Stochastic dual dynamic programming for gas supply-demand equilibrium

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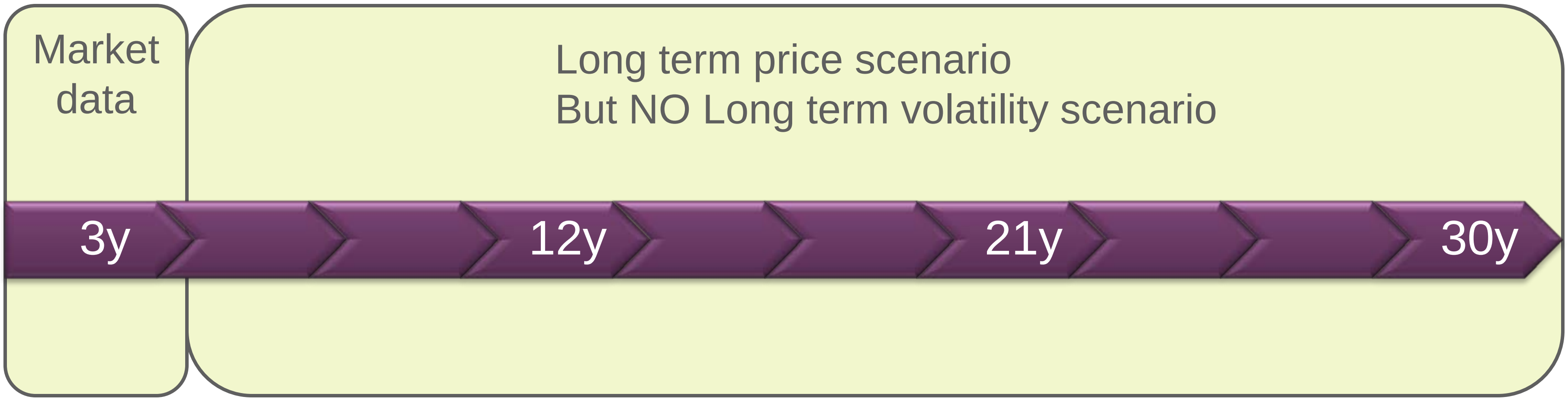
