Managing risk in emission markets

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Emission trading: Examples

Several market are in operation

- Vouluntary: UK Emission Trading Scheme
- Sulfur Dioxide Trading System (US Acid Rain Program) http://www.epa.gov/airmarkt/trading/factsheet.html
- European Union Emission Trading Scheme (EU)
  - Two periods 2005-2007, 2008-2012
  - Allowances: EUA, cover one tonne of CO_2 equivalent emission
  - Contracts: EUA-Futures, European Call options on EUA-futures

Principle:

Reduction by Cap-and-Trade-Mechanism

- Administrator
  - distributes allowances
  - sets penalty for non-compliance
  - defines compliance dates
- Emissions sources reduce penalty payments
  - by abatement
    - technological changes,
    - production shut down,
    - re-schedule of the production
  - Emission trading
    - physical (Spot)
    - financial (Forwards/Futures)

EUA 2007

Source: EEX

EUA 2012 can reach 100 Euro

Source: EEX

Modeling approaches

- econometric models
  (time series models for certificate prices)
- equilibrium models
  (agent's preferences, strategies, uncertainties)
- risk-neutral models (no-arbitrage certificate dynamics)
  - hybrid approach
    (some equilibrium results from risk-neutral viewpoint)
  - reduced-form approach
    (pure martingale modeling targeted on closed-form option formulas)

In this talk

Reduced-form approach, we gradually move from

- one compliance period $[0, T]$
  (allowance price $(A_t)_{t \in [0,T]}$ finishes at 0 or π)
- two compliance periods $[0, T]$, $[T, T']$
  (allowance prices $(A_t)_{t \in [0,T]}$ $(A_t)_{t \in [0,T']}$ show relation depending on regulations)

Important: results from one-period modeling are used as building blocks in many-period models.

Target: Simple, easy-to-calibrate models, providing easy-to-calculate option formulas

One-compliance period $[0, T]$

- consider emission certificate price evolution $(A_t)_{t\in[0,T]}$
- assume that $(A_t)_{t\in[0,T]}$ is futures price with maturity T written on physical emission allowance price at compliance date T
- assume that there are two outcomes only
  - $A_T = 0$ total emissions are within the target
  - $A_T = \pi$ market missed the target
- for no-arbitrage reasons, make sure that $(A_t)_{t\in[0,T]}$ follows a martingale (with respect to spot martingale measure)

Idea

all we need is to specify on

filtered probability space $(\Omega, \mathcal{F}, \mathbb{Q}, (\mathcal{F}_t)_{t \in [0,T]})$

the event $N \in \mathcal{F}_T$ of non-compliance settles the terminal futures price as

$$A_T = \pi 1_N. \quad (1)$$

giving

$$A_t = \pi \mathbb{E}^{\mathbb{Q}}(1_N \,|\mathcal{F}_t), \quad t \in [0, T].$$

What is important for practitioner?

What is important for practitioner?

- flexibility: having observed at time τ

  recent price, price fluctuations

  one needs a model which allows for to match of both by

  $$A_\tau, \qquad d[A]_\tau$$

- calibration: the identification of parameters for this match should be reliable and fast
- calculation: valuation of options should be fast

To have more structure,

introduce the non-compliance case

$$N = \{\Gamma_T \geq 1\}.$$

as an event where a hypothetic Γ_T exceeds the boundary 1.

This is convenient:

- imagine that the total pollution E_T exceeds the total allocation γ

$$N = \{\underbrace{E_T/\gamma}_{\Gamma} > \gamma\}$$

- remember digital options

$$A_t = \pi \mathbb{E}^{\mathbb{Q}}(1_{\{\Gamma_T \geq 1\}} | \mathcal{F}_t), \quad t \in [0, T]$$

With this approach,

focus on

$$\Gamma_T = \Gamma_0 e^{\int_0^T \sigma_s dW_s - \frac{1}{2}\int_0^T \sigma_s^2 ds}, \qquad \Gamma_0 \in (0, \infty)$$

where

$$(\sigma_s)_{s \in]0,T[} \text{ is positive, continuous, squate-integrable}$$

Given: allowance price is digital option on Geometric Brownian Motion

To do: adjust allowance price volatility by that of underlying Brownian Motion

Result: not easy!

