Recursive utility and dynamic consistency : A literature review

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1. Does time consistency matter ?
2. Wonderful world of time consistent agents
3. If irrational agents are more than expected...

Plan

Example 1 : Fatty food or balanced diet ?

Here, a very simple example of time inconsistency based on **time distance** and **spineless agent**.

When asking an agent if she prefers eating a Big Mac or a salad, the answer depends on when the meal occurs :

- Right now $\longrightarrow$ Big Mac (short term gratification, long term pain)
- One week later $\longrightarrow$ Salad (reverse)

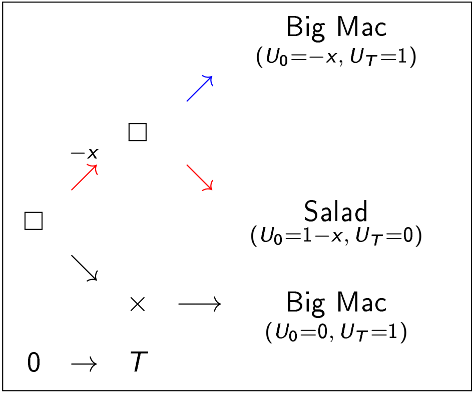
Ask an agent if she prefers eating a Big Mac or a salad for a meal taking place in one week. Then one week later, before the meal begins, ask if she wants to change her mind.

The answer will be **YES** !

Example 1 : Never mind if I change my mind...

Change your mind can be costly especially when your are involved in an optimization problem. Let U_t measure the agent's satisfaction at time t.

- —, agent's strategy at time 0.
- —, agent's deviation at time T,



Example 2 : Mean-Variance criteria

We consider a producer who is subject to production uncertainty and price variation for maturity T_2. At each time t, he is allowed to partially hedge his exposure with T_2-forward contracts (F_{t,T_2}) on a market with transaction costs.
For the sake of simplicity, we assume a two period model, ie $t \in \{0, T_1, T_2\}$

Notations/Assumptions

- Q_{t,T_2} : prevision at time t of the production for maturity T_2.
- Q_t^H : position in T_2-forward contracts at time t^-, $Q_0^H = 0$.
- $q_t F_{t,T_2} h^2$:transaction cost at time t for a transaction of size h.
- $q_{T_2} = 0$.

Mean-Variance allocation criteria

At each time $t \in \{0, T_1, T_2\}$, the producer follows the first stage of the optimal strategy $\left(\pi_s^{*,t}\right)_{s\in\{0,T_1,T_2\}\cap[t,T_2]}$ related to :

$$V_t^* = \max_{\pi^t \in \{0,1\}^{T_2 - t} \times \{1\}} \underbrace{\mathbb{E}_t\left[P_{t,T_2}^{\pi^t}\right] - \gamma Var_t\left(P_{t,T_2}^{\pi^t}\right)}_{V_t^{\pi^t}}$$

with :

$$\begin{aligned} P_{t,T_2}^{\pi^t} &= \sum_{s=t}^{T_2} \pi_s^t \left(F_{s,T_2}\left(Q_{s,T_2} - Q_s^{H,t}\right) - q_s F_{s,T_2}\left(Q_{s,T_2} - Q_s^{H,t}\right)^2 \right) \\ Q_{s+1}^{H,t} &= Q_s^{H,t} + \pi_s^t\left(Q_{s,T_2} - Q_s^{H,t}\right), \; Q_t^{H,t} = Q_t^H \end{aligned}$$

What have I done wrong ?

