A simple model for commodities markets

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Bibliographie

Our work defines a partial equilibrium framework for commodities markets. Some of the aspects have been covered in earlier papers:

- Anderson & Danthine (1983)
- Hirschleifer (1988)
- Deaton & Laroque (1992)
- Guesnerie & Rochet (1993)

A simple model of a commodity market

- two periods, $t = 1$ and $t = 2$. All decisions are taken at $t = 1$ and a source of uncertainty (Ω, P) operates between $t = 1$ and $t = 2$.
- one commodity, produced in quantity ω_1 at $t = 1$ and $\tilde{\omega}_2$ at $t = 2$
- two spot markets, at $t = 1$ and $t = 2$. These are *physical* markets: only positive quantities can be traded.
- one futures markets. Contracts are bought at $t = 1$ and settled at $t = 2$. This is a *financial* market: negative positions are allowed.

There are therefore three prices: two spot prices P_1 and $\tilde{P}_2$, and a future price P_f. Note that P_1 and P_f are observed at time $t = 1$, but $\tilde{P}_2$ is not. The aim of the paper is to determine $P_1, \tilde{P}_2$ and P_f by equilibrium conditions (all markets clear).

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To make things simple, interest rate is set to 0.

- The market is in *contango* (report) if $P_f > P_1$, and in *backwardation* (déport) if $P_f < P_1$If inventory is not zero, then arbitrage theory (cash-and-carry) should imply that the market is in contango. However, backwardation is sometimes observed with non-zero inventory (convenience yield).

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- If $P_f \neq E\left[\tilde{P}_2\right]$, the futures market is *biased*. Keynes argues that futures markets have a systematic downwards bias: $P_f < E\left[\tilde{P}_2\right]$ (producers and processors of commodities are more prone to hedge their price risk than consumers or speculators, so the latter insure the former)

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- Does the existence of a financial market influence prices on the physical markets ? Some excellent economists say yes, and other excellent economists say no.

The agents

- **Spot traders**, who intervene only on the spot markets.
- Industrial users, or **processors**, who use the commodity to produce other goods which they sell to consumers. Because of the inertia of the production process or because they sell their production forward, they have to decide at $t = 1$ how much to produce at $t = 2$. They cannot store the commodity, so they have to buy all of their input at $t = 2$.
- **Inventory holders**, which have storage capacity, and who can use it to buy the commodity at $t = 1$ and release it at $t = 2$.
- Money managers, or **speculators**, who do not trade on the physical markets, they trade only in futures.

Rational behaviour

All agents have mean-variance utility: if they make a profit $\tilde{\pi}$ they derive utility:

$$E[\tilde{\pi}] - \frac{1}{2}\alpha \mathrm{Var}[\tilde{\pi}], \text{ with } \alpha = \alpha_I, \alpha_P, \alpha_S$$

They make optimal decisions at $t = 1$, based on the conditional expectation of $\tilde{P}_2$, which will be determined in equilibrium. All of them (except the spot traders) take positions on the futures market, either for hedging or for speculating.

Rational behaviour

- **Spot traders**. If price at time $t = 1, 2$ is P_t, the demand from spot traders is $\mu_t - m_t P_t$. To simplify the analysis, we will allow negative prices (so that spot traders are paid to hold the commodity)
- **Inventory holders**. Storage is costly: holding a quantity x costs $\frac{1}{2}Cx^2$. If they buy $x \geq 0$ on the spot market at $t = 1$, resell it on the spot market at $t = 2$, and take a position f_I on the futures market, the resulting profit is:

$$\pi_I(x, f_I) = x(\tilde{P}_2 - P_1) + f_I(\tilde{P}_2 - P_F) - \frac{1}{2}Cx^2.$$

The optimal positions are:

$$x^* = \max\left\{\frac{P_F - P_1}{C}, 0\right\}, \quad f_I^* = \frac{\mathrm{E}[\tilde{P}_2] - P_F}{\alpha_I \mathrm{Var}[\tilde{P}_2]} - x^*.$$

The storer holds inventory if the futures price is higher than the current spot price.

