Econometrics of share auctions

Philippe Février, Raphaële Préget and Michael Visser

Uniform and discriminatory auctions

- Uniform and discriminatory auctions are used to sell Treasury bonds.
  - Bidders submit a function : price/quantity
  - The equilibrium price equalizes demand and offer (fixed)
  - In the uniform format, bidders pay this equilibrium price for all units that they acquire
  - In the discriminatory format, bidders pay the marginal price they have bid
- Uniform auctions are used on electricity markets

Common value versus independent values

Common value : the (unknown) value is the same for all bidders. They only have a signal about this value **Private values** : each bidder has his own valuation of the good

- Theoretical models usually consider a common value model.
- Empirically, both are usually considered reasonable to describe Treasury auctions : private needs / resale market
- In electricity, independent values seem the relevant model

Literature

Literature since Friedman (1960) : which format generates the higher revenue ?

- **Theory** (Wilson 1979, Back and Zender (1993), Wang and Zender (2001)). No clear predictions.
- **Experiment** (Smith, 1967, and Abbink, Brandts and Pezanis-Christou, 2001)
- **Empirical** : based on natural experiment-type data (Umlauf, 1993, Simon 1994, and Berg, Boukai and Landsberger, 1998).

Literature

What about the **structural approach** ?

- Hortaçsu (2002/2011)
  - The IPV paradigm
  - The discriminatory auction is revenue-superior to the uniform auction format using data from Turkey.
- In our paper :
  - Wilson's (1979) share auction model : common value model.
  - Data from France in 1995 : discriminatory data.
- A paper by Armantier and Sbai (2003)
  - The CV paradigm with asymmetry and risk-aversion
  - The uniform auction is revenue-superior to the discriminatory auction format using French data from 1998 to 2000.
- A paper by Kastl (2008)in which he shows that the uniform format performs quite well

Outline

1. Share auction theory
2. Identification and estimation
3. Data and results

Model

- A divisible good is auctioned.
- $n \geq 2$ risk-neutral bidders.
- Value of good V, with distr. function $F_V(v)$.
- Each bidder $i = 1, ..., n$ receives private signal about value of the good : S_i.

  Bidder's signals $S_1, ..., S_n$ are i.i.d. given V, with distr. function $F_{S|V}(s|v)$.
- S_i is only observed by bidder i. Number of bidders n, and distr. functions $F_V(\cdot)$ and $F_{S|V}(\cdot|\cdot)$ are common knowledge.

Model

- Each bidder i submits a demand function $x(p, s_i)$ (symmetry).
- The equilibrium price is the price that equals supply (1) and demand.
- In the uniform auction, each bidder pays the equilibrium price for each quantity of the good that he receives.
- In the discriminatory auction, each bidder pays the marginal price that he bids for each quantity of the good that he receives.

Model

Consider the discriminatory auction.

- Let $x(\cdot,\cdot)$ designate the optimal strategy.
- Suppose that all bidders except i use the strategy $x(\cdot,\cdot)$, and that i uses the strategy $y(\cdot,\cdot)$.
- Let p^0 denote the equilibrium price, i.e. p^0 is the price such that
$$\sum_{j\neq i} x(p^0, s_j) + y(p^0, s_i) = 1. \tag{1}$$

Model

- Bidders do not know (ex ante) p^0, but they know distr. function of P^0.

  Bidder i can thus determine

$$
\begin{aligned}
H(p; v, y) &= \Pr\{P^0 \leq p | V = v, y(p, s_i) = y, S_i = s_i\} \\
&= \Pr\left\{\sum_{j \neq i} x(p, S_j) \leq 1 - y | V = v, S_i = s_i\right\} \\
&= \Pr\left\{\sum_{j \neq i} x(p, S_j) \leq 1 - y | V = v\right\}.
\end{aligned}
$$

Model

- When bidder i uses the strategy $y(\cdot,\cdot)$, and if the value of the good and equilibrium price are respectively v and p^0, his profit is

$$(v - p^0)y(p^0, s_i) - \int_{p^0}^{p^{\max}} y(u, s_i)du,$$

where $p^{\max}$ is the largest price for which demand $y(\cdot, s_i)$ is non-negative.