Remark

The mean variance criteria in a multi period setting results in time inconsistency of investment policies.

$$V_0^{\pi^0} = \pi_0^0 \left(F_{0,T_2} \left(Q_{0,T_2} - Q_0^H \right) - q_0 F_{0,T_2} \left(Q_{0,T_2} - Q_0^H \right)^2 \right) + \mathbb{E}_0 \left[V_{T_1}^{\pi^0_{T_1}} \right] \underbrace{- \gamma Var_0 \left(\mathbb{E}_{T_1} \left[P_{T_1}^{\pi^0_{T_1}} \right] \right)}_{\textit{time inconsistency}}$$

Numerical example

- $T_1 = 1,\ T_2 = 2,\ q_0 = 0.4,\ q_1 = 0.39,\ \gamma = 4$

$$F_{t,2}: \quad \begin{array}{ccccc} & & & & 1.2 \\ & & & \nearrow & \\ 1.0 & \rightarrow & 1.0 & & \\ & & & \searrow & \\ & & & & 0.8 \end{array} \qquad\qquad Q_{t,2}: \quad \begin{array}{ccccc} & & 1.1 & \rightarrow & 1.1 \\ & \nearrow & & & \\ 1.0 & & & & \\ & \searrow & & & \\ & & 0.9 & \rightarrow & 0.9 \end{array}$$

Results

- $\pi^{*,0} = (1, \{1, 0\}),\ \pi^{*,1} = 1$
- The optimal strategy conditionally to producer's future behaviour is $\pi = (0, \{1, 1\})$. (hedging cost : $\{0.43, 0.35\}$)
- The strategy really followed by the producer is $\pi = (1, \{1, 1\})$. (hedging cost : $\{0.44, 0.44\}$)

Time consistency

Definition (Time inconsistency)

Our interpretation of time inconsistency is that our behaviour may change over time in a inconsistent way.

Time-consistency and optimization problem

In general, time-inconsistent preferences/risk measures can not be incorporated in a consistent way in a multi-stage decision problem, ie the dynamic programming principle does not hold.

How can we deal with time inconsistency ?

There is at least four different ways of handling a time inconsistent problem :

- pretending nothing's wrong → absurd
- allowing commitment → not so realistic at the level of firm (commitment devices exist for consumers[Lai97a])

What I am going to talk about :

- requiring rational behaviour → preference/risk measure theory
- looking for equilibrium → game theory

Plan

Risk profiles - a lottery framework

- Let Ω be a finite set of states and $\mathcal{B}(\mathbb{R})$ be the space of bounded functions from Ω to the real line $\mathbb{R}$. An element of $\mathcal{B}(R)$ is viewed as the net loss (risk) incurred in a period.
- The space of t-period risk profiles is constructed recursively. Let $D_0 = \mathcal{B}(\mathbb{R})$:

$$\begin{cases} D_0 = \mathcal{B}(\mathbb{R}) \\ D_t = \mathcal{B}(\mathbb{R} \times D_{t-1}), \; t \geq 1 \end{cases}$$

- The space D [EZ89, Wan99, ML78] of risk profiles is defined as follow :

$$D = \{(d_1, d_2, ...) : d_t \in D_t \; and \; d_{t-1} = f_t(d_t)\}$$

where $f_t : D_t \to D_{t-1}$ is the "tail-cut" operator.

Bellman's Principle of Optimality

Denote by d^{π} the risk profile associated to a portfolio controlled by the policy π, and V_t our optimization criteria at time t. At time t, the agent faces the following optimization problem :

$$\max_{\pi^t=(\pi_t,\ldots,\pi_T)} V_t\left(d^{\pi^t}\right), \forall t$$

In general **the Bellman's Principle of Optimality does not hold**, ie :

$$\pi^{t,*} \in \underset{\pi^t}{argmax}\, V_t\left(d^{(\pi^t)}\right) \not\Rightarrow \left(\pi_s^{t,*}, \ldots, \pi_T^{t,*}\right) \in \underset{\pi^s}{argmax}\, V_s\left(d^{(\pi^s)}\right)$$

How can we choose (V_t) in order to assure that the DPP hold ?

Definition (Dynamic risk measure)

$V : D \to \mathbb{R}$ is called a dynamic risk measure if $V(x, 0) = x$.

