Ambit processes in energy markets

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Overview

1. Brief introduction to electricity markets as the prime example of energy markets
2. Lévy semi-stationary (LSS) models for power spot prices
3. Empirical example from the EEX
4. Derivation of forward prices leading to ambit processes

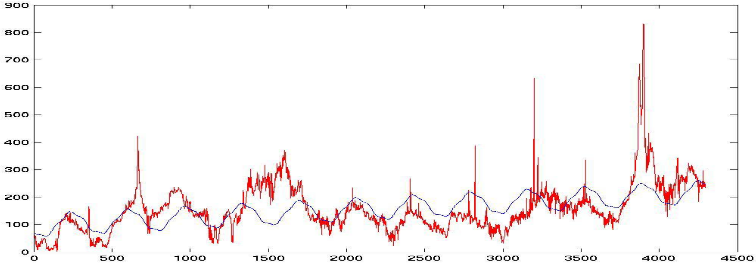
Power markets

- Typically, power markets organize trade in
  - Hourly spot electricity, next-day delivery
  - Forward and futures contracts on the spot
  - European options on forwards
- Examples: Powernext (EPEX), EEX, NordPool in Nordic region

The spot market

- An hourly market with physical delivery of electricity
- Participants hand in bids before noon *the day ahead*
  - Volume and price bids for each of the 24 hours next day
  - Maximum amount of bids within technical volume and price limits
- The exchange creates demand and production curves for each hour of the next day

- The spot price is the equilibrium
  - Price for delivery of electricity at a specific hour next day
  - The *daily* spot price is the average of the 24 hourly prices
- Reference price for the forward market
- Historical spot price at NordPool from the beginning in 1992 (NOK/MWh)



The forward and futures market

- Contracts with "delivery" of electricity over a period
  - Financially settled: The money-equivalent of receiving electricity is paid to the buyer
  - The reference is the hourly system price in the delivery period
- Delivery periods: next day, week, month, quarter, year
- Base and peak load contracts
- European call and put options on these forwards

Lévy semi-stationary models of power spot prices

Classical power spot models

- Lucia-Schwartz model for (log-) spot price

$$S(t) = \Lambda(t) + X(t) + Y(t)$$

  - $\Lambda(t)$ seasonality function
  - $X(t)$ short-term variations (stationary)
  - $Y(t)$ long-term non-stationary variations

$$dX(t) = -\alpha X(t)\,dt + \sigma\,dB(t) \qquad dY(t) = \mu\,dt + \eta\,dW(t)$$

- Lévy process driving the stationary $X(t)$
  - Cartea and Figueroa
- Multi-factor Lévy driven OU-processes, separating spike and "normal" variations
  - B., Kallsen and Meyer-Brandis
- CARMA-model driven by alpha-stable processes
  - Klüppelberg et al.
- For these models

$$X(t) = X(0)g(t) + \int_0^t g(t-s)\,dL(s)$$

- g is a known function:

$$g(t) = \exp(-\alpha t)\,, \text{or}\, g(t) = \mathbf{b}' \exp(A(t-s))\mathbf{e}_p$$

Lévy semi-stationary processes

- LSS process:

$$Z(t) = \mu + \int_{-\infty}^{t} g(t-s)\sigma(s-)\,dL(s) + \int_{-\infty}^{t} q(t-s)\eta(s)\,ds$$

  - μ constant, g, q non-negative deterministic functions, $g(t) = q(t) = 0$ for $t \leq 0$
  - L two-sided Lévy process
  - σ, η cadlag processes independent of L

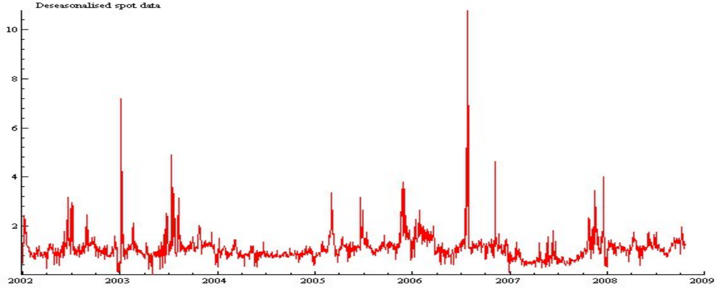
- Integrability assumptions on integrands assumed

- For σ, η stationary, Z is stationary
- Note that Z is generalizing all the models discussed above (in stationarity)
  - But provides a rich class of new models as well....
- We focus on models with $q = 0$.

