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Exact solutions to binary equilibrium problems with compensation and the power market uplift problem

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Introduction – Optimization and markets

What we learn at university… may lead us astray!

- In *Optimization 101*, we learn how to solve linear programs
- In *Economics 101*, we learn that the dual variables from a linear program can be interpreted as market-clearing prices

  ⇒ these prices support a Nash equilibrium in competitive markets!
- In *Engineering 101*, we learn that many technical aspects cannot be described adequately without binary or integer decision variables

  e.g., "lumpy" capacity investment, power plant dispatch, learning

So we combine our knowledge, build integer optimization problems, and interpret the dual variables as prices…

⇒ unfortunately, this is wrong!

⇒ there are no dual variables in mixed-integer programs

⇒ the "prices" reported by your solver do not clear the market!

Some perspective on power market efficiency gains

Even small improvements in power market operation can have huge societal benefits due to increased efficiency

Over the past decade, power market operation was greatly improved by using Mixed-Integer Programs (MIP) instead of Linear Programs (LP)

⇒ *In 2004, PJM implemented MIP in its day-ahead market [...], with savings estimated at $100 million/year.*

⇒ *On April 1, 2009, the California ISO (CAISO) implemented its Market Redesign and Technology Update (MRTU), [...] achieving an estimated $52 million in annual estimated savings using MIP.*

Quoted from: "Recent ISO Software Enhancements and Future Software and Modelling Plans", FERC Staff Report, 2011, pages 3-4

Introduction – The power market uplift problem

In electricity markets, marginal-cost pricing may yield incentive-incompatible dispatch and losses for generators

- Generators may earn negative profits based on market-clearing prices derived from least-cost unit commitment & dispatch

  ⇒ generators leave the market or do not follow the optimal dispatch prescribed by the ISO (“self-scheduling”)

- More generally, in markets with non-convexities & indivisibilities, using duals as market-clearing (Walrasian) prices doesn’t quite work

  ⇒ there may not even exist any Nash equilibrium in many cases

- In practice, the ISO pays out compensation to “make whole” individual generators after market clearing (“no-loss rule”)

  ⇒ potential for gaming and issues of acceptance by consumers

Current approaches to find binary equilibria

There exists no scalable approach to find Nash equilibria in non-cooperative games with binary decision variables

Current approaches for equilibria in binary games:

- neglect the gaming aspects altogether
- compute all permutations of the binary variables
  - ⇒ quickly grows beyond the bounds of computational tractability
- use a two-stage approach such as "make-whole" payments or convex-hull pricing and the minimum uplift problem
- relax/linearize the binary variables, apply equilibrium methods
  see Gabriel, Conejo, Ruiz, and Siddiqui (2013)
  - ⇒ doesn't yield exact solution, not clear whether it's truly a NE
- mixed-strategy games with discontinuous pay-off functions
  see Wang, Shanbhag, and Meyn (mimeo)

A solution method for equilibria in binary games

We propose an exact solution approach to find equilibria in non-cooperative games with binary variables

Outline of the proposed solution methodology:

- ⇒ derive KKT conditions for *both states* of each binary variable
- ⇒ add an objective function as *equilibrium selection* mechanism
- ⇒ include compensation payments to ensure incentive compatibility as decision variables of the equilibrium selection problem

The reformulated problem…

- is a *multi-objective bi-level* optimization program
- allows to incorporate the trade-off between market efficiency and compensation payment budget
- can be solved as a mixed-binary linear program

The math – Obtaining "duals" in integer programming

What are duals in integer or binary programming, and why can (or can't) we use them as market-clearing prices?

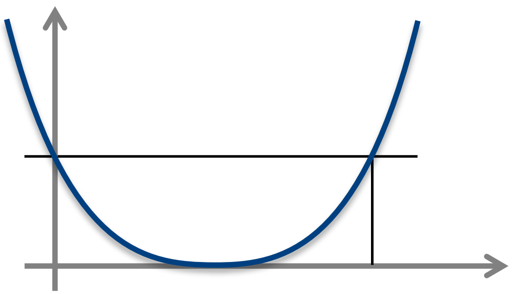
Using optimization for determining prices relies on *strong duality*

⇒ but duality in integer programs is difficult to establish

⇒ the notion of a "marginal relaxation" doesn't quite make sense

Look at a stylized example:

$$\begin{aligned} \min_x \quad & (x-0.5)^2 \\ \text{s.t.} \quad & x \in \{0,1\} \end{aligned}$$



If you solve this program in GAMS (or any numerical solvers), you will get some "duals" reported

⇒ where do these values come from?

