$ at each time step with linear regression between the volatility functions vector and the returns vector.
4. Linear regressions between (ΔX_t) and (X_t) to determine the matrices Π and $\Sigma\Sigma^*$.
5. Computation of the function $(\eta_t)_t$?

   This is the constant term in the previous linear regression.

   In practice, this term is not estimated accurately.

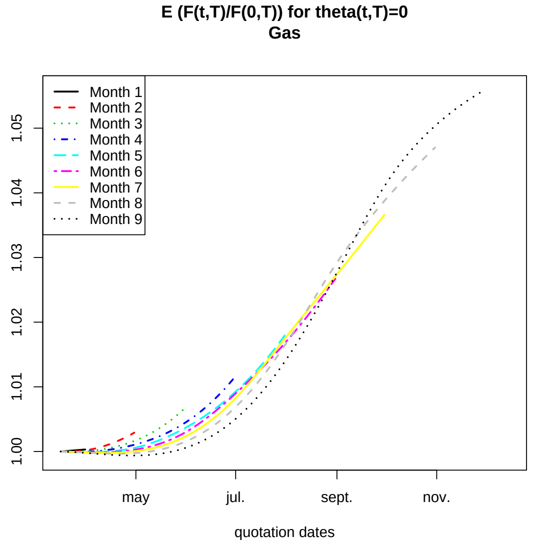
Adjustement of η_t

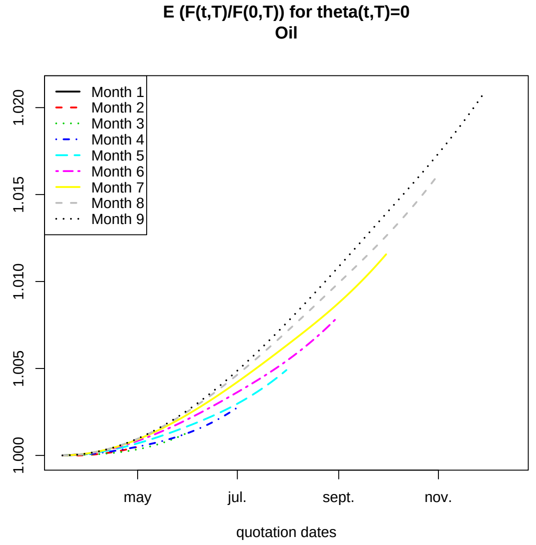
η_t can be chosen to adjust the expected forward curve $\mathbb{E}_{\mathbb{P}}(F^e(t,T))$ for any t and T.

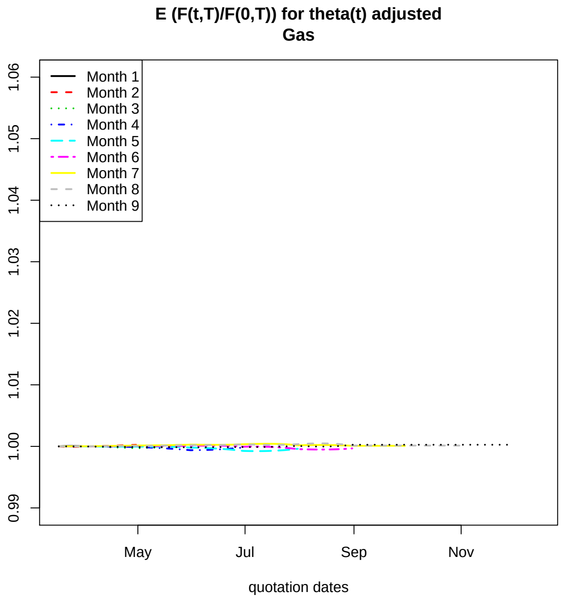
Exemple: one can center the expected forward curve:

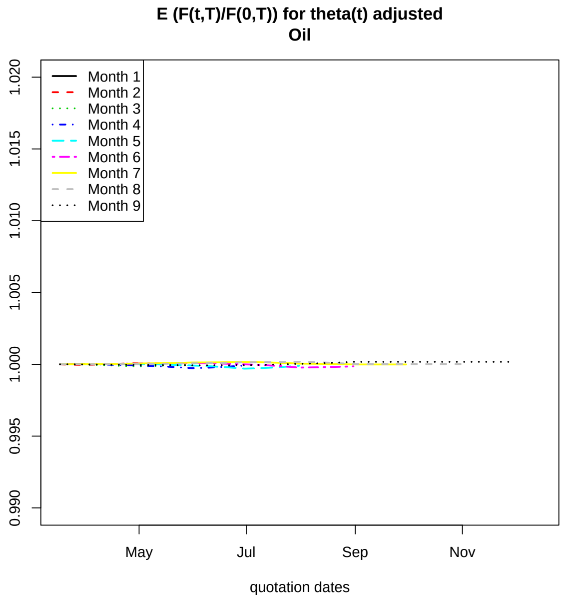
$$\mathbb{E}_{\mathbb{P}}(F^e(t,T)) = F^e(0,T).$$

Can be made explicitly since the model is Gaussian (formula avalaible).

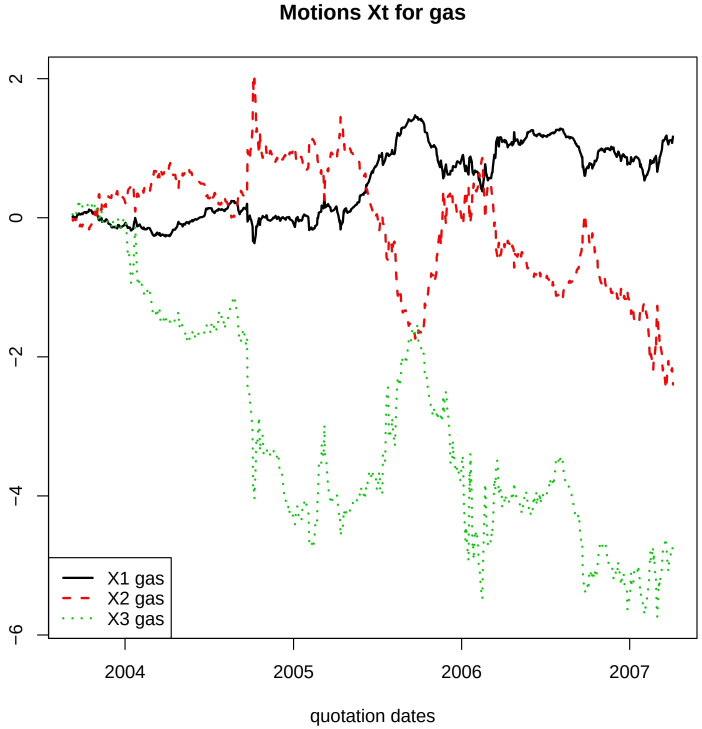

E (F(t,T)/F(0,T)) for theta(t,T)=0
Gas
Month 1
Month 2
Month 3
Month 4
Month 5
Month 6
Month 7
Month 8
Month 9
1.00
1.01
1.02
1.03
1.04
1.05
may
jul.
sept.
nov.
quotation dates

