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Information Risk Premia in Energy Markets

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Explaining the Spot-Forward Relationships

- Classical Theory
- Market Risk Premium

Information Approach

- The Information Premium – Motivation
- The Information Premium – Modelling

Empirical Study

- CO_2 Emission Certificates Introduction
- The German Moratorium
- Additional Regressors

Options on Electricity Futures and Additional Information

- Mathematical Framework
- Vanilla Call Option
- Discussion
- Stylised Example

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Agenda

Explaining the Spot-Forward Relationships
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Spot-Forward Relationship: Classical theory

Under the no-arbitrage assumption we have the spot-forward relationship

$$F(t,T) = S(t)e^{(r-y)(T-t)} \tag{1}$$

where r is the interest rate at time t for maturity T and y is the convenience yield.

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Spot-Forward Relationship: Classical theory

- In a stochastic model this means

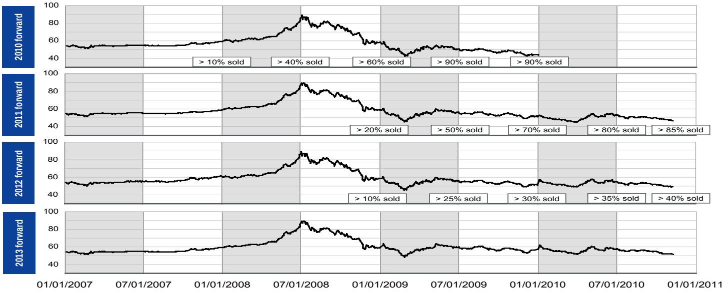
$$F(t,T) = \mathbb{E}_{\mathbb{Q}}(S(T)|\mathcal{F}_t)$$

  where $\mathcal{F}_t$ is the accumulated available market information (in most models the information generated by the spot price).
- $\mathbb{Q}$ is a risk-neutral probability
  - discounted spot price is a $\mathbb{Q}$-martingale
  - fixed by calibration to market prices or a market price of risk argument

Forward Selling Activity I

Forward selling[1] by RWE Power in the German market

(Base-load forwards in €/MWh)



(average realised price for 2008 forward: €58/MWh, for 2009 forward: €70/MWh)

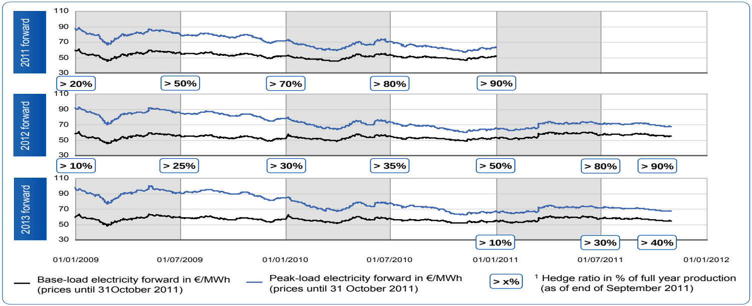
[1] Forward prices until November 8, 2010; hedge ratio as of Sept. 30, 2010.

RWE The energy to lead

Forward Selling Activity II

Forward selling[1] by RWE Power in the German market

(Base-load & peak-load forwards in €/MWh)



(Average realised price for 2010 forward: €67/MWh)

RWE The energy to lead

RWE AG | Q1-Q3 2011 Conference Call | 10 November 2011 22

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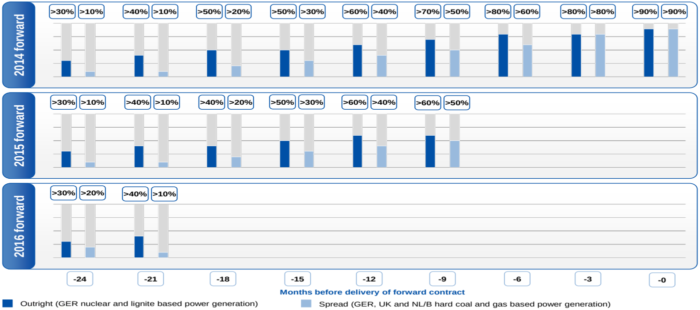
Forward Selling Activity III

