Ancillary Service to the Grid Using Intelligent Deferrable Loads

Séminaire FIME

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INVENTEURS DU MONDE NUMÉRIQUE

ENS

COGNITION & CONTROL IN COMPLEX SYSTEMS

Electrical and Computer Engineering
University of Florida

Outline

1. Challenges of Renewable Energy Integration
2. Demand Dispatch
3. Control of Deferrable Loads
4. Local Markovian Dynamics and Mean Field Model
5. Design for Intelligent Loads
6. Conclusions and Extensions

Some of the Challenges

1. Large sunk cost (decreasing!)

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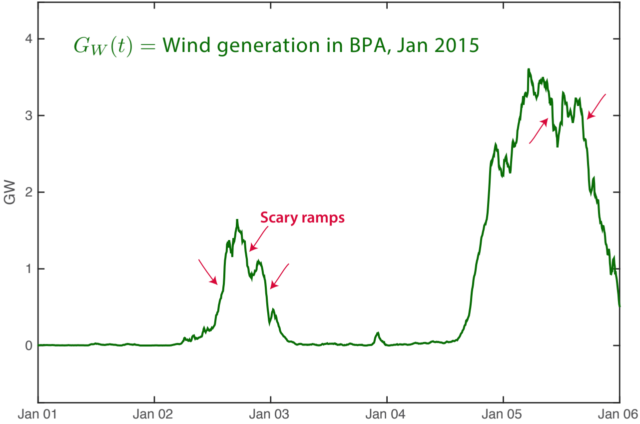
2. Engineering uncertainty
3. Policy uncertainty
4. Volatility

Start at the bottom...

Some of the Challenges

What is scary about volatility?

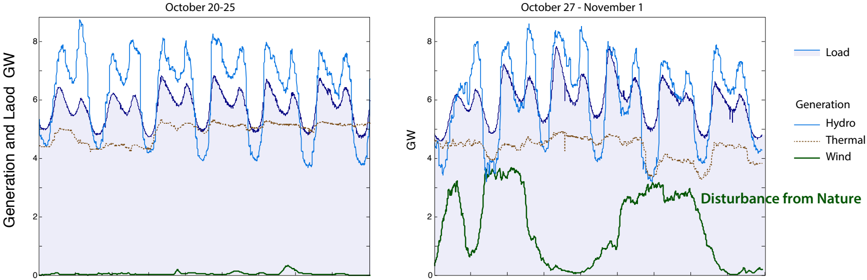
4. Volatility



Some of the Challenges

What is scary about volatility?

4. Volatility $\Longrightarrow$ greater regulation needs



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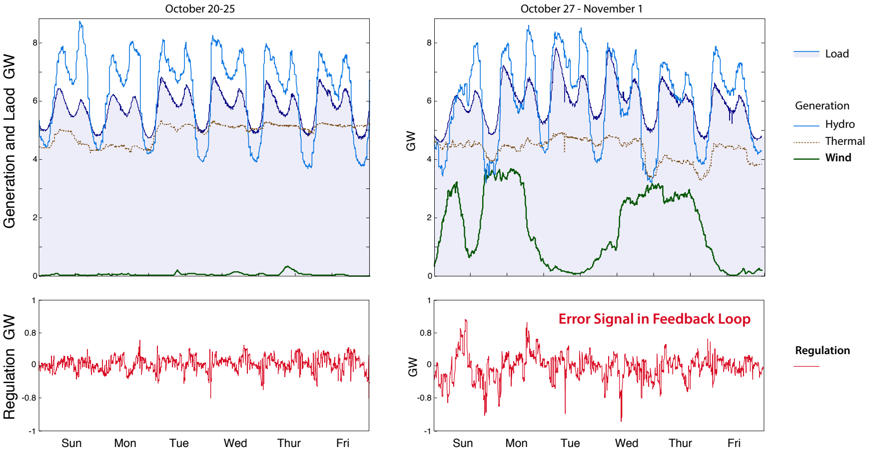
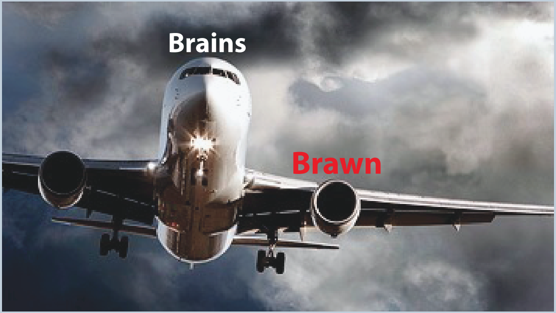


Comparison: Flight control

How do we fly a plane through a storm?