If V is a risk measure, define $\tilde{V}(d) : \Omega \to \mathbb{R}$ by

$$\tilde{V}(d)(\omega) = V\left(\tilde{x}_0(\omega), \tilde{d}_1(\omega)\right)$$

Assumptions

Continuity (C) : For all $\{d_n\}_n$, $\{d'_n\}_n \in D$, $d_n \to d, d'_n \to d'$,

$$V(d_n) \geq V(d'_n) \Rightarrow V(d) \geq V(d')$$

Risk separability (RS) :For all $x, x' \in \mathbb{R}$, $d, d' \in D$,

$$V(x, d) \geq V(x, d') \Leftrightarrow V(x', d) \geq V(x', d')$$

Consistency(TC) : For all (x_i, d_i) and $(x'_i, d'_i) \in \mathbb{R} \times D$, $i = 1, ..., n$, and all partitions $\{A_i,\ i = 1, ..., n\}$ of Ω, if $V(x_i, d_i) \geq V(x'_i, d'_i)$ for all i, then

$$V\left(\sum_i (x_i, d_i) \mathbf{1}_{A_i}\right) \geq V\left(\sum_i (x'_i, d'_i) \mathbf{1}_{A_i}\right)$$

Stationarity (S) : For some $\overline{x} \in \mathbb{R}$ and all $(d, d') \in D^2$,

$$V(\overline{x}, d) \geq V(\overline{x}, d') \Leftrightarrow V(d) \geq V(d')$$

Theorem ([EZ89, Wan99])

The risk measure V satisfies properties (C),(RS),(TC),(S) if and only if

$$V(d) = \mu\left(\tilde{V}(d)\right)$$

and

$$V(x, d) = W(x, V(d))$$

where μ is a certainty equivalent and $W : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is continuous and strictly monotone

Epstein Framework

Epstein specification [EZ89]

$$W(x,z) = \phi^{-1}(\phi(x) + \beta\phi(z))$$

for some strictly increasing function $\phi : \mathbb{R} \to \mathbb{R}$ with $\phi(0) = 0$, where β is a constant.

Example : *Epstein and Zin recursive utility* [EZ89, Cam92]
If $\mu(X) = \beta\mathbb{E}_{\mathbb{P}}\left[X^{\theta}\right]^{\frac{1}{\theta}}$ and $\phi(x) = x^{\frac{1-\gamma}{\theta}}$,then

$$V(d) = \left(x_0^{\frac{1-\gamma}{\theta}} + \beta\mathbb{E}_{\mathbb{P}}\left[\tilde{V}(d)^{1-\gamma}\right]^{\frac{1}{\theta}}\right)^{\frac{\theta}{1-\gamma}}$$

In such framework, γ represents the risk aversion, while $\psi = 1 - \frac{1-\gamma}{\theta}$ is the inter-temporal elasticity of substitution.

Wang Framework

Wang specification [Wan99]

$$V(x,d) = \phi^{-1}\left(\phi(x) + \beta\psi^{-1}\left[\int \psi\left[\phi\left(\tilde{V}(d)\right)\right] d\nu\right]\right)$$

where ν is a monotonic set function and ϕ and ψ are strictly increasing functions such that $\phi(0) = \psi(0) = 0$ and

$$\psi^{-1}\left(\int \psi\left[\phi(\beta\tilde{x})\right] d\nu\right) = \beta\psi^{-1}\left(\int \psi\left[\phi(\tilde{x})\right] d\nu\right)$$

Example : *The worst-case likelihood measure*

Let $q\left(\{\tilde{V}(d) \geq z\}\right) = \sup_{\mathbb{P}\in\mathcal{P}} \mathbb{P}\left(\{\tilde{V}(d) \geq z\}\right)$, let $\beta = 1$ and let $\phi(x) = \psi(x) = x$.

$$V(x,d) = x + Q_{q,p}\left(\tilde{V}(d)\right)$$

Dynamic risk measures

Let's $T \in \mathbb{N}^*$ and $\left(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t\in[0,T]\cap\mathbb{N}}, \mathbb{P}\right)$ be filtered probability space that satisfies the usual assumptions.
Let define :