Proposition

$$\textit{Assume} \quad \Gamma_T = \Gamma_0 e^{\int_0^T \sigma_s dW_s - \frac{1}{2}\int_0^T \sigma_s^2 ds}, \qquad \Gamma_0 \in (0,\infty)$$

with continuous and square-integrable $(0,T) \ni t \hookrightarrow \sigma_t$, *then*

$$a_t = \mathbb{E}^{\mathbb{Q}}(1_{\{\Gamma_T \geq 1\}} \,|\, \mathcal{F}_t) \quad t \in [0,T] \tag{2}$$

is given by

$$a_t = \Phi\left(\frac{\Phi^{-1}(a_0)\sqrt{\int_0^T \sigma_s^2 ds} + \int_0^t \sigma_s dW_s}{\sqrt{\int_t^T \sigma_s^2 ds}}\right) \tag{3}$$

and it solves

$$da_t = \Phi'(\Phi^{-1}(a_t))\sqrt{z_t} dW_t \tag{4}$$

with positive-valued

$$z_t = \sigma_t^2 / \int_t^T \sigma_u^2 du, \qquad t \in (0,T). \tag{5}$$

There is a problem

with constant volatility

$$\sigma_s \equiv \bar{\sigma} \in (0, \infty) \quad \text{for all } \; s \in [0, T]$$

since independently on its level, we have the same

$$\begin{aligned} a_t &= \Phi\left(\frac{\Phi^{-1}(a_0)\sqrt{T} + W_t}{\sqrt{T-t}}\right). \\ da_t &= \Phi'(\Phi^{-1}(a_t))\frac{1}{\sqrt{T-t}}dW_t \end{aligned}$$

With $\bar{\sigma} \in]0, \infty[$, we can match both, the recent allowance price and its fluctuation intensity.

How about non-constant $(\sigma_s)_{s\in[0,T]}$?

There are many positive-valued functions, a parameterized family would be appropriate.

Try a slight correction, introducing only two degrees of freedom

$$da_t = \Phi'(\Phi^{-1}(a_t))\sqrt{\beta(T-t)^{-\alpha}}dW_t \tag{6}$$

with $\alpha \in \mathbb{R},\ \beta \in (0, \infty)$. ($\alpha = 1, \beta = 1$ gives $\bar{\sigma}$-model)

- is it possible to find a corresponding $(\sigma_s)_{s\in[0,T]}$?
- which α and β describe observed allowance prices
- how to determine them?

Parameterized function family is

$$(z_t(\alpha,\beta) = \beta(T-t)^{-\alpha})_{t\in(0,T)}, \qquad \beta > 0 \quad \text{and} \quad \alpha \in \mathbb{R}.$$

($\alpha \geq 1$ gives divergence of the integral to ensure $A_T \in \{0,\pi\}$).

The corresponding $(\sigma_t(\alpha,\beta))_{t\in(0,T)}$ are

$$\begin{aligned}
\sigma_t(\alpha,\beta)^2 &= z_t(\alpha,\beta)e^{-\int_0^t z_u(\alpha,\beta)du} \qquad (7)\\
&= \begin{cases} \beta(T-t)^{-\alpha}e^{\beta\frac{T^{-\alpha+1}-(T-t)^{-\alpha+1}}{-\alpha+1}} & \text{for } \beta > 0, \alpha > 1 \\ \beta(T-t)^{\beta-1}T^{-\beta} & \text{for } \beta > 0, \alpha = 1 \end{cases}
\end{aligned}$$

Consider observations of the futures prices $(A_t)_{t\in[0,T]}$ which we transform as

$$\xi_t = \Phi^{-1}(a_t) = \Phi^{-1}\left(\frac{1}{\pi}A_t\right), \qquad t \in [0,t] \tag{8}$$

It turns out that

$$d\xi_t = \frac{1}{2}z_t\xi_t dt + \sqrt{z_t}dW_t$$

The objective measure $\mathbb{P}$ can be recovered from the spot martingale measure $\mathbb{Q}$ via its Radon-Nikodym density

$$\frac{d\mathbb{P}}{d\mathbb{Q}} = e^{\int_0^T H_t dW_t - \frac{1}{2}\int_0^T H_t^2 dt}.$$

We assume that the market price of risk process $(H_t)_{t\in[0,T]}$ is constant and deterministic, $H_t \equiv h$ for $t \in [0, T]$, for some fixed $h \in \mathbb{R}$.

