# Electricity Pay-as-bid Market: The Best Response of a Producer

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# Outline

# Project context

- Project PGMO-IROE "*Nash equilibrium problems for the valorization of daily offers: the point of view of the producer*" (2012-2015)
- Ph.D. student: Miroslav Pistek - co-supervising with Jiri Outrata (Czech Academy of Sciences, Prague, Czech Rep.)
- EDF partner: Pascale Bendotti (Osiris)

Aim:

*In a deregulated electricity market, given an estimation of the bid of the other players, provide the best bid for a fix producer and study the stability of this best response.*

# Modeling an Electricity Markets

- electricity market consists of
  - i) generators/consumers $i \in \mathcal{N}$ respect their own interests in competition with others
  - ii) market operator (ISO) who maintain energy generation and load balance, and protect public welfare

- the ISO has to consider:
  - ii) quantities $q_i$ of generated/consumed electricity
  - iii) electricity dispatch $t_e$ with respect to transmission capacities

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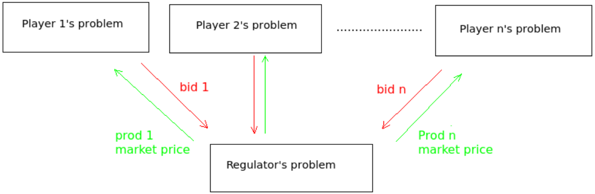
  - iii) electricity dispatch $t_e$ with respect to transmission capacities
- since 1990s, Nash equilibrium problem is the most popular way of modeling spot electricity markets

# Scientific context

In this project, we consider, at a first glance, a simplified model:

- Bilevel model, that is, Multi-leader-common-follower game



# Scientific context

In this project, we consider, at a first glance, a simplified model:

- Bilevel model, that is, Multi-leader-common-follower game
- no transmission losses
- no production capacity constraint or at least the production bounds are not reached
- no transmission capacity

In order to simplify the notations, we aggregate the total demand and consider only producers

# Some references on the topic:

- **Electricity markets without transmission losses:**
  *X. Hu & D. Ralph*, Using EPECs to Model Bilevel Games in Restructured Electricity Markets with Locational Prices, *Operations Research (2007).*
  *bid-on-a-only*

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- **Electricity markets with transmission losses:**
  - *Henrion, R., Outrata, J. & Surowiec, T.*, Analysis of M-stationary points to an EPEC modeling oligopolistic competition in an electricity spot market, *ESAIM: COCV (2012). M-stationary points*
  - *D. A., R. Correa & M. Marechal* Spot electricity market with transmission losses, *J. Industrial Manag. Optim (2013).*
    *existence of Nash equil., case of a two island model*

# Some references on the topic (cont.)

- **Electricity markets with transmission losses:**
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- **Best response in electricity markets:**
  - *E. Anderson, P. Holmberg and A. Philpott, Mixed strategies in discriminatory divisible-good auctions, The RAND Journal of Economics (2013). necessary optimality cond. for local best response*
  - *E. Anderson and A. Philpott*, Optimal Offer Construction in Electricity Markets, *Mathematics of Operations Research (2002). necessary optimality cond. for local best response in time dependent case*

# Description of the model

# Notations

Let consider a fixed time instant and denote

- $D > 0$ be the overall energy demand of all consumers
- $\mathcal{N}$ be the set of producers
- $q_i \geq 0$ be the production of $i$-th producer, $i \in \mathcal{N}$

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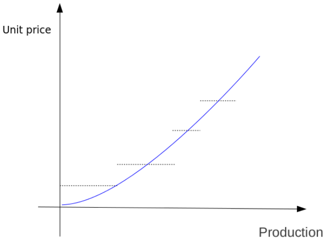
Similarly, let $A_i q_i + B_i q_i^2$ be the true production cost of $i$-th producer with $A_i \geq 0$ and $B_i > 0$ reflecting the increasing marginal cost of production.

# Why a quadratic bid?




Smooth approximation of box bids

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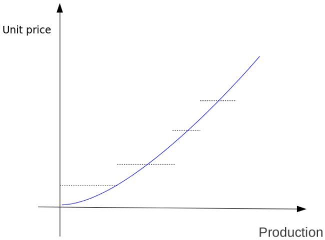
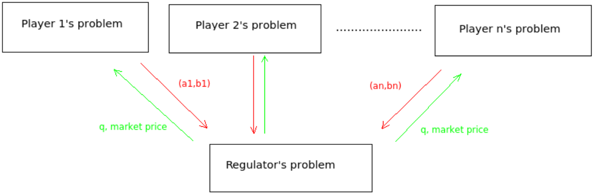



Smooth approximation of box bids

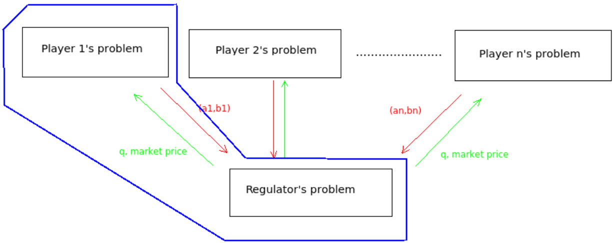
This approach has already been used in Hu-Ralph (bid-on-*a*-only), ACM and HOS..

