Limits of Limits of Arbitrage
Theory and Evidence

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motivation 1/2

- basic question: **why/when do asset prices deviate from fundamentals?**
- economic literature on "limits to arbitrage" (Shleifer&Vishny, Gromb&Vayanos, Brunnermeier&Pedersen)
  1. prices deviate
  2. arbitrageurs lose capital (equity)
  3. they unwind their positions
  4. prices deviate further

  $\longrightarrow$ *why not increase positions if arbitrage deviates?*
- **key assumption**: arbitrageurs cannot set-up contingent financing
  this paper: assumes ex ante optimal contracting / derives testable predictions / tests them

motivation 2/2

- **hedge funds**
  1. lock-up periods: 21% of funds have 1 year lock-up
  2. redemption periods: monthly (50%), quarterly (30%)
  3. notice period: 1 month (30%)
  4. side pockets, gates
- **to some extent: private equity funds, closed end funds**
- *LTCM: 3 year lock-up, $1bn credit facility*
  $\Rightarrow$ not useful to withstand the crisis, but better for short term shocks
  (rather: the 98 or '04 convertible arb meltdown)

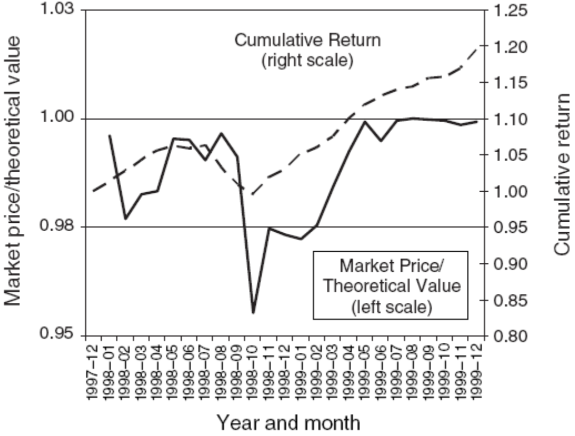

FIGURE 3. PRICE-TO-THEORETICAL-VALUE OF CONVERTIBLE BONDS, AND RETURN OF CONVERTIBLE BOND HEDGE FUNDS (1997/12-1999/12)

what we do

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  - contingent financing contract solve the effort-making problem
  - in bad states, assets are underpriced & (past) returns are lower
  - some funds ("illiquid") receive capital in bad states, others ("liquid") don't

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  - in bad states, assets are underpriced & (past) returns are lower
  - some funds ("illiquid") receive capital in bad states, others ("liquid") don't
- predictions on HF returns
  - returns of "illiquid" funds rebound *more* when past performance is low

    $\longrightarrow$ test on HF data ("illiquid" = impediments to withdrawal)

related literature 1/2

- **theory literature on limits to arbitrage**
  1. *Shleifer&Vishny, Gromb&Vayanos, Acharya&Viswanathan, Brunnermeier&Pedersen*: endogenous prices lead to destabilizing feedback
     we have optimal capital structure choice $\Longrightarrow$ stabilizing feedback in our model
  2. *Stein(05)*: prices not endogenous:
     we have endogenous asset prices $\Longrightarrow$ this makes arbitrage easier to sustain
  3. *Stein(09):* endogenous capital structure of arbitrageurs
     we endogenize the cost of contingent financing (getting capital in bad state of nature depends on how you deal with it)
     $+$ predictions on fund returns, that we test.
  4. *Campbell&Viceira*: long term investors should buy mean reverting assets
     in our model, this is true in equilibrium

related literature 2/2

- **empirical literature on (mostly hedge) funds**
  1. *Coval&Stafford*: fire sales by mutual funds depress prices
     we look at the impact on performance & avoid 13fs
     we look at funds that do not have to fire sell
  2. *Agarwal&al, Aragon*: impediments to withdrawal ⇒ illiquidity premium for investors
     we ask how issuers deliver this premium: they provide liquidity
     (evidence from convertible arb by *Agarwal&al*, *Choi&al*)
  3. *Aragon, Ding&al, Liang&Park*: lock-ups ⇒ smooth HF returns
     we have opposite results, because we work @ annual frequency

outline of the Talk

1. motivation
2. model
3. tests
4. conclusion

model 1/3

- an asset in supply $= 1$, which pays off V at the last date
- competitive risk neutral investors
- continuum of fund manager, equity A
- financing contract is **optimal**
  - organizes capital allocation
  - contingent on date / state of nature / past fund returns

model 2/3

t=0: contracting stage

- contract = funds entrusted I $(t = 1)$ and I_U, I_M, I_D $(t = 2)$

model 2/3

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- managers buy asset using contractual cash I.
- asset market clears at price P