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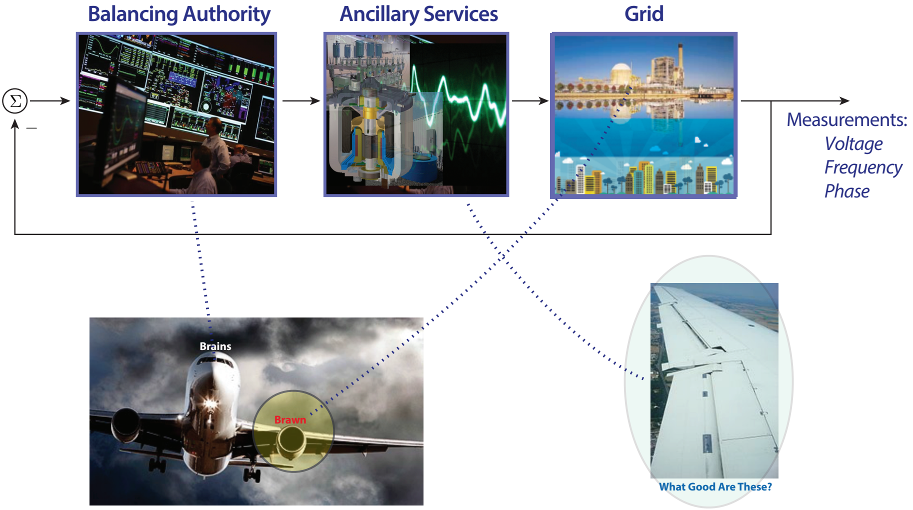
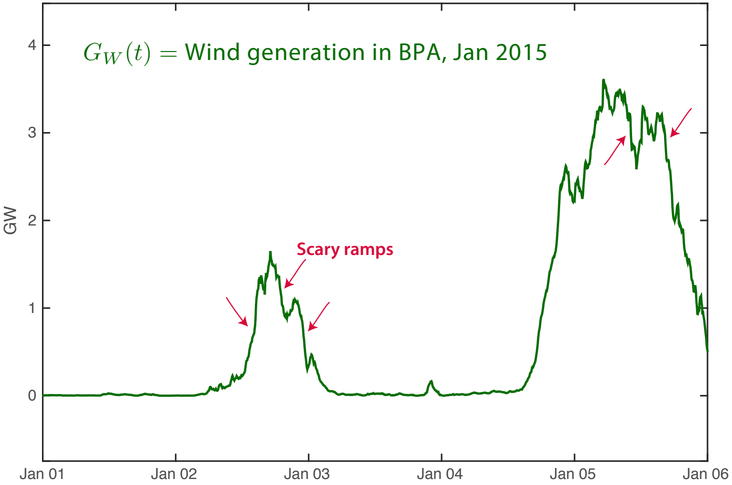
How do we operate the grid in a storm?



How do we operate the grid in a storm?

Disturbance decomposition



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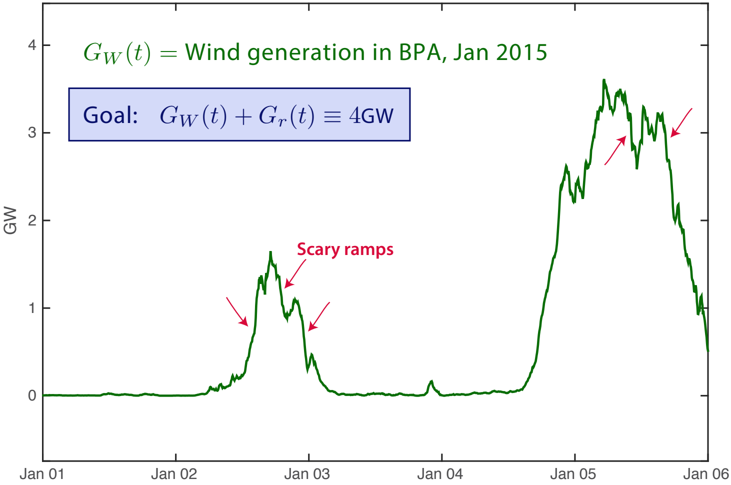

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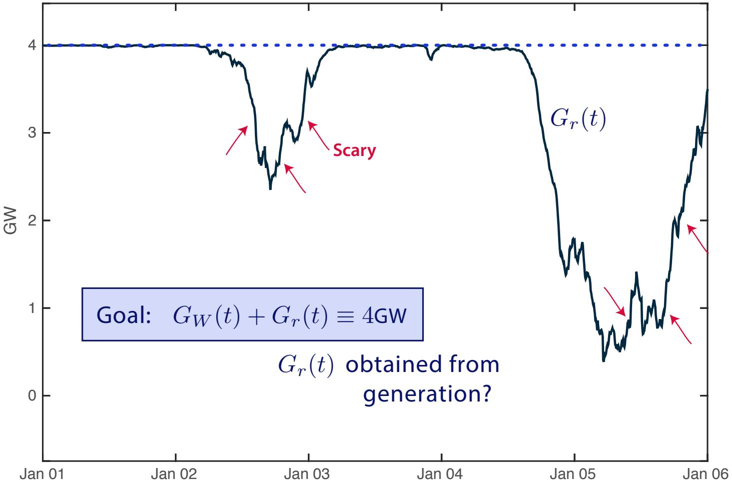