# Multi-Leader-Common-Follower game



# Multi-Leader-Common-Follower game

Our focus in this work



# Multi-Leader-Common-Follower game

Peculiarity of electricity markets is their bi-level structure:

$$P_i(a_{-i}, b_{-i}, D) \qquad \max_{a_i, b_i} \max_{q_i} \quad a_i q_i + b_i q_i^2 - (A_i q_i + B_i q_i^2)$$
$$\text{such that} \quad \begin{cases} a_i, b_i \geq 0 \\ (q_j)_{j \in \mathcal{N}} \in Q(a, b) \end{cases}$$

where set-valued mapping $Q(a, b)$ denotes solution set of

$$ISO(a, b, D) \qquad Q(a, b) = \underset{q}{argmin} \quad \sum_{i \in \mathcal{N}} (a_i q_i + b_i q_i^2)$$
$$\text{such that} \quad \begin{cases} q_i \geq 0 \ , \ \forall i \in \mathcal{N} \\ \displaystyle\sum_{i \in \mathcal{N}} q_i = D \end{cases}$$

Is the above model well-posed/coherent?

- from the point of view of dispatch quantities/flows
- from the point of view of Market price

# Optimistic case

$$P_i(a_{-i}, b_{-i}, D) \qquad \max_{a_i,b_i} \max_{q_i} \quad a_i q_i + b_i q_i^2 - (A_i q_i + B_i q_i^2)$$
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# Pessimistic case

$$P_i(a_{-i}, b_{-i}, D) \qquad \max_{a_i, b_i} \min_{q_i} \quad a_i q_i + b_i q_i^2 - (A_i q_i + B_i q_i^2)$$

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# Uniqueness for the ISO's problem

Knowing overall demand $D > 0$ and bid vectors $(a, b) \in \mathbb{R}_+^{2N}$ provided by producers, the ISO computes $q \in \mathbb{R}_+^N$ in order to minimize the total generation cost.

$$\min_q \sum_{i \in \mathcal{N}} (a_i q_i + b_i q_i^2)$$
$$\text{s.t.} \begin{cases} q_i \geq 0, \ \forall i \in \mathcal{N} \\ b_i > 0, \ \forall i \in \mathcal{N} \\ \sum_{i \in \mathcal{N}} q_i = D \end{cases} \quad .$$

Assumption used in Hu-Ralph, ACM and HOS

This problem has a unique solution.

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$$

$$
\text{s.t.} \begin{cases} q_i \geq 0, \ \forall i \in \mathcal{N} \\ (a_i, b_i) = (a_j, b_j) \Longrightarrow q_i = q_j, \forall i, j \in \mathcal{N} \quad \text{(H)} \\ \sum_{i \in \mathcal{N}} q_i = D \end{cases}
$$

Assumption called Equity property

This problem also has a unique solution.

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Assumption called Equity property

This model allows linear (**Crucial point**, see conclusion part)

Let us consider different cases for the revenue function:

- Pay-as-bid market: $\pi_i(a_i, b_i) = a_i.q_i + b_i.q_i^2$
- Marginal price with production capacity: $\pi_i(a_i, b_i) = \lambda_i.q_i$
- Marginal price without production capacity: $\pi_i(a_i, b_i) = (a_i + 2b_i.q_i).q_i$

$$P_i(a_{-i}, b_{-i}) \quad \max_{a_i, b_i, q, t} \quad \pi_i(a, b, q, t) - (A_i q_i + B_i q_i^2)$$
$$\text{s.t.} \quad \begin{cases} \underline{A}_i \leq a_i \leq \overline{A}_i, \\ \underline{B}_i \leq b_i \leq \overline{B}_i, \\ (q, t) \text{ solves } ISO(a, b), \end{cases} \tag{1}$$

where $ISO(a, b)$ stands for the following ISO's problem

$$ISO(a, b) \quad \min_{q,t} \sum_{i \in \mathcal{N}} (a_i q_i + b_i q_i^2)$$
$$\text{s.t.} \begin{cases} q_i \geq 0, \ \forall i \in \mathcal{N} \\ q_i \leq \overline{Q}_i, \ \forall i \in \mathcal{N} \\ q_i + \sum_{e \in \mathcal{L}} \left( \delta_{ie} t_e - \frac{L_e |\delta_{ie}|}{2} t_e^2 \right) \geq D_i, \forall i \in \mathcal{N} \\ t_e \geq \underline{T}_e, \ \forall e \in \mathcal{L} \\ t_e \leq \overline{T}_e, \forall e \in \mathcal{L} \end{cases} \tag{2}$$