$$Z(t) = \mu + \int_{-\infty}^{t} g(t-s)\sigma(s-)\,dL(s)$$

Empirical example: EEX

- Daily peak load spot prices from EEX ranging from 01.01.2002 till 21.10.2008. In total 1775 observations
- Goal: find suitable g, σ and L fitting *de-seasonalized* data



- Spot model

$$S(t) = \Lambda(t) \times Z(t)$$

- Seasonal function

$$\ln \Lambda(t) = \beta_0 + \beta_1 t + \beta_2 \cos\left(\frac{\tau_1 + 2\pi t}{261}\right) + \beta_3 \cos\left(\frac{\tau_2 + 2\pi t}{5}\right)$$

- Fitting to (log-)prices by least squares

- The sample mean is approximately 1, and we substract it to end up with a model

$$\frac{S(t)}{\Lambda(t)} - 1 = \int_{-\infty}^{t} g(t-s)\sigma(s-)\,dL(s) := X(t)$$

- Autocorrelation function (ACF) of $X(t)$

$$\mathrm{Corr}(X(t), X(t+h)) = \frac{\int_0^\infty g(x+h)g(x)\,dx}{\int_0^\infty g^2(x)\,dx}$$

- Follows from an assumption of L having mean zero

- Propose g to be a sum of two exponential functions

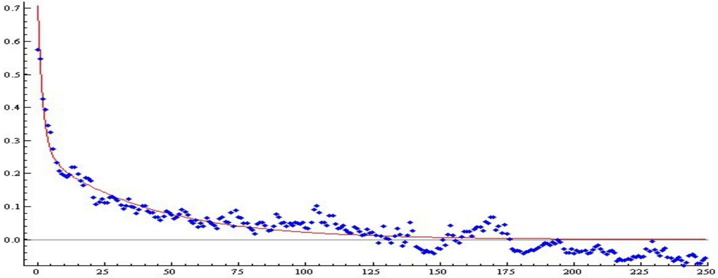
$$g(x) = w\exp(-\alpha_1 x) + (1-w)\exp(-\alpha_2 x) \quad 0 < w < 1, \alpha_i > 0$$

- ACF:

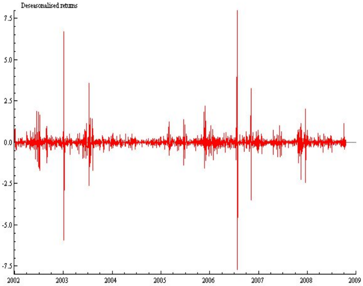
$$\mathrm{Corr}(X(t), X(t+h)) = w^*\exp(-\alpha_1 h) + (1-w^*)\exp(-\alpha_2 h)$$

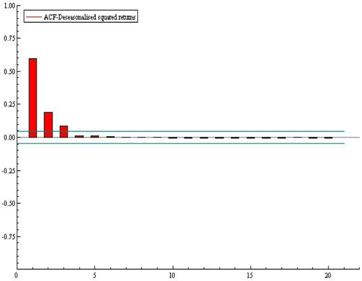
- Note: same ACF as in a 2-factor OU-processes

- Fitted ACF by a sum of two exponentials
- Note the fast decay for short lags, slow for longer lags

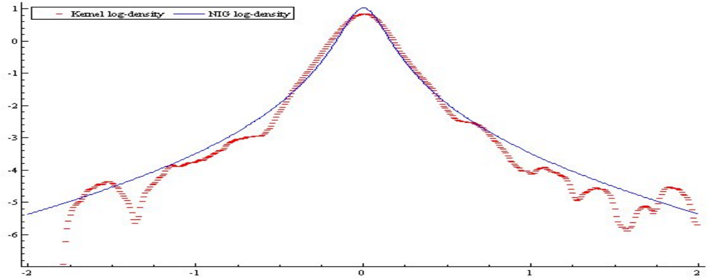


- Signs of stochastic volatility in squared return data





- Returns data nicely fitted by the normal inverse Gaussian (NIG) distribution



- Two possible models:
  1. No stochastic volatility, and choose L to be NIG
  2. Model $\sigma(t)$ as an OU-process with inverse Gaussian stationary distribution and $L = B$
- 2nd choice known as the BNS-stochastic volatility model
- Note that we have a *one*-factor model, explaining the ACF function by a function g
  - ...rather than a mixture of two OU-processes

Some remarks

- Model under stationarity:
- Today's spot price $S(t)$ is an *observation* from the model
  - We do not condition on that the dynamics is equal to the observation today
  - Hence, all historical prices are treated equivalently as observations
- $S(t)$ (or rather $Z(t)$) is in general *not* a semimartingale
  - It is if $g(0)$ is well-defined and g being differentiable
  - The spot is not financially tradeable, so no "arbitrage-problems" with these models