Girsanov's theorem ensures that

$$\tilde{W}_t = W_t - ht \text{ for } t \in [0, T)$$

is a Brownian motion with respect to the objective measure $\mathbb{P}$, and under this measure, ξ_t satisfies:

$$d\xi_t = (\frac{1}{2} z_t \xi_t + h\sqrt{z_t})dt + \sqrt{z_t} d\tilde{W}_t$$

with solution

$$\xi_\tau = e^{\frac{1}{2}\int_t^\tau z_s ds} \xi_t + h \int_t^\tau e^{\frac{1}{2}\int_s^\tau z_u du} \sqrt{z_s} ds + \int_t^\tau e^{\frac{1}{2}\int_s^\tau z_u du} \sqrt{z_s} d\tilde{W}_s.$$

for $0 \leq t \leq \tau \leq T$

Having observed $(\xi_{t_i})_{i=1}^n$, the conditional distribution of ξ_{t_i} given $\xi_{t_{i-1}}$ is Gaussian with mean μ_i and variance σ_i^2

$$\mu_i(h,\alpha,\beta) = e^{\frac{1}{2}\int_{t_{i-1}}^{t_i} z_s ds}\xi_{t_{i-1}} + h\int_{t_{i-1}}^{t_i} e^{\frac{1}{2}\int_s^{t_i} z_u du}\sqrt{z_s}ds,$$

$$\sigma_i^2(h,\alpha,\beta) = \int_{t_{i-1}}^{t_i} z_s e^{\int_s^{t_i} z_u du} ds,$$

For a given realization $(\xi_{t_i})_{i=1}^n \in \mathbb{R}^n$, the log-likelihood is

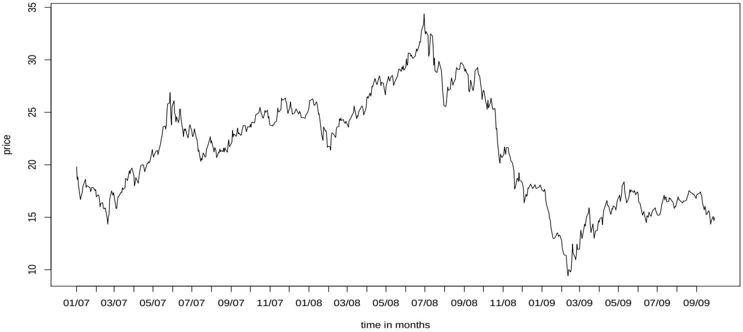
$$L_{\xi_{t_1},\ldots,\xi_{t_n}}(h,\alpha,\beta) = \sum_{i=1}^n \left(-\frac{(\xi_{t_i} - \mu_i(h,\alpha,\beta))^2}{2\sigma_i^2(\alpha,\beta)} - \ln(\sqrt{2\pi\sigma_i^2(\alpha,\beta)}) \right)$$

for all $h,\alpha,\beta \in \mathbb{R}$.

There is no closed-form estimate for the parameters $h, \alpha, \beta \in \mathbb{R}$.

However, the maximum of the likelihood function can be determined numerically.

Given the historical futures prices of EUA with maturity in December 2012,



we use the integral approximations

$$\begin{aligned}\mu_i(h,\alpha,\beta) &\sim e^{\frac{1}{2}(t_i-t_{i-1})z_{t_{i-1}}}\xi_{t_{i-1}} + h(t_i-t_{i-1})\sqrt{z_{t_{i-1}}}e^{\frac{1}{2}(t_i-t_{i-1})z_{t_{i-1}}}\\ \sigma_i^2(h,\alpha,\beta) &\sim (t_i-t_{i-1})z_{t_{i-1}}e^{(t_i-t_{i-1})z_{t_{i-1}}}\end{aligned}$$

to calculate the log-likelihood.

