An optimal trading problem in intraday electricity markets

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A bit of context

Development of renewable energy in continental Europe

- windfarm: Germany 31 GW of 177 GW total installed capacity, Spain 22 GW of 105 GW
- solar power: Germany 32 GW, Italy 16 GW of 124 GW of installed capacity.
- source: Department Of Energy, Energy Information Agency

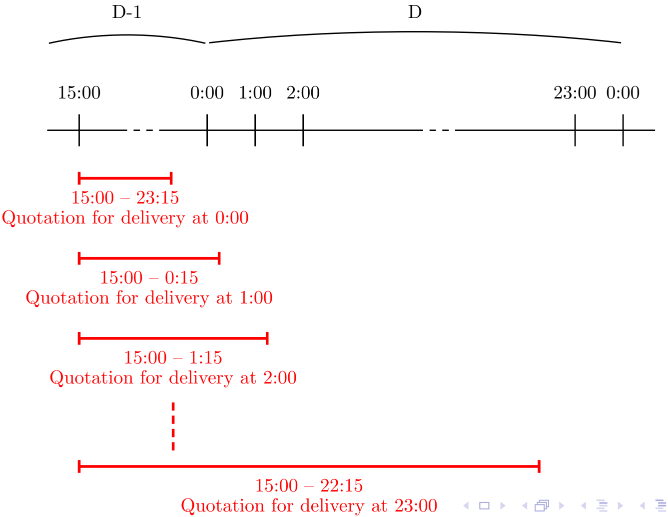
Effects on generation management and trading

- Increasing forecasting error on short time horizon.
- Root mean square error (RMSE) of the error forecast for the production of a wind farm in six hours can reach 20% of its installed capacity (Giebel et al. (2011)).
- Producers / retailers endure imbalance costs. Imbalance: difference between generation (plus purchases) and consumption (plus sales).
- They are penalized for their imbalances, and thus look for ways to balance their short-term position.
- Development of intraday electricity market: the exchanged volume on this market has grown from 2 TWh (2008) to 25 TWh (2013) in Germany.

Short-term electricity markets

- EPEX (European Power EXchange) day-ahead fixing auction.
- EPEX Spot intraday market, organized in continuous trading:
    - Opens at 15:00 the day before.
    - Possibility to buy/sell physical delivery contracts for the 24 periods 0:00 – 1:00, ..., 23:00 – 24:00.
    - Closes 45 minutes before beginning of delivery.

Short-term electricity markets : intraday market



Short-term electricity markets : intraday market

Goal of a power producer

Minimize the "total cost of production and trading":

- Cost of production using
  - Thermal plants (oil, gas, nuclear): expensive but accurate. Must be anticipated.
  - Renewable energy (wind, solar): cheaper, but forecasting is difficult
- Power traded
- Trading costs
- Penalization of difference between furniture/bought delivery and demand.

References

- Forecasting accuracy: Giebel et al. (2011)
- How a producer can cope with forecasting errors thanks to the intraday market: Henriot (2014).
- Almgren–Chriss (2000) and other references on optimal execution problems.

With respect to Almgren–Chriss: random target, as we only have a forecast of final residual demand (total final demand minus quantity agreed upon during the auction time).

Scope of our study

Although 24 products may coexist, we only care for delivery at time T: intraday market is open between dates 0 and T.

- Final residual demand D_T is unknown. Yet we have access to a forecast $(D_t)_{0 \leq t \leq T}$ on $(\Omega, \mathcal{F}, (\mathcal{F}_t)_t, \mathbf{P})$.
- Thermal production ξ is chosen at time $T - h$, at the cost $c(\xi) = \frac{\beta}{2}\xi^2$, $\beta > 0$. Length h is the delay in production.
- Producer has inventory X_t on the intraday market, defined as

$$X_t = X_0 + \int_0^t q_s ds,$$

with $(q_s)_s$ being the instantaneous buying/selling rate on the intraday market. This is the control.

Price processes

At time t, the price that is quoted on the market is

$$Y_t = Y_0 + \widehat{P}_t + \int_0^t g(q_s) ds,$$

where

- $(\widehat{P}_t)_t$ is the unaffected price,
- g is the permanent price impact function, introducing impact after our action $\mathbf{q}$.

The price at which a transaction of q_t is realized is

$$P_t(\mathbf{q}) = Y_t + f(q_t),$$

where f is the temporary price impact function.