Rational behaviour

- **Processors** decide at time $t = 1$ how much input y to buy at $t = 2$, and which position f_P to take on the futures market. The input y results in an output $cy - \frac{\beta}{2}y^2$ which is sold at a price P_0 which is known at time $t = 1$. The resulting profit is:

$$\pi_P(y_2, f_P) = P_0\left(cy - \frac{\beta}{2}y^2\right) - y\tilde{P}_2 + f_P(\tilde{P}_2 - P_F).$$

  The optimal positions are:

$$f_P^* = \frac{\mathrm{E}[\tilde{P}_2] - P_F}{\alpha_P \mathrm{Var}[\tilde{P}_2]} + y^*, \quad y^* = \max\left\{\left(c - \frac{P_F}{P_0}\right)\frac{1}{\beta}, 0\right\}.$$

- **Speculators**.The profit resulting from a futures position f_S is:

$$\pi_S(f_S) = f_S(\tilde{P}_2 - P_F),$$

$$f_S^* = \frac{\mathrm{E}[\tilde{P}_2] - P_F}{\alpha_S \mathrm{Var}[\tilde{P}_2]}.$$

- **Spot market at** $t = 1$. On the supply side the harvest ω_1 and on the other side we have the inventory $N_I x^*$ bought by the storers, and the demand of the spot traders.

$$\begin{aligned} \omega_1 &= N_I x^* + \mu_1 - m_1 P_1, \\ P_1 &= \frac{1}{m_1} \left(\mu_1 - \omega_1 + N_I x^* \right) \end{aligned}$$

- **Spot market at time 2**. On the supply side, the harvest $\tilde{\omega}_2$, and the inventory $N_S x_S^*$ sold by the storers, and, on the other side, the input $N_P y_P$ bought by the processors and the demand of the spot traders.

$$\tilde{\omega}_2 + N_I x^* = N_P y^* + \tilde{\mu}_2 - m_2 \tilde{P}_2,$$

$$\tilde{P}_2 = \frac{1}{m_2} \left(\tilde{\mu}_2 - \tilde{\omega}_2 - N_I x^* + N_P y^* \right)$$

Markets clearing

- **Futures market.** Positions can be positive or negative:

$$N_S f_S^* + N_P f_P^* + N_I f_I^* = 0.$$

$$P_F = \mathrm{E}[\tilde{P}_2] + \frac{\mathrm{Var}[\tilde{P}_2]}{\frac{N_P}{\alpha_P} + \frac{N_I}{\alpha_I} + \frac{N_S}{\alpha_S}} \left(N_P y^* - N_I x^*\right)$$

The equilibrium equations

$$\text{Market Characteristic: } A = 1 + m_2 \frac{\mathrm{Var}[\tilde{\mu}_2 - \omega_2]}{\frac{N_P}{\alpha_P} + \frac{N_I}{\alpha_I} + \frac{N_S}{\alpha_S}}$$

Substituting the value for $\tilde{P}_2$ into the equations for P_1 and P_f (which are just numbers, not random variables) we get the system:

$$m_1 P_1 - N_I \max\left\{\frac{P_F - P_1}{C}, 0\right\} =$$
$$\mu_1 - \omega_1$$
$$m_2 P_F + A\left(N_I \max\left\{\frac{P_F - P_1}{C}, 0\right\} - N_P \max\left\{\left(c - \frac{P_F}{P_0}\right)\frac{1}{\beta}, 0\right\}\right) =$$
$$E[\tilde{\mu}_2 - \tilde{\omega}_2]$$

which is a system of two (nonlinear) equations for two unknowns P_1 and P_f. If we can solve this system we derive $\tilde{P}_2$ by substituting.

Solving for the equilibrium

We solve by investigating the piecewise linear map:

$$F\left(P_1, P_F\right) = \left(\begin{array}{c} m_1 P_1 - N_I \max\left\{\frac{P_F - P_1}{C}, 0\right\} \\ m_2 P_F + m_3 \left(N_I \max\left\{\frac{P_F - P_1}{C}, 0\right\} - N_P \max\left\{\left(c - \frac{P_F}{P_0}\right)\frac{1}{\beta}, 0\right\}\right) \end{array} \right)$$

and showing that it is surjective.