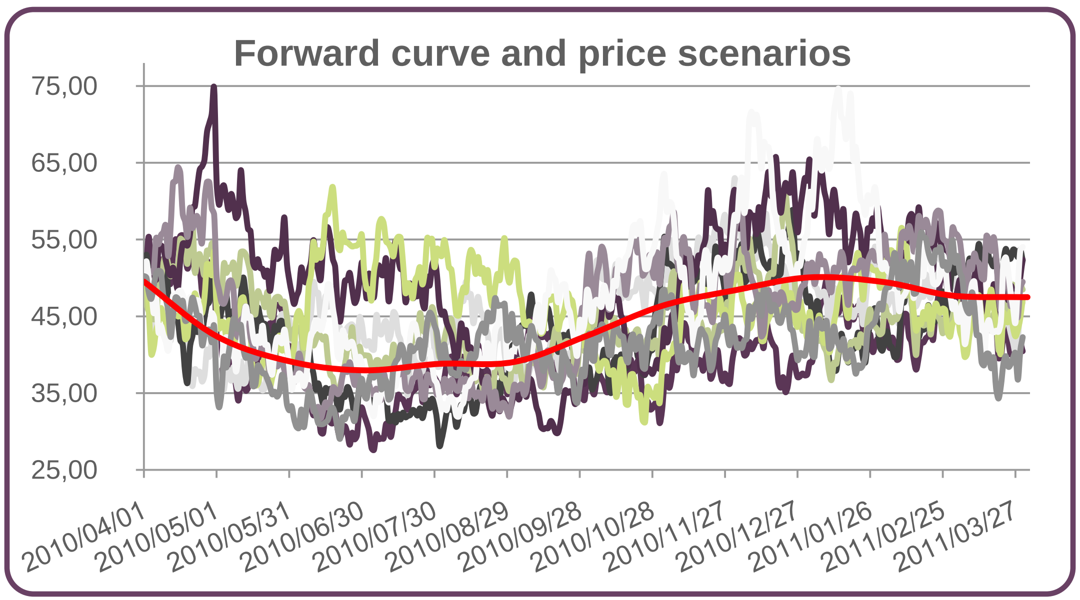
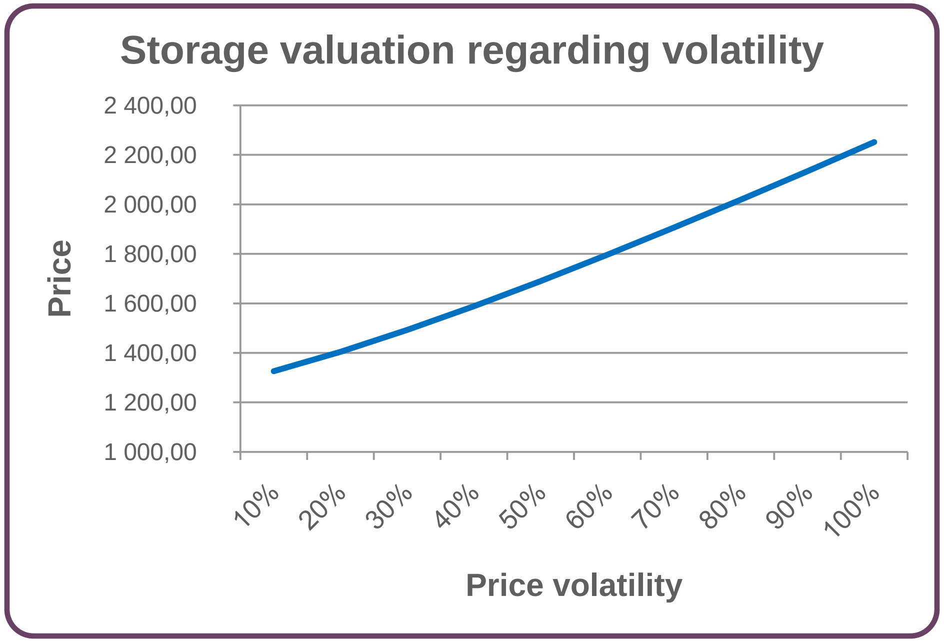
Inadequate market data for valuation on 30 years