- Bidder's i expected profit is therefore

$$E\left\{\int_0^{\infty}\left[(V - p)y(p, s_i) - \int_{p}^{p^{\max}} y(u, s_i)du\right] dH(p; V, y(p, s_i))|S_i = s_i\right\}. \tag{2}$$

Model

- Strategy $x(\cdot, \cdot)$ is optimal if the maximum is attained at $y(\cdot, \cdot) = x(\cdot, \cdot)$.
- Euler condition in this case is :

$$0 = E \left\{ (V - p) \partial H(p; V, y) / \partial p - H(p; V, y) | S_i = s_i \right\} \quad (3)$$

where the partial derivatives of H with respect to p and y are evaluated at $y = x(p, s_i)$.

Model

- H and its derivatives are quite nasty (implicit dependence on the equilibrium strategy $x(\cdot,\cdot)$.) ... Taking the expectation with respect to V, S_i, and then integrating over p, gives

$$\begin{aligned} 0 &= E\left\{(n-1)\cdot\left(E\left(V|S_1=s_1,...,S_n=s_n\right)-p\right)\cdot\mathbf{1}\left\{P^0\leq p\right\}\right\} \\ &\quad -E\left\{\left(p-P^0\right)\cdot\mathbf{1}\left\{P^0\leq p\right\}\right\} \end{aligned} \tag{4}$$

The condition must hold for all $p\in[0,\infty)$.

Model

Extensions :

- The first order condition given by equation (4) is still valid in the case of *asymmetric* bidders.
- This equation can be easily modified to incorporate some *noise* in the quantity offered by the seller.

Outline

1. Share auction theory
2. Identification and estimation
3. Data and results

Identification

- The model is not identified non-parametrically (Laffont and Vuong (1996), Athey and Haile (2002)).
- We rely on parametric assumption to obtain identification (Paarsch (1992)). We obtain identification results for two classes of functions (Gamma distributions and normal distributions).

Estimation

- Suppose there are L auctions and let l index the l-th auction.
- The goods sold in the different auctions are different + number of bidders varies.
  - z_l : characteristics of good at auction l.
  - n_l : number of bidders at auction l.

Estimation

- The distr. functions are specified parametrically.
  - $F_{V|Z}(\cdot|z;\theta_1)$: cond. distr. function of V_l given $Z_l = z$.
  - $F_{S|V,Z}(\cdot|v,z;\theta_2)$: cond. distr. function of S_{il} given $V_l = v$ and $Z_l = z$.
  - These distr. functions determine $F_{S|Z}(\cdot|z;\theta)$. With

    $\theta = (\theta_1', \theta_2')'$.

Estimation

The objective is to find an estimator of θ^0.

- Euler condition with auction-specific variables becomes

$$
\begin{aligned}
0 \quad = \quad & E\left\{(n_l - 1)\cdot\left(E\left(V_l | S_{1l} = s_{1l}, \ldots, S_{n_l l} = s_{n_l l}, Z_l = z_l\right) - p\right)\cdot 1\left\{P_l^0 \leq p\right\} | Z_l = z_l\right\} \\
& - E\left\{(p - P_l^0)\cdot 1\left\{P_l^0 \leq p\right\} | Z_l = z_l\right\} \qquad (5)
\end{aligned}
$$

Estimation

Let us next find an empirical counterpart for the above moment condition.

- Problem : signals $s_{1l}, ..., s_{n_l l}$ not observed.
- But we know that

$$s_{il} = x^{-1}(x_{ilp}, p, n_l, z_l; \theta^0) = F^{-1}_{S|Z}(1 - G(x_{ilp}|n_l, z_l; p)|z_l; \theta)$$