Starting with initial parameter $h := 0$, $\alpha = 0.5$, $\beta = 0.5$, a numerical maximization method returned the maximizer

$$\alpha^* = 0.332, \qquad \beta^* = 0.161, \qquad h^* = -0.078.$$

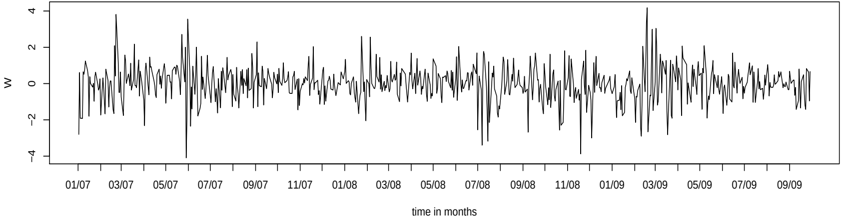
To verify the validity of our procedure, we determine the *residuals*

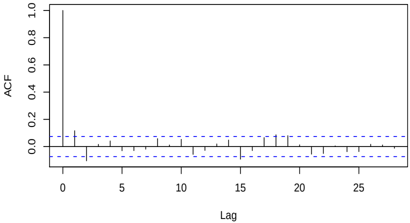
$$w_i = \frac{\xi_{t_i} - \mu_i(h^*, \alpha^*, \beta^*)}{\sqrt{\sigma_i^2(h^*, \alpha^*, \beta^*)}}, \qquad i = 1, \ldots, n.$$

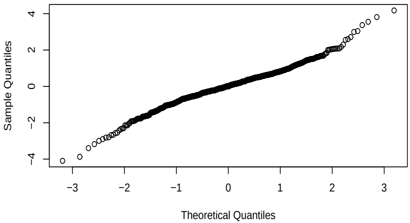
Under the model assumptions, this series must be a realization of independent standard normal random variables.

Thus, standard statistical can be applied to verify the the quality of the model fit.

Statistical analysis of residuals







Option pricing

The price of a European call with maturity $\tau \in [0, T]$ *and strike* $K \geq 0$ *written on an allowance futures maturing at the end* T *of the compliance period is given at time* $t \in [0, \tau]$ *by*

$$C_t = e^{-\int_t^\tau r_s ds} \int_{\mathbb{R}} (\pi\Phi(x) - K)^+ N(\mu_{t,\tau}, \sigma_{t,\tau}^2)(dx) \qquad (9)$$

with $\mu_{t,\tau}$ *and* $\sigma_{t,\tau}^2$ *given by formulas below.*

$$\mu_{t,\tau}(\alpha,\beta) = \begin{cases} \xi_t \left(\frac{T-t}{T-\tau}\right)^{(\beta/2)} & \text{if } \alpha = 1 \\ \xi_t \exp\left[\frac{\beta}{2(1-\alpha)}[(T-t)^{1-\alpha} - (T-\tau)^{1-\alpha}]\right] & \text{if } \alpha \neq 1. \end{cases}$$