Cost minimization

The producer aims to minimize the expectation of his total cost:

$$\inf_{\substack{\mathbf{q}\in\mathcal{A} \\ \xi\in L^0_+(\mathcal{F}_{T-h})}} \mathbf{E}\left(\int_0^T q_s P_s(\mathbf{q}) ds + \underbrace{\frac{\beta}{2}\xi^2 + \frac{\eta}{2}(D_T - X_T - \xi)^2}_{=:C(D_T - X_T, \xi)} \right)$$

where

- $\int_0^T q_s P_s(\mathbf{q}) ds$ takes into account the price of electricity bought on the intraday market and the cost of trading;
- $C(D_T - X_T, \xi)$ is the cost of holding X_T while the final demand is D_T and the production ξ has been chosen. It includes production cost and a square penalization term.

Model assumptions

As in Almgren and Chriss, we assume linear impact functions:

- Permanent impact function $g : x \mapsto \nu x$, $\nu > 0$.
- Temporary impact function $f : x \mapsto \gamma x$, $\gamma > 0$.

Model assumptions

In this section, $h = 0$: we assume that thermal production can be decided instantaneously, therefore the production decision has to be made at final time T.

The unaffected price process is

$$\widehat{P}_t = \widehat{P}_0 + \sigma_0 W_t, \text{ thus } dY_t = \nu q_t dt + \sigma_0 dW_t,$$

and the residual demand forecast has dynamics

$$dD_t = \mu dt + \sigma_d dB_t,$$

with $\mu \in \mathbf{R}$, $\sigma_0 > 0, \sigma_d > 0$ and $d < W, B >_t = \rho dt$, $\rho \in [-1, 1]$.

Dynamic formulation

The value function associated with the minimization problem is

$$v(t,x,y,d) = \inf_{\substack{\mathbf{q}\in\mathcal{A}_t \\ \xi\in L^0_+(\mathcal{F}_T)}} J(t,x,y,d;\mathbf{q},\xi)$$

where

$$J(t,x,y,d;\mathbf{q},\xi) = \mathbf{E}\Big(\int_t^T q_s(Y_s^{t,y}+\gamma q_s)ds + C(D_T^{t,d}-X_T^{t,x},\xi)\Big),$$

$\mathcal{A}_t$ being the set of adapted processes $\mathbf{q}$ such that $\mathbf{E}\big(\int_t^T q_s^2 ds\big) < +\infty$.

Solving the problem

The production quantity ξ is chosen at time T, after the choice of the whole trajectory $\mathbf{q}$. Therefore,

$$\inf_{\substack{\mathbf{q}\in\mathcal{A}_t \\ \xi\in L^0_+(\mathcal{F}_T)}} J(t,x,y,d;\mathbf{q},\xi)$$

$$= \inf_{\mathbf{q}\in\mathcal{A}_t} \mathbf{E}\Big(\int_t^T q_s(Y_s^{t,y} + \gamma q_s) ds + \inf_{\xi\in L^0_+(\mathcal{F}_T)} C(D_T^{t,d} - X_T^{t,x}, \xi) \Big)$$

Thus,

$$\xi_T^* = \operatorname*{argmin}_{\xi\geq 0} \Big(\frac{\beta}{2}\xi^2 + \frac{\eta}{2}(D_T^{t,d} - X_T^{t,x} - \xi)^2 \Big)$$

$$= \frac{\eta}{\eta+\beta}(D_T^{t,d} - X_T^{t,x}) \mathbf{1}_{D_T^{t,d} - X_T^{t,x} \geq 0} =: \hat{\xi}^+(D_T^{t,d} - X_T^{t,x}).$$

Solving the problem

Then

$$
\begin{aligned}
& v(t,x,y,d) \\
&= \inf_{\mathbf{q}\in\mathcal{A}_t} \mathbf{E}\Big(\int_t^T q_s(Y_s^{t,y} + \gamma q_s)ds \\
&\qquad + \frac{\beta}{2}\hat{\xi}^+(D_T^{t,d} - X_T^{t,x})^2 \\
&\qquad + \frac{\eta}{2}(D_T^{t,d} - X_T^{t,x} - \hat{\xi}^+(D_T^{t,d} - X_T^{t,x}))^2 \Big)
\end{aligned}
$$

Solving the problem

Then

$$
\begin{aligned}
& v(t,x,y,d) \\
&= \inf_{\mathbf{q}\in\mathcal{A}_t} \mathbf{E}\Big(\int_t^T q_s(Y_s^{t,y} + \gamma q_s)ds \\
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&\qquad + \frac{\eta}{2}(D_T^{t,d} - X_T^{t,x} - \hat{\xi}^+(D_T^{t,d} - X_T^{t,x}))^2 \Big)
\end{aligned}
$$

We do not expect to get an explicit formula, due to the indicator function.