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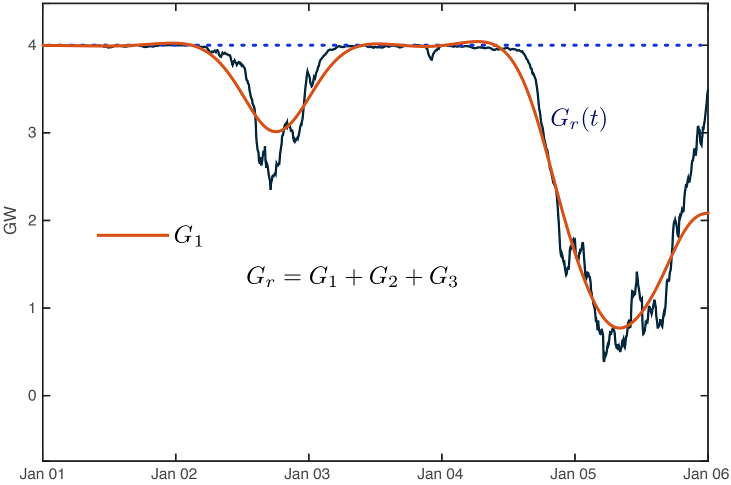

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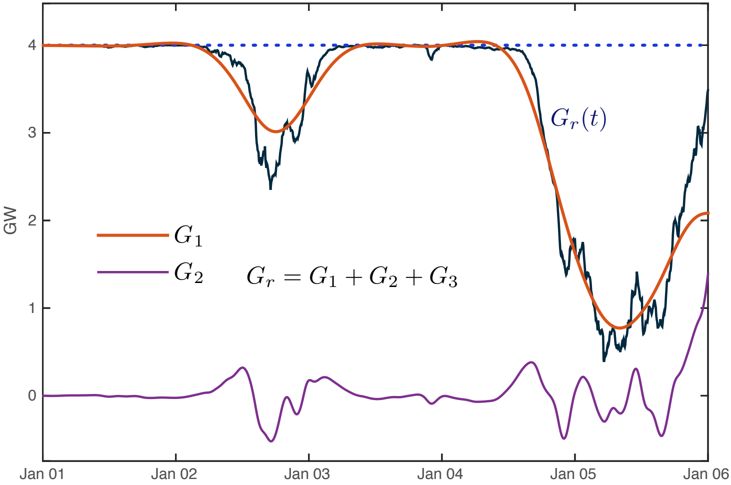

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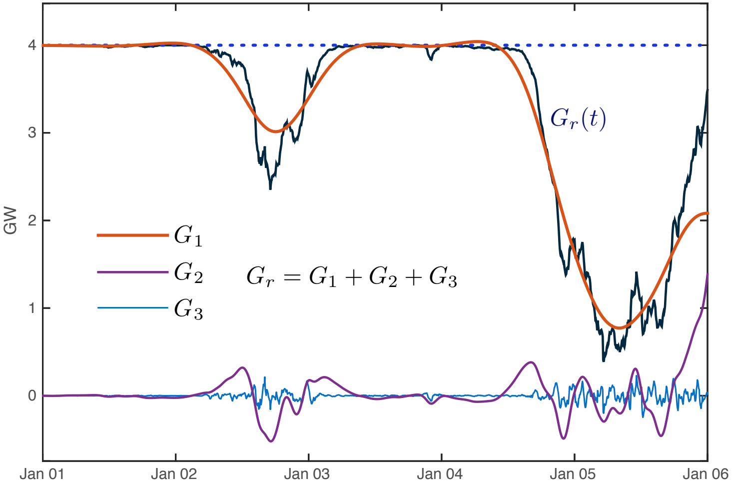

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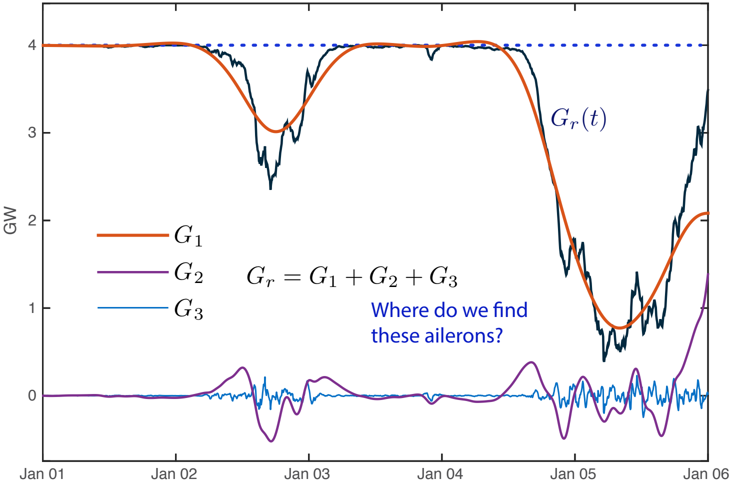
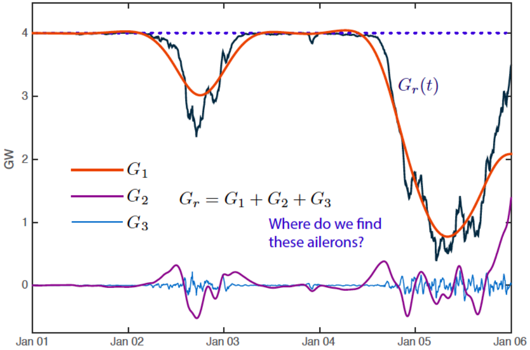




Demand Dispatch

Goal: Responsive Regulation

Demand Dispatch the Answer?