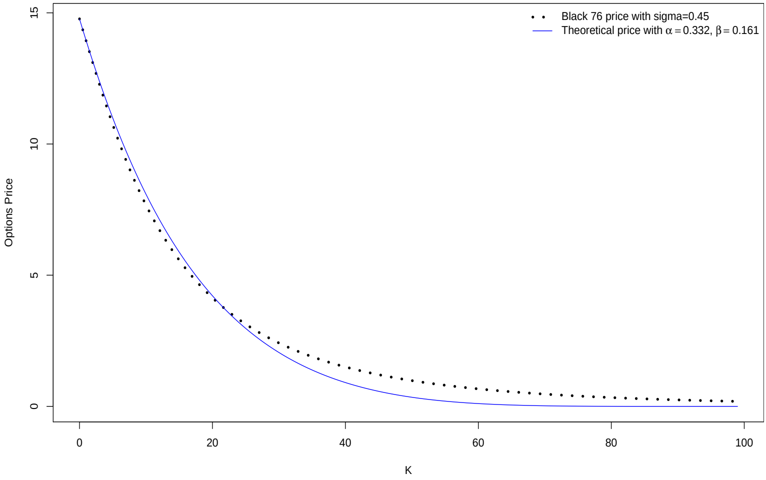
$$\sigma^2_{t,\tau}(\alpha,\beta) = \begin{cases} \left(\frac{T-t}{T-\tau}\right)^{\beta} - 1 & \text{if } \alpha = 1 \\ \exp\left[\frac{\beta}{1-\alpha}[(T-t)^{1-\alpha} - (T-\tau)^{1-\alpha}]\right] - 1 & \text{if } \alpha \neq 1. \end{cases}$$

Our valuation technique differs from the traditional Black 76 formula.

However, the difference can be moderate, for parameters relevant to current situation of the EU ETS and for low strike prices.

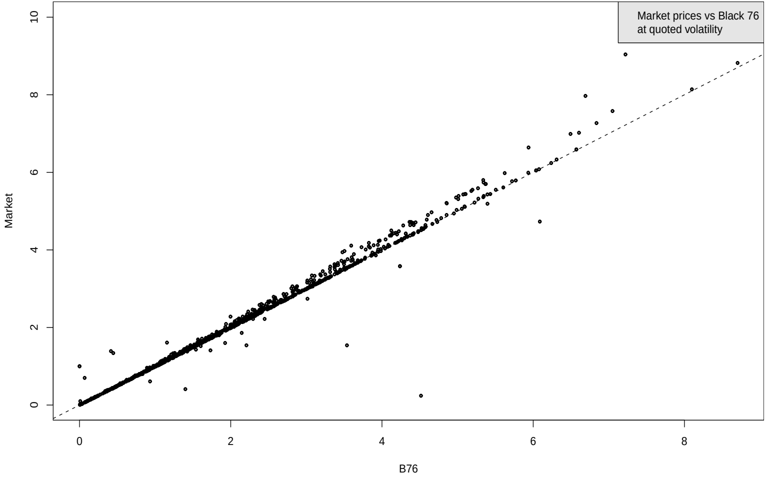
Compare Call option price with maturity $\tau_o = 3.44$ depending on the strike K, calculated at $t = 0$ for the underlying futures price of $A_0(\tau_f) = 17.54$ and supposing that the futures contract matures at $\tau_f = 3.46$. Suppose that the time to compliance date is $T = 5$ and set the short rate as $r = 0.07$.

Black 76 versus our model

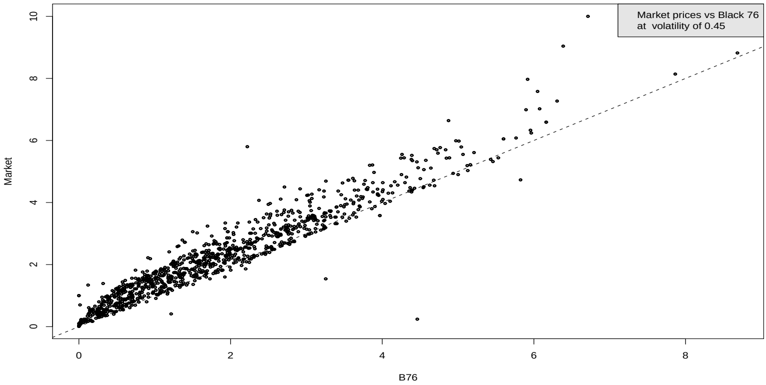


Quoted B76 volatilities

show indeed that the ECX prices EUA calls by B76.



However, a constant B76 volatility does not explain the history of daily call option prices quoted at ECX during 4th of January 2007 to 30th of September of 2009.



To compare our technique to the market option prices, we decided toto fit our model to the paid option prices.

To do so, we suggest to determine those parameters α and β which minimize the sum of squared deviations between historical market prices and their theoretical values, based on our model.