Estimation

- Idea : replace s_{il} by

$$\tilde{x}^{-1}(x_{ilp}, p, n_l, z_l; \theta) = F_{S|Z}^{-1}(1 - \widehat{G}(x_{ilp}|n_l, z_l; p)|z_l; \theta),$$

and consider the following empirical counterpart :

$$\begin{aligned} & m(x_{11p}, \ldots, x_{n_L Lp}, n_1, \ldots, n_L, z_1, \ldots, z_L, p; \theta) \qquad (6) \\ = & \sum_{l=1}^{L} [\left(E\left(V_l | S_{1l} = \tilde{x}^{-1}(x_{1lp}, p, n_l, z_l; \theta), \ldots, S_{n_l l} = \tilde{x}^{-1}(x_{n_l lp}, p, n_l, z_l; \theta), Z_l = z_l \right) - p \right) \\ & \times (n_l - 1) 1 \left\{ p_l^0 \leq p \right\} - \left(p - p_l^0 \right) 1 \left\{ p_l^0 \leq p \right\}]. \end{aligned}$$

Estimation

Two-step estimator. **First step :** A nonparametric estimate of G is

$$\hat{G}(x|n,z;p) = \frac{\sum_{l=1}^{L} \frac{1}{n_l} \sum_{i=1}^{n_l} 1\left\{x_{ilp} \leq x\right\} K\left(\frac{n-n_l}{h_N}, \frac{z-z_l}{h_Z}\right)}{\sum_{l=1}^{L} K\left(\frac{n-n_l}{h_N}, \frac{z-z_l}{h_Z}\right)} \qquad (7)$$

Estimation

second step :

- Given T values for p, $p_1, ..., p_T$, the estimation method exploits that the Euler condition must hold at these prices.
- Second step of estimation procedure consists in minimizing over θ the sum of the T squared empirical moments :

$$\hat{\theta} = Arg \min_{\theta} \sum_{t=1}^{T} m^2(x_{11p_t}, ..., x_{n_L L p_t}, n_1, ..., n_L, z_1, ..., z_L, p_t; \theta). \qquad (8)$$

Estimation

Choice of moments conditions

- Our estimator of θ^0 belongs to the class of semiparametric two-step estimators considered by Newey and McFadden.
- The estimator is $\sqrt{L}$-consistent, and it is asymptotically normally distributed.

Outline

1. Share auction theory
2. Identification and estimation
3. Data and results

DATA

Analysis is based on all auctions (held in 1995)of

- The *Bons du Trésor à taux Fixe et à intérêts ANnuels* (the BTANs) ; these are tradable fixed-rate medium-term Treasury notes with interest paid annually and with maturities of two or five years.
- The *Obligations Assimilables du Trésor* (the OATs) ; these are fungible Treasury bonds with maturities ranging between 7 and 30 years.

The auction is as follow :

- The Treasury announces the amount of securities offered.
- The Treasury uses a discriminatory auction.
- A bid consists of price/quantity pairs. Bidders are allowed to submit as many bids as they wish.

Data

Here are some statistics about the data :

Table 1. Overall information about the auctions

Number of auctions	45
OAT	25 (56%)
BTAN	20 (44%)
Number of bidders	937
Number of bids	2677
Totally served	1 016 (38%)
Partialy served	423 (16%)
Not served	1 238 (46%)
Total amount issued by the Treasury (FFr millions)	464 579
competitive bids (FFr millions)	423 720 (91%)
ONC1 (FFr millions)	4 831 (1%)
ONC2 (FFr millions)	36 028 (8%)

Data

Table 2. Summary statistics per auction

Variables	Mean	Std. dev.	Min	Max	Obs
Number of bidders	20.82	1.71	15	23	45
Number of bids	59.49	17.41	28	102	45
Amount issued by Treasury (FFr millions)	10 324	5 922	2 052	21 849	45
Winning competitive bids (FFr millions)	9 416	5 335	1 800	19 125	45
ONC1 (FFr millions)	107	121	0	496	45
ONC2 (FFr millions)	801	820	0	2 553	45
Auction coverage	2.25	075	1.29	5.18	45
Maturity of security (in days)	3 749	3 227	586	11 231	45
Nominal yield (%)	7.31	0.80	5.75	8.50	45
Secondary market price	98.07	9.29	71.33	108.50	45
Stop-out price	97.94	9.40	70.88	108.16	45
Highest price bid - lowest price bid	0.32	0.13	0.10	0.68	45
Auction scatter (average price - stop-out price)	0.03	0.02	0.00	0.16	45