A partial list of the needs of the grid operator, and the consumer:

Goal: Responsive Regulation

Demand Dispatch the Answer?

A partial list of the needs of the grid operator, and the consumer:

- High quality AS? (Ancillary Service)
  Does the deviation in power consumption accurately track the desired deviation target?

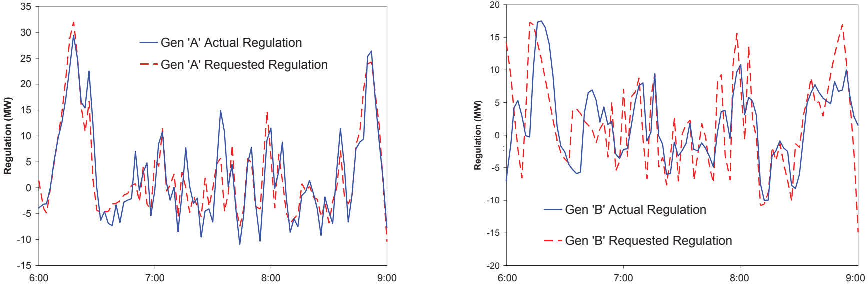
Goal: Responsive Regulation

Demand Dispatch the Answer?

A partial list of the needs of the grid operator, and the consumer:

- High quality AS? (Ancillary Service)

Fig. 10. Coal-fired generators do not follow regulation signals precisely....
Some do better than others



Regulation service from generators is not perfect

Frequency Regulation Basics and Trends — Brendan J. Kirby, December 2004

Goal: Responsive Regulation

Demand Dispatch the Answer?

A partial list of the needs of the grid operator, and the consumer:

- High quality AS?
- Reliable?
  Will AS be available each day?
  It may vary with time, but capacity must be predictable.

Goal: Responsive Regulation

Demand Dispatch the Answer?

A partial list of the needs of the grid operator, and the consumer:

- High quality AS?
- Reliable?
- Cost effective?
  This includes installation cost, communication cost, maintenance, and environmental.

Goal: Responsive Regulation

Demand Dispatch the Answer?

A partial list of the needs of the grid operator, and the consumer:

- High quality AS?
- Reliable?
- Cost effective?
- Is the incentive to the consumer reliable?
  If a consumer receives a $50 payment for one month, and only $1 the next, will there be an explanation that is clear to the consumer?

Goal: Responsive Regulation

Demand Dispatch the Answer?

A partial list of the needs of the grid operator, and the consumer:

- High quality AS?
- Reliable?
- Cost effective?
- Is the incentive to the consumer reliable?
- Customer QoS constraints satisfied?
  The pool must be clean, fresh fish stays cold, building climate is subject to strict bounds, farm irrigation is subject to strict constraints, data centers require sufficient power to perform their tasks.

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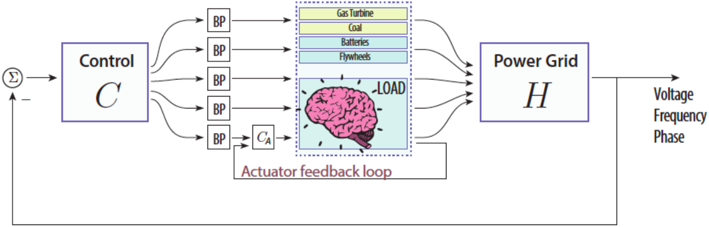
Can demand dispatch do all of this?

Control Architecture

Frequency Decomposition for Demand Dispatch

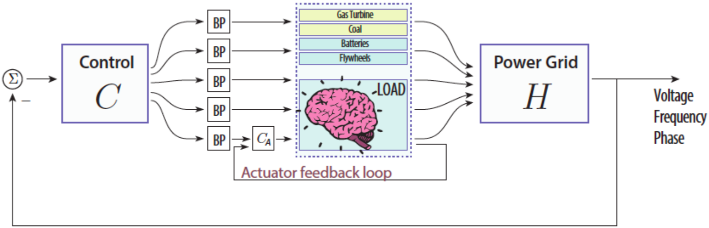


Today: PJM decomposes regulation signal based on bandwidth,

RegA + RegD

Control Architecture

Frequency Decomposition for Demand Dispatch



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Proposal: Each class of DR (and other) resources will have its own bandwidth of service, based on QoS constraints and costs.

Control Architecture

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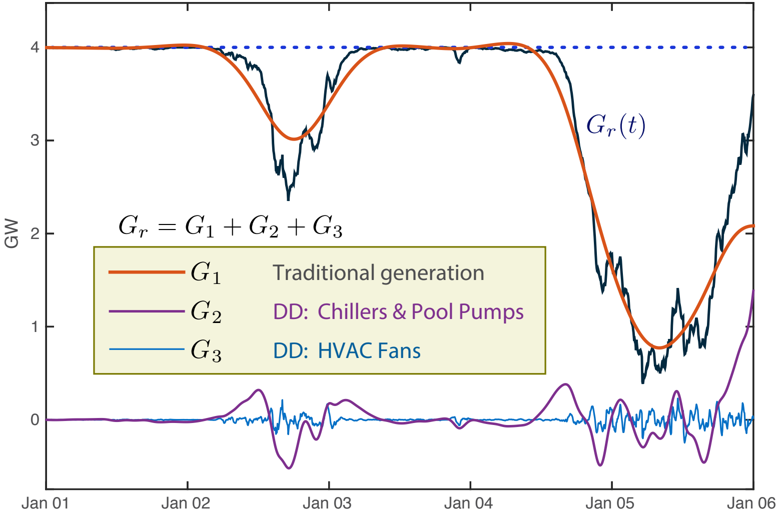
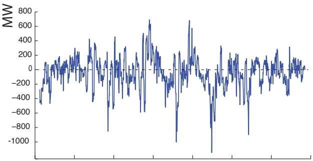