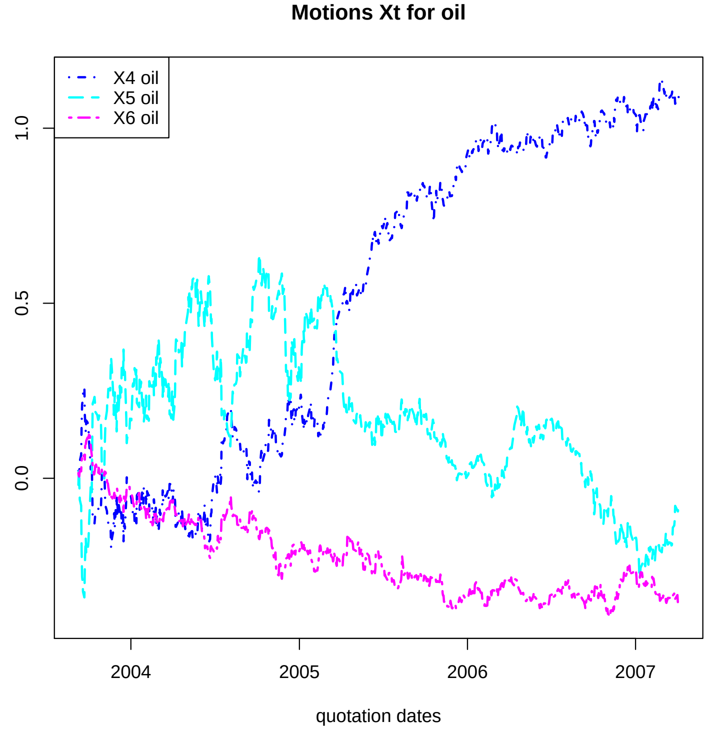

E (F(t,T)/F(0,T)) for theta(t,T)=0
Oil
Month 1
Month 2
Month 3
Month 4
Month 5
Month 6
Month 7
Month 8
Month 9
1.000
1.005
1.010
1.015
1.020
may
jul.
sept.
nov.
quotation dates


E (F(t,T)/F(0,T)) for theta(t) adjusted
Gas
Month 1
Month 2
Month 3
Month 4
Month 5
Month 6
Month 7
Month 8
Month 9
0.99
1.00
1.01
1.02
1.03
1.04
1.05
1.06
May
Jul
Sep
Nov
quotation dates


E (F(t,T)/F(0,T)) for theta(t) adjusted
Oil
Month 1
Month 2
Month 3
Month 4
Month 5
Month 6
Month 7
Month 8
Month 9
0.990
0.995
1.000
1.005
1.010
1.015
1.020
May
Jul
Sep
Nov
quotation dates

Numerical results

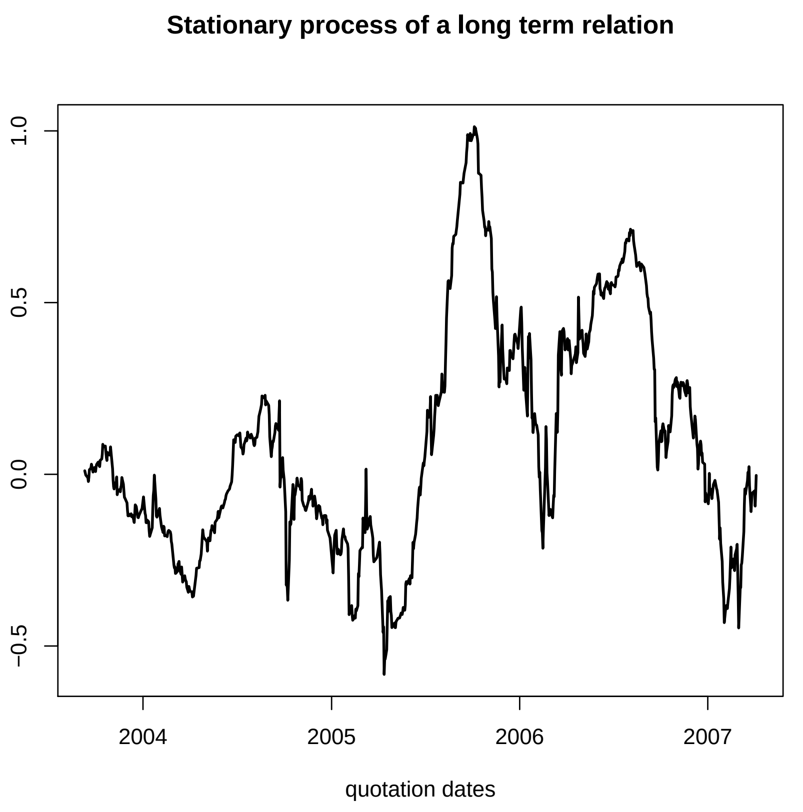




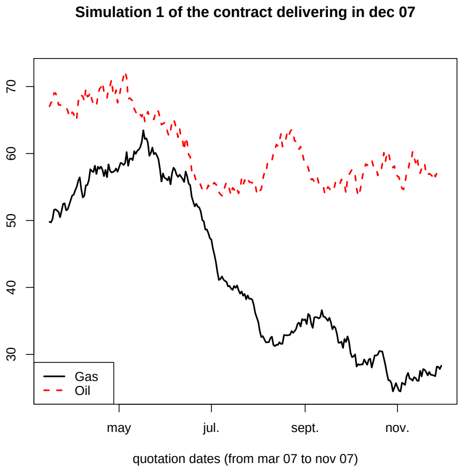
Motions $(X_t)_t$ for natural gas and crude oil.

