Optimal regulations-pricing rules for a wholesale electricity market

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Outline

- Introduction and motivation
- Modeling and Market Power
- Efficient regulations and mechanism design
- Conclusions

Since the liberalization of the energy markets in the 80´s, they have been modified or improved to avoid market power

ISO


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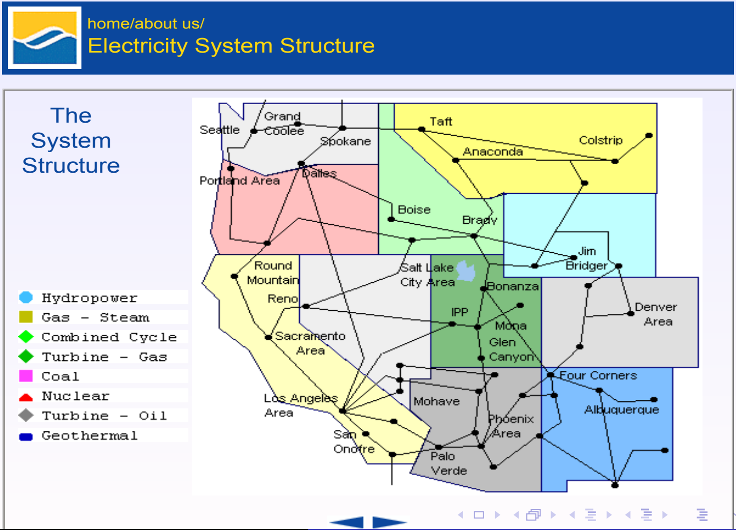
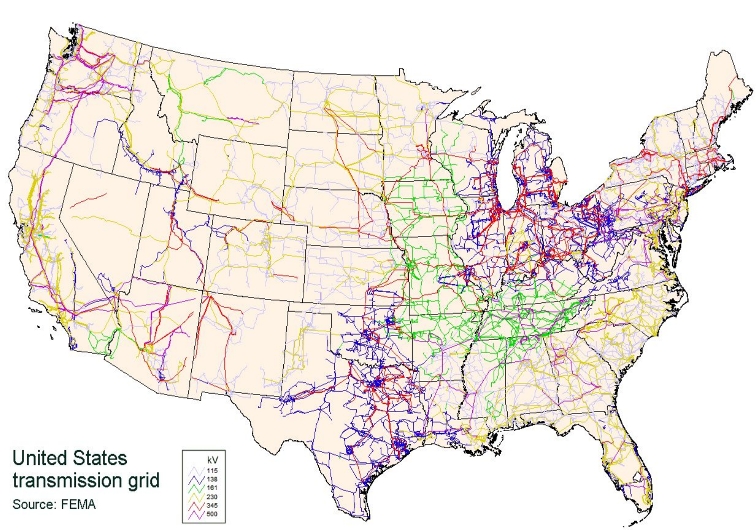
How It Works
POWER PLANT
CALIFORNIA ISO
ISO-CONTROLLED TRANSMISSION SYSTEM
Open market electricity carried across a power grid that connects power plants to substations.
POWER PLANT
HIGH VOLTAGE TRANSMISSION LINES
Buttressed by 100-150 foot towers spanning long distances.
SUBSTATION
Power is "stepped down" in voltage for distribution by local utilities to homes & businesses
DISTRIBUTION SYSTEM
Transports electricity to neighborhoods. Power lines often supported by wooden poles. Sometimes they are built underground.

