A structural risk neutral approach for energy option pricing and hedging

– Work in progress –

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Introduction I : Spot energy market

- Spot market : the spot price is hourly (or half-hourly, e.g. Amsterdam) day ahead, prices determined once per day
- Spikes on spot prices, due to very high demand (e.g. unexpexted very high/low temperature) or very low offer (e.g. plants failures)
- Seasonality of spot prices
- Electricity is not storable, thus buy-and-hold strategies on the spot are just not feasible

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- Forward/Futures contracts : spot price is the underlying, trading on a continuous basis, with physical delivery or financially settled.
- Delivery periods forward contracts: next day, week or month ; quarterly ; yearly
- Due to discrete-time (spot) vs continuous-time (forward), let us assume that the spot can be embedded in a continuous-time process (see, e.g., Benth et al. book for details)
- European options on forward (quarterly, yearly)
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Introduction III : Motivations

- In standard stock financial markets : $F_t(T) = P_t e^{r(T-t)}$
- This equality relies heavily on asset's costless storability, it breaks down when P_t is spot price of electricity
- A priori, no relations between spot and forward at least in a market composed of electricity and bank account (see, e.g., Geman-Vasicek (2001)), so that literature splitted into two main streams :
  - Models for spot and $F_t(T) := \mathbb{E}_Q[P_T \mid \mathcal{F}_t]$
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Introduction IV : Toy model

- Consider a fictitious economy where electricity is produced only out of coal, so that $P_t = h_c S_t^c$ and agents can trade coal, buy electricity and have a bank account
- Assume no-arbitrage in the market of coal : there exists a risk-neutral measure $\mathbb{Q}$ for $\tilde{S}_t^c = e^{-rt} S_t^c$
- A forward contract on spot electricity $P_T = h_c S_T^c$ can be viewed as a contract on coal, so that

$$F_0^e(T) = \mathbb{E}_{\mathbb{Q}}[P_T] = \mathbb{E}_{\mathbb{Q}}[S_T^c] = h_c F_0^c(T)$$

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The Model I : Fuels and demand

- Riskless asset : $S_t^0 = e^{-rt}$, $r > 0$ constant for simplicity.
- **Fuels** : $n \geq 1$ fuels (as coal, gas, ...) whose prices S^i to produce 1 MWh of electricity follows

$$dS_t^i = S_t^i(\mu_t^i dt + \sigma_t^i dW_t^{S,i})$$

  where $W^{S,i}$ are correlated BMs and coeffs are s.t. $S^1 < \ldots < S^n$ (model spreads $Y^i = S^{i+1} - S^i$ as independent geometric BMs)
- Assume NA and completeness for fuels
- Convenience yields and storage costs are zero for simplicity.
- **Electricity demand:** $dD_t = a(t, D_t)dt + b(t, D_t)dW_t^D$ with W^D BM independent of S

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The Model II : Capacities

- Capacities : $dC_t^i = \alpha_i(t, C_t^i)dt + \beta_i(t, C_t^i)dW^{C,i}$ where $W^{C,i}$ are indep BMs, indep of D and S as well
- Capacity of first i fuels : $\overline{C}_t^i := \sum_{k=1}^i C_t^k$, $C_t^{\max} = \overline{C}_t^n$ maximal capacity
- To choose the technology, the producer looks at D_t

$$D_t \in I_t^i := \left[\overline{C}_t^{i-1}, \overline{C}_t^i\right) \Rightarrow P_t = h_i S_t^i$$

convention: $I_t^1 := (-\infty, C_t^1)$ and $I_t^n := [\overline{C}_t^{n-1}, +\infty)$

- In Aïd et al. (2009) if forget permutations among fuels

$$P_t = \sum_{i=1}^n S_t^i \mathbf{1}_{\{D_t \in I_t^i\}}$$

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The Model III : function g and spikes

- Consider the function

$$g(x) = \min\left\{M, \frac{\gamma}{x^{\nu}}\right\} \mathbf{1}_{x>0} + M\mathbf{1}_{x\leq 0}$$