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t=2: state of nature $\in \{U, M, D\}$ revealed + second purchase

- in states M and D: a "wrong" asset with PV 0 appears
  state M: "right" asset selected with proba μ
  state D: "right" asset selected by good managers only
- managers trade assets, have contractual cash I_U, I_M, or I_D
- asset market clears at P_U, P_M, or P_D

model 2/3

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t=3: "right" asset payoff:

- V
- state M, D : only $V - B$ pledgeable to investor (unmodelled M.H.)

model 3/3

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1. for given expected prices P, P_U, P_M, P_D,
   we find the optimal contract I, I_U, I_M, I_D
2. write the market clearing conditions to solve for prices

optimal contract 1/2

- maximize fund NPV given prices P, P_U, P_M, P_D:

$$\max_{\text{effort},I,I_U,I_M,I_D} I \underbrace{\left[\lambda_U P_U + \lambda_M P_M + \lambda_D P_D - P\right]}_{\text{date 1 NPV}}$$

$$+\lambda_U I_U \underbrace{\left[V - P_U\right]}_{\text{date 2/U NPV}} + \lambda_M I_M \underbrace{\left[\mu V - P_M\right]}_{\text{date 2/M NPV}} + \lambda_D I_D \underbrace{\left[\rho V - P_D\right]}_{\text{date 2/D NPV}}$$

where $\rho = \mu$ if high effort or $\mu - \Delta\mu$ if low effort

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- and (IC) managers supposed to exert effort do so:

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- *at equilibrium:*

$P = \lambda_U P_U + \lambda_M P_M + \lambda_D P_D$ otherwise $I = \pm\infty$
$P_U = V$ otherwise $I_U = \pm\infty$
$P_M, P_D \in (\mu(V - B), \mu V]$ otherwise $I_D, I_M = \pm\infty$

optimal contract 2/2

- fund managers want to raise infinite funds, but pledgeable income is scarce
  $\Longrightarrow$ not for I or I_U: no agency problem
  $\Longrightarrow$ but allocate it between state M and state D:

  1. $P_M < P_D \rightarrow$ all funds invest in M only: $I_D = 0$
  2. $\frac{\mu}{\mu - \Delta\mu} P_D < P_M \rightarrow$ all funds invest in D only: $I_M = 0$
  3. $P_D < P_M < \frac{\mu}{\mu - \Delta\mu} P_D \rightarrow$ high effort funds have $I_D > 0$; low effort have $I_M > 0$

  $\Rightarrow$ high (low) effort funds have comparative advantage in state D (M)
  $\Rightarrow$ only case 3 can be an equilibrium
  $\Rightarrow$ two types of funds exist at equilibrium

asset market equilibrium 1/3

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- write down asset market equilibrium in all states:

$$
\begin{aligned}
P &= \lambda_U . P_U + \lambda_M . P_M + \lambda_D . P_D \\
P_U &= V \\
P_M &= \mu\left(V - B\right) + \frac{\mu(1-\alpha)A}{\lambda} \\
P_D &= \mu(V - B) + \frac{\mu\alpha A}{\varepsilon}
\end{aligned}
$$