The math – Obtaining "duals" in integer programming

Reported duals in integer programs are determined by solving the linearized problem in a two-step solution procedure

O'Neill et al. (2005) proposed a two-step approach:

⇒ solve the MIP using standard methods (Problem 1)

⇒ solve the linearized LP model (Problem 2), fixing discrete/binary variables at optimal level x^* as determined by Problem (1)

$$\begin{aligned} \min_{x,y} \quad & f(x,y) && (1) \\ \text{s.t.} \quad & g(x,y) \le 0 \\ & x \in \{0,1\}^n \\ & y \in \mathbb{R}^m \end{aligned}$$

$$\begin{aligned} \min_{x,y} \quad & f(x,y) && (2) \\ \text{s.t.} \quad & g(x,y) \le 0 && (\lambda) \\ & x = x^* && (\mu) \\ & (x,y) \in \mathbb{R}^{n+m} \end{aligned}$$

The dual variables (λ^*, μ^*) to Problem (2) can be interpreted as market-clearing, Walrasian prices!

⇒ but these are not actually the correct prices for Problem (1)!

The relevance of the O'Neill approach in reality

There is no (significant) power market that implements the prices obtained from the O'Neill method in actual operation

- All European power markets are "energy-only" markets

  $\Rightarrow$ there is no centralised dispatch and no side payments
- In the US, system operators solve a mixed-integer least cost problem

$$
\begin{aligned}
\min_{x,y} \quad & f(x,y) \\
\text{s.t.} \quad & g(x,y) = d \qquad (p) \\
& x \in \{0,1\}^n, (y) \in \mathbb{R}^m
\end{aligned}
$$

Nodal energy balance $\Rightarrow$ Locational marginal prices

The relevance of the O'Neill approach in reality

There is no (significant) power market that implements the prices obtained from the O'Neill method in actual operation

- All European power markets are "energy-only" markets
  - ⇒ there is no centralised dispatch and no side payments
- In the US, system operators solve a mixed-integer least cost problem
  - ⇒ but O'Neill prices may be confiscatory or "*payment for no service*"

$$\min_{x,y} \; f(x,y)$$

$$\text{s.t.} \quad g(x,y) = d \qquad (p)$$

Nodal energy balance ⇒ Locational marginal prices

$$x = x^* \qquad ~~(\mu)~~$$

Duals from linearized problem for on-off decisions

$$(x,y) \in \mathbb{R}^{n+m}$$

⇒ instead, uplift payments set by administrative fiat (e.g., no-loss rule)

⇒ loss of incentive-compatibility: potential gaming & self-scheduling!

⇒ no guarantee for revenue adequacy!

⇒ no guarantee that costs + uplift payments are indeed minimal!

An exact solution method – The "switch value"

Rather than focusing on relaxations of binary variables, let's look at the loss from deviation ("switch value")

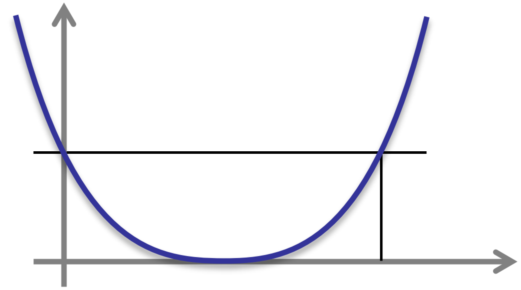
We introduce the term "switch value κ" to describe the absolute (not marginal) loss when deviating from the optimal value of x^*:

$$f\left(x^*, y^*\right) = f\left(x^\times, y^\times\right) - \kappa$$

$$\text{where } x^\times = 1 - x^* \text{ and } y^\times = \arg\min_y f\left(x^\times, y\right)$$

Applying this idea to the simple example:

$$\begin{aligned} \min_x \quad & (x - 0.5)^2 \\ \text{s.t.} \quad & x \in \{0,1\} \end{aligned}$$



$$\mu(x^* = 0) = -1$$
$$\mu(x^* = 1) = 1$$
$$\kappa = 0$$

⇒ the switch value can be interpreted as a *binary shadow price*!