- $\mathcal{R}^{\infty}_{t,T} = \{1_{\{t\leq .\}}X : [0, T] \cap \mathbb{N} \times \Omega \to \mathbb{R},\ Prog,\ \|X^*\|_\infty\}$
- $\mathcal{A}^1 = \{a : [0, T] \cap \mathbb{N} \times \Omega \to \mathbb{R} | \mathbb{F} - adapted,\ Var(a) \in L^1(\mathbb{P})\}$
- $\forall a \in \mathcal{A}^1, X \to \langle X, a\rangle_{t,T} = \mathbb{E}\left[\sum_{s\geq t} X_s \Delta a_s | \mathcal{F}_t\right]$
- $\mathcal{D}_{t,T} = \{1_{\{t\leq .\}}a | a \in \mathcal{A}^1_+ | \langle 1, a\rangle_{t,T} = 1\}$

Conditional risk measure

For all $X, Y \in \mathcal{R}^{\infty}_{t,T}$, $\lambda \in L^{\infty}_{+}(\mathcal{F}_t)$

Monetary risk measure

$\rho : \mathcal{R}^{\infty}_{t,T} \to L^{\infty}(\mathcal{F}_t)$ is a conditional monetary risk measure if :

- (Normalisation) $\rho(0) = 0$
- (Monotonicity) $X \leq Y \Rightarrow \rho(X) \geq \rho(Y)$
- ($\mathcal{F}_t$-Translation) $\rho\left(X + m\mathbf{1}_{[t,T]}\right) = \rho(X) - m$, $\forall m \in L^{\infty}(\mathcal{F}_t)$

Furthermore, ρ is

- **convex** if $\rho(\lambda X + (1-\lambda) Y) \leq \lambda\rho(X) + (1-\lambda)\rho(Y)$,$\lambda \leq 1$
- **coherent** iff ρ is convex and satisfies that $\rho(\lambda X) = \lambda\rho(X)$
- T-**relevant** iff $A \subset \{\rho\left(-\epsilon\mathbf{1}_A\mathbf{1}_{[s,T]}\right) > 0\}\, \forall\epsilon > 0$, $A \in \mathcal{F}_s$

Representation Theorem

Theorem (Dual representation [PCK04, PCK03, Del00])

Let $\rho_{t,T} : \mathcal{R}^{\infty}_{t,T} \to \mathbb{R}$ be a conditional monetary risk measure $(\Leftrightarrow)$:

- *There exists a $\sigma^* \left(\mathcal{A}^1, \mathcal{R}^{\infty}\right)$, $\mathcal{F}_t$-closed convex set $\mathcal{Q} \in \mathcal{D}_{t,T}$:*

$$\rho_{t,T}(X) = -\underset{a \in \mathcal{Q}}{ess.\inf} \langle X, a \rangle_{t,T} \, \forall X \in \mathcal{R}^{\infty}_{t,T}$$

- *$\rho_{t,T}$ satisfies the Fatou property.*

Example :if $\rho(X) = -\mathbb{E}_{\mathbb{P}}[\inf_n X_n]$, then ρ is a coherent risk measure with representation :

$$\rho(X) = -\inf_{A^{\tau} \in \mathcal{A}} \mathbb{E}_{\mathbb{P}} \left[\sum X_n \left(A^{\tau}_n - A^{\tau}_{n-1} \right) \right]$$

$$\mathcal{A} = Hull \left(\{ A^{\tau} | A^{\tau}_n = A^{\tau}_{n-1} + \mathbb{E}_{\mathbb{P}} \left[\mathbf{1}_{\{\tau = n\}} | \mathcal{F}_n \right], \tau \text{ random time} \} \right)$$

Definition (Time consistency)

A monetary risk measure $(\rho_{t,T})_{t\in[S,T]}$ is time-consistent (⇔)

- ***Consistency :***
  $\forall X, Y \in \mathcal{R}^{\infty}_{t,T}$ *s.t.*

$$X\mathbf{1}_{[\tau,\theta)} = Y\mathbf{1}_{[\tau,\theta)} \text{ and } \rho_{\theta,T}(X) \le \rho_{\theta,T}(Y)$$

  then

$$\rho_{\tau,T}(X) \le \rho_{\tau,T}(Y)$$

- ***Bellman principle :***

$$\rho_{t,T}(X) = \rho_{t,T}\left(X\mathbf{1}_{[t,\theta)} - \rho_{\theta,T}(X)\,1_{[\theta,T]}\right)$$

  for each $t \in [S,T]$, every finite $\mathcal{F}_t$-stopping time θ such that $t \le \theta \le T$ and all processes $X \in \mathcal{R}^{\infty}_{t,T}$