Forward pricing

- Suppose log-price model for spot, i.e.,

$$S(t) = \Lambda(t) \exp(Z(t))$$

- $Z(t)$ BSS model

$$Z(t) = \int_{-\infty}^{t} g(t-s)\sigma(s)\, dB(s)$$

- LSS volatility model (U being a subordinator)

$$\sigma^2(t) = \int_{-\infty}^{t} h(t-s)\, dU(s)$$

- Forward price $F(t,T)$ at time $t \leq T$ for a contract delivering at time T is

$$F(t,T) = \mathbb{E}_Q\left[S(T) \,|\, \mathcal{F}_t\right]$$

- Q a pricing measure, in general *any* $Q \sim P$
- Introduce Q by Girsanov on B, unchanged σ: for θ a deterministic function,

$$dB(t) = dW(t) + \frac{\theta(t)}{\sigma(t)}\,dt$$

- σ intepreted as volatility, and thus $\sigma > 0$

- Recalling σ independent of B, double conditioning

$$
\begin{aligned}
F(t,T) = \Lambda(T)\Theta(t,T) \\
&\times \exp\left(\int_{-\infty}^{t} g(T-s)\sigma(s)\,dW(s)\right) \\
&\times \exp\left(\frac{1}{2}\int_{-\infty}^{t}\int_{t}^{T} g^2(T-v)h(v-s)\,dv\,dU(s)\right)
\end{aligned}
$$

- Θ a function involving θ, g, h and the log-moment generating function of $U(1)$.

Some properties of the forward price

- Forward price is *not* explicitly dependent on spot:

$$\ln F(t,T) \sim \int_{-\infty}^{t} g(T-s)\sigma(s)\,dW(s) + \ldots$$

$$\ln S(t) \sim \int_{-\infty}^{t} g(t-s)\sigma(s)\,dW(s) + \ldots$$

- But, forward and spot will be dependent/correlated
- Can represent the (log-)forward as a regression on the (log-)spot
  - Very unlike forward prices for "all" other one-factor spot models
  - There forward prices are explicit functions of the spot

- For stationary (mean reverting OU) models, the forward prices are constant in the long end: for time-to-maturity $T - t$ large,

$$F(t, T) \sim \text{const}$$

- But market prices are not constant
- In our model, one can obtain random forward prices in the long end by choosing g "non-stationary"

- To be more specific, we consider Volterra model for the spot

$$g(t-s) \longrightarrow g(t,s) := g_1(t)g_2(s)$$

- All results on forward remain the same, with $g(t-s)$ substituted by $g_1(t)g_2(s)$
- Suppose constant volatility and $g_1(T) \to g_2(\infty) > 0$ when $T \to \infty$,

$$\lim_{T\to\infty} \ln F(t,T)/\Lambda(T)\Theta(t,T) = (\ln S(t) - \ln \Lambda(t))\frac{g_1(\infty)}{g_1(t)}$$

- Conclusion: forward prices vary as the spot in the long end

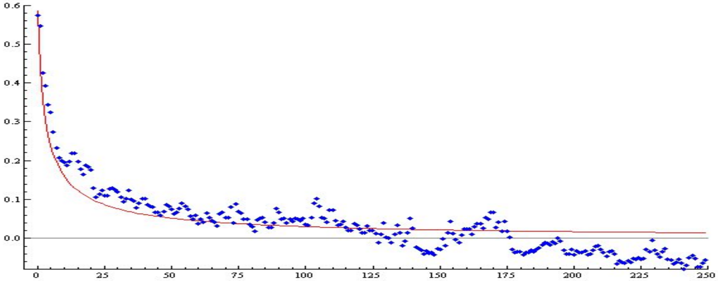
Affine structure of the forward

Theorem

Forward price is affine in $Z(t)$ and $\sigma^2(t)$ if and only if $g(t,s) = g_1(t)g_2(s)$ and $h(t,s) = h_1(t)h_2(s)$

- Requires some mild differentiability assumption on g_1 and h_1
- Proof similar to classical arguments for forward rate models in fixed-income theory (see Carverhill)

- A non-example: the Bjerksund-model
- Choose constant volatility and $g(x) = a/x + b$
  - Steep increase in ACF close to maturity
- Forward is not affine



Forward price dynamics

- Dynamics of $F(t,T)$

$$\frac{dF(t,T)}{F(t-,T)} = g(T,t)\sigma(t)\,dW(t) + \frac{1}{2}\int_t^T g^2(T,s)h(s,t)\,ds\,d\widetilde{U}(t)$$