Solution: consider an auxiliary problem

We relax the positivity constraint on the production ξ:

$$\tilde{v}(t,x,y,d) = \inf_{\substack{\mathbf{q}\in\mathcal{A}_t \\ \xi\in L^0(\mathcal{F}_T)}} J(t,x,y,d;\mathbf{q},\xi).$$

Also,

$$\hat{\xi}(D_T^{t,d} - X_T^{t,x}) = \frac{\eta}{\eta+\beta}(D_T^{t,d} - X_T^{t,x})$$

and

$$\tilde{v}(t,x,y,d) = \inf_{\mathbf{q}\in\mathcal{A}_t} \mathbf{E}\left(\int_t^T q_s(Y_s^{t,y} + \gamma q_s)ds + \frac{1}{2}\frac{\eta\beta}{\eta+\beta}(D_T^{t,d} - X_T^{t,x})^2 \right).$$

This is a linear-quadratic problem!

Value function of the auxiliary problem

$$
\begin{aligned}
\tilde{v}(t,x,y,d) = {} & \frac{r(\eta,\beta)(\frac{\nu}{2}(T-t)+\gamma)}{(r(\eta,\beta)+\nu)(T-t)+2\gamma}\left((d-x)^2 + 2\mu(T-t)(d-x)\right) \\
& + \frac{T-t}{(r(\eta,\beta)+\nu)(T-t)+2\gamma}\left(-\frac{y^2}{2} + r(\eta,\beta)\mu(T-t)y\right) \\
& + \frac{r(\eta,\beta)(T-t)}{(r(\eta,\beta)+\nu)(T-t)+2\gamma}(d-x)y \\
& + \gamma\frac{\sigma_0^2 + \sigma_d^2 r^2(\eta,\beta) - 2\rho\sigma_0\sigma_d r(\eta,\beta)}{\big(r(\eta,\beta)+\nu\big)^2}\ln\left(1 + \frac{(r(\eta,\beta)+\nu)(T-t)}{2\gamma}\right) \\
& + \frac{\sigma_d^2 r(\eta,\beta)\nu + 2\rho\sigma_0\sigma_d r(\eta,\beta) - \sigma_0^2}{2\big(r(\eta,\beta)+\nu\big)}(T-t) \\
& + \frac{r(\eta,\beta)\mu^2(T-t)^2(\frac{\nu}{2}(T-t)+\gamma)}{(r(\eta,\beta)+\nu)(T-t)+2\gamma}
\end{aligned}
$$

where $r(\eta,\beta) := \frac{\eta\beta}{\eta+\beta}$.

Optimal strategy in the auxiliary problem

At time t, $\hat{q}_t$ is given by

$$\hat{q}_t = \frac{r(\eta,\beta)(\mu(T-t) + D_t - \widehat{X}_t) - \widehat{Y}_t}{(r(\eta,\beta)+\nu)(T-t) + 2\gamma}$$

Interpretation:

- Incentive to buy more when anticipated demand grows;
- Incentive to buy less when quoted price increases, and as the inventory gets closer to demand.

Interpretation of the optimal strategy

Introduce

$$\hat{\xi}_s = \frac{\eta}{\eta + \beta}\left(D_s + \mu(T - s) - \widehat{X}_s - \hat{q}_s(T - s)\right),$$

which is seen as the forecast final production seen from time $t \leq s \leq T$. Then we see that

$$\widehat{Y}_s + \nu \hat{q}_s(T - s) + 2\gamma \hat{q}_s = c'\left(\hat{\xi}_s\right).$$

We tend to make the intraday price equal to the marginal production cost!

Martingale property in the auxiliary problem

By applying Itô to $\hat{q}_s = \hat{q}\big(T - s, D_s - \widehat{X}_s, \widehat{Y}_s\big)$,

$$d\hat{q}_s = \big(D_2\hat{q}\big)\sigma_d dB_s + \big(D_3\hat{q}\big)\sigma_0 dW_s.$$

The optimal control is therefore a martingale!

- The expected inventory is thus a linear function of time:

$$\mathbf{E}\big(\widehat{X}_s\big) = X_0 + \hat{q}_0 s.$$

- Interesting extension to Almgren and Chriss.
- Previous “forecasts keeping the same control” are thus pertinent.

Back to the original problem

Now the production ξ has to be positive. Suggested strategy:

1. First follow strategy $(\hat{q}_s)_{t\leq s\leq T}$ of the auxiliary problem.
2. Then choose production level $\hat{\xi}^+$ $(= \hat{\xi}\mathbf{1}_{\hat{\xi}\geq 0})$.

If the controlled inventory is not very likely to become larger then the demand, then the approximation should be correct.