# Market price: uniqueness

**Proposition**

*Assume that for all producers $i \in \mathcal{N}$, one has $a_i \neq 0$ or $b_i \neq 0$, and, for all lines $e \in \mathcal{L}$, $L_e > 0$. Then $ISO(a, b)$ admits a unique solution $(q^*, t^*)$.*

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## Proposition

*Let $(a, b) \in \mathcal{A} \times \mathcal{B}$ be such that for all producers $i \in \mathcal{N}$, one has $a_i \neq 0$ or $b_i \neq 0$ and there exists a unique $(q^*, t^*)$ solving $ISO(a, b)$. Further, suppose that for all $e \in \mathcal{L}$, $\underline{T}_e < t^*_e < \overline{T}_e$ and that there exists a node $i_0 \in \mathcal{N}$ satisfying $q^*_{i_0} \in (0, \overline{Q}_{i_0})$ . Then for each $i \in \mathcal{N}$ there exist unique Lagrange multipliers $\lambda^*_i$, $\mu^*_i$, $\bar{\mu}^*_i$ and for each $e \in \mathcal{L}$ there exist unique Lagrange multipliers $\beta^*_e$ and $\bar{\beta}^*_e$.*

D.A., M. Cervinka, M. Maréchal, RAIRO, under revision

# Another alternative to fix well-posedness of market price

Another alternative has been used in Escobar-Jofré, *Equilibrium analysis of electricity auctions*, preprint (2014) to fix the problem of well-posedness of market price: they use a selection of the set of Lagrange multipliers.

# Analytic solution of ISO's problem

# Critical Parameters

$$\lambda^m(a) = \min_{i\in\mathcal{N}} a_i \tag{3}$$

Since we allow $b_i = 0$, we need to introduce more parameters a critical market price $\lambda^c(a, b)$, a critical value of the overall demand $D^c(a, b)$, and a set of producers bidding critical (linear) bids $\mathcal{N}^c(a, b) \subset \mathcal{N}$

$$\begin{aligned}
\lambda^c(a, b) &= \min_{i:b_i=0} a_i \\
D^c(a, b) &= \sum_{i:a_i<\lambda^c(a,b)} \frac{\lambda^c(a, b) - a_i}{2b_i} \\
\mathcal{N}^c(a, b) &= \{i \in \mathcal{N} \ : \ a_i = \lambda^c(a, b) \text{ and } b_i = 0\}
\end{aligned}$$

# Critical Parameters

(a) On one hand, if the price is strictly below $\lambda^c(a, b)$ then only truly quadratically bidding producers will be active in the market. On the other hand, if price equals $\lambda^c(a, b)$, there is some linearly bidding producer $(b_i = 0)$ that can formally provide arbitrary amount of electricity at price $\lambda^c(a, b)$.

(b) $\mathcal{N}^c(a, b)$ is the set of all the critical producers - that is, producers bidding linearly and at the critical price - that may possibly be active in the market.

(c) $D^c(a, b)$ will be later identified with the overall amount of electricity produced by sub-critical producers, i.e., those bidding with $b_i > 0$

(d) From the definition of $\lambda^c(a, b)$ we clearly have that $a_i < \lambda^c(a, b)$ immediately implies $b_i > 0$. This means that if the linear term of the bid of producer $i$ is strictly smaller than the critical market price, then this producer is bidding quadratically.

(e) We note that condition $D^c(a, b) = 0$ means that no sub-critical producer, i.e. producer bidding $b_i > 0$, can be active in the market. Moreover, this condition may be equivalently stated as $\lambda^m(a) = \lambda^c(a, b)$.

# Market Price

Next we define

$$\lambda(a,b,D) = \begin{cases} \lambda \in \mathbb{R}_+ \text{ s.t. } \sum_{i:a_i<\lambda} \frac{\lambda - a_i}{2b_i} = D \text{ if } D \in ]0, D^c(a,b)[ \\ \lambda^c(a,b) \text{ if } D \geq D^c(a,b) \end{cases}$$

which is continuous, piece-wise linear, and non-decreasing in $D$.