Having implemented the function describing the sum of squared deviations depending on model parameters α and β, a numerical procedure, based on Nelder-Mead method was applied to determine the minimizer

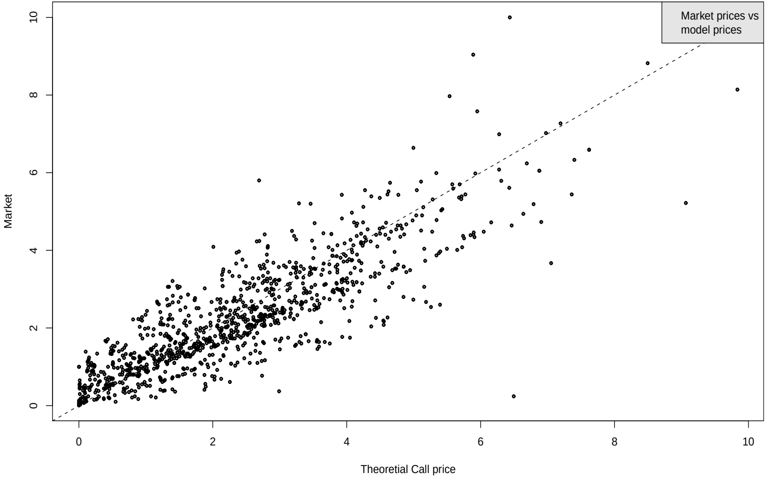
$$\alpha^\star \approx 0.318 \qquad \beta^\star \approx 0.144 \tag{10}$$

Which is very close to the parameters obtained from the historical parameter estimation

$$\alpha^* = 0.332, \qquad \beta^* = 0.161$$

Such a fit could be interpreted in favor of model validity.

Still, even for implicitly calibrated parameters, the deviation of market prices from their theoretical values is strong.



From this perspective, our model behaves not better that the Black 76 pricing scheme.

This is simply because market has been pricing emission allowance options in terms of Black 76 model until now.

We expect a change in the future, when market participants realize differences between allowance price evolution and model assumptions underlying Black 76 approach.

The differences become obvious closer to the compliance date.

Multi periods markets

So far, we focused on one compliance period.

This is a simplification since real-world markets are operating in a multi-period framework

Usually, periods are connected by regulations.

Three regulatory mechanisms

- Borrowing
- Banking
- Withdrawal

Three regulatory mechanisms

- Borrowing allows for the transfer of a (limited) number of allowances from the next period into the present one;
- Banking allows for the transfer of a (limited) number of (unused) allowances from the present period into the next;
- Withdrawal penalizes firms which fail to comply in two ways: by penalty payment for each unit of pollutant which is not covered by credits and by withdrawal of the missing allowances from their allocation for the next period.

Seemingly policy makers tend to admit unlimited banking, forbid borrowing, and apply withdrawal rule.

Two period model

without borrowing, with unlimited banking and with withdrawal.

- two periods $[0, T]$ and $[T, T']$
- two processes $(A_t)_{t\in[0,T]}$, $(A'_t)_{t\in[0,T]}$ for futures contracts with maturities at compliance dates T, T' written on allowance prices from the first and the second period respectively.

Two period model

In the case of the first-period compliance the allowance price drops

$$A_T 1_{\Omega \setminus N} = \kappa A'_T 1_{\Omega \setminus N},$$

where $\kappa \in (0, \infty)$ stands for discount factor

$$\kappa = e^{-\int_T^{T'} r_s ds}.$$

In the case of first period non-compliance agent pays penalty in addition to next-period certificate which must be withdrawn at spot price

$$A_T 1_N = \kappa A'_T 1_N + \pi 1_N.$$

The difference

$$A_t - \kappa A'_t = \mathbb{E}^{\mathbb{Q}}(A_T - \kappa A'_T \,|\, \mathcal{F}_t) = \mathbb{E}^{\mathbb{Q}}(\pi \mathbf{1}_N \,|\, \mathcal{F}_t) \qquad t \in [0, T]$$

is a digital martingale, use the same methodology as in one period model

$$A_t - \kappa A'_t = \pi \Phi(X_t^1) \qquad t \in [0, T],$$

where $(X_t^1)_{t\in[0,T]}$ is Gaussian process with $(\sigma_s)_{s\in[0,T]}$ in parameterized form and driven by a Brownian motion $(W_t^1, \mathcal{F}_t)_{t\in[0,T']}$.