Denote

$$\begin{aligned}\mathcal{E}_1(t,x,y,d) &= J\big(t,x,y,d;\hat{\mathbf{q}},\hat{\xi}^+\big) - v(t,x,y,d),\\ \mathcal{E}_2(t,x,y,d) &= v(t,x,y,d) - \tilde{v}(t,x,y,d).\end{aligned}$$

Measuring the quality of approximation

For all $(t, x, y, d) \in [0, T] \times \mathbf{R} \times \mathbf{R} \times \mathbf{R}$, we have

$$0 \leq \mathcal{E}_i(t, x, y, d) \leq \frac{\eta r(\eta, \beta)}{2\beta} V(T-t)\psi\Big(\frac{m(T-t, d-x, y)}{\sqrt{V(T-t)}}\Big), \quad i = 1, 2,$$

where

$$\psi(z) := (z^2+1)\Phi(-z) - z\phi(z), \quad z \in \mathbf{R}, \quad \to 0 \text{ as } z \to \infty$$

with $\phi = \Phi'$ the density of the standard normal distribution, and

$$m(t, d, y) := \frac{(\nu t + 2\gamma)(\mu t + d) + yt}{(r(\eta, \beta) + \nu)t + 2\gamma},$$

$$V(t) := \int_0^t \frac{\sigma_0^2 s^2 + \sigma_d^2(\nu s + 2\gamma)^2 + 2\rho\sigma_0\sigma_d s(\nu s + 2\gamma)}{\big[(r(\eta, \beta) + \nu)s + 2\gamma\big]^2} ds \geq 0.$$

Simulation (1/3)

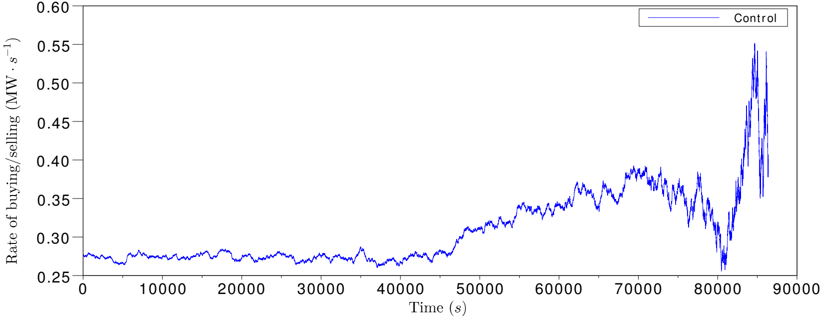

Figure: Evolution of the trading rate control $\hat{\mathbf{q}}$

Simulation (2/3)

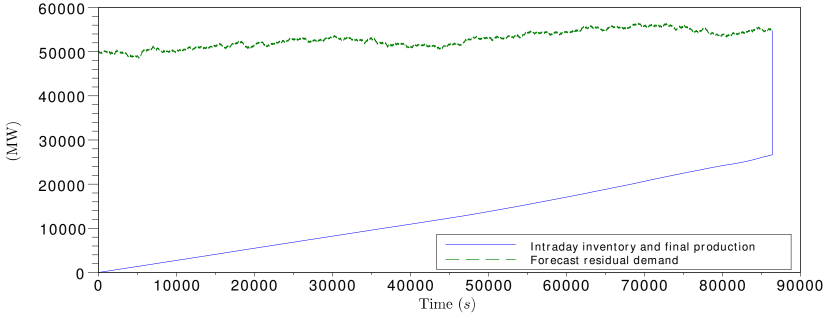

Figure: Evolution of the controlled inventory $\widehat{X}$

Simulation (3/3)

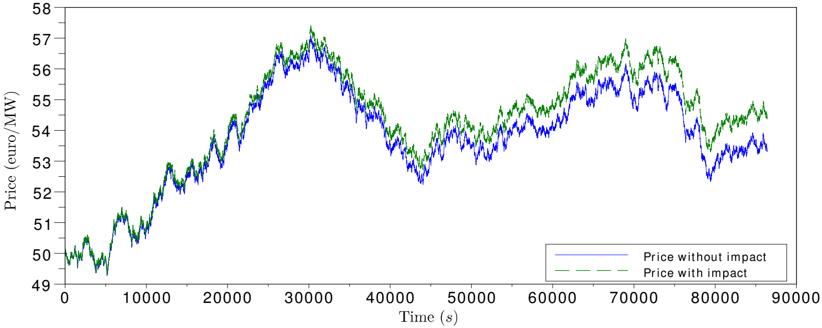

Figure: Evolution of the unaffected and impacted prices

Incorporation of jumps on demand

The prediction of renewable energy production is subject to uncertainties, due to the errors while forecasting temperature, wind speed, hours of sunshine.
How does this impact the previous strategies?

We add a compound Poisson process $(N_t^+, N_t^-)_t$ with intensity λ, counting positive and negative jumps, to model sudden changes in residual demand forecast. At each jump time t,

- with probability p^+, $(N_t^+)_t$ has a jump.
- with probability $p^- = 1 - p^+$, $(N_t^-)_t$ has a jump.