# Analytic Solution to ISO(a,b,D) Problem

## Theorem

*Let $D > 0$ and $(a, b) \in \mathbb{R}_+^{2N}$, then ISO$(a, b, D)$ admits a unique solution obeying the equity property (H) with $q(a, b, D)$ given by*

$$q_i(a, b, D) = \begin{cases} \frac{\lambda(a,b,D)-a_i}{2b_i} & \text{if } a_i < \lambda(a, b, D) \\ \frac{D-D^c(a,b)}{N^c(a,b)} & \text{if } a_i = \lambda(a, b, D), b_i = 0 \\ 0 & \text{if } a_i = \lambda(a, b, D), b_i > 0 \\ 0 & \text{if } a_i > \lambda(a, b, D) \end{cases}$$

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Denoting $C(a, b, D)$ the overall cost of production, it holds

$$\lambda(a, b, D) = \partial_D C(a, b, D).$$

# Active bidders/producers

One can deduce that for a given demand $D > 0$ and a given bid vector $(a, b)$, the active producers are:

(a) either the quadratically bidding producers ($b_i > 0$) for whom the linear term coefficient $a_i$ of the bid is strictly less than the market price $\lambda(a, b, D)$,

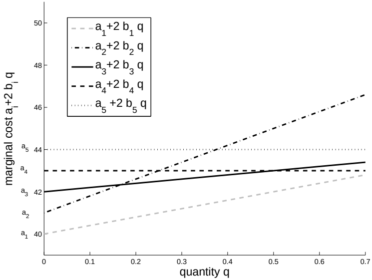
(b) or the linearly bidding producers ($b_i = 0$) who bid exactly the market price $\lambda(a, b, D)$.

# A toy market

Consider a market with 5 producers, $\mathcal{N} = \{1, \ldots, 5\}$, having bid functions given by

| $i \in \mathcal{N}$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $(a_i, b_i)$ | $(40, 2)$ | $(41, 4)$ | $(42, 1)$ | $(43, 0)$ | $(44, 0)$ |

Respective marginal bid functions $a_i + 2b_i q_i$ of such producers are



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Then, we have

- critical market price $\lambda^c(a, b) = 43$
- critical value of the overall demand $D^c(a, b) = 1.5$
- set of producers bidding critical (linear) bids $\mathcal{N}^c(a, b) = \{4\}$

# A toy market (cont.)

| $i \in \mathcal{N}$ | 1 | 2 | 3 | 4 | 5 |
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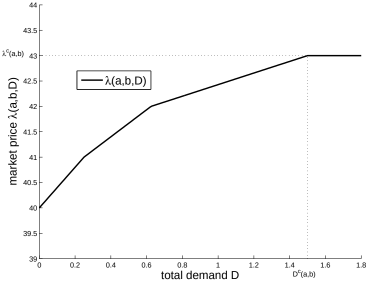


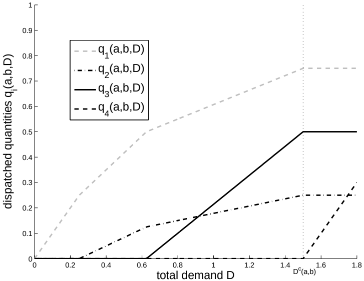

# Best response

Aim : given the aggregated demand $D > 0$ and the bids of "the other players" $(a_{-i}, b_{-i})$, determine, if exists, the best response $(\tilde{a}_i, \tilde{b}_i)$ of producer $i$ that solves

$$P_i(a_{-i}, b_{-i}, D) \qquad \tilde{\pi}_i := \sup_{a_i, b_i \geq 0} \pi_i(a_i, a_{-i}, b_i, b_{-i}, D)$$

where

$$\pi_i(a, b, D) = \left[a_i.q_i(a, b, D) + b_i.q_i(a, b, D)^2\right] - \left[A_i.q_i(a, b, D) + B_i.q_i(a, b, D)^2\right]$$

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Our conclusions will be:

- A linear bid is the best response!!

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Our conclusions will be:

- A linear bid is the best response!!
- But it's often better not to search for the best response!!!

Similarly to previous notation: on a market consisting only of producers in $\mathcal{N} \setminus \{i\}$: we define

$$\lambda^c(a_{-i}, b_{-i}) = \min_{j \in \mathcal{N} \setminus \{i\}, b_j = 0} a_j,$$

and similarly also the other critical parameters $\mathcal{N}^c(a_{-i}, b_{-i})$, $D^c(a_{-i}, b_{-i})$ of E-ISO$(a_{-i}, b_{-i}, D)$.