Long term prices & volatilities required for investment decisions

Feed back from short term pricing

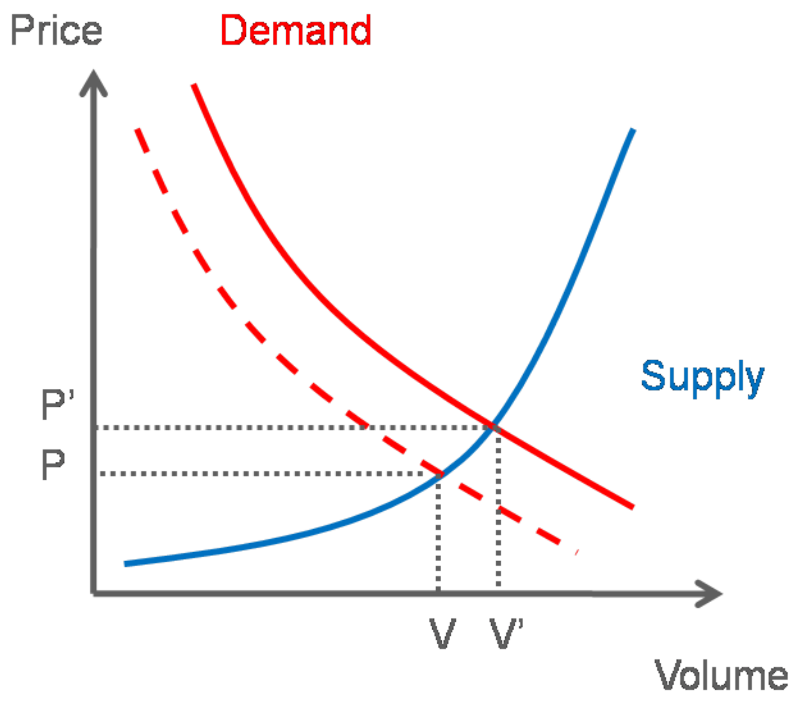


Should we invest in this gas storage ?