Dynamics of demand and price

Dynamics of demand:

$$dD_t = \mu dt + \sigma_d dB_t + \delta^+ dN_t^+ + \delta^- dN_t^- .$$

with $\delta^+ > 0$ and $\delta^- < 0$.

Impact on intraday price:

$$dY_t = \nu q_t dt + \sigma_0 dW_t + \pi^+ dN_t^+ + \pi^- dN_t^- ,$$

with $\pi^+ > 0$ and $\pi^- < 0$.

Let $\delta := p^+\delta^+ + p^-\delta^-$ and $\pi := p^+\pi^+ + p^-\pi^-$.

Value function and auxiliary problem

We define the value function in this problem with jumps:

$$v^{(\lambda)}(t,x,y,d) = \inf_{\substack{\mathbf{q}^{(\lambda)} \in \mathcal{A}_t \\ \xi \in L^0_+(\mathcal{F}_T)}} J(t,x,y,d;\mathbf{q},\xi).$$

As we did before, we work on an auxiliary problem, relaxing the constraint of positivity of the production:

$$\tilde{v}^{(\lambda)}(t,x,y,d) = \inf_{\substack{\mathbf{q}^{(\lambda)} \in \mathcal{A}_t \\ \xi \in L^0(\mathcal{F}_T)}} J(t,x,y,d;\mathbf{q},\xi).$$

The solution to that problem is explicit.

Value function to the auxiliary problem

$$\begin{aligned}
\tilde{v}^{(\lambda)}(t,x,y,d) &= \tilde{v}^{(0)}(t,x,y,d) + \frac{\lambda}{2}\frac{r(\eta,\beta)(T-t)\big(\pi(T-t)+2\delta(\nu(T-t)+2\gamma)\big)}{(r(\eta,\beta)+\nu)(T-t)+2\gamma}(d-x) \\
&- \frac{\lambda}{2}\frac{(T-t)^2(\pi-2r(\eta,\beta)\delta)}{(r(\eta,\beta)+\nu)(T-t)+2\gamma}y \\
&+ \lambda\gamma\frac{p^+(\pi^+-r(\eta,\beta)\delta^+)^2+p^-(\pi^--r(\eta,\beta)\delta^-)^2}{(r(\eta,\beta)+\nu)^2}\ln\left(1+\frac{(r(\eta,\beta)+\nu)(T-t)}{2\gamma}\right) \\
&- \frac{\lambda}{2}\frac{p^+((\pi^+)^2-r(\eta,\beta)\delta^+(2\pi^++\nu\delta^+))+p^-((\pi^-)^2-r(\eta,\beta)\delta^-(2\pi^-+\nu\delta^-))}{r(\eta,\beta)+\nu}(T-t) \\
&+ \frac{\lambda r(\eta,\beta)}{2}\frac{2\nu\mu\delta+\lambda((p^+)^2\delta^+(\pi^++\nu\delta^+)+(p^-)^2\delta^-(\pi^-+\nu\delta^-))}{r(\eta,\beta)+\nu}(T-t)^2 \\
&+ \lambda^2\gamma r(\eta,\beta)\frac{r(\eta,\beta)\delta^2+2\nu p^+p^-\delta^+\delta^- - ((p^+)^2\delta^+\pi^+ + (p^-)^2\delta^-\pi^-)}{(r(\eta,\beta)+\nu)((r(\eta,\beta)+\nu)(T-t)+2\gamma)}(T-t)^2 \\
&+ \frac{2\lambda\gamma r^2(\eta,\beta)\mu\delta}{(r(\eta,\beta)+\nu)((r(\eta,\beta)+\nu)(T-t)+2\gamma)}(T-t)^2 - \frac{\lambda^2\pi^2}{48\gamma}(T-t)^3 \\
&+ \frac{\lambda^2 p^+p^- r(\eta,\beta)}{2}\frac{2\nu\delta^+\delta^- + \delta^-\pi^+ + \delta^+\pi^-}{(r(\eta,\beta)+\nu)(T-t)+2\gamma}(T-t)^3 \\
&+ \frac{1}{8}\frac{4r(\eta,\beta)\mu\lambda\pi-\lambda^2\pi^2}{(r(\eta,\beta)+\nu)(T-t)+2\gamma}(T-t)^3.
\end{aligned}$$

Optimal control in the auxiliary problem

$$\hat{q}_s^{(\lambda)} = \hat{q}_s^{(0)} + \lambda \frac{r(\eta,\beta)\delta(T-s) + \frac{\pi}{4\gamma}(r(\eta,\beta)+\nu)(T-s)^2}{(r(\eta,\beta)+\nu)(T-s)+2\gamma},$$

We prove that

$$\left(q_s^{(\lambda)} + \frac{\lambda\pi}{2\gamma}(s-t)\right)_{t\leq s\leq T}$$

is a martingale:

- if $\pi > 0$, then $(q_s^{(\lambda)})_s$ is a supermartingale.
- if $\pi < 0$, then $(q_s^{(\lambda)})_s$ is a submartingale.