### Lemma

*Consider demand $D > 0$ and bid vector $(a, b) \in \mathbb{R}_+^{2N}$. Then*

(a) $\lambda(a, b, D) \leq \lambda(a_{-i}, b_{-i}, D)$,

(b) $a_i \leq \lambda(a, b, D)$ *if and only if* $a_i \leq \lambda(a_{-i}, b_{-i}, D)$,

(c) *if* $b_i > 0$, *then,* $a_i < \lambda(a, b, D)$ *if and only if* $a_i < \lambda(a_{-i}, b_{-i}, D)$.

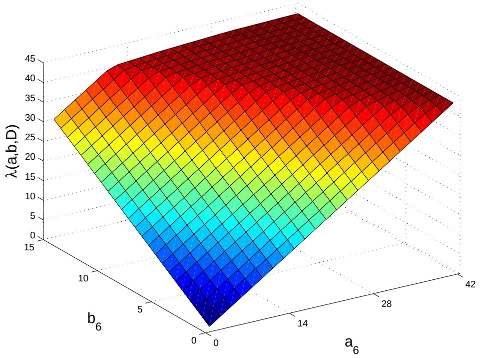
An economical interpretation:

(a) it states that the price in the market including producer $i$ is always less or equal to the price in the market without producer $i$

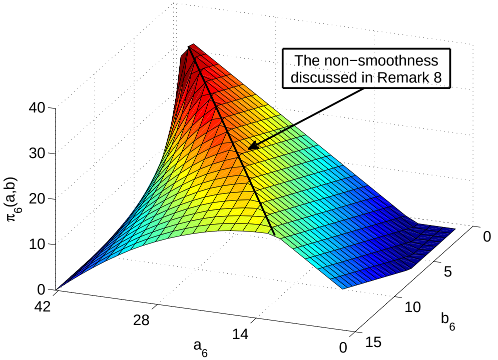
(b) (respectively (c)) it enlightens that if producer $i$ would have been active with a linear bid (respectively with a quadratic bid) in the market without him then he will be active in the market with him.

# Extended toy market: a new producer $i = 6$

Let us consider another producer $i = 6$ in the toy market described above. Then the price curve in the market with producer 6, its real production cost coefficients $(A_6, B_6)$ and $D = 1$ is:



# Example: the Profit of Producer $i = 6$, $D = 1$



**Theorem**

*Assume $D > 0$ and take $(a_{-i}, b_{-i}) \in \mathbb{R}_+^{2N-2}$. Then, considering the unique solution $q(a,b,D)$ to the regulator's problem E-ISO(a,b,D), the i-th producer profit $\pi_i(a,b,D)$ satisfies one of the following statements:*

(a) *for $a_i \leq \lambda(a_{-i}, b_{-i}, D)$ and $b_i > 0$,*

$$\pi_i(a,b,D) = \frac{\lambda(a,b,D) - a_i}{4b_i^2}\left[a_i b_i - 2A_i b_i + a_i B_i + \lambda(a,b,D)(b_i - B_i)\right],$$

(b) *for $a_i < \lambda(a_{-i}, b_{-i}, D)$ and $b_i = 0$ (and so $a_i = \lambda^c(a,b)$ and $\mathcal{N}^c(a,b) = \{i\}$),*

$$\pi_i(a,b,D) = (\lambda^c(a,b) - A_i)(D - D^c(a,b)) - B_i(D - D^c(a,b))^2,$$

(c) *for $a_i = \lambda(a_{-i}, b_{-i}, D)$ and $b_i = 0$ (and so $a_i = \lambda^c(a,b)$ and $i \in \mathcal{N}^c(a,b)$),*

$$\pi_i(a,b,D) = (\lambda^c(a,b) - A_i)\frac{D - D^c(a,b)}{N^c(a,b)} - B_i\left(\frac{D - D^c(a,b)}{N^c(a,b)}\right)^2,$$

(d) *for $a_i > \lambda(a_{-i}, b_{-i}, D)$ it holds $\pi_i(a,b,D) = 0$.*

Note that $a_i$ is compared to $\lambda(a_{-i}, b_{-i}, D)$

# Ideal production for producer $i$

We introduce a level of production

$$q_i^\star(a_{-i}, b_{-i}) = \frac{\lambda^c(a_{-i}, b_{-i}) - A_i}{2B_i}$$

having a significant economic meaning for producer $i \in \mathcal{N}$:

*Let $(a_{-i}, b_{-i}) \in \mathbb{R}^{2N-2}$, $a_i = \lambda^c(a_{-i}, b_{-i})$ and $b_i = 0$ be fixed for some $i \in \mathcal{N}$. Then, if we consider $q_i$ as a free variable for the moment, the profit of producer $i$ is given by $\pi_i^c(q_i) : q_i \to (\lambda^c(a_{-i}, b_{-i}) - A_i)\, q_i - B_i\, q_i^2$. Then, the maximum of $\pi_i^c(q_i)$ is attained for $q_i = q_i^\star(a_{-i}, b_{-i})$, thus corresponding to a kind of ideal production rate for producer $i$. This follows from $B_i > 0$, then for production quantity higher than $q_i^\star(a_{-i}, b_{-i})$ the additional production cost will be higher than the respective additional gain. Finally, we note that $q_i^\star > 0$ and $\pi_i^c(q_i^\star) > 0$ provided $A_i < \lambda^c(a_{-i}, b_{-i})$.*

## Proposition

*Let* $(a_{-i}, b_{-i}) \in \mathbb{R}_+^{2N-2}$, $D > 0$ *and* $b_i = 0$ *be fixed. Then,* $\pi_i(a_i, a_{-i}, 0, b_{-i}, D)$ *is strictly quasiconcave in* $a_i$ *on* $[0, \lambda(a_{-i}, b_{-i}, D)[$, *and problem*

$$\hat{P}_i(a_{-i}, b_{-i}, D) \qquad \sup_{a_i \in [0, \lambda(a_{-i}, b_{-i}, D)[} \pi_i(a_i, a_{-i}, 0, b_{-i}, D)$$

*admits a solution if and only if one of the following alternatives holds:*

(a) $A_i < \lambda(a_{-i}, b_{-i}, D) < \lambda^c(a_{-i}, b_{-i})$ *(implying* $\lambda^m(a_{-i}) < \lambda(a_{-i}, b_{-i}, D)$*),*

(b) $\lambda^m(a_{-i}) < \lambda(a_{-i}, b_{-i}, D) = \lambda^c(a_{-i}, b_{-i})$ *and* $q_i^c(a_{-i}, b_{-i}) > D - D^c(a_{-i}, b_{-i})$.

*Moreover, if a solution exists, it is unique. Denoting it by* $\tilde{a}_i$, *it is given by*

$$\begin{cases} \tilde{a}_i = \lambda^m(a_{-i}) & \text{if} \quad D \leq q_i^m(a_{-i}, b_{-i}), \\ \dfrac{\tilde{a}_i - A_i}{2B_i + m^-(a_{-i}, b_{-i}, \tilde{a}_i)} \leq D - F(a_{-i}, b_{-i}, \tilde{a}_i) \leq \dfrac{\tilde{a}_i - A_i}{2B_i + m^+(a_{-i}, b_{-i}, \tilde{a}_i)} & \text{if} \quad D > q_i^m(a_{-i}, b_{-i}), \end{cases}$$

*and satisfies* $\tilde{a}_i \in [\lambda^m(a_{-i}), \lambda^c(a_{-i}, b_{-i})[$. *Moreover, the respective maximal profit is positive,* $\pi_i(\tilde{a}_i, a_{-i}, 0, b_{-i}, D) > 0$. *Additionally, if a solution does not exist, then* $\pi_i(a, b, D)$ *is strictly increasing in* $a_i$ *on* $[0, \lambda(a_{-i}, b_{-i}, D)[$.

# Best response?

**Theorem**

*Let $D > 0$, $(a_{-i}, b_{-i}) \in \mathbb{R}_+^{2N-2}$ for some $i \in \mathcal{N}$ and consider the problem*

$$\tilde{\pi}_i := \sup_{a_i, b_i \geq 0} \pi_i(a_i, a_{-i}, b_i, b_{-i}, D).$$

*Then either $A_i \geq \lambda^c(a_{-i}, b_{-i})$ and $\tilde{\pi}_i \leq 0$, or one of the following alternatives holds:*

(a) *if $D \in ]0, q_i^0(a_{-i}, b_{-i})]$ then $\tilde{\pi}_i \leq 0$,*

(b) *if $D \in ]q_i^0(a_{-i}, b_{-i}), D^c(a_{-i}, b_{-i}) + q_i^c(a_{-i}, b_{-i})[$ then $\tilde{\pi}_i > 0$ and there is a unique best response $(\tilde{a}_i, \tilde{b}_i)$ given by $\tilde{b}_i = 0$, and $\tilde{a}_i < \lambda^c(a_{-i}, b_{-i})$ satisfying*

$$\begin{cases} \tilde{a}_i = \min_{i \in \mathcal{N}} a_i & \text{if} \quad D \leq q_i^m(a_{-i}, b_{-i}), \\ \frac{\tilde{a}_i - A_i}{2B_i + m^-(a_{-i}, b_{-i}, \tilde{a}_i)} \leq D - F(a_{-i}, b_{-i}, \tilde{a}_i) \leq \frac{\tilde{a}_i - A_i}{2B_i + m^+(a_{-i}, b_{-i}, \tilde{a}_i)} & \text{otherwise} \end{cases}$$