Second-period price

can be modeled only if a continuation of the system is specified.

If there is no decision yet the process $(A'_t)_{t\in[0,T]}$ should be modeled exogenously

Simplest choice is to suppose that the system will be terminated, hence

$$A'_t = \pi\Phi(X_t^2) \qquad t \in [0, T'].$$

with a $\{\sigma_s^2\}_{s\in[0,T]}$ in parameterized form, and driven by another Brownian motion $(W_t^2, \mathcal{F}_t)_{t\in[0,T']}$.

Option pricing

with these regulatory assumptions, consider European Call with strike price $K \geq 0$ and maturity $\tau \in [0, T]$ written on futures price of allowance from the first period

the payoff is

$$C_\tau = (A_\tau - K)^+ \qquad \text{at time } \tau \in [0, T].$$

the fair price is

$$C_0 = e^{-\int_0^\tau r_s ds} \mathbb{E}^{\mathbb{Q}}((A_\tau - K)^+)$$

Using decomposition

$$(A_\tau - K)^+ = (A_\tau - \kappa A'_\tau + \kappa A'_\tau - K)^+,$$

we express terminal payoff as

$$(A_\tau - K)^+ = (\pi\Phi(X_\tau^1) + \kappa\pi\Phi(X_\tau^2) - K)^+$$

and fair price is found as

$$\begin{aligned} C_0 &= e^{-\int_0^\tau r_s ds}\mathbb{E}^{\mathbb{Q}}((A_\tau - K)^+) \\ &= e^{-\int_0^\tau r_s ds}\mathbb{E}^{\mathbb{Q}}((\pi\Phi(X_\tau^1) + \kappa\pi\Phi(X_\tau^2) - K)^+) \end{aligned}$$

where for $\beta_1 > 0$, $\beta_2 > 0$ we have

$$
\begin{aligned}
X^1_\tau &= \Phi^{-1}\left(\frac{A_0 - \kappa A'_0}{\pi}\right)\sqrt{\left(\frac{T}{T-\tau}\right)^{\beta_1}} + \beta_1^{\frac{1}{2}} \frac{\int_0^\tau (T-u)^{\frac{\beta_1 - 1}{2}} W^1_u du}{(T-\tau)^{\frac{\beta_1}{2}}} \\
X^2_\tau &= \Phi^{-1}\left(\frac{\kappa A'_0}{\pi}\right)\sqrt{\left(\frac{T'}{T'-\tau}\right)^{\beta_2}} + \beta_2^{\frac{1}{2}} \frac{\int_0^\tau (T'-u)^{\frac{\beta_2 - 1}{2}} W^2_u du}{(T'-\tau)^{\frac{\beta_2}{2}}}.
\end{aligned}
$$

with correlated Brownian motions

$$[W^1, W^2]dt = \rho dt, \qquad \rho \in [-1, 1],$$

apply the same argumentation to calculate the price

Proposition

Price of Call with strike price $K \geq 0$ and maturity $\tau \in [0, T]$ written on first-period allowance futures price at time $t \in [0, \tau]$ is

$$C_t = e^{-\int_t^\tau r_s ds} \int_{\mathbb{R}^2} (\pi\Phi(x_1) + \kappa\pi\Phi(x_2) - K)^+ N(\mu_{t,\tau}, \nu_{t,\tau})(dx_1, dx_2)$$

$$\begin{aligned} \mu^1_{t,\tau} &= \Phi^{-1}\left(\frac{A_t - \kappa A'_t}{\pi}\right)\sqrt{\left(\frac{T-t}{T-\tau}\right)^{\beta_1}} \\ \mu^2_{t,\tau} &= \Phi^{-1}\left(\frac{\kappa A'_t}{\pi}\right)\sqrt{\left(\frac{T'-t}{T'-\tau}\right)^{\beta_2}} \end{aligned}$$