⇒ It can be used as a measure of "disequilibrium" (Çelebi & Fuller)

Collecting definitions for binary games and equilibria

We introduce the notion of a binary quasi-equilibrium to describe incentive-compatible outcomes with compensation

Definition: *Binary game*

We have a set of players $i \in I$,
each seeking to solve a binary problem

$$\left\{ \begin{array}{lll} \min\limits_{\substack{x_i \in \{0,1\} \\ y_i \in \mathbb{R}^m}} & f_i\left(x_i, y_i, y_{-i}\left(x_{-i}\right)\right) & \\ \text{s.t.} & g_i\left(x_i, y_i\right) \leq 0 & \left(\lambda_i\right) \end{array} \right.$$

Definition: *Equilibrium in a binary game*

A (Nash) equilibrium in binary variables is a feasible vector $(x_i^*, y_i^*)_{i \in I}$
such that $f_i\left(x_i^*, y_i^*, y_{-i}^*\left(x_{-i}^*\right)\right) \leq f_i\left(x_i^\times, y_i^\times, y_{-i}^*\left(x_{-i}^*\right)\right) \quad \forall\, i \in I$

Definition: *Quasi-equilibrium in a binary game with compensation*

A (Nash) equilibrium in binary variables is a feasible vector $(x_i^*, y_i^*)_{i \in I}$
and a vector of compensation payments $(\zeta_i^*)_{i \in \mathrm{I}}$
such that $f_i\left(x_i^*, y_i^*, y_{-i}^*\left(x_{-i}^*\right)\right) - \zeta_i \leq f_i\left(x_i^\times, y_i^\times, y_{-i}^*\left(x_{-i}^*\right)\right) \quad \forall\, i \in I$

The central idea of our solution approach

We compute the optimal value w.r.t. the continuous variables for both states of the binary variable simultaneously

Assume that KKT conditions are necessary and sufficient w.r.t. continuous variables y_i for fixed binary x_i and given rivals actions y_{-i}

…we compute the optimal response of y_i for both states of variable x_i:

… assuming x_i =1:

$$0 = \nabla_{y_i} f_i\Big(\mathbf{1}, \widetilde{y}_i^{(\mathbf{1})}, y_{-i}(x_{-i})\Big) + \widetilde{\lambda}_i^{(\mathbf{1})} \nabla_{y_i} g_i\big(\mathbf{1}, \widetilde{y}_i^{(\mathbf{1})}\big) \quad , \quad \widetilde{y}_i^{(\mathbf{1})} \text{ (free)}$$

$$0 \geq g_i\big(\mathbf{1}, \widetilde{y}_i^{(\mathbf{1})}\big) \quad \perp \quad \widetilde{\lambda}_i^{(\mathbf{1})} \geq 0$$

… and assuming x_i=0:

$$0 = \nabla_{y_i} f_i\Big(\mathbf{0}, \widetilde{y}_i^{(\mathbf{0})}, y_{-i}(x_{-i})\Big) + \widetilde{\lambda}_i^{(\mathbf{0})} \nabla_{y_i} g_i\big(\mathbf{0}, \widetilde{y}_i^{(\mathbf{0})}\big) \quad , \quad \widetilde{y}_i^{(\mathbf{0})} \text{ (free)}$$

$$0 \geq g_i\big(\mathbf{0}, \widetilde{y}_i^{(\mathbf{0})}\big) \quad \perp \quad \widetilde{\lambda}_i^{(\mathbf{0})} 1 \geq 0$$

⇒ And then, check which strategy is optimal by comparing pay-offs:

$$f_i\big(\mathbf{1}, \widetilde{y}_i^{(\mathbf{1})}, y_{-i}(x_{-i})\big) \lesseqgtr f_i\big(\mathbf{0}, \widetilde{y}_i^{(\mathbf{0})}, y_{-i}(x_{-i})\big)$$

The central idea of our solution approach (II)

We use the switch value to the incentive-compatibility check to replace the cumbersome "if-then" conditions