(c) *$D \in [D^c(a_{-i}, b_{-i}) + q_i^c(a_{-i}, b_{-i}), D^c(a_{-i}, b_{-i}) + q_i^\star(a_{-i}, b_{-i})]$ then $\tilde{\pi}_i > 0$ and a limiting best response $(\tilde{a}_i^k, \tilde{b}_i^k)_k$ is given by $\tilde{a}_i^k \nearrow \lambda^c(a_{-i}, b_{-i})$ and $\tilde{b}_i^k = 0$,*

**Theorem**

*Let* $D > 0$, $(a_{-i}, b_{-i}) \in \mathbb{R}_+^{2N-2}$ *for some* $i \in \mathcal{N}$ *and consider the problem*

$$\tilde{\pi}_i := \sup_{a_i, b_i \geq 0} \pi_i(a_i, a_{-i}, b_i, b_{-i}, D).$$

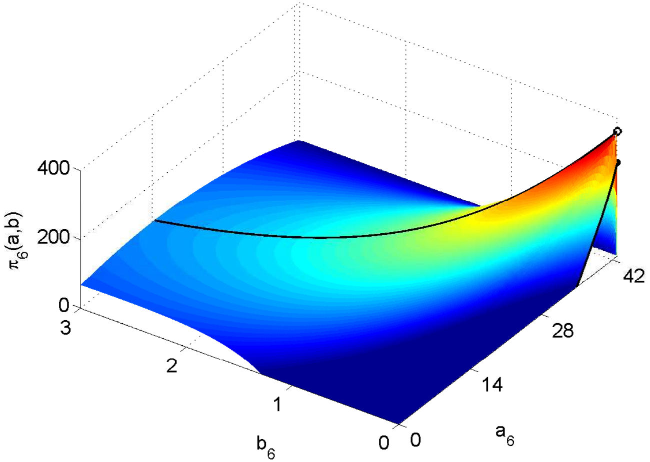
*Then either* $A_i \geq \lambda^c(a_{-i}, b_{-i})$ *and* $\tilde{\pi}_i \leq 0$, *or one of the following alternatives holds:*

(c) *if* $D \in ]D^c(a_{-i}, b_{-i}) + q_i^\star(a_{-i}, b_{-i}), +\infty[$ *and* $D \neq D^c(a_{-i}, b_{-i}) + (N^c(a_{-i}, b_{-i}) + 1)\, q_i^\star(a_{-i}, b_{-i})$ *then* $\tilde{\pi}_i > 0$ *and a limiting best response* $(\tilde{a}_i^k, \tilde{b}_i^k)_k$ *is given by* $\tilde{a}_i^k \nearrow \lambda^c(a_{-i}, b_{-i})$ *and* $\tilde{b}_i^k \searrow 0$ *satisfying*

$$\tilde{a}_i^k = \frac{A_i \tilde{b}_i^k + B_i\, \lambda^c(a_{-i}, b_{-i})}{\tilde{b}_i^k + B_i}$$

(d) *if* $D = D^c(a_{-i}, b_{-i}) + (N^c(a_{-i}, b_{-i}) + 1)\, q_i^\star(a_{-i}, b_{-i})$ *then* $\tilde{\pi}_i > 0$ *and there is a unique best response* $(\tilde{a}_i, \tilde{b}_i) = (\lambda^c(a_{-i}, b_{-i}), 0)$.

# Example: the Profit of Producer $i$, $D = 30$



**Corollary**

*Let $D > 0$, $i \in \mathcal{N}$, $b_i = 0$, $(a_{-i}, b_{-i}) \in \mathbb{R}_+^{2N-2}$ and denote $\xi_i := \xi(a_{-i}, b_{-i})$. Then, one of the following alternatives has to be satisfied:*

(a) *if* $\lambda(a_{-i}, b_{-i}, D) < \lambda^c(a_{-i}, b_{-i})$ *then*

$$\lim_{a_i \nearrow \lambda(a_{-i}, b_{-i}, D)} \pi_i(a_i, a_{-i}, 0, b_{-i}, D) = \pi_i(\lambda(a_{-i}, b_{-i}, D), a_{-i}, 0, b_{-i}, D),$$