California

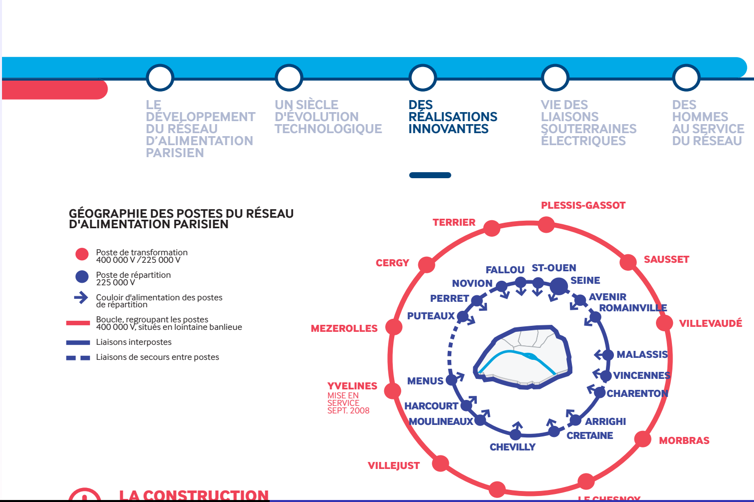
Electricity System Structure



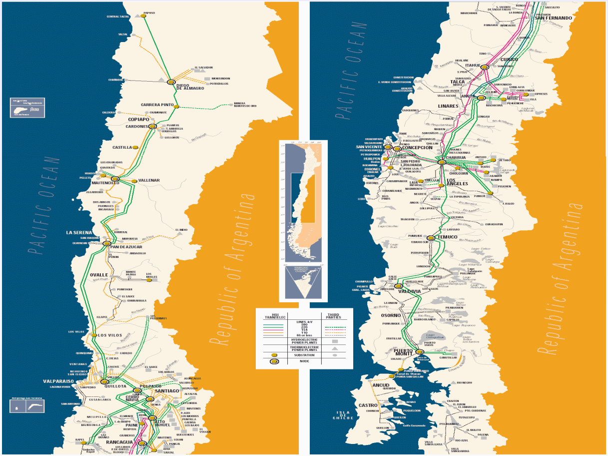
Transmission US



Transmission Japan



Transmission Chile



A generation-transmission pool market

- Short term: for example day-ahead markets
- Today: generators reveal *generation cost functions* taking into account an estimation of the demand. Generators bid *increasing piece-wise linear cost functions or equivalently piece-wise constant "price"*. Even general convex cost functions.
- Tomorrow: the (ISO) using this information and knowing a realization of the demand, minimizes the sum of the costs to satisfy demands at each node considering all the transmission constraints: "dispatch problem".
- The (ISO) sends back to generators the optimal quantities and "prices" (multipliers associated to supply = demand balance equation at each node)

ISO problem

The (ISO) knows a realization of the demand $d \in \boldsymbol{R}^V$, receives the costs functions bid $(c_i)_{i \in G}$ and compute how much each generator will produce $(q_i)_{i \in G}$ and the system of "prices-multipliers" $(p_i)_{i \in G}$ solving the following "dispatch" problem:

$$DP(c,d) \quad OPT(c,d) = \min_{(h,q) \in \Omega(d)} \sum_{i \in G} c_i(q_i).$$

In which, for each demand vector d, we encapsulate the supply $\geq$ demand and capacity constraints in:

$$\Omega(d) \subset \boldsymbol{R}^E \times \boldsymbol{R}^G,$$

defined by the following constraints (1)-(2).

modeling ISO: supply-demand constraints

A dispatch $(h, q) \in \boldsymbol{R}^E \times \boldsymbol{R}^G$ is feasible when (Node balance)

$$\sum_{e \in K_i} \frac{r_e}{2} h_e^2 + d_i \leq q_i + \sum_{e \in K_i} h_e sgn(e, v), \quad v \in G \tag{1}$$

It is called DC approximation.

ISO: Capacity on generation and transmission lines constraints

$$q_i \in [0, \bar{q}_i], \quad v \in G, \tag{2}$$

where $\bar{q}_i > 0$.

$$0 \leq h_e \leq \overline{h}_e$$

.

We denote $Q(c, d) \subset \boldsymbol{R}^G$ the generation component of the optimal solution set associated to each cost vector submitted $c = (c_i)$ and demand d.
We denote $\Lambda(c, d) \subset \boldsymbol{R}^G$ the set of multipliers associated to the node balance (1) in the problem $DP(c, d)$

Modeling Generators

At each node $v \in G$ we have a generator with payoff

$$u_i(p, q) = pq - \bar{c}_i(q)$$

, in which $\bar{c}_i$ is the real cost. The strategic set for $v \in G$ denoted S_i is the set of functions $c_i : \boldsymbol{R} \to \boldsymbol{R}_+$ convex, nondecreasing with bounded slope- subgradients in $[0, p^*]$ where p^* is a *price cap*.