Theorem ([PCK04, PCK03, Nad06])

Let $(\rho_{t,T})$ be a time consistent dynamic coherent risk measure such that $\rho_{0,T}$ is T-relevant and continuous for bounded decreasing sequences. Then the sets

$$\mathcal{Q}^0_{0,T} = \{a \in \mathcal{D}_{0,T} | \rho^\sharp(a) = 0\} \text{ and } \mathcal{Q}^{0,rel}_{0,T} = \mathcal{Q}^0_{0,T} \cap \mathcal{D}^{rel}_{0,T},$$

*are **stable** [Del03], and for every finite stopping time $\tau \leq T$ and $X \in \mathcal{R}^\infty_{\tau,T}$,*

$$\rho_{\tau,T}(X) = -\operatorname{ess.inf}_{a \in \mathcal{Q}^{0,rel}_{0,T}} \frac{\langle X, a\rangle_{\tau,T}}{\langle 1, a\rangle_{\tau,T}}$$

Example

Let $\mathcal{Q}$ be a stable subset of $\mathcal{D}_{0,T} \cap \{Z_T | Z_T \geq 0, \in \mathbb{E}_{\mathbb{P}}[Z_T] = 1\}$.

- **Example :** *Terminal value risk measures [Rie03, BR02]*
  The dynamic risk measure ρ defined by $\mathcal{Q}$ can be identified with :
  $$\rho_{t,T}(X) = -\underset{\mathbb{P}\in\mathcal{P}}{ess\,\inf} \mathbb{E}_{\mathbb{P}}[X_T|\mathcal{F}_t]$$
- **Example :** *Snell Envelop [PA04]*
  $\rho_t(X) = -ess.\inf\{\mathbb{E}_{\mathbb{Q}}[X_\tau|\mathcal{F}_t] \,|\tau \geq t, \, stopping\ time, \, \mathbb{Q} \in \mathcal{Q}\}$
  coincides with $\overline{\rho}$ defined recursively by :

$$\begin{cases} \overline{\rho}_T(X) = -X_N \\ \overline{\rho}_t(X) = -\left(X_t \wedge ess.\inf_{\mathbb{Q}\in\mathcal{P}^e} \mathbb{E}_{\mathbb{Q}}\left[\overline{\rho}_{t+1}(X)\,|\mathcal{F}_t\right]\right), \, 0 \leq t < T \end{cases}$$

Entropic risk measure

Theorem ([KS09])

The family $(\rho_t)_{t\in\mathbb{N}_0}$ is law invariant, time consistent, relevant dynamic risk measure if and only if there is $\gamma \in (-\infty, +\infty]$ such that :

$$\rho_t(X) = \frac{1}{\gamma} ln \mathbb{E}\left[\exp\left(-\gamma X\right)|\mathcal{F}_t\right]$$

The limiting cases $\gamma = 0$ and $\gamma = \infty$ are defined such as :

$$\rho_t(X) = \begin{cases} \mathbb{E}\left[-X|\mathcal{F}_t\right] & \gamma = 0 \\ ess.\sup_{Z\in\mathcal{P}_t} \mathbb{E}\left[Z(-X)\right] & \gamma = \infty \end{cases}$$

where $\mathcal{P}_t$ denotes the set of all positive integrable random variables Z with $\mathbb{E}\left[Z|\mathcal{F}_t\right] = 1$ In addition, the dynamic risk measure $(\rho_t)_{t\in\mathbb{N}_0}$ is convex (resp coherent) iff $\gamma \in [0,\infty]$(resp $\gamma \in \{0,\infty\}$)

Plan

What does a time-inconsistent decision problem look like ?