What profits over the next 30 years?

How about gas prices in the next 30 years ?

Volatility from demand side: uncertainty on the daily supply-demand equilibrium



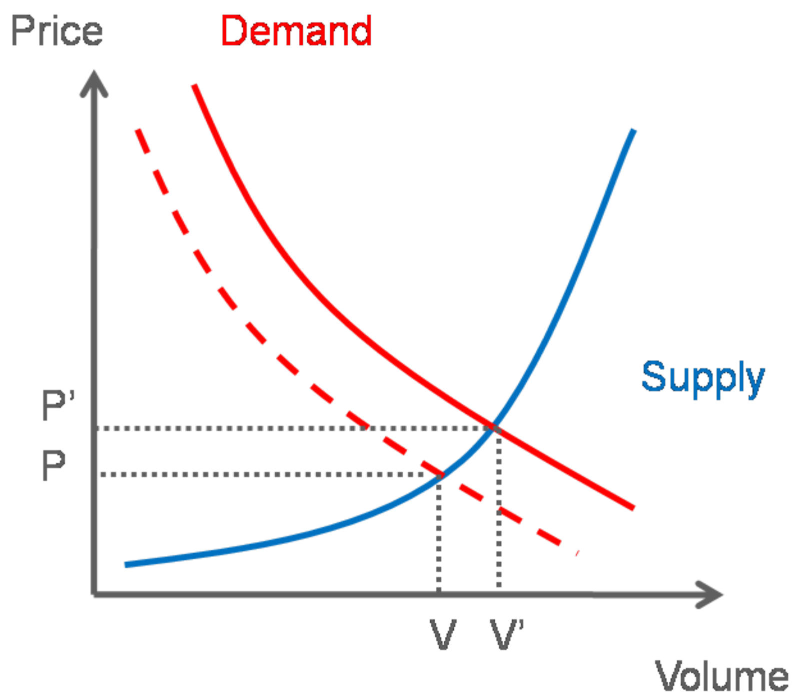
Simulate supply-demand equilibrium

- With daily time step
- In a non anticipative way

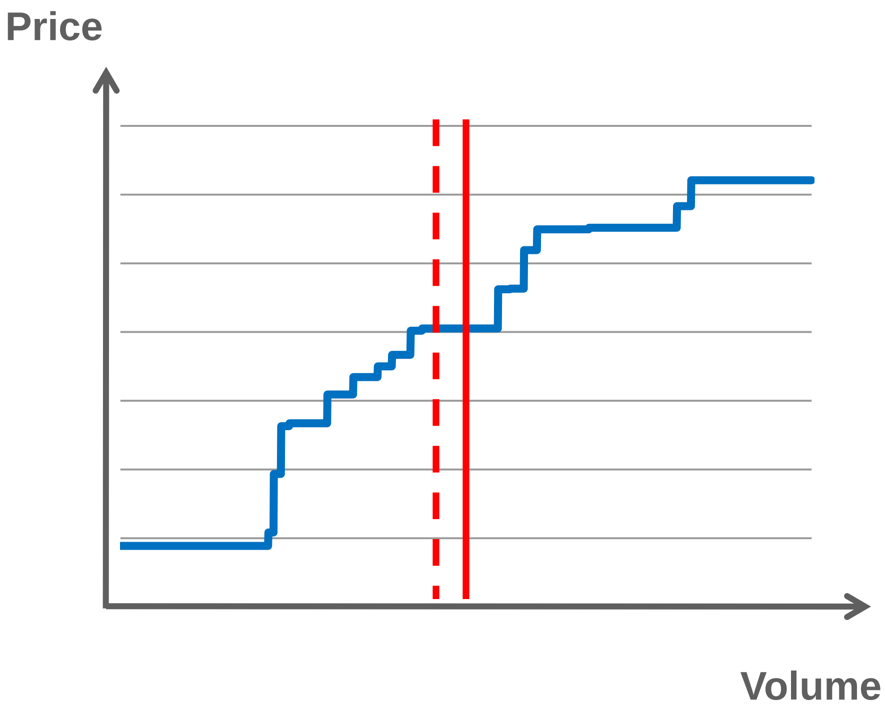
Market price = marginal cost on demand constraint