Equilibrium

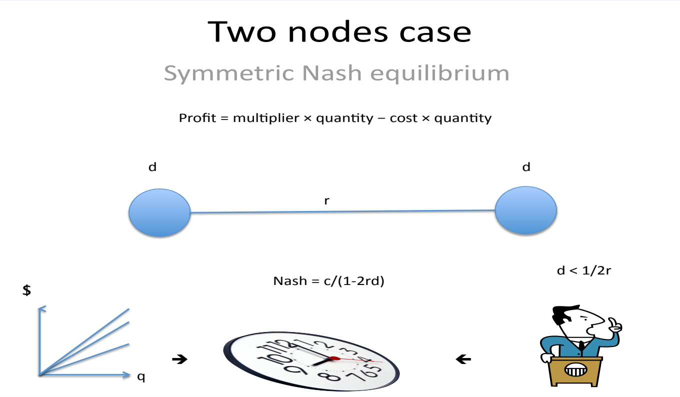
An equilibrium is (q, λ, m) such that q is a selection of $Q(\cdot, \cdot)$ and λ is a selection of $\Lambda(\cdot, \cdot)$ and $m = (m_i)_{i \in G}$ is a mixed-strategy equilibrium of the generator game in which each generator submits costs $c_i \in S_i$ with a payoff

$$\mathbb{E}u_i(\lambda_i(c, \cdot), q_i(c, \cdot)) = \int_D u_i(\lambda_i(c, d), q_i(c, d)) d\mathbb{P}(d),$$

where

$$u_i(\lambda_i(c, d), q_i(c, d)) = \lambda_i(c, d) q_i(c, d) - \bar{c}_i(q_i(c, d))$$

Two-node case



Introduction and Motivation: the ISO Problem

Given that each generator reveals a cost c_i, the (ISO) solves the following minimization problem whose optimal value is denoted $OPT(c,d)$

$$\begin{array}{ll} \min\limits_{q,h} & \sum\limits_{i=1}^{2} c_i q_i \\ s.t. & q_i - h_i + h_{-i} \geq \frac{r}{2}[h_1^2 + h_2^2] + d \text{ for } i = 1,2 \\ & q_i, h_i \geq 0 \text{ for } i = 1,2 \end{array}$$

Introduction and Motivation: result

- Escobar and J. (ET (2010) and MOR (2009)), in a symmetric model with complete information, establish that in the presence of transmission costs, equilibrium exists but producers charge a price above marginal cost with the current regulation.
- demand d, r resistance, $c(q) = cq, \bar{c}$ is the real cost,

$$Nash = \bar{c}/(1 - 2rd)$$

Economic Intuition

- This model might also be viewed as an extension of a Bertrand Game
- The losses in the transmission lines induce a product differentiation among generators: Q is continuous.
- **But**... Q_i is non-differentiable. So, Λ is a set valued map and therefore when cannot use the usual tools of oligopoly theory.

Sensitivity formula

Proposition

Let $c \in \prod_{i \in G} S_i$ *and* $c_i - \hat{c}_i$ *a Lipschitz function with constant* κ*. Then,*

$$|Q_i(c,d) - Q_i(\hat{c}_i, c_{-i}, d)| \leq \kappa\eta,$$

where $\eta = 2\frac{(1+r_i\bar{h}_i)^2}{\min_{i \in G} r_i c_i^+(0)} \in]0, +\infty[$ *and* $c_i^+(0) = \lim_{y \to 0+} \frac{c_i(y) - c_i(0)}{y}$.

Why? Because of the losses, the second-order growth condition is satisfied.

Market Power formula

Dangerous incentive: If the number of generators is small or the topology of the network isolates some demand nodes then the generators will play strategically with the ISO exercising market power.

Market Power formula

Proposition

The equilibrium prices p_i satisfy

$$\mathbb{E}|p_i - \gamma| \geq \frac{\mathbb{E}[Q_i(p_i, p_{-i}, d)]}{\bar{\eta}}$$

where $\bar{\eta}_i = 2\frac{|K_i|^2\left(1+\max\{r_e\bar{h}_e \,:\, e\in K_i\}\right)^2}{p_* \min_{e\in K} r_e}$ *and* $\gamma(p_{-i}, d)$ *is a measurable selection of the subdifferential* $\partial\bar{c}_i(Q_i(p_i, p_{-i}, d))$. *If for example the true costs are linear,* $\bar{c}_i(q) = \bar{c}_i q$, *then*

$$p_i - \bar{c}_i \geq \frac{\mathbb{E}[Q_i(p_i, p_{-i}, d)]}{\bar{\eta}}.$$

The Questions

In an electric network with transmission costs and private information:

- Does the usual (price equal Lagrange multiplier) regulation mechanism minimize costs for the society?
- If not, what is the mechanism that achieves this objective?
- How does the performance of both systems compare?