Simulation with dominant positive jumps: $\pi > 0$ (1/2)

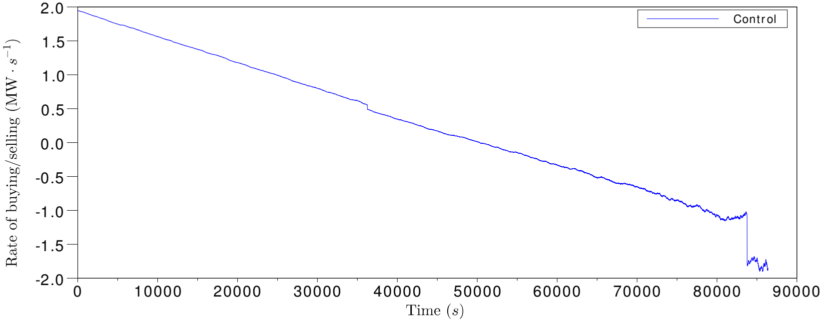

Figure: Evolution of the trading rate control $\hat{\mathbf{q}}$

Simulation with dominant positive jumps: $\pi > 0$ (2/2)

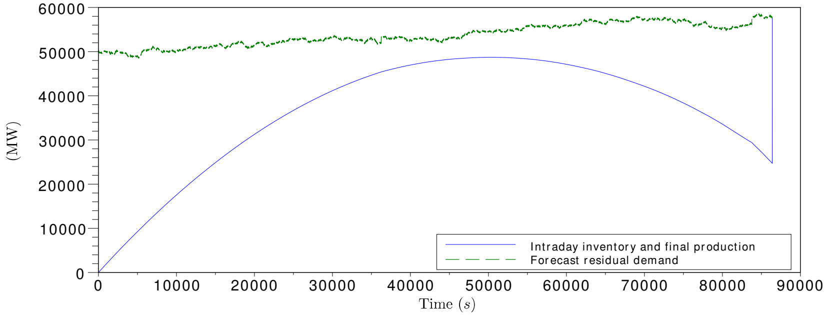

Figure: Evolution of the controlled inventory $\widehat{X}$

Simulation with dominant negative jumps: $\pi < 0$ (1/2)

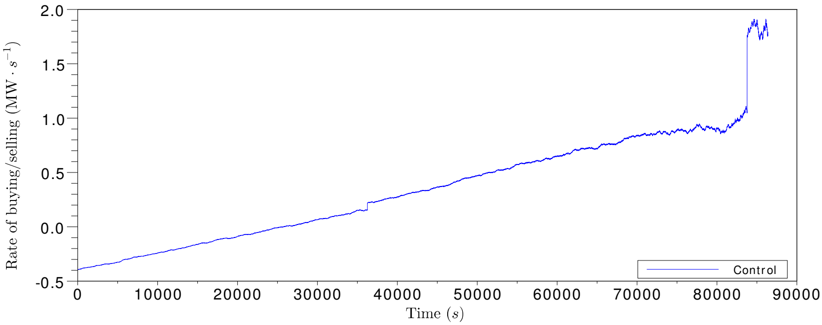

Figure: Evolution of the trading rate control $\hat{\mathbf{q}}$

Simulation with dominant negative jumps: $\pi < 0$ (2/2)

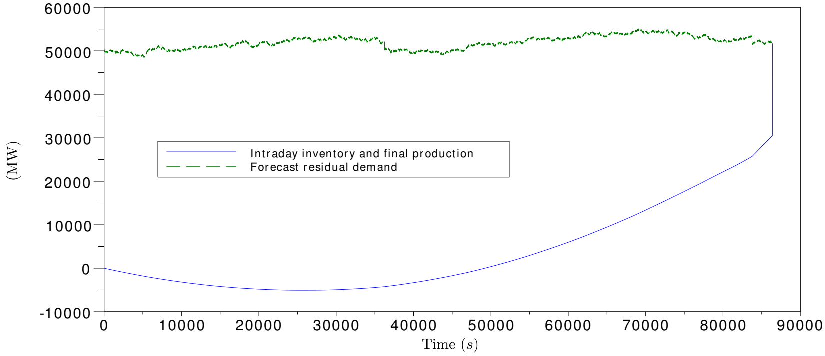

Figure: Evolution of the controlled inventory $\widehat{X}$

Incorporating delay in production

Thermal production decision has to be made a few hours in advance. Let h be that delay.