- $d\widetilde{U}(t) = dU(t) - \frac{d}{dx}\phi_U(x)|_{x=0}\,dt$
- Theoretical implication: spot has continuous paths, while forward has jumps from volatility
- Note the "Samuelson effect" in the volatility of the forward price

Ambit processes and forward price modelling

Definition of ambit process

$$Y(t,x) = \int_{-\infty}^{t} \int_{\mathbb{R}_+} k(t-s,x,y)\sigma(s,y)\, L(ds,dy)$$

- L is a *Lévy basis*
- k non-negative deterministic function, $k(u,x,y) = 0$ for $u < 0$.
- Stochastic volatility process σ independent of L, stationary

- L is a *Lévy basis* on $\mathbb{R}^d$ if
  1. the law of $L(A)$ is infinitely divisible for all bounded sets A
  2. if $A \cap B = \emptyset$, then $L(A)$ and $L(B)$ are independent
  3. if $A_1, A_2, \ldots$ are disjoint bounded sets, then

$$L(\cup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} L(A_i)\,, a.s$$

- We restrict to zero-mean, and square integrable Lévy bases

- Stochastic integration in ambit process: use the Walsh-definition
  - Extension of Itô integration theory to temporal-spatial setting
  - In time: integration "as usual"
  - In space: exploit independence and additivity properties
  - Isometry by square-integrability hypothesis
- Suppose k and σ integrable
  - Essentially square-integrability in time and space

Forward modelling by ambit processes

- Extension of the HJM approach
  - by direct modelling rather than as the solution of some dynamic equation
- Simple arithmetic model could be (in the risk-neutral setting)

$$F(t,x)=\int_{-\infty}^{t}\int_{0}^{\infty} k(t-s,x,y)\sigma(s,y)L(dy,ds)$$

- x is "time-to-maturity"

Martingale condition

- Forwards are tradeable
- $t \mapsto F(t, T-t)$ must be a martingale

Theorem

$F(t, T-t)$ is a martingale if and only if

$$k(t-s, T-t, y) = \widetilde{k}(s, T, y)$$

- Example: exponential damping function (motivated by OU spot models)

$$k(u,x,y)=\exp\left(-\alpha(u+x+y)\right)$$

- Satisfies the martingale condition

$$k(t-s,T-t,y)=\exp\left(-\alpha(y+T-s)\right)=:\widetilde{k}(s,T,y)$$

- Another example: the Musiela SPDE specification
  - $L = W$, Wiener case for simplicity
  - No spatial dependency in W

$$df(t,x) = \frac{\partial f(t,x)}{\partial x}\,dt + h(t,x)\,dW(t)$$

- Solution of the SPDE

$$f(t,x) = f_0(x+t) + \int_0^t h(s, x+(t-s))\,dW(s)$$

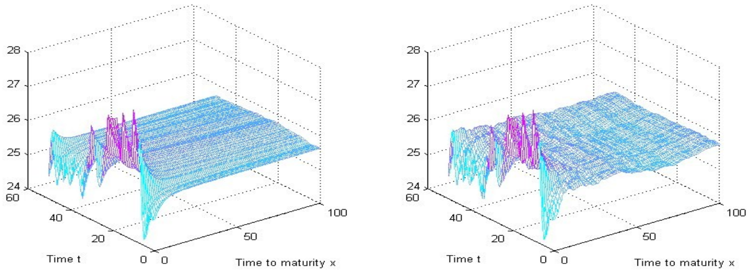
- Note: forward price $f(t,x)$ is an ambit process

- Letting $x = T - t$,

$$f(t, T - t) = f_0(T) + \int_0^t h(s, T - s)\, dW(s)$$

- Martingale condition is satisfied

- Simulation example: Forward prices in Musiela parametrization $f(t,x)$
- Spot-implied model vs. HJM model
- Parameters taken from EEX study
    - Random field generated as conditional Gaussian field, with variance given by inverse Gaussian
    - Exponential spatial correlation



Conclusions

- Used LSS processes to model spot prices of energy
  - General class, encompassing existing models
  - Provides a flexible framework for modelling
  - Analytically tractable
  - Model "in stationarity"
- Empirical example on EEX data
- Derivation of forward prices
  - Assumed "non-stationary" Volterra form of kernel
  - Classified affine structures
  - Long-term behaviour of forward prices analysed

- Direct modelling of forward prices using ambit processes
  - An explicit form of HJM modelling
  - Derived martingale condition
- Future work
  - Empirical studies of forward prices
  - Deeper investigations into stochastic volatility in spots and forwards
  - Simulation and estimation of ambit processes

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