Control Architecture

Frequency Decomposition for Demand Dispatch

Balancing Reserves from Bonneville Power Administration:



BPA Reg signal
(one week)

Control Architecture

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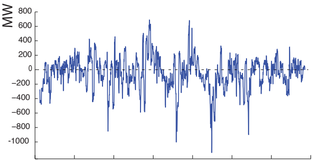

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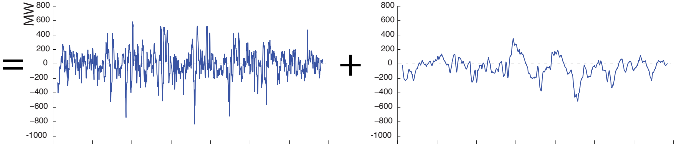

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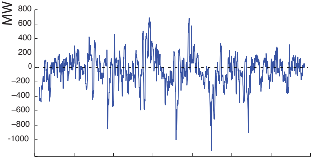

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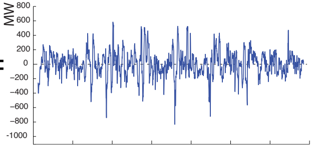

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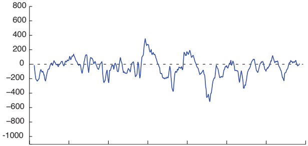
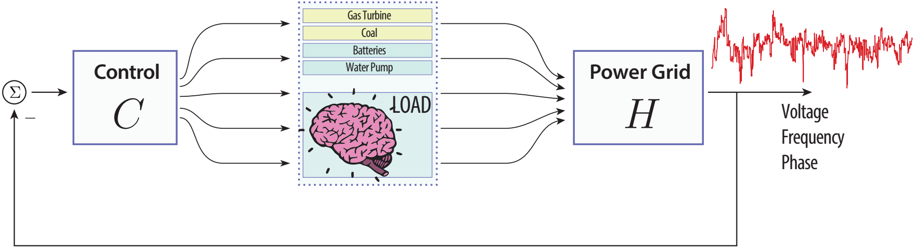


= HVAC + On/Off loads



Control of Deferrable Loads

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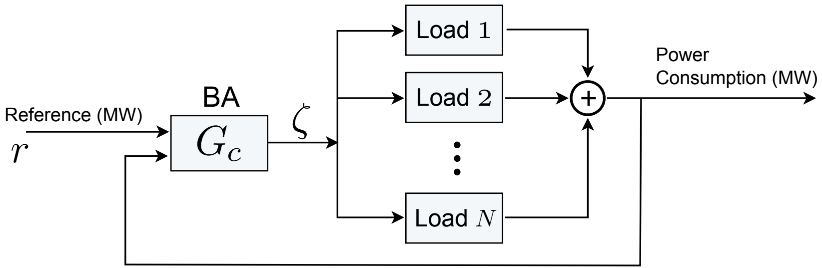
Control Goals and Architecture



Context: Consider population of similar loads that are *deferrable*.

Examples: Chillers in HVAC systems, water heaters, residential TCLs, ...
... residential pool pumps

Control of Deferrable Loads

Randomized Control Architecture

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Constraints: Grid operator demands reliable ancillary service; Consumer demands reliable service

Control strategy

Requirements:

1. **Minimal communication.** Each load should know the needs of the grid, and the status of the service it is intended to provide.

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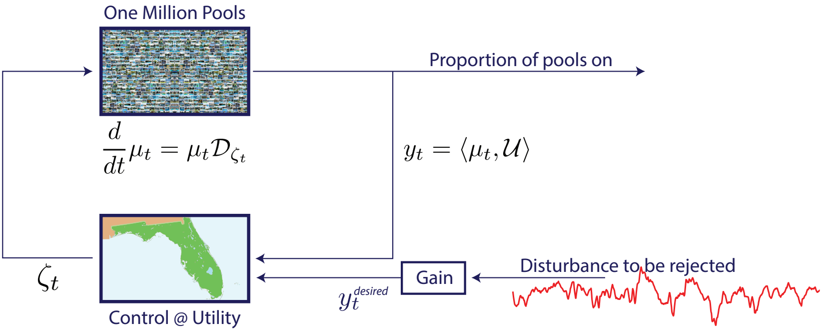
$\Longrightarrow$ **Randomization**

Need: *A practical theory for distributed control based on this architecture*



Intelligent Appliances

General Model

Controlled Markovian Dynamics

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- Continuous time: ith load $X^i(t)$ evolves on finite state space X
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Signal $\zeta = \{\zeta_t\}$ is broadcast to all loads

- Controlled Markovian rate-matrix: For any two states $x^-, x^+ \in$ X,

$$\mathsf{P}\{X^i(t+s) = x^+ \mid X^i(t) = x^-\} \approx s\mathcal{D}_{\zeta_t}(x^-, x^+) + O(s^2)$$

General Model

Aggregate model: N loads running independently, each under the command ζ.