Methodology:

- Bayesian Game Theory
- Mechanism Design

The framework

Framework

- Two-node network with demand d at each node.
- One producer at each node, with marginal cost of production $c_i \sim F_i[\underline{c}_i, \overline{c}_i]$.
- Transmission costs rh^2, with h the amount sent from one node to another.

Benchmark

The ISO Problem

Given that each generator reveals a cost c_i, the dispatcher solves:

$$\begin{aligned} \min_{q,h} \quad & \sum_{i=1}^{2} c_i q_i \\ s.t. \quad & q_i - h_i + h_{-i} \geq \frac{r}{2}[h_1^2 + h_2^2] + d \text{ for } i = 1,2 \\ & q_i, h_i \geq 0 \text{ for } i = 1,2 \end{aligned}$$

The Solution for ISO problem

If we define

$$H(x,y) = d + \frac{1}{2r}\left(\frac{x-y}{x+y}\right)^2 - \frac{1}{r}\left(\frac{x-y}{x+y}\right)$$

and

$$\overline{q} = 2\left[\frac{1-\sqrt{1-2dr}}{r}\right]$$

then the solution to this problem can be written as

$$q_i(c_i,c_{-i}) = \begin{cases} H(c_i,c_{-i}) & \text{if } H(c_i,c_{-i}) \geq 0 \text{ and } H(c_{-i},c_i) \geq 0 \\ \overline{q} & \text{if } H(c_{-i},c_i) < 0 \\ 0 & \text{if } H(c_i,c_{-i}) < 0 \end{cases}$$

$$\lambda_i(c_i,c_{-i}) \equiv p_i(c_i,c_{-i}) = c_i \text{ if } H(c_i,c_{-i}) \geq 0$$

The Bayesian Game

The game:

- 2 players. Strategies $c_i \in C_i = [\underline{c}_i, \overline{c}_i]$, i=1,2.
- Payoff $u_i(c_i, c_{-i}) = (p_i(c_i, c_{-i}) - \mathbf{c}_i) q_i(c_i, c_{-i})$,

where $\mathbf{c}_i$ is the real cost. The Equilibrium:

- A strategy $b : [\underline{c}_i, \overline{c}_i] \longrightarrow [\underline{c}_i, \overline{c}_i]$.
- In a Nash equilibrium

$$\bar{b}(c) \in arg \max_x \int_{C_{-i}} [p_i(x, \bar{b}(c_{-i})) - c] q_i(x, \bar{b}(c_{-i})) f_{-i}(c_{-i}) dc_{-i} \tag{3}$$

Numerical Approximation

- For simplicity $C_i = [1, 2]$.
- Let $k \in \{0, ..., n-1\}$, and $b(c) = b_k$ for $c \in [\frac{k}{n}, \frac{k+1}{n}]$.
- The weight of each interval is given by $w_k = F(\frac{k+1}{n}) - F(\frac{k}{n})$.
- The approximate equilibrium is characterized by:

$$b_k \in arg \max_x \sum_{l=0}^{n-1} [p_i(x, b_l) - r_k] q_i(x, b_l) w_l \text{ for all } k \in \{0, ..., n-1\} \tag{4}$$

Optimal Mechanism

Mechanisms

- A *direct revelation mechanism* $M = (q, h, x)$ consists of an *assignment rule* $(q_1, q_2, h_1, h_2) : C \longrightarrow R^4$ and a *payment rule* $x : C \longrightarrow R^2$.
- The ex-ante expected utility of a buyer of type c_i when he participates and declares c_i' is

$$U_i(c_i, c_i'; (q, h, x)) = E_{c_{-i}}[x_i(c_i', c_{-i}) - c_i q_i(c_i', c_{-i})]$$

- A mechanism (q, h, x) is feasible iff:

$$\begin{aligned} U_i(c_i, c_i; (q, h, x)) &\geq U_i(c_i, c_i'; (q, h, x)) \text{ for all } c_i, c_i' \in C_i \\ U_i(c_i, c_i; (q, h, x)) &\geq 0 \text{ for all } c_i \in C_i \\ q_i(c) - h_i(c) + h_{-i}(c) &\geq \frac{r}{2}[h_1^2(c) + h_2^2(c)] + d \text{ for all } c \in C \\ q_i(c), h_i(c) &\geq 0 \text{ for all } c \in C \end{aligned}$$