Now, ξ belongs to $L^0_+(\mathcal{F}_{T-h})$.

Let us observe that

$$X_T^{0,X_0} + \xi = X_T^{T-h, X_{T-h}^{0,X_0} + \xi},$$

we therefore expect to cut the problem into

- one problem between dates 0 and $T - h$, followed by a production decision;
- one problem between dates $T - h$ and T, with no production leverage.

Indeed, if v_h is the value function with $\xi \in L^0_+(\mathcal{F}_{T-h})$,

$$v_h = \inf_{\substack{\mathbf{q}\in\mathcal{A} \\ \xi\in L^0_+(\mathcal{F}_{T-h})}} \mathbf{E}\Big[\int_0^{T-h} q_s\big(Y_s + \gamma q_s\big)ds \\ + \frac{\beta}{2}\xi^2 + v_{NP}\big(T-h, X_{T-h}+\xi, Y_{T-h}, D_{T-h}\big)\Big],$$

v_{NP} being the very first problem with no production: $\beta \to +\infty$.
Let

$$C_h(x, y, d, \xi) := \frac{\beta}{2}\xi^2 + v_{NP}\big(T-h, X_{T-h}+\xi, Y_{T-h}, D_{T-h}\big).$$

Choosing production at time $T-h$

$$\begin{aligned}\hat{\xi}^{h,+}_{T-h} =& \operatorname*{argmin}_{\xi\geq 0} C_h(X_{T-h}, Y_{T-h}, D_{T-h}, \xi)\\ =& \hat{\xi}^h(D_{T-h}-X_{T-h}, Y_{T-h})\mathbf{1}_{\hat{\xi}^h(D_{T-h}-X_{T-h}, Y_{T-h})\geq 0},\end{aligned}$$

where

$$\hat{\xi}^h(d,y) := \frac{\eta}{\eta+\beta}\frac{(\nu h+2\gamma)(\mu h+d)+hy}{(r(\eta,\beta)+\nu)h+2\gamma}$$

Relaxation of the positivity constraint

Now,

$$v_h = \inf_{\mathbf{q}\in\mathcal{A}} \mathbf{E}\Big[\int_0^{T-h} q_s\big(Y_s+\gamma q_s\big)ds + \frac{\beta}{2}\big(\hat{\xi}^{h,+}_{T-h}\big)^2 + v_{NP}\big(T-h, X_{T-h}+\hat{\xi}^{h,+}_{T-h}, Y_{T-h}, D_{T-h}\big)\Big].$$

If we relax the positivity constraint:

$$\tilde{v}_h = \inf_{\mathbf{q}\in\mathcal{A}} \mathbf{E}\Big[\int_0^{T-h} q_s\big(Y_s+\gamma q_s\big)ds + \frac{\beta}{2}\big(\hat{\xi}^{h}_{T-h}\big)^2 + v_{NP}\big(T-h, X_{T-h}+\hat{\xi}^{h}_{T-h}, Y_{T-h}, D_{T-h}\big)\Big].$$

Solution to the relaxed problem (with no jumps)

$$\tilde{v}_h = \inf_{\mathbf{q}\in\mathcal{A}} \mathbf{E}\Big[\int_0^{T-h} q_s\big(Y_s+\gamma q_s\big)ds + \tilde{v}_0\big(T-h, X_{T-h}, Y_{T-h}, D_{T-h}\big)\Big] + K_h,$$

where K_h is the constant, positive term

$$K_h := \frac{\eta^2}{2}\frac{\sigma_0^2+\sigma_d^2\nu^2+2\rho\sigma_0\sigma_d\nu}{(\eta+\beta)(\eta+\nu)(r(\eta,\beta)+\nu)}h + \gamma\frac{\sigma_0^2+\sigma_d^2\eta^2-2\rho\sigma_0\sigma_d\eta}{(\eta+\nu)^2}\ln\left(1+\frac{(\eta+\nu)h}{2\gamma}\right) - \gamma\frac{\sigma_0^2+\sigma_d^2 r^2(\eta,\beta)-2\rho\sigma_0\sigma_d r(\eta,\beta)}{(r(\eta,\beta)+\nu)^2}\ln\left(1+\frac{(r(\eta,\beta)+\nu)h}{2\gamma}\right).$$

Solution to the relaxed problem (with no jumps)

By dynamic programming principle,

$$\tilde{v}_h = \tilde{v}_0 + K_h,$$

and as usual, a bound for approximation error can be derived.

We follow this strategy:

1. Before $T - h$, follow $(\hat{q}_t)_t$, given by the problem with no jumps and no delay.
2. Choose production level $\hat{\xi}^{h,+}_{T-h}$.
3. Between $T - h$ and T, follow $(\hat{q}^{NP}_t)_t$, given by the problem with no jumps and no production.