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- $\alpha \longleftrightarrow$ indifferent (high effort + $I_D > 0$) & (low effort + $I_M > 0$):

$$\frac{\varepsilon}{\alpha} - \frac{C}{B} = \frac{\lambda}{1-\alpha}$$

asset market equilibrium 2/3

1. underpricing in state D: $P_D < P_M$
   even though same expected payoff $= \mu V$
   else, no manager acquires information
   both prices are lower than P_U

asset market equilibrium 2/3

1. underpricing in state D: $P_D < P_M$
   even though same expected payoff $= \mu V$
   else, no manager acquires information
   both prices are lower than P_U
2. at equilibrium, two types of funds coexist
   low effort funds with $I_M > 0$
   high effort funds with $I_D > 0$
   $\longrightarrow$ high effort = receiving funds when past performance $P_D - P$ is very low
   = impediment to withdrawals
   $\longrightarrow$ in the model: ex ante & ex post optimal

asset market equilibrium 3/3

testable implication

- mean reversion stronger in high effort funds
  $E(R_3|R_2 \text{ is low}, I_D > 0) > E(R_3|R_2 \text{ is low}, I_D = 0$
  $E(R_3|R_2 \text{ is high}, I_D > 0) = E(R_3|R_2 \text{ is high}, I_D = 0)$
  $\Rightarrow$ ΔMR is asymmetric (only is past returns are low)
  $\Rightarrow$ does not depend on $\mu < 1$, also true for $\mu = 1$
  (because $P_D < P_M$)

data

- **EurekaHedge:** ~6,000 funds, 1993-2007
  annual net-of-fee returns
  annual net-of-fee AUMs
  lock-up period (Yes=1), Redemption+notice period (⩾3months=1)

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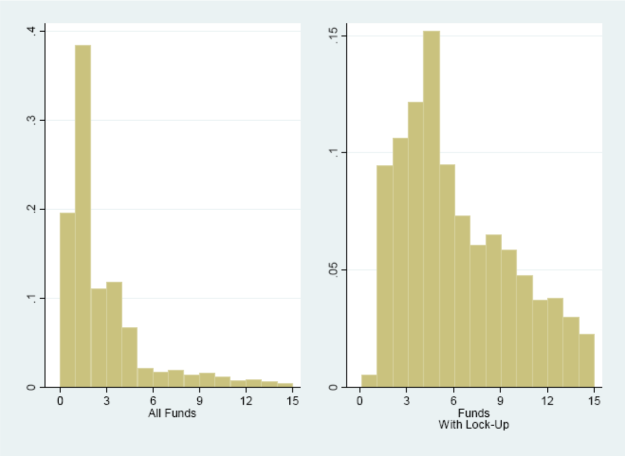
  for fund i at data t.
- we restrict ourselves to funds with AUM>$20m
  21% have lock-ups
  38% have redemption+notice > 3 months
  noisy info (no side letters)

duration of capital: descriptive statistics

mean earliest possible withdrawal of AUM =
notice + (redemption/2) + past inflows x remaining period under lock-up



step 1: impediment to withdrawals prevent outflows

- we run the following regression (table 2)

$$\text{net outflow}_{it} = \alpha_i + \beta.1_{\left\{r_{it-1}<r^f_{t-1}\right\}} + \gamma.1_{\left\{r_{it-1}<r^f_{t-1}\right\}} \times \text{impediment}_i + \varepsilon_{it}$$

for fund i at date t. ε_{it} are assumed correlated by t or i.
$1_{\left\{r_{it-1}<r^{rf}_{t-1}\right\}} = 1$ if the fund's return at $t-1$ was below the risk free rate
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- γ is positive statistically significant: *after low performance*
  - without lock up: outflows = 13% AUM
  - with lock up: outflows = 8% AUM
  - with > quarterly redemption: flows = 9% AUM

mean reversion in returns stronger with impediments to withdrawal 1/3

- first, we run the following regression:

$$r_{it} = \alpha_i + \beta.1_{\{r_{it-1}<r^f_{t-1}\}} + \gamma.1_{\{r_{it-1}<r^f_{t-1}\}} \times \text{Impediment}_i + \varepsilon_{it}$$

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- γ negative and statistically significant. Following bad performance
  - without lock-up: return $= +3$ ppt
  - with lock up: return $= +8$ ppt
  - with long redemption period: return $= +7$ppt
    (robust to accounting for fund death)