The "if-then" conditions to determine the individually optimally binary decision are very painful to compute in large-scale problems:

$$
\begin{aligned}
f_i\big(\mathbf{1}, \widetilde{y}_i^{(\mathbf{1})}, y_{-i}(x_{-i})\big) &< f_i\big(\mathbf{0}, \widetilde{y}_i^{(\mathbf{0})}, y_{-i}(x_{-i})\big) \quad &\Rightarrow \quad & x_i^* = 1 \\
f_i\big(\mathbf{1}, \widetilde{y}_i^{(\mathbf{1})}, y_{-i}(x_{-i})\big) &> f_i\big(\mathbf{0}, \widetilde{y}_i^{(\mathbf{0})}, y_{-i}(x_{-i})\big) \quad &\Rightarrow \quad & x_i^* = 0 \\
f_i\big(\mathbf{1}, \widetilde{y}_i^{(\mathbf{1})}, y_{-i}(x_{-i})\big) &= f_i\big(\mathbf{0}, \widetilde{y}_i^{(\mathbf{0})}, y_{-i}(x_{-i})\big) \quad &\Rightarrow \quad & x_i^* = \{0,1\}
\end{aligned}
$$

We use the switch value κ and introduce a compensation payment ξ_i:

$$
\begin{aligned}
f_i\Big(\mathbf{1}, \widetilde{y}_i^{(\mathbf{1})}, y_{-i}\Big) + \kappa_i^{(\mathbf{1})} - \zeta_i^{(\mathbf{1})} - \kappa_i^{(\mathbf{0})} + \zeta_i^{(\mathbf{0})} &= f_i\Big(\mathbf{0}, \widetilde{y}_i^{(\mathbf{0})}, y_{-i}\Big) \\
\kappa_i^{(\mathbf{1})} + \zeta_i^{(\mathbf{1})} &\le x_i \; \widetilde{K} \\
\kappa_i^{(\mathbf{0})} + \zeta_i^{(\mathbf{0})} &\le (1 - x_i) \; \widetilde{K} \\
\kappa_i^{(\mathbf{1})}, \kappa_i^{(\mathbf{0})}, \zeta_i^{(\mathbf{1})}, \zeta_i^{(\mathbf{0})} &\in \mathbb{R}_+
\end{aligned}
$$

The central idea of our solution approach (III)

We solve for a binary equilibrium using a two-stage problem by introducing an upper-level “market operator” player

We introduce an additional player called “market operator”

- ⇒ not exclusively related to electricity, but rather a “coordinator”
- ⇒ can also be interpreted as an equilibrium selection mechanism

The market operator acts as an upper-level player, optimizing:

$$\min \quad F\left(\left(x_i, y_i\right)_{i \in I}\right) + G\left(\left(\zeta_i\right)_{i \in I}\right)$$

while guaranteeing feasibility, optimality, and incentive compatibility for each player.

- ⇒ This player can effectively consider the trade-off between market efficiency and compensation payments!

The two-stage program to obtain binary quasi-equilibria

The market operator incorporates the trade-off of efficiency vs. compensation, subject to a binary quasi-equilibrium

$$\min_{\substack{x_i, y_i, \widetilde{y}_i^{(\mathbf{\overline{x}_i})}, \widetilde{\lambda}_i^{(\mathbf{\overline{x}_i})} \\ \kappa_i^{(\mathbf{\overline{x}_i})}, \zeta_i^{(\mathbf{\overline{x}_i})}}} \quad F\Big((x_i, y_i)_{i \in I}\Big) + G\Big(\big(\zeta_i^{(\mathbf{\overline{x}_i})}\big)_{i \in I}\Big)$$

Multi-objective function of equilibrium selection mechanism

$$\text{s.t.} \quad \nabla_{y_i} f_i\Big(\mathbf{1}, \widetilde{y}_i^{(\mathbf{1})}, y_{-i}\Big) + \big(\widetilde{\lambda}_i^{(\mathbf{1})}\big)^T \nabla_{y_i} g_i\Big(\mathbf{1}, \widetilde{y}_i^{(\mathbf{1})}\Big) = 0$$

$$0 \leq -g_i\Big(\mathbf{1}, \widetilde{y}_i^{(\mathbf{1})}\Big) \perp \widetilde{\lambda}_i^{(\mathbf{1})} \geq 0$$