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Limiting model:

$$\frac{d}{dt}\mu_t(x') = \sum_{x\in\mathsf{X}} \mu_t(x)\mathcal{D}_{\zeta_t}(x,x'), \quad y_t := \sum_x \mu_t(x)\mathcal{U}(x)$$

via Law of Large Numbers for martingales

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Mean-field model:

$$\frac{d}{dt}\mu_t = \mu_t \mathcal{D}_{\zeta_t}, \qquad y_t = \mu_t(\mathcal{U})$$

$$\zeta_t = f_t(\mu_0, \dots, \mu_t) \quad \textit{by design}$$

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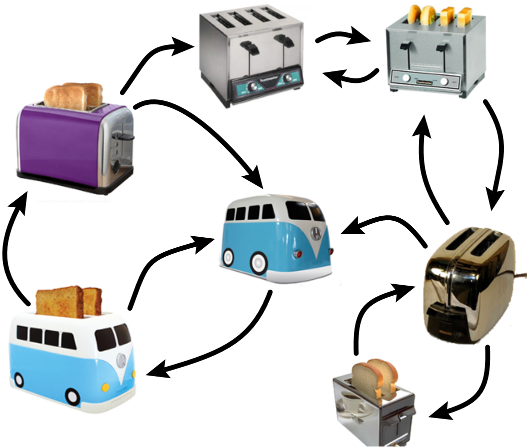
Question: *How to design D_ζ?*



Design

Mean Field Model

Goal: Construct a family of rate-matrices $\{\mathcal{D}_\zeta : \zeta \in \mathbb{R}\}$

Design: Consider first the finite-horizon control problem:

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Choose distribution p_ζ to *maximize*

$$\frac{1}{T}\Big(\zeta \mathsf{E}\Big[\int_{t=0}^{T} \mathcal{U}(X_t)\Big] - D(p_\zeta \| p_0)\Big)$$

Expectation is w.r.t. p_ζ.
D denotes relative entropy.
p_0 denotes nominal Markovian model.

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Explicit solution for finite T:

$$p_\zeta^*(x_0^T) \;\propto\; \exp\Big(\zeta \int_{t=0}^{T} \mathcal{U}(x_t)\, dt\Big) p_0(x_0^T)$$

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$$p^*_\zeta(x_0^T) \;\propto\; \exp\Bigl(\zeta \int_{t=0}^{T} \mathcal{U}(x_t)\, dt\Bigr) p_0(x_0^T)$$

Markovian, but not time-homogeneous.

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$$\mathcal{D}_\zeta(x,y) = \frac{v(y)}{v(x)}\Bigl[\zeta\mathcal{U}(x) - \Lambda + \mathcal{D}_0(x,y)\Bigr]$$

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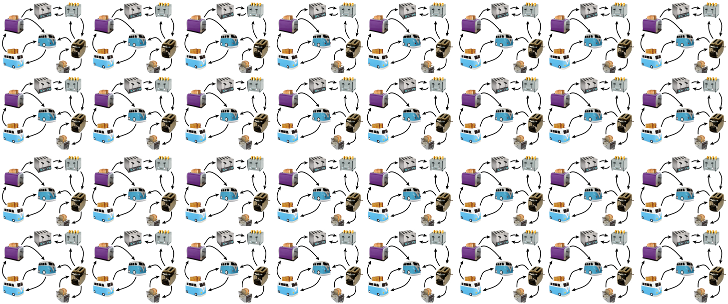
where $\widehat{\mathcal{D}} v = \Lambda v, \qquad \widehat{\mathcal{D}}(x,y) = \zeta\mathcal{U}(x) + \mathcal{D}_0(x,y)$

Extension/reinterpretation of [Todorov 2007] + [Kontoyiannis & M 200X]



$$\frac{d}{dt}\mu_t = \mu_t \mathcal{D}_{\zeta_t}$$
$$y_t = \langle \mu_t, \mathcal{U} \rangle$$

$$\frac{d}{dt}\Phi_t = A\Phi_t + B\zeta_t$$
$$\gamma_t = C\Phi_t$$

Linearized Dynamics

Mean Field Model

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Mean-field model: $\frac{d}{dt}\mu_t = \mu_t \mathcal{D}_{\zeta_t}, \qquad y_t = \mu_t(\mathcal{U})$

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$$\Phi_t(x) \approx \mu_t(x) - \pi(x),\ x \in \mathsf{X}$$

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- $\gamma_t \approx y_t - y^0$; deviation from nominal steady-state