- Spot price becomes

$$P_t = g(C_t^{max} - D_t) \sum_{i=1}^{n} S_t^i \mathbf{1}_{\{D_t \in I_t^i\}}$$

- We add g to include
  - non-convex constraints of generation (start up cost, minimal run time, minimal production level ...)
  - peak load plant production are not sold at short term marginal cost – need to cover fixed cost for few hours during the year
  - market power
- Typically $M = 50$, $\gamma \sim 10000$, $\nu \sim 1.10$

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The Model IV: No Arbitrage

1. By assumption, there exists a unique risk-neutral $\mathbb{Q} \sim \mathbb{P}$ for fuels
2. Market information flow is generated by fuels S, energy demand D and capacities C, i.e. $\mathcal{F}^{S,D,C}$ right filtration
3. Demand and capacities makes the market incomplete
4. Perfect hedging is not possible, many criteria in the literature ...
5. We choose local risk minimization

Local risk minimization I

Roughly speaking, let X a multidimensional (discounted) price process

- Introduced by Föllmer-Schweizer (1991)
- Under regularity condition, any payoff H with maturity T

$$H = H_0 + \int_0^T \theta_t^H dX_t + L_T^H$$

where $H_0 \in \mathbb{R}$ and L^H martingale orthogonal to X.

- $\int_0^T \theta dX$ is the hedgeable part, L_T^H the residual risk, $H_0 + L_T^H$ the cost of the strategy
- How to compute H_0, θ^H, L^H? Easy when X is a martingale, difficult when it is not

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Local risk minimization II

- When X is not a martingale but not far from being so ...
- Föllmer-Schweizer (1991): there exists a risk-neutral $\widehat{\mathbb{Q}}$ for X s.t.

$$H = \widehat{\mathbb{E}}[H] + \int_0^T \widehat{\theta}_t^H dX_t + \widehat{L}_T^H$$

- $H_0 = \widehat{\mathbb{E}}[H]$, $\theta^H = \widehat{\theta}^H$, $L^H = \widehat{L}^H$
- $\widehat{\mathbb{Q}}$ is called minimal equivalent martingale measure
- $\widehat{\mathbb{Q}}$ exists in our model and S, D, C are still independent under $\widehat{\mathbb{Q}}$ and C, D have the same law as under $\mathbb{P}$
- Hobson (2005) : in diffusion models with non-tradable assets, $\widehat{\mathbb{E}}[H]$ is an upper bound for U-indifference bid price.

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Pricing of energy futures

- Energy future final payoff $P_T = \varphi(S_T, C_T, D_T)$
- Thus $F^e_t(T) = \widehat{\mathbb{E}}[P_T | \mathcal{F}_t]$, so

$$F^e_t(T) = \sum_{i=1}^{n} h_i F^i_t(T) \mathbb{E}\left[g(C^{max}_T - D_T) \mathbf{1}_{\{D_T \in I^i_T\}} | \mathcal{F}^{D,C}_t \right]$$

where $F^i_t(T)$ is T-future of fuel i, h_i heat rate

- Set $G_i(t, C_t, D_t) = \mathbb{E}\left[g(C^{max}_T - D_T) \mathbf{1}_{\{D_T \in I^i_T\}} | \mathcal{F}^{D,C}_t \right]$
- We have

$$dF^e_t(T) = \theta^S_t dW_t + \theta^C_t dW^C_t + \theta^D_t dW^D_t$$

for adapted processes $\theta^S, \theta^C, \theta^D$ explicitly computable

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- Set $G_i(t, C_t, D_t) = \mathbb{E}\left[g(C^{max}_T - D_T)\mathbf{1}_{\{D_T \in I^i_T\}}|\mathcal{F}^{D,C}_t\right]$
- We have

$$dF^e_t(T) = \theta^S_t dW_t + \theta^C_t dW^C_t + \theta^D_t dW^D_t$$

  for adapted processes $\theta^S, \theta^C, \theta^D$ explicitly computable

Pricing of energy futures

- Energy future final payoff $P_T = \varphi(S_T, C_T, D_T)$
- Thus $F_t^e(T) = \widehat{\mathbb{E}}[P_T|\mathcal{F}_t]$, so

$$F_t^e(T) = \sum_{i=1}^{n} h_i F_t^i(T)\mathbb{E}\left[g(C_T^{max} - D_T)\mathbf{1}_{\{D_T \in I_T^i\}}|\mathcal{F}_t^{D,C}\right]$$