Optimal decision y_i for each player i given $x_i = 1$

$$\nabla_{y_i} f_i\Big(\mathbf{0}, \widetilde{y}_i^{(\mathbf{0})}, y_{-i}\Big) + \big(\widetilde{\lambda}_i^{(\mathbf{0})}\big)^T \nabla_{y_i} g_i\Big(\mathbf{0}, \widetilde{y}_i^{(\mathbf{0})}\Big) = 0$$

$$0 \leq -g_i\Big(\mathbf{0}, \widetilde{y}_i^{(\mathbf{0})}\Big) \perp \widetilde{\lambda}_i^{(\mathbf{0})} \geq 0$$

Optimal decision y_i for each player i given $x_i = 0$

$$f_i\Big(\mathbf{1}, y_i^{(\mathbf{1})}, y_{-i}\Big) + \kappa_i^{(\mathbf{1})} - \zeta_i^{(\mathbf{1})} - \kappa_i^{(\mathbf{0})} + \zeta_i^{(\mathbf{0})} = f_i\Big(\mathbf{0}, y_i^{(\mathbf{0})}, y_{-i}\Big)$$

Comparison of pay-off for each player

$$\kappa_i^{(\mathbf{1})} + \zeta_i^{(\mathbf{1})} \leq x_i \, \widetilde{K}$$

$$\kappa_i^{(\mathbf{0})} + \zeta_i^{(\mathbf{0})} \leq (1 - x_i) \, \widetilde{K}$$

$$\widetilde{y}_i^{(\mathbf{0})} - x_i \, \widetilde{K} \leq y_i \leq \widetilde{y}_i^{(\mathbf{0})} + x_i \, \widetilde{K}$$

$$\widetilde{y}_i^{(\mathbf{1})} - (1 - x_i) \, \widetilde{K} \leq y_i \leq \widetilde{y}_i^{(\mathbf{1})} + (1 - x_i) \, \widetilde{K}$$

Translation mechanism of individually optimal strategy to equilibrium outcome

$$x_i \in \{0, 1\}, \big(y_i, \widetilde{y}_i^{(\mathbf{\overline{x}_i})}\big) \in \mathbb{R}^{3m}, \big(\lambda_i^{(\mathbf{\overline{x}_i})}, \kappa_i^{(\mathbf{\overline{x}_i})}, \zeta_i^{(\mathbf{\overline{x}_i})}\big) \in \mathbb{R}_+^{2k+4}$$

Properties of the market operator's problem

The market operator's problem is an exact solution method for binary equilibria – and in many cases, it's a linear program!

Theorem: *Exact reformulation of a binary equilibrium program*

Any feasible solution of the market operator's problem is an equilibrium to the binary game with compensation (quasi-equilibrium)

⇒ we can use the market operator's objective towards "optimality"

Theorem: *Quadratic mixed-binary program with linear constraints*

The market operator's problem can be reformulated as a mixed-binary linear or quadratic program (under certain conditions)

⇒ These conditions hold for the power market problem, fossil fuel & resource markets, agriculture, investment games, etc.

Theorem: *Dominance of the binary equilibrium over current practice*

The optimal solution to the market operator's problem weakly dominates the current practice using a two-step procedure (under certain conditions)

An application: the power market uplift problem (I)

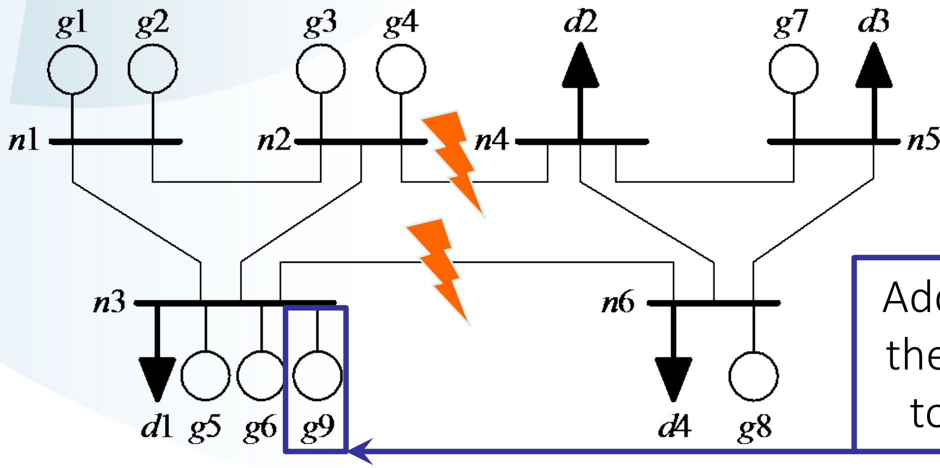
Liberalized power markets are the natural area of application for non-cooperative binary equilibrium problems