Volatility from supply side: stepwise gas supply leads to a large dimension problem

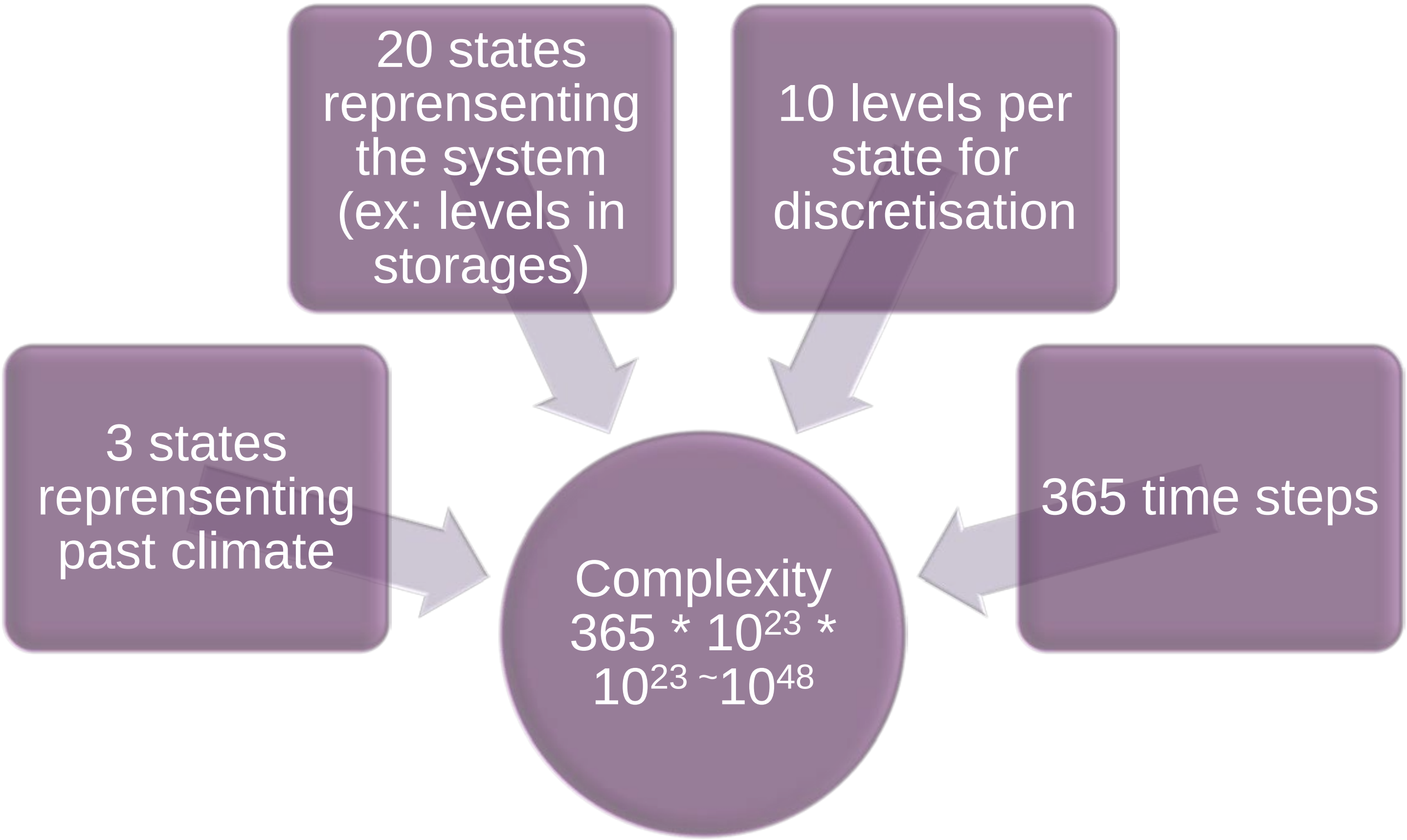
« Theorical » supply & demand curves



Modelized supply & demand curves



A difficult problem to solve where « direct » dynamic programming is not tractable



Mathematical modelisation

Decision process

State at time t → Hazard (consumption) → Decision → Costs / State at time t+1

x_t ω_t u_t

Costs: $c_t u_t$

State at time t+1: $x_{t+1} = F\left(x_t, u_t\right)$

Cost function

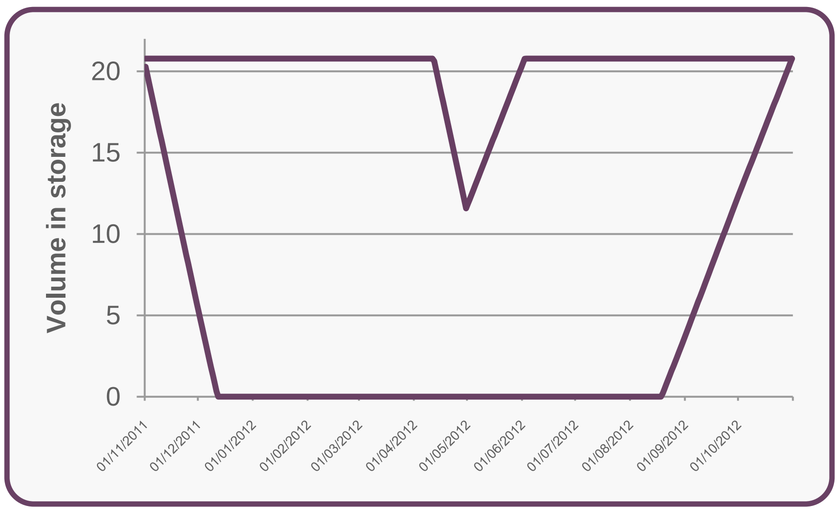
$$V_t(x_t) = \mathbb{E}_{\text{aléa}}\left[\min_{u \in U_t(x_t,\omega_t)} \underbrace{c_t u}_{\text{Instant costs}} + \underbrace{V_{t+1}\left(F(x_t,u)\right)}_{\text{Futur expected costs}}\right]$$