  $\Longrightarrow$ but this is no evidence of asymmetry $\rightarrow$

mean reversion in returns stronger with impediments to withdrawal 2/3

asymmetry: only present conditional on bad states of nature

Dependent variable		r_{it}	
Impediment to withdrawal	None	Lock Up	Quart. Red.
	(1)	(2)	(3)
$(r_{it-1} < r^{rf}_{t-1})$	3.0*	1.8	2.8
	(1.7)	(1.9)	(1.7)
$(r_{it-1} > 20\%)$	-2.7*	-2.9*	-2.1
	(1.5)	(1.5)	(2.0)
$(r_{it-1} < r^{rf}_{t-1})$	-	5.0***	2.8***
$\times$ Impediment$_i$		(1.4)	(0.8)
$(r_{it-1} > 20\%)$	-	0.5	-1.0
$\times$ Impediment$_i$		(1.0)	(2.1)
Fund FE	Yes	Yes	Yes
Observations	4,541	4,412	3,902
Adj. R^2	0.48	0.48	0.49

mean reversion in returns stronger with impediments to withdrawal 3/3

not driven by attrition / fund death

Table 4: Probability of Exit and Impediments to Withdrawal

Dependent variable	Exit_{it}			
Impediment to withdrawal	Lockup	Quarterly Redemption	Lockup	Quarterly Redemption
	(1)	(2)	(3)	(4)
Impediment_i	-	-	-0.01**	-0.01*
			(0.00)	(0.00)
$(r_{it-1} < r^{rf}_{t-1})$	0.06***	0.07***	0.08***	0.09***
	(0.01)	(0.01)	(0.02)	(0.02)
$(r_{it-1} < r^{rf}_{t-1})$	-0.05**	-0.05*	-0.03	-0.04
$\times \text{Impediment}_i$	(0.02)	(0.02)	(0.03)	(0.03)
Fund FE	Yes	Yes	No	No
Observations	4,707	4,171	4,707	4,171
Adj. R^2	0.62	0.61	0.02	0.02

relation to hedge fund literature 1/3

- **opposite to Lo, Aragon, Ding&al, Liang&Park**
  - autocorrelation in returns prevalent (us: mean reversion, rather)
  - stronger with "illiquid" funds (us: weaker with illiquid funds)
  - signs of earning smoothing / illiquid assets

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  - stronger with "illiquid" funds (us: weaker with illiquid funds)
  - signs of earning smoothing / illiquid assets
- **reason: we work @ annual frequency, not monthly**
  - to avoid accounting/smoothing issues
  - in our data, @ monthly freq: mean autocorr is 0.09 (=Lo)
  - in our data, @ monthly freq: lock-up and autocorr have correlation of 0.08 (=Ding&al)

  $\Rightarrow$ *not a data difference*

  $\rightarrow$ we run our regressions at the monthly, and quarterly frequencies

relation to hedge fund literature 2/3

monthly data / less liquid assets → our results disappear, become like lit.

Dep. Variable		r_{it}	
Panel A: Monthly frequency			
	All	Long short equity	Fixed Income
	(1)	(2)	(3)
$(r_{it-1} < r^{rf}_{t-1})$	-0.38**	-0.44**	-0.45***
	(0.18)	(0.21)	(0.14)
$(r_{it-1} < r^{rf}_{t-1})$	-0.14*	0.14	-0.49***
× Lock-Up$_i$	(0.08)	(0.13)	(0.14)
Fund FE	Yes	Yes	Yes
Observations	120,734	51,963	6,929
Adj. R^2	0.06	0.05	0.10

relation to hedge fund literature 3/3

quarterly data / more liquid assets → our results re-appear.

Panel B: Quarterly frequency

	All	Long short equity	Fixed Income
	(1)	(2)	(3)
$(r_{it-1} < r^{rf}_{t-1})$	-0.10	-0.42	-0.49
	(0.42)	(0.58)	(0.40)
$(r_{it-1} < r^{rf}_{t-1})$	0.53**	1.37***	-1.24**
$\times$ Lock-Up$_i$	(0.26)	(0.24)	(0.55)
Fund FE	Yes	Yes	Yes
Observations	34,447	14,828	1,989
Adj. R^2	0.15	0.15	0.18

conclusion

- we investigate the effect of LT financing of arbitrageurs on their "market making" ability
- a model of optimal arbitrageur capital structure, to derive equilibrium predictions
- key empirical result: mean reversion in HF return larger when they are LT financed (i.e. with lock ups)
- not applicable to current crisis.
- bridges with the strategic asset allocation literature: long term investor should buy mean reverting assets