We use the nodal-pricing power market uplift problem example from Gabriel, Conejo, Ruiz, and Siddiqui (2013):

⇒ 6 nodes, 9 generators, 4 load units, 2 periods (high/low demand)

⇒ each generator has on-off decisions, start-up/shut-down costs, and minimum generation constraints (if active)

⇒ two zones, transmission bottlenecks on lines *N2-N4* and *N3-N6*



Additional generator compared to the model by Gabriel et al. (2013) to have a more salient example

An application: the power market uplift problem (II)

There are different market rules in real-world power markets and our approach is flexible to implement a variety of those

The power market uplift problem:

⇒ can be reformulated and solved as a mixed-binary linear program

⇒ a variety of market rules can be implemented as linear constraints

We compare three different market rule implementations:

⇒ Standard optimization approach:

- Two-step method following O'Neill et al. (2005), with a no-loss rule (ex-post compensation payment) for every generator

⇒ New methodology:

- A game-theoretic solution (binary equilibrium with compensation)
- A regulatory framework that no generator should loose money, but only active generators receive compensation (no-loss & active)

Illustrative results

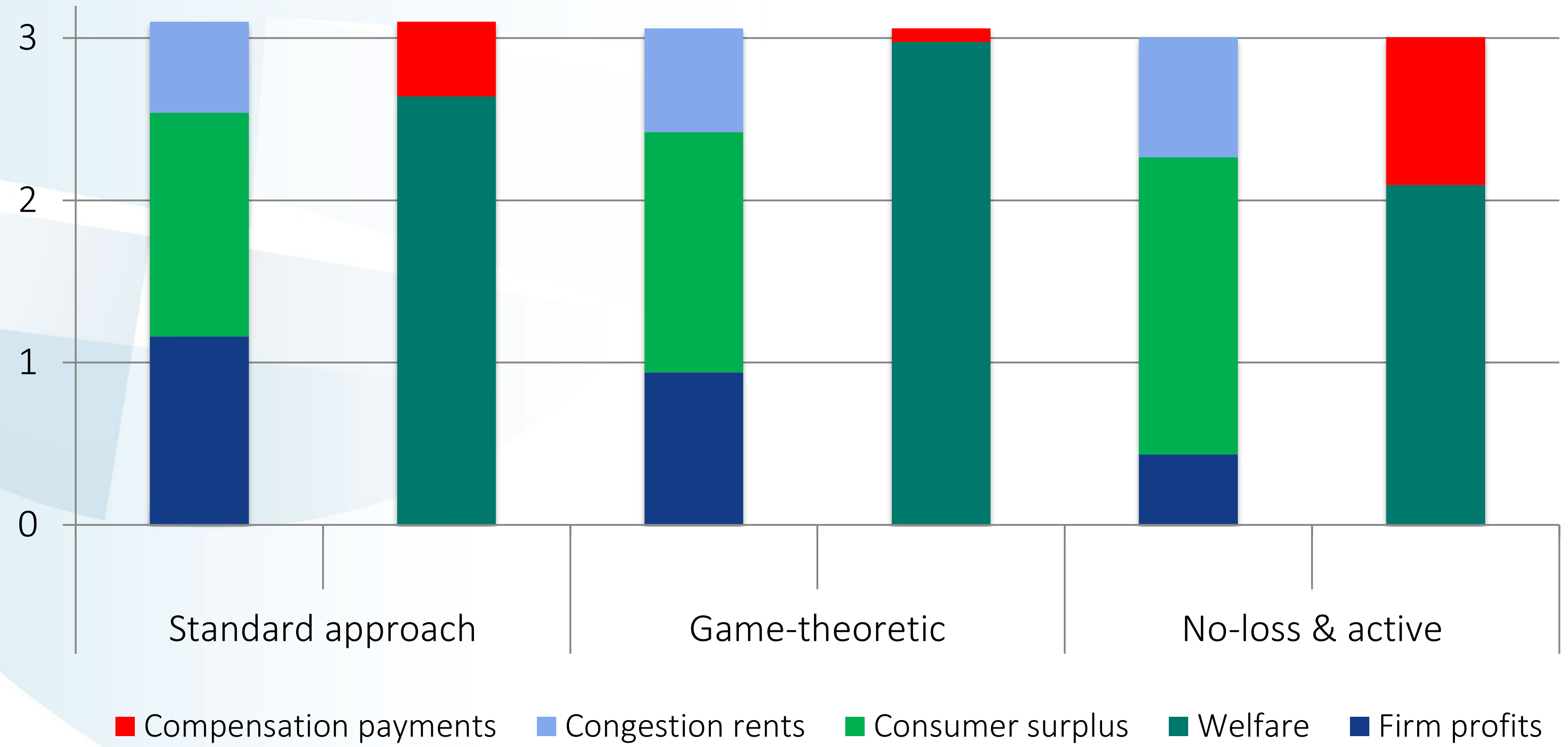
Compensation isn't necessary for incentive-compatibility and overly restrictive market rules reduce overall efficiency