$$\begin{aligned} \nu^{1,1}_{t,\tau} &= \mathrm{Var}(X^1_\tau) = \left(\frac{T-t}{T-\tau}\right)^{\beta_1} - 1 \\ \nu^{2,2}_{t,\tau} &= \mathrm{Var}(X^2_\tau) = \left(\frac{T'-t}{T'-\tau}\right)^{\beta_1} - 1 \\ \nu^{1,2}_{t,\tau} = \nu^{2,1}_{t,\tau} &= \beta_1^{\frac{1}{2}}\beta_2^{\frac{1}{2}} \frac{\int_t^\tau (T-u)^{\frac{\beta_1-1}{2}}(T'-u)^{\frac{\beta_2-1}{2}}\rho du}{(T-\tau)^{\frac{\beta_1}{2}}(T'-\tau)^{\frac{\beta_2}{2}}}. \end{aligned}$$

Implementation in R

```
Call1<-function(ta, Tmat1, Tmat2 , A1,A2, K, r, beta1, beta2, rho)
```

- `ta` corresponds to $\tau - t$, time to options maturity
- `Tmat1` stands for $T_1 - t$, time to the first period compliance
- `Tmat2` stands for $T_2 - t$, time to the second period compliance
- `A1` stands for A_t, first-period allowance futures price
- `A2` stands for A'_t, second-period allowance futures price
- `beta1`, `beta2` stands for β and β' respectively
- `rho` denotes the correlation ρ
- `penalty`, `K`, `r`, correspond to the model parameters π, K, r

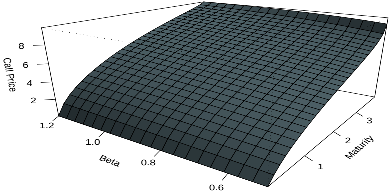
Implementation in R

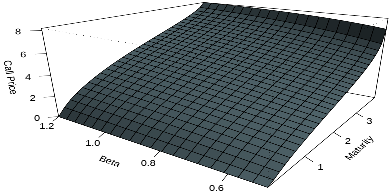
```
Call2<-function(ta, Tmat1, Tmat2 , A1,A2, K, r, beta1, beta2, rho)
{
kapp<-exp(-r*(Tmat2-Tmat1))
mu1<-qnorm((A1-kapp*A2)/penalty)*(Tmat1/(Tmat1-ta))^(beta1/2)
mu2<-qnorm((kapp*A2)/penalty)*(Tmat2/(Tmat2-ta))^(beta2/2)
nu11<- (Tmat1/(Tmat1-ta))^(beta1) -1
nu22<- (Tmat2/(Tmat2-ta))^(beta2) -1
g<-function(u)
{ (Tmat1-u)^((beta1-1)/2)*(Tmat2-u)^((beta2-1)/2)
}
nu12<-sqrt(beta1*beta2/((Tmat1-ta)^(beta1)*
(Tmat2-ta)^(beta2)))*integrate(g, 0, ta)$value*rho
nu1c<-nu11-(nu12)^2/nu22

GG<-function(x)
{
       mu1c<-mu1+(x-mu2)*(nu12/nu22)
       Kc<-K-kapp*penalty*pnorm(x)
      if (Kc>=penalty)
           result<-0
      if (Kc<=0)
         result<-penalty*pnorm(mu1c/sqrt(1+nu1c))-Kc
      if ((0<Kc)&(Kc<penalty))
      {
       f<-function(x)
       {(penalty*pnorm(x)-Kc)*dnorm(mean=mu1c, sd=sqrt(nu1c), x)}
      result<-integrate(f, qnorm(Kc/penalty), Inf)$value
      }
      return(result*dnorm(mean=mu2, sd=sqrt(nu22),x))
}

GGG<-Vectorize(GG)
return(exp(-r*ta)*integrate(GGG, -Inf, Inf)$value)
}
```

Parameter dependence





Thank you!