(b) *if* $\lambda(a_{-i}, b_{-i}, D) = \lambda^c(a_{-i}, b_{-i})$ *and* $q_i^\star(a_{-i}, b_{-i}) = \xi_i(D - D^c(a_{-i}, b_{-i}))$ *then*

$$\lim_{a_i \nearrow \lambda^c(a_{-i}, b_{-i})} \pi_i(a_i, a_{-i}, 0, b_{-i}, D) = \pi_i(\lambda^c(a_{-i}, b_{-i}), a_{-i}, 0, b_{-i}, D),$$

(c) *if* $\lambda(a_{-i}, b_{-i}, D) = \lambda^c(a_{-i}, b_{-i})$ *and* $q_i^\star(a_{-i}, b_{-i}) > \xi_i(D - D^c(a_{-i}, b_{-i}))$ *then*

$$\lim_{a_i \nearrow \lambda^c(a_{-i}, b_{-i})} \pi_i(a_i, a_{-i}, 0, b_{-i}, D) > \pi_i(\lambda^c(a_{-i}, b_{-i}), a_{-i}, 0, b_{-i}, D),$$

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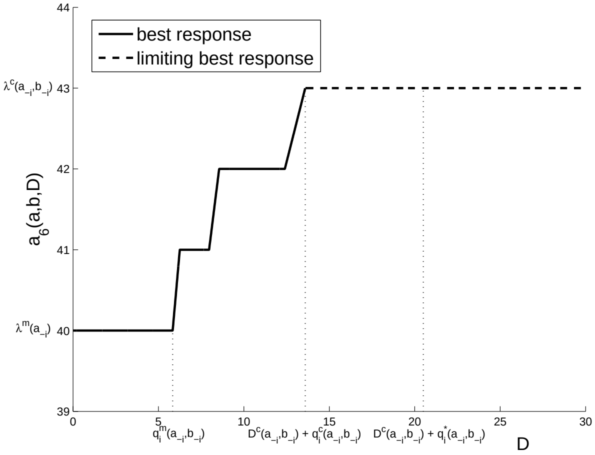
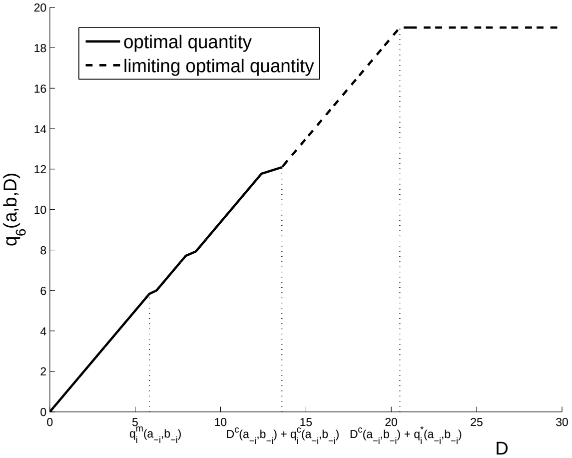
$$\lim_{a_i \nearrow \lambda^c(a_{-i}, b_{-i})} \pi_i(a_i, a_{-i}, 0, b_{-i}, D) < \pi_i(\lambda^c(a_{-i}, b_{-i}), a_{-i}, 0, b_{-i}, D).$$

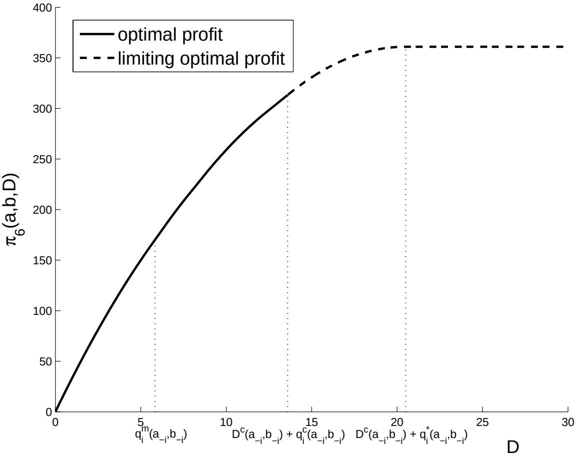
# The Best Response of Producer $i \in \mathcal{N}$



# Production quantity yielded by the best response



# The Optimal Profit of Producer $i \in \mathcal{N}$



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- we provided (eventually limiting) best response of a producer discussed with respect to $D$
- we shown that assuming $b_i > 0$ alone is not consistent from a game-theoretical point of view

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There are several possible extensions of the proposed model/technique

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Thank you for your attention.

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- D. Aussel, R. Correa & M. Marechal, *Spot electricity market with transmission losses*, J. Industrial Manag. Optim. 9 (2013), 275-290.
- D. Aussel, R. Gupta & A. Mehra, *Evolutionary Variational Inequality Formulation of Generalized Nash Equilibrium Problem*, (2014), 14 pp.
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