Player		Standard approach			Game-theoretic			No-loss & Active		
	x^{init}	(x_{1i}, x_{2i})	π_i	ξ_i	(x_{1i}, x_{2i})	π_i	ξ_i	(x_{1i}, x_{2i})	π_i	ξ_i
g3	1	(0,0)	-300	300	(0,0)	-300	15	(1,1)	-525	525
g4	1	(1,1)	-160	160	(1,1)	-250	65	(1,0)	-335	335
g5	1	(1,1)	50		(1,1)	-50		(1,1)	-50	50
g6	1	(1,1)	200		(1,1)	100		(1,1)	100	
g7	0	(1,1)	690		(1,1)	715		(1,1)	690	
g8	0	(1,1)	680		(1,1)	730		(1,1)	550	
g9	0	(0,0)	0		(1,1)	-5	5	(0,0)	0	
All			1160	460		940	85		435	910
Other			1940			2120			2570	
Total			3100	460		3060	85		3005	910

Generators *g1* and *g2* are not active in any case; "Other": consumer & congestion rent

Illustrative results (II)

In this example, the market rules to "protect" generators actually induce a substantial welfare shift towards consumers



Rents by stakeholder group in the three settings in 1000 $ (method/market setting cases)

Computational aspects of the exact solution approach

The number of binary variables increases only linearly in the number of generators and hours

The main challenge in computing equilibria in binary variables is the exponential increase in the number of binary variables

⇒ size of trial problem: 6 nodes, 9 generators, 4 load units, 2 hours

- Standard approach (welfare-optimal unit-commitment)
  - ⇒ number of binary variables: $|T| \cdot |I| = 18$
  - ⇒ Question: negative profits, no-loss rule, or incentive-compatibility?
- Brute-force enumeration of equilibria
  - ⇒ number of equilibrium problems: $2^{|T| \cdot |I|} > 262\text{k}$
- Exact binary quasi-equilibrium solution method
  - ⇒ number of binary variables: $|T|\left(2\left(|I|+|J|+|L|+|N|-1\right)+|I|\right)=122$

Summary

We propose a novel and very flexible numerical method to find Nash equilibria in binary games with compensation

We develop a method to find solutions to binary equilibrium problems

We introduce the term "quasi-equilibrium" for market results that are incentive-compatible only if compensation is paid to some players

Under very general conditions, the problem can be solved as a mixed-binary linear/quadratic program using standard methods

The method allows to include a multitude of market rules and regulations to replicate real-world settings and considerations

⇒ No-loss rules, compensation only for active generators, etc.

The GAMS code is publicly available under a Creative Commons license at http://danielhuppmann.github.io/binary_equilibrium/

Outlook on future research

The method opens up a host of future research opportunities towards real-world applications and new algorithms

Applying the methodology to real-world size ISO market data

- ⇒ illustrate the trade-off between efficiency & compensation
- ⇒ can our method realize welfare gains similar to the switch to MIP?
- ⇒ numerical experiments indicate potential for savings!

Adapting the methodology to European power market design

- ⇒ integration with the primal-dual framework proposed by Madani and Van Vyve (2015)

Yield a better understanding of gaming opportunities in power sector

- ⇒ our method can explicitly compare game-theoretic aspects!

Extend the method to Generalized-Nash games, integer variables, etc.

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Thank you very much for your attention!

The GAMS (and maybe soon Python) codes are available under a *Creative Commons Attribution License 4.0* at http://danielhuppmann.github.io/binary_equilibrium

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