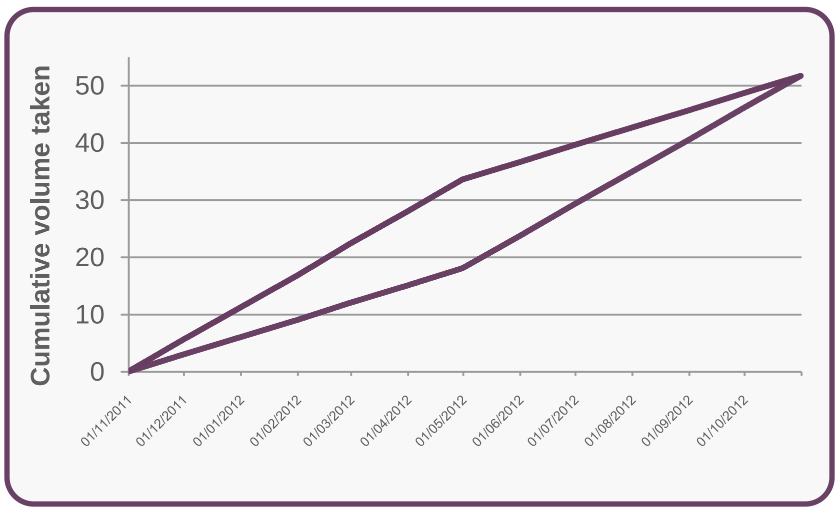
Linear constraints & costs

Convex problem

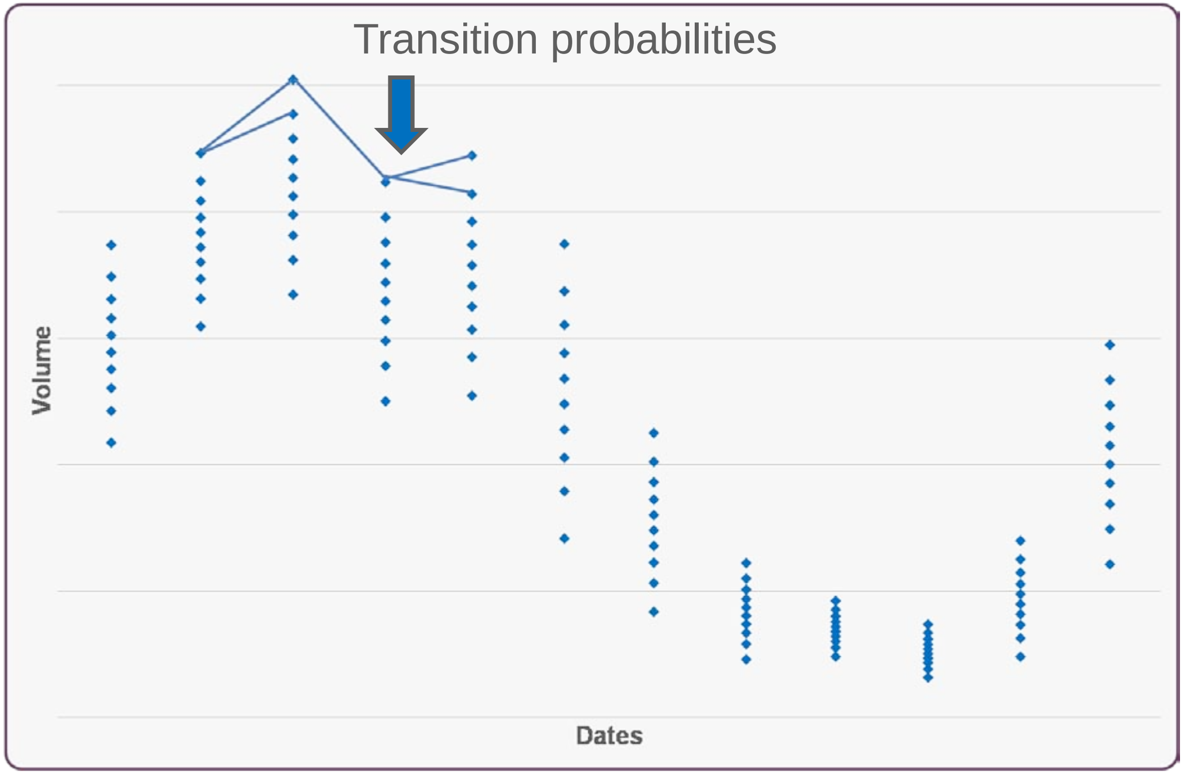
Constraints’ envelope

Decision constraints + Quota or level contraints = Constraints’ envelope





Consumption is dicretized using optimal quantization



Autroregressive gaussian process

$$X_{t+dt} = A(dt) \cdot X_t + T(dt) \cdot G$$

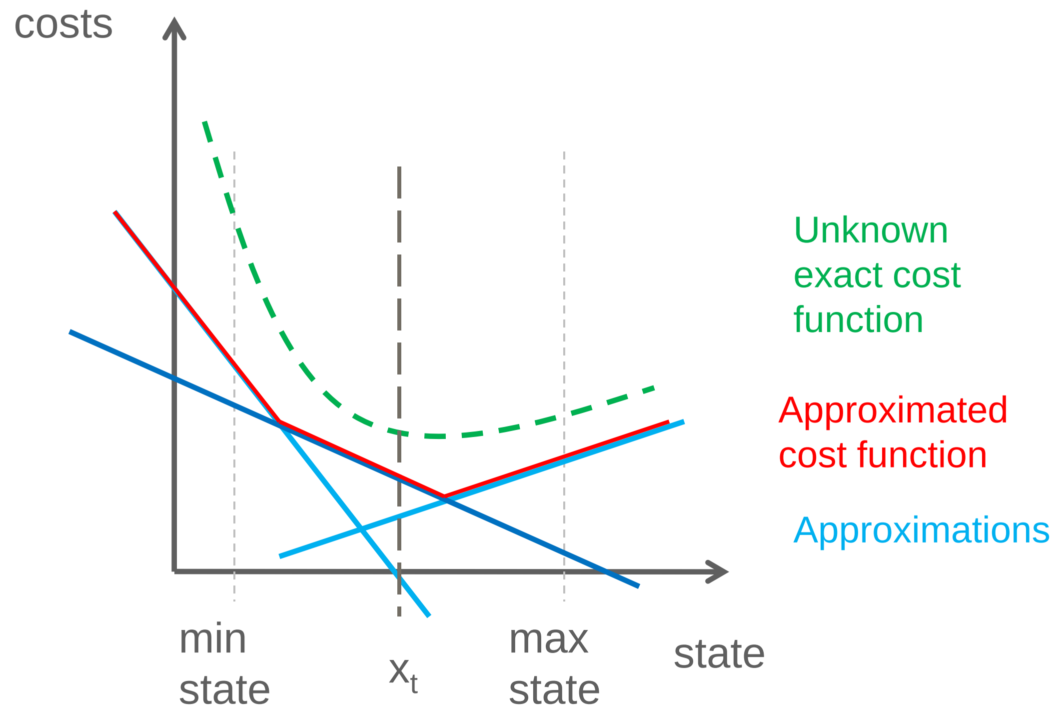
Normal law grid from www.quantize.maths-fi.com

An alternative to dynamic programing: SDDP