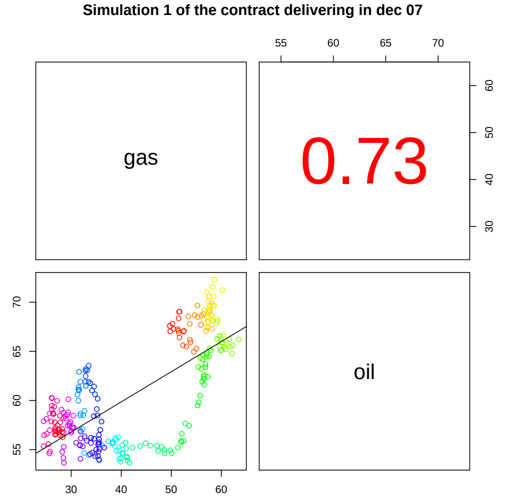
According to the Phillips-Ouliaris cointegration tests, the motions $(X_t)_t$ are cointegrated.

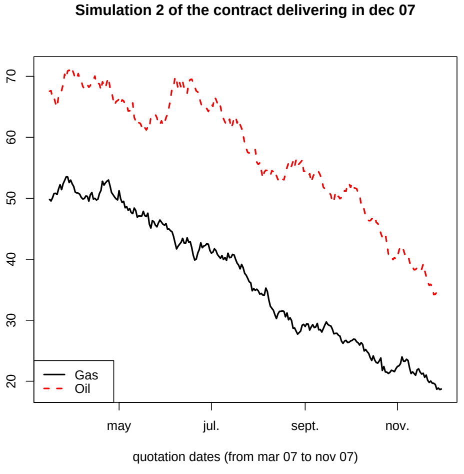
The long term relation presented here is $X_t^1 - 0.97X_t^4 = u_t$ where (u_t) is a stationnay process.

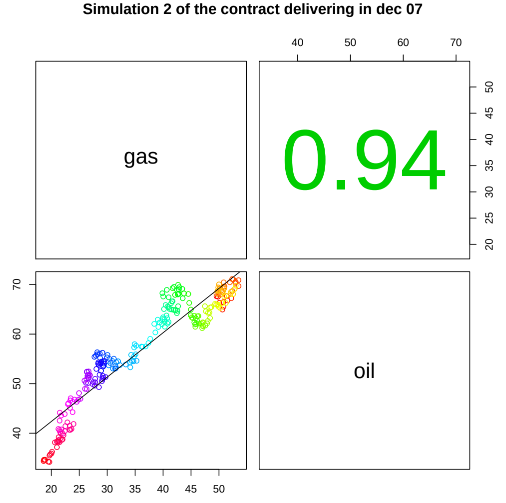


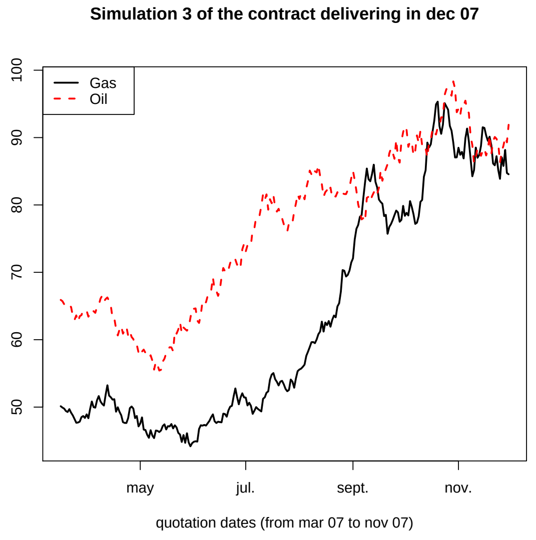
Examples of simulations for contracts delivering in one month