Simulation with $T = 24$ h and $h = 4$ h (1/2)

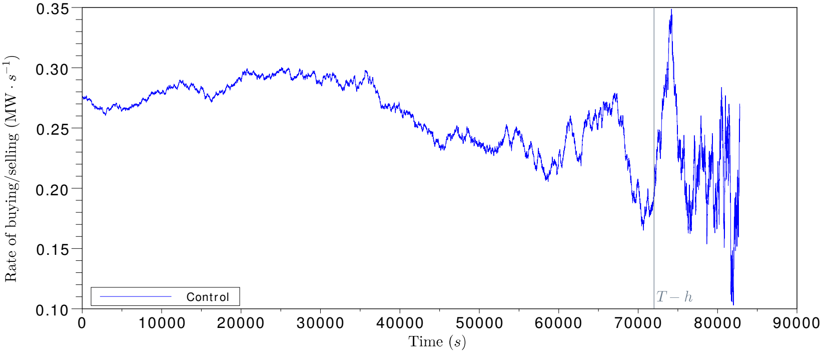

Figure: Evolution of the trading rate control $\hat{\mathbf{q}}$ (without the last hour)

Simulation with $T = 24$ h and $h = 4$ h (2/2)

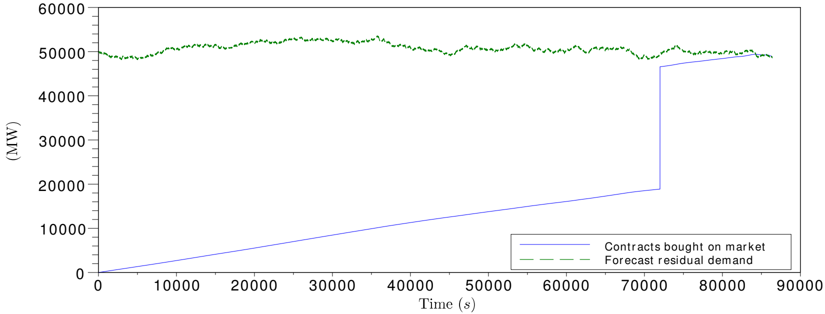

Figure: Evolution of the controlled inventory $\widehat{X}$

Conclusion

Conclusion

- Analysis of electricity intraday trading with a small and tractable stochastic control model.
- Extension of Almgren and Chriss (2000) optimal execution model with linear impact to stochastic target.
- Confort the operational strategy.

Perspective

- Intraday price models.
- Statistical arbitrage.
- Risk management.

Numerical tests (1/2)

T (h)	$\mathbf{P}[\hat{X}_T > D_T]$	$\tilde{v}(0, X_0, Y_0, D_0)$ (€)	$\bar{\mathcal{E}}(T, D_0 - X_0, Y_0)$ (€)
1	$< 10^{-16}$	1.88×10^6	$< 10^{-16}$
8	$< 10^{-16}$	1.88×10^6	$< 10^{-16}$
24	$< 10^{-16}$	1.89×10^6	4.16×10^{-12}
50	7.72×10^{-13}	1.90×10^6	2.48×10^{-4}

Table: $Y_0 = 50$ €$\cdot(\text{MW})^{-1}$ and $D_0 = 50000$ MW

D_0 (MW)	$\mathbf{P}[\hat{X}_T > D_T]$	$\tilde{v}(0, X_0, Y_0, D_0)$ (€)	$\bar{\mathcal{E}}(T, D_0 - X_0, Y_0)$ (€)
500	$< 10^{-16}$	-5.86×10^5	4.16×10^{-12}
5000	$< 10^{-16}$	-3.62×10^5	4.16×10^{-12}
50000	$< 10^{-16}$	1.89×10^6	4.16×10^{-12}
500000	$< 10^{-16}$	2.44×10^7	4.16×10^{-12}

Table: $T = 24$ h and $Y_0 = 50$ €$\cdot(\text{MW})^{-1}$

Numerical tests (2/2)

Y_0 (€$\cdot$(MW)$^{-1}$)	$\mathbf{P}[\hat{X}_T > D_T]$	$\tilde{v}(0, X_0, Y_0, D_0)$(€)	$\bar{\mathcal{E}}(T, D_0 - X_0, Y_0)$ (€)
500	$< 10^{-16}$	2.51×10^6	$< 10^{-16}$
50	$< 10^{-16}$	1.89×10^6	4.16×10^{-12}
40	9.51×10^{-15}	1.61×10^6	3.80×10^{-4}
30	4.57×10^{-10}	1.29×10^6	1.30×10^{-2}
20	2.23×10^{-5}	9.13×10^5	1.26×10^3

Table: $T = 24$ h and $D_0 = 50000$ MW