No discretisation a priori of the states

Cost function approximated where needed

Simulation with current approximation



Errors in Sddp

Error propagation towards initial date

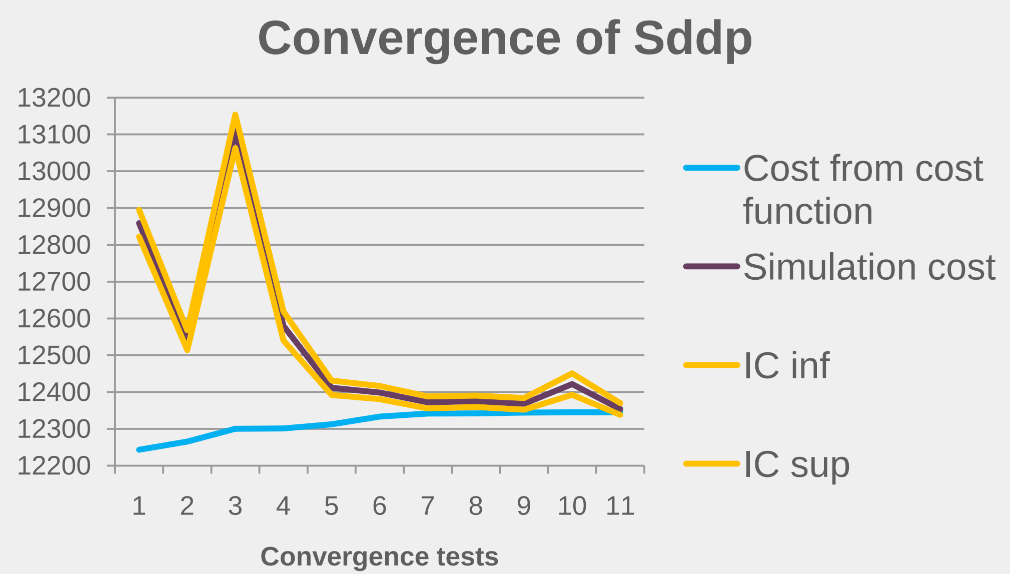
$$\begin{aligned}
\bar{V}_t(x_t) &= \mathbb{E}\left[\min_u c_t.u + \bar{V}_{t+1}\left(x_{t+1}\right)\right] \\
&= \mathbb{E}\left[\min_u c_t.u + V_{t+1}\left(x_{t+1}\right) + error_{t+1}\left(x_{t+1}\right)\right] \\
&= V_t(x_t) + error_t(x_t)
\end{aligned}$$