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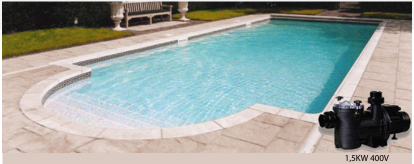
- $A = \mathcal{D}_0^{\intercal}$, $C_i = \mathcal{U}(x^i)$, and input dynamics linearized:

$$B^{\intercal} = \frac{d}{d\zeta}\pi\mathcal{D}_\zeta\Big|_{\zeta=0}$$

Example: One Million Pools in Florida

How Pools Can Help Regulate The Grid



Needs of a single pool

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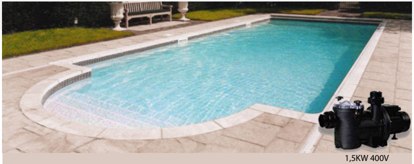

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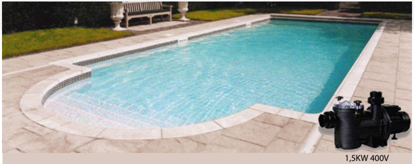

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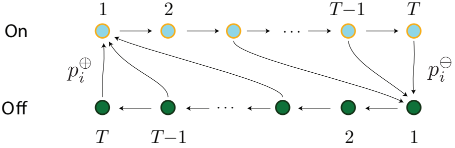
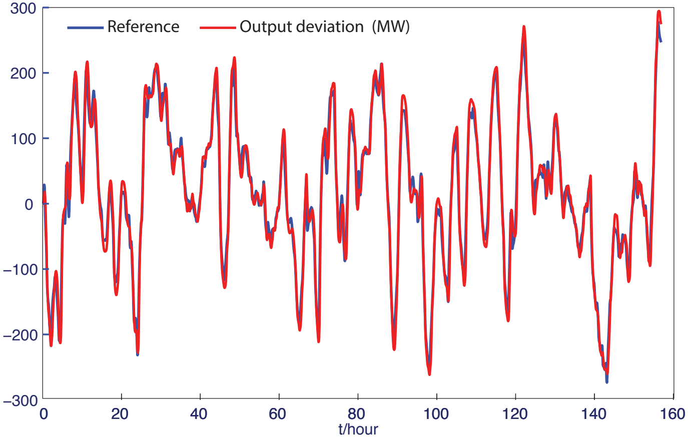
- Filtration system circulates and cleans: Average pool pump uses 1.3kW and runs 6-12 hours per day, 7 days per week
- Pool owners are oblivious, until they see *frogs and algae*
- Pool owners do not trust anyone: *Privacy is a big concern*

Single pool dynamics:



Pools in Florida Supply G_2 – BPA regulation signal*

Stochastic simulation using $N = 10^5$ pools



*transmission.bpa.gov/Business/Operations/Wind/reserves.aspx

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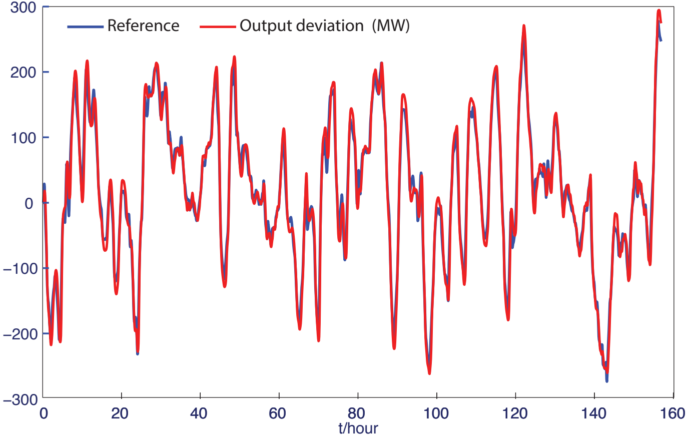

Each pool pump turns on/off with probability depending on
1) its internal state, and 2) the BPA reg signal

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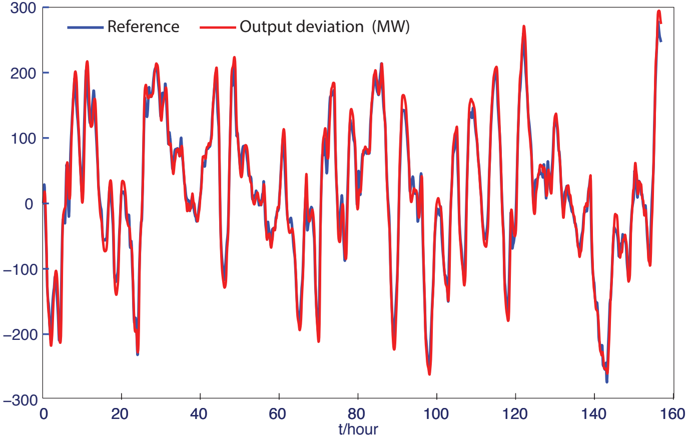

Mean-field model: Input-output system *stable?* *Passive?*

Linearized Dynamics

Transfer Function

Linear state space model:

$$\frac{d}{dt}\Phi_t = A\Phi_t + B\zeta_t$$

$$\gamma_t = C\Phi_t \qquad A = \mathcal{D}_0^{\mathsf{T}}, \quad C_i = \mathcal{U}(x^i), \quad B^{\mathsf{T}} = \frac{d}{d\zeta}\pi\mathcal{D}_\zeta\Big|_{\zeta=0}$$