Data

Table 3. Summary statistics per bidder or per bid

Variable	Mean	Std. dev.	Min	Max	Obs
Number of bids	2.86	1.58	1	9	937
Demanded quantity per bid (FFr millions)	326	328	10	2500	2677
Price bid	98.54	7.93	70.54	108.26	2677
Highest price bid - lowest price bid	0.07	0.07	0	0.54	937

Data

Differences between the theoretical model and the application :

- Bids are discrete points instead of a function. We suppose that the points given by the bidders belong to the equilibrium.
- Not an isolated market : secondary and when-issued markets.
- We suppose that bidders are symmetric.

Results

- The secondary market price, the nominal yield and the maturity of the security (divided by 1000) sold at the l-th auction are the variables included in the vector z_l.
- V_l given $Z_l = z_l$ has the distribution function

$$F_{V|Z}(v|z_l;\theta_1) = \int_0^v \gamma u^{\gamma-1} \frac{\beta_l^{\alpha_l}}{\Gamma(\alpha_l)} u^{\gamma(\alpha_l-1)} \exp\left[-\beta_l u^{\gamma}\right] du \quad (9)$$

where $\alpha_l = (1, z_l) \cdot \alpha$, $\beta_l = (1, z_l) \cdot \beta$ and $\Gamma(\cdot)$ is the gamma function, α and β are vectors (of dimension 4 by 1) of parameters, and γ is a scalar parameter.

Results

- The signal S_{il} given $V_l = v_l$ and $Z_l = z_l$ follows an exponential distribution :

$$F_{S|V,Z}(s|v_l, z_l; \theta_2) = 1 - \exp\left[-s v_l^{\gamma}\right] \tag{10}$$

- The complete vector of parameters is $\theta' = (\alpha', \beta', \gamma)$.
- With these specifications, it is possible to prove that the model is **identified**.
- We choose $T = 45$, and the prices $p_1, ..., p_T$ are equal to the observed stop-out prices.

Results

Table 4. Second-step estimate of θ (est. standard error)

Estimate of α :	
Constant	8596.67 (114.87)**
Secondary market price	-104.30 (1.24)**
Nominal yield	340.41 (6.77)**
Maturity of security (in days/1000)	-2.04 (1.57)
Estimate of β :	
Constant	-15848.93 (320.72)**
Secondary market price	66.67 (3.48)**
Nominal yield	1617.29 (8.76)**
Maturity of security (days/1000)	142.24 (6.96)**
γ	12.28 (0.0085)**

Revenue comparison

We compare the actual income of the French Treasury with the hypothetical income the Treasury would have earned had it adopted the uniform share auction mechanism.

- In 1995 the total actual income is FFr421.453 billion for the discriminatory auction.
- The calculation of the hypothetical total income under the uniform auction format is less straightforward.

Revenue comparison

- First we need to determine an explicit optimal bidding strategy in the uniform auction format. Given our parametric specifications of the distribution functions, we show that

$$x(p, s_{il}, n_l, z_l; \theta) = \left[1 - \left\{\frac{\beta_l}{n_l} + s_{il}\right\}\left\{\frac{\Gamma(n_l + \alpha_l)}{\Gamma(n_l + \alpha_l + 1/\gamma)}\frac{1+\gamma}{\gamma}p\right\}^{\gamma}\right] / (n_l - 1) . \quad (11)$$

- Replacing θ by $\hat{\theta}$, and for all i s_{il} by $\hat{s}_{il}$, gives the estimated stop-out price in auction l.
- The hypothetical revenue equals FFr400.421 billion. The 95% confidence interval is $[398.210; 402.632]$.
- The hypothetical revenue with discrete bids equals FFr400.061 billion.

Revenue comparison

Comparison with the literature

- As in Hortaçsu (2002), discriminatory auction is revenue-superior to uniform auction.

  Our estimated revenue loss of 5% is smaller though Hortaçsu's one (14%).

- Castellanos and Oviedo (2002) used this method on Mexican data and found that uniform auctions dominate discriminatory auction.

- We also estimate the model with **normal distributions** (Kyle 1989) that confirm our results