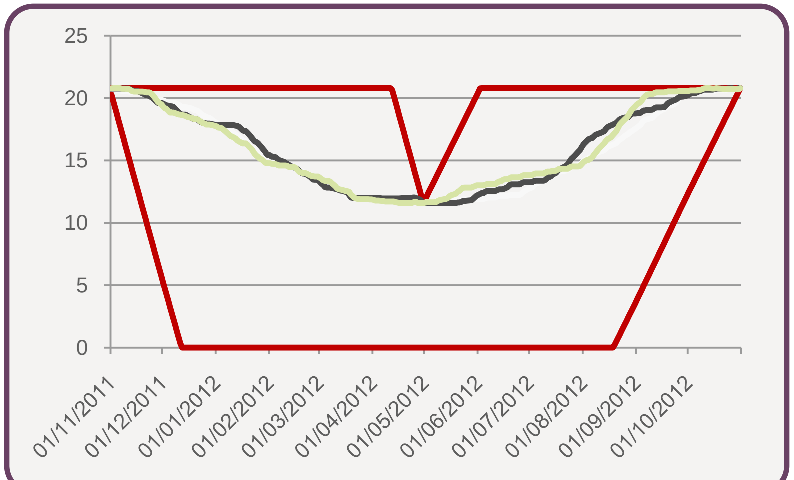
Error can be measured

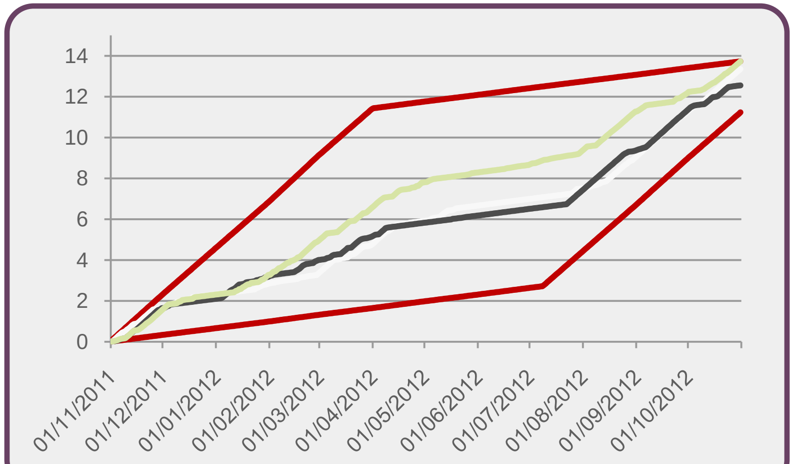
Lower bound from cost function

Upper bound from simulations

Sddp’s outputs







- 19 assets
- 5 consumption levels per time step
- 365 time steps

➢ Computation time: ~ 2 hours