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$$= CR_s^{\intercal}B$$

TF for L-MFM ⇆ Resolvent for one load

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Theorem: *Reversibility* $\implies$ *Passivity*

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Then its linearization satisfies,

$$\mathrm{Re}\, G(j\omega) = \mathsf{PSD}_Y(\omega), \qquad \omega \in \mathbb{R}\,,$$

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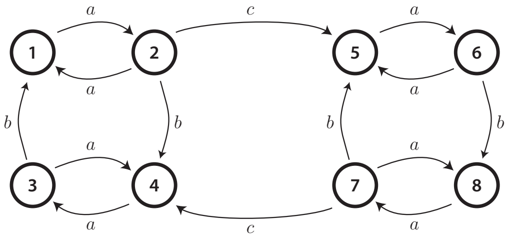
Implication for control: $G(s)$ *is positive real*

Linearized Dynamics

Example Without Passivity

Example: Eight state model

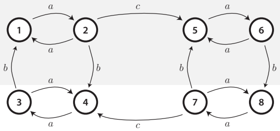
Utility function $\mathcal{U}(x^i) = i$.



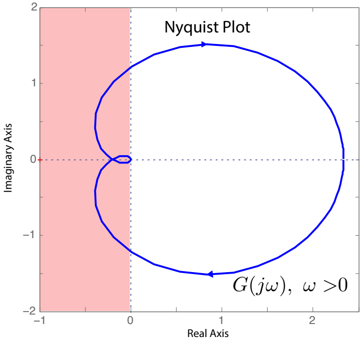
Not reversible

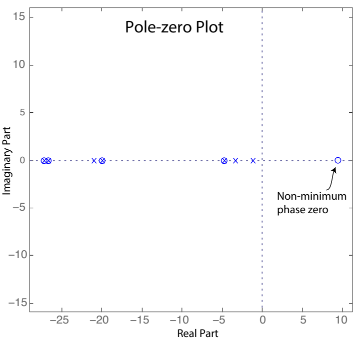
Linearized Dynamics

Example Without Passivity

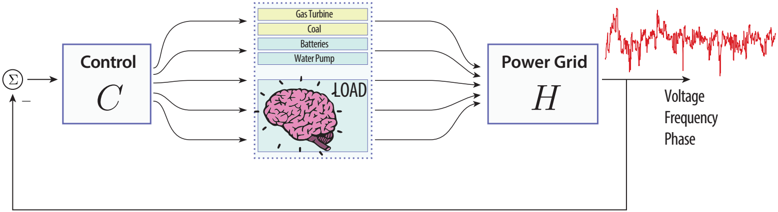


Example: Eight state model $a = c = 10,\ b = 1$





$G(s) = C[Is - A]^{-1}B$ *not positive real*



Conclusions and Extensions

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A particular approach to distributed control is proposed

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The grid level control problem is simple because:

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- ⋆ LTI approximation is **positive real**

Analysis:

Transfer function $\leftrightarrows$ Resolvent for one load

not the Kalman–Yakubovich–Popov lemma

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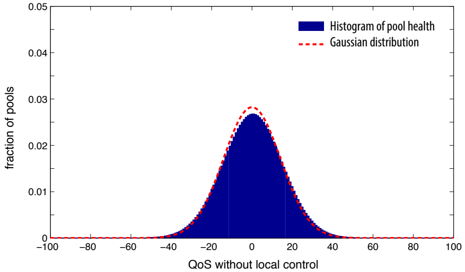

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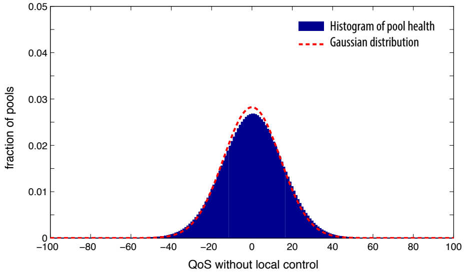

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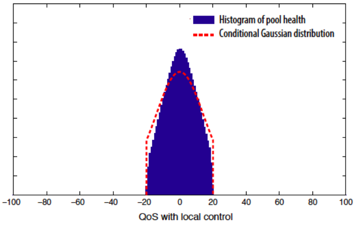
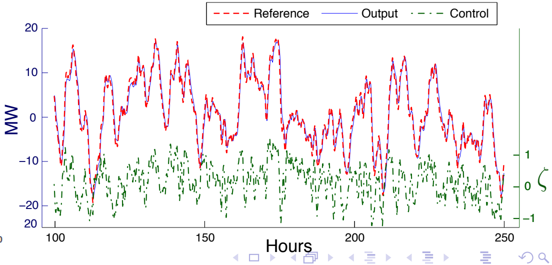
Conclusions

Extensions

- The minimum phase condition was observed in all of our applications, even though the nominal model was not reversible — *mystery*
- Performance for an individual load: Gaussian approximations for QoS
  There will be "rare events" in which QoS is poor.

Remedy: Additional layer of control at each load
$\Longrightarrow$ hard constraints on performance can be assured.





Conclusions

Thank You!

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