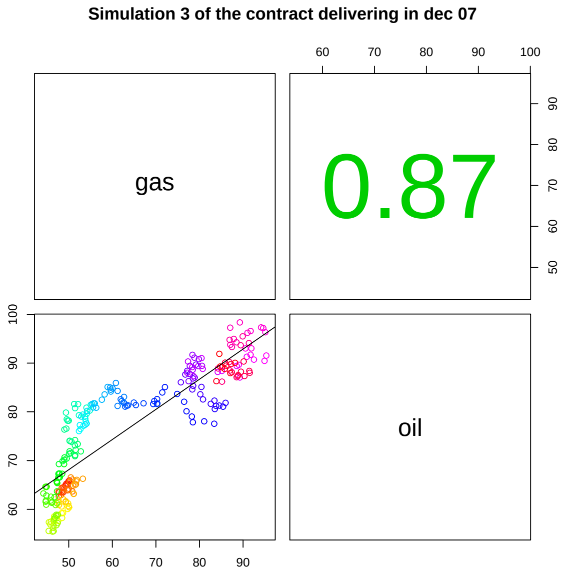












The model conveys the long term relation between natural gas and crude oil contracts.

Application to risk management

Spread option:

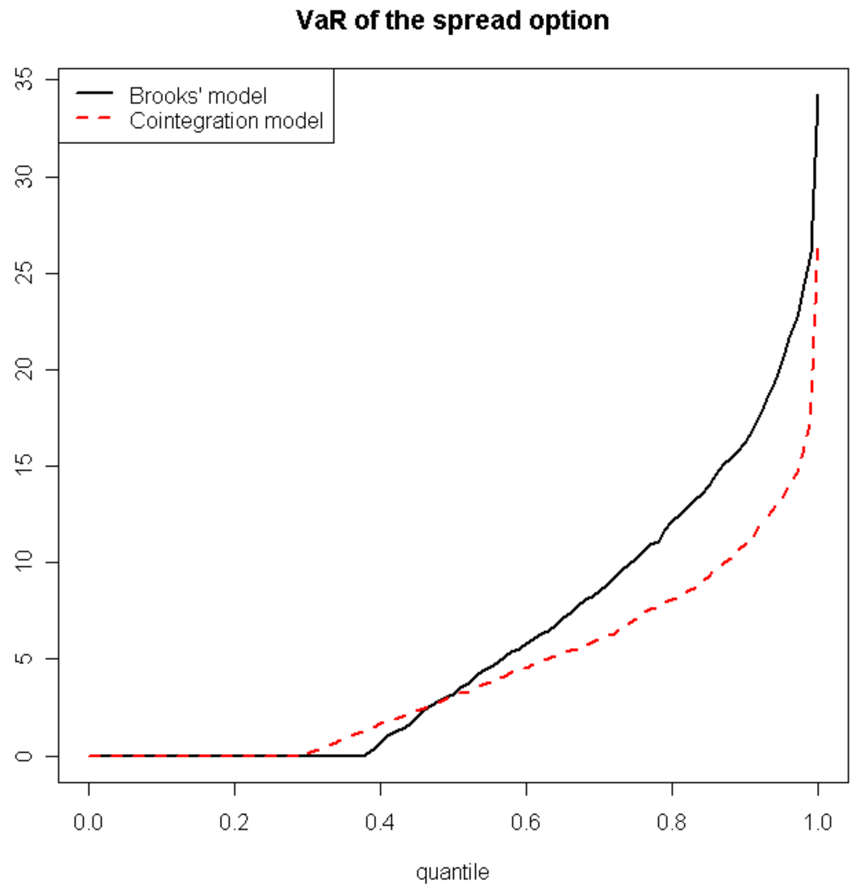
$$VaR_\alpha((F^g(t,T) - aF^c(t,T) + b)_+)$$

where

- $t = 15/03/07$
- $t = 15/06/07$
- $a = 0.9$ and $b = -20$

Simple approach for the risk management of a long term contract.

VaR Result



- Brooks' model: VaR(95%)=20.4
- This model: VaR(95%)=13.4

Conclusion

- Continuous time model for gas and oil future markets
  - Long term link
  - Free of arbitrage
- This model can be used for Risk management (Value at Risk,)
  Coherent with a risk neutral model for option pricing
- Possible improvements
  - By incorporating derivatives prices in the calibration set
  - By adding seasonality in volatility
  - Especially regarding gas