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Call on spread

Two fuels, $n = 2$. Consider $(P_T - S^1_T - K)_+$, strike $K > 0$

- $\pi_0 = \sum_i \widehat{\mathbb{E}}\left[(g(C^{max}_T - D_T)S^i_T - S^1_T - K)_+ \mathbf{1}_{\{D_T \in I^i_T\}}\right]$
- S, C, D independent under $\widehat{\mathbb{Q}}$, then

$$\begin{aligned}\pi_0 &= \int_{\mathbb{R}^3} \left\{\phi_1(c,z)\mathbf{1}_{\{z<c_1\}} + \phi_2(c,z)\mathbf{1}_{\{z \geq c_1\}}\right\} \\ &\quad \times f_{D_T}(z) f_{C^1_T}(c_1) f_{C^2_T}(c_2)\, dz\, dc_1\, dc_2\end{aligned}$$

where $\phi_1 = (g-1)BS_0(\sigma_1, K)\mathbf{1}_{\{g>1\}}$ and ϕ_2 explicit as a mixture of BS formulae. We set $g := g(c_1 + c_2 - z)$.

Call on energy futures

Two fuels, $n = 2$. Consider $(F^e_T(T^*) - K)_+$, strike $K > 0$ and $T^* > T$.

- Same arguments lead to

$$\pi_0^F = \int_{\mathbb{R}^3} \psi_0(c,z) f_{D_T}(z) f_{C^1_T}(c_1) f_{C^2_T}(c_2) dz\, dc_1 dc_2$$

where $\psi_0(c,z)$ equals

$$(w_1 + w_2) \int_0^{K/w_2} \hat{f}_{Y^2_T}(y_2) dy_2 BS_0 \left(\sigma_1, \frac{K - w_2 y_2}{w_1 + w_2}\right)$$
$$+(w_1 + w_2) Y^1_0 \widehat{\mathbb{Q}} \left(Y^2_T > \frac{K}{w_2}\right) + w_2 BS_0 \left(\sigma_2, \frac{K}{w_2}\right)$$

and $w_i = h_i G_i$.

Hedging with futures on electricity

Consider a derivative $H = \varphi(F^e_T(T^*), F_T(T^*), C_T, D_T)$ with $T^* > T$. We drop T^*

- By Markov, its $\widehat{\mathbb{Q}}$-price in t is $\phi(t, F^e_t, F_t, C_t, D_t)$ with $\phi(t, x, y, c, z)$ regular
- H's decomposition hedgeable part/residual risk

$$H = \widehat{\mathbb{E}}[H] + \int_0^T \xi_t dF_t + \int_0^T \xi^e_t dF^e_t + L_T$$

where

$$\begin{aligned}
\xi^e_t &= \partial_x \phi + \left(\sum_i \theta^{C,i}_t \beta_i \partial_{c_i} \phi + \theta^D_t b \partial_z \phi \right) \frac{\mathbf{1}_{\{||\theta_t||>0\}}}{||\theta_t||^2} \\
\xi^i_t &= \partial_{y_i} \phi
\end{aligned}$$