Following [BM10], we introduce the quit general time-inconsistent optimization problem :

$$\begin{cases} V(t,x) = \max_{u \in \mathcal{U}} \left\{ \mathbb{E}_{t,x}\left[\int_t^T C(t,x,s,X_s^x,u_s)\,ds + F(t,x,X_T^x) \right] \right. \\ \qquad\qquad\qquad \left. + G(t,x,\mathbb{E}_{t,x}[X_T^x]) \right\} \\ dX_s^{t,x} = \mu(s,X_s^{t,x},u_s)\,ds + \sigma(t,X_s^{t,x},u_s)\,dW_s \\ X_t^{t,x} = x \end{cases}$$

This kind of problems is **time inconsistent** in the sense that **Bellman Optimality Principle** does not hold.

- Consumption problem with impatience rate :

$$\begin{cases} \max_{u=(c,\theta)} \mathbb{E}\left[\int_t^T h(s-t)\,u(c_s)\,ds\right] \\ dX_s^{t,x} = \left(rX_s^{t,x} + \theta_s(\mu - r) - c_s\right)ds + \theta_s\sigma dW_s \\ X_t^{t,x} = x \end{cases}$$

Exemple : *Hyperbolic discount factor* [EA08, EL10, EL08]
Let u be of CRRA form and h such that :

$$h(s-t) = \delta e^{-\rho_0(s-t)} + (1-\delta)\,e^{-\rho_\infty(s-t)}$$

- Portfolio allocation under mean variance criteria [BM10, TBZ11, BC11] :

$$\begin{cases} \max_{\theta} \mathbb{E}_t\left[X_T^{t,x}\right] - \gamma Var_t\left(X_T^{t,x}\right) \\ dX_s^{t,x} = \left(rX_s^{t,x} + \theta_s(\mu - r)\right)ds + \theta_s\sigma dW_s \\ X_t^{t,x} = x \end{cases}$$

Irrational behaviour and Game theory

Optimization problem under time-inconsistent behaviour can be viewed as a sequential game, where the players are several successive "selves" $\Rightarrow$ Game theory problem

Equilibrium

Nash equilibrium [PM72]

A strategy is a Nash equilibrium, if at time t, the agent has nothing to gain by changing unilaterally his strategy on period $[t, t+1]$.

$\Downarrow$ sequential game

Perfect Sub-game Nash equilibrium [Gol80, R.H55]

A strategy is a Perfect Sub-game Nash equilibrium, if it is constructed by backward induction as follow : at time t the decision maker guesses what her successors are planning to do, and decides her optimal plan accordingly. (uniqueness ?)

Recursive approach

Recursive optimization technique dominates many areas of economic dynamics.

- Sub-game perfection → arbitrary beliefs about breaking the indifference
- Recursive approach [CL04] → leaving the breaking of indifference to the initial self

Indeed the agent chooses :

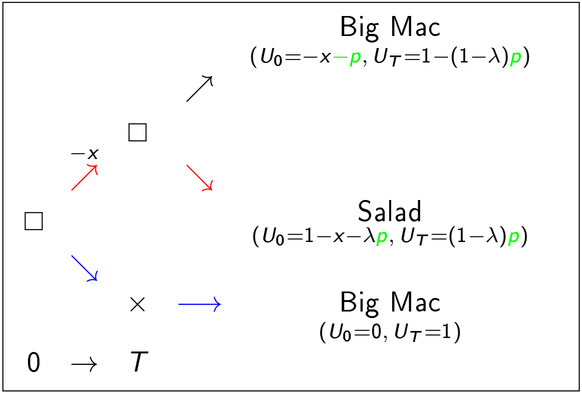
- an action today
- an optimal continuation plan from the set of optimal continuation plans

Example 1 : Fatty food vs balanced diet with incitations

We suppose that the agent is able to incite his future incarnation thanks to a premium p :