The Regulator's Problem

Using the revelation principle, the regulator's problem can be written as:

$$\min \int_C \sum_{i=1}^{2} x_i(c) f(c) dc \tag{5}$$

subject to (q, h, x) being "feasible"

The Regulator's Problem (II)

It can be rewritten as

$$\begin{aligned} \min \quad & \int_C \sum_{i=1}^{2} q_i(c)[c_i + \tfrac{F_i(c_i)}{f_i(c_i)}]f(c)dc \\ \text{s.t} \quad & \int_{C_{-i}} q_i(c_i, c_{-i})f_{-i}(c_{-i})dc_{-i} \text{ is non-increasing in } c_i \\ & q_i(c) - h_i(c) + h_{-i}(c) \geq \tfrac{r}{2}[h_1^2(c) + h_2^2(c)] + d \text{ for all } c \in C \\ & q_i(c), h_i(c) \geq 0 \text{ for all } c \in C \end{aligned}$$

We denote by $J_i(c_i) = c_i + \frac{F_i(c_i)}{f_i(c_i)}$ the virtual cost of agent i. We assume it is increasing (Monotone likelihood ratio property: true for any log concave distribution)

Solution

An optimal mechanism is given by

$$\hat{q}_i(c_i, c_{-i}) = \begin{cases} H(J_i(c_i), J_{-i}(c_{-i})) & \text{if } H(J_i(c_i), J_{-i}(c_{-i})) \geq 0 \text{ an} \\ \overline{q} & \text{if } H(J_{-i}(c_{-i}), J_i(c_i)) < 0 \\ 0 & \text{if } H(J_i(c_i), J_{-i}(c_{-i})) < 0 \end{cases}$$

$$\hat{x}_i(c_i, c_{-i}) = c_i \hat{q}_i(c_i, c_{-i}) + \int_{c_i}^{\overline{c}_i} \hat{q}_i(s, c_{-i}) ds$$

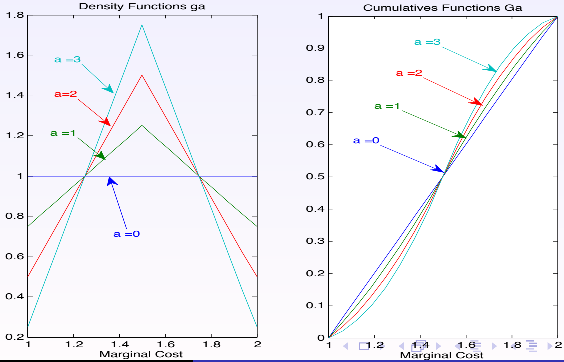
Such a mechanism is dominant strategy incentive compatible.

Comparison

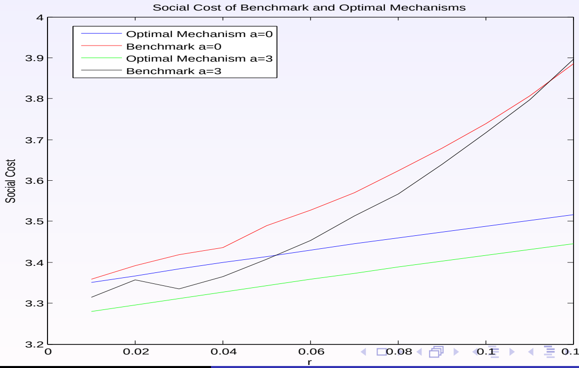
We consider the family of distributions with densities

$$f_a(x) = \begin{cases} a(x-1) + (1 - \frac{a}{4}) & \text{if } x \leq 1.5 \\ -a(x-1) + (1 + \frac{3a}{4}) & \text{if } x \geq 1.5 \end{cases}$$

Asymmetric information



Social costs for different mechanisms



Robustness and Practical Implementation

- The optimal mechanism is detail free. If the designer is wrong about common beliefs, then the mechanism is still not bad:

$$||X_f - X_{\tilde{f}}|| \leq ||x||_1 ||f - \tilde{f}||_\infty \leq \bar{c}\bar{q}||f - \tilde{f}||_\infty$$

- The assignment rule is computationally simple to implement. It requires solving **once** the dispatcher problem, with modified costs.
- However, the payments are computationally difficult

$$c_i \hat{q}_i(c_i, c_{-i}) + \int_{c_i}^{\bar{c}_i} \hat{q}_i(s, c_{-i}) ds$$

- The integral requires solving infinitely many dispatcher problems. But it can be approximated using the risk neutrality of agents.

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