L_T can be computed explicitly as well

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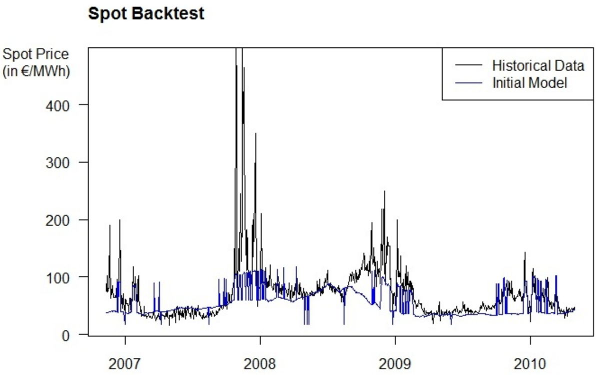
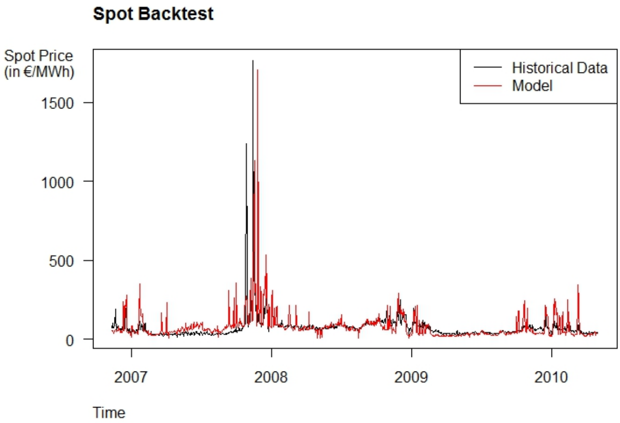
Backtest I : Spot without g

Prices capped at 500€ /MWh, $g \equiv 1$
spot Powernext 19^{th} hours from 13/11/2006 to 04/06/2010
Both D, C follow Ornstein-Uhlenbeck



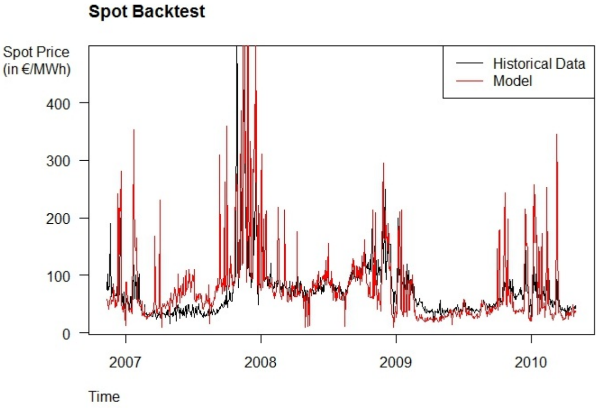
Backtest II : Spot with g

Same data

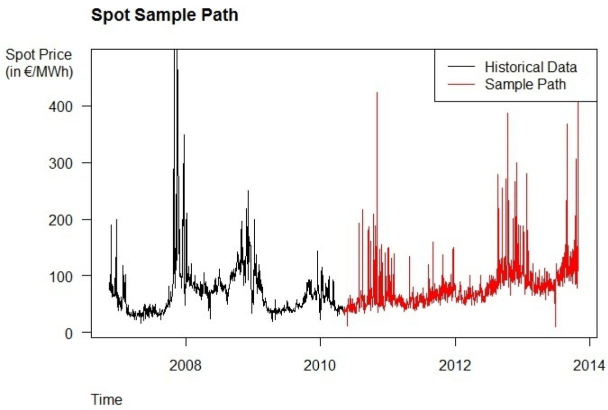


Backtest III : Spot with g (cont'd)

Same data, prices capped at 500€ /MWh



Spot simulation



Few references

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- Barlow (2002) : Electricity spot price is defined as an equilibrium between demand and production
- Fleten and Lemming (2003) : Forward curve reconstruction method, relies on external structural model
- Cartea and Villaplana (2008) : $P_t = \varphi(C_t, D_t)$, with φ exp of affine function, compute forward/future prices, forward premium.
- Aïd et al. (2009) : Spot defined via fuels, capacities and demand, $g \equiv 1$, order of fuels may vary
- Coulon, Howison (2009): Stochastic Behaviour of the Electricity Bid Stack

Conclusions

- We considered an enlarged market of energy **and fuels**, to justify the use of risk-neutral approach as in Aïd et al. (2009)
- Function g introduces spikes in our model
- The market is incomplete, use local risk minimization
- Explicit formulae for hedging strategies and errors
- Trading energy futures (partially) covers the risk coming from demand and capacities
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In progress ...

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