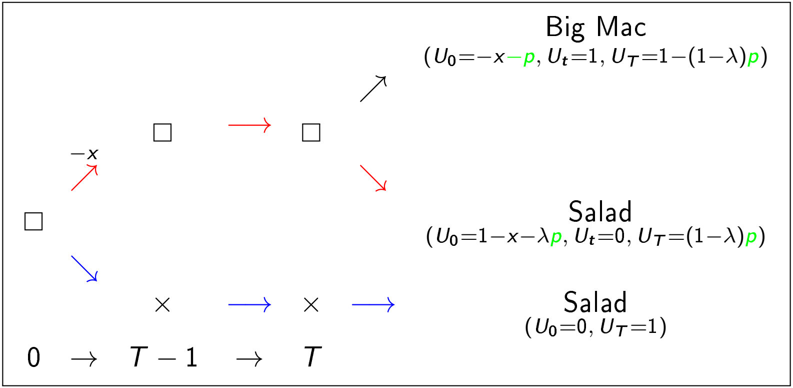
At the equilibrium, $p = \frac{1}{2(1-\lambda)}$:

- —, agent's recursive equilibrium.
- — + —, agent's PSNE.



Example 2 : Fatty food vs balanced diet with renegotiation

We modify the previous example by adding an intermediate renegotiation time. The recursive equilibrium dismisses.



Continuous time perfect sub-game Nash equilibrium

Consider a control law $\widehat{u}$. Choose a fixed $u \in \mathcal{U}$, a fixed number $h > 0$. Also fix an arbitrary chosen initial point (t, x). Define the control law u_h by :

$$u_h(s) = \begin{cases} u, \ t \leq s \leq t+h \\ \widehat{u}_s, \ t+h \leq s \leq T \end{cases}$$

If

$$\liminf_{h \to 0} \frac{J(t, x, \widehat{u}) - J(t, x, u_h)}{h} \geq 0$$

for all $u \in \mathcal{U}$, we say that $\widehat{u}$ **is an equilibrium control law** [EA08]. The equilibrium value function V is defined by :

$$V(t, x) = J(t, x, \widehat{u})$$

Extended HJB system

The extended HJB system of equations for the Nash equilibrium problem is defined as follows [BM10] :

$$\begin{cases} 0 = \sup_{u \in \mathcal{U}} \left\{ \mathcal{L}^u V(t,x) + C(t,x,t,x,u) + h(t,x,u,\widehat{u}) \right\} \\ V(T,x) = F(T,x,x) + G(T,x,x) \end{cases}$$

Remark

- Verification theorem
- Fixed point problem
- Equivalent time consistent problem

Example - Portfolio allocation

The investor's wealth process X follows :

$$dX_t = (rX_t + \theta_t(\mu - r))\,dt + \theta_t \sigma dW_t,\ X_0 = x$$

At each time t, the investor faces the optimization problem :

$$\max_{\theta} \mathbb{E}_t[X_T] - \frac{\gamma}{2} Var_t(X_T)$$

Results

- $\theta_t^* = \frac{\mu - r}{\gamma\sigma^2} e^{-r(T-t)} \xi_t e^{\left(\frac{\mu - r}{\sigma}\right)^2 (T-t) + rt}$ *(Pre-commitment)*
- $\widehat{\theta}_t = \frac{\mu - r}{\gamma\sigma^2} e^{-r(T-t)}$ *(Nash subgame equilibrium)*
- $\max_{\theta} \mathbb{E}_t\left[-\epsilon_T e^{-\gamma X_T}\right]$ *(Equivalent time-consistent problem)*

$$d\epsilon_t = -\frac{\gamma^2}{2}\epsilon_t Var_t\left(dX_t^{\widehat{u}} e^{r(T-t)}\right) \text{ and } d\xi_t = \xi_t\left(-rdt - \frac{\mu - r}{\sigma} dW_t\right)$$

Conclusion-Goals

We have present to way of handling time-inconsistency :

- time consistent preferences (representation theorems + DP)
- game theory approach (unchanged preferences + BI)

At EDF, we attempt to study time-inconsistency in managing a commodities portfolio on a market with transaction costs and margin calls. We try to compare and to understand each of the two approach lists above.

Imperative

Because we work with the firm's financial management, we must deduce rules that have a clear meaning (ex : CAPM approach).

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