RWE's forward hedging of conventional electricity production (German, Dutch and UK portfolio)

As of 31 March 2014



RWE The energy to lead

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Spot-Forward Relationship

- Backwardation: Forward prices are below spot price – Producers accept paying a premium for securing future production because of hedging pressure for long term investments
- Most models give either backwardation or contango: No stochastic change of sign (even for jump models)
- In electricity markets one normally observes that,
  - for 'long' dated forward contracts, markets are in normal backwardation (forward below expected spot)
  - for 'shorter' maturities the markets are in normal contango (forward above expected spot).

Market Risk Premium

- The *market risk premium* or *forward bias* $\pi(t, T)$ relates forward and expected spot prices.
- It is defined as the difference, calculated at time t, between the forward $F(t, T)$ at time t with delivery at T and expected spot price:

$$\pi(t, T) = F(t, T) - \mathbb{E}^{\mathbb{P}}[S(T)|\mathcal{F}_t]. \tag{2}$$

Here $\mathbb{E}^{\mathbb{P}}$ is the expectation operator, under the historical measure $\mathbb{P}$, with information up until time t and $S(T)$ is the spot price at time T.

Market Risk Premium – Explanations

- Equilibrium approach Benth, F.E.; Cartea, A.; Kiesel, R. *JBF*: Consider producers and retailers and their long- and short-term hedging pressures.
- Determine bounds for forward price by producer and retailer indifference prices.
- Actual forward price is found by using market power.
- The model is able to reproduce a changing sign of the market risk premium

Literature – Bessembinder/Lemmon

Bessembinder, H.and Lemmon, M.: Equilibrium Pricing and Optimal Hedging in Electricity Forward Markets, *J. Finance*, 2002.

- Use an equilibrium model to analyse predictive power of forward price
- Find a downward bias if expected power demand is low and demand risk is moderate
- Forward premium increases if either expected demand or demand variance is high
- Some empirical evidence

Literature – Benth/Meyer-Brandis

Benth, F. and Meyer-Brandis, T.: The Information Premium in Electricity Markets; *Journal Energy Markets*

- Discuss assumption that information filtration is generated by spot prices
- Use 'enlargement of filtration' to incorporate future information on the spot (such as power plant maintenance)
- Theoretical argument that a significant part of the market price of risk is due to an information premium

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Information Approach

- As electricity is non-storable future predictions about the market will not affect the current spot price, but will affect forward prices.
- Stylized example: planned outage of a power plant in one month
- Market example: in 2007 the market knew that in 2008 CO_2 emission costs will be introduced; this had a clearly observable effect on the forward prices!
- German moratorium 2011: shut-down 7 nuclear power plants for 3 months with possible complete shut-down.

German Moratorium I

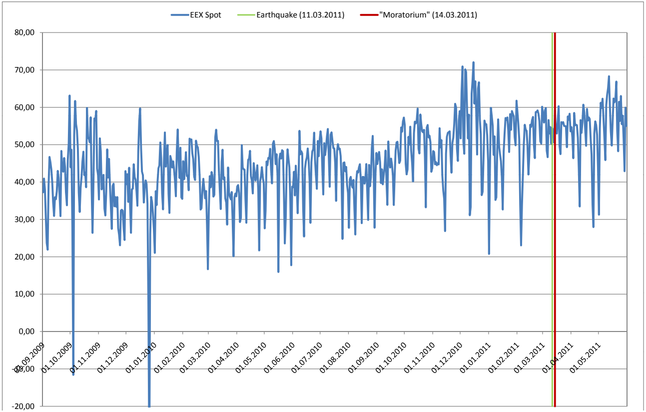

Figure : EEX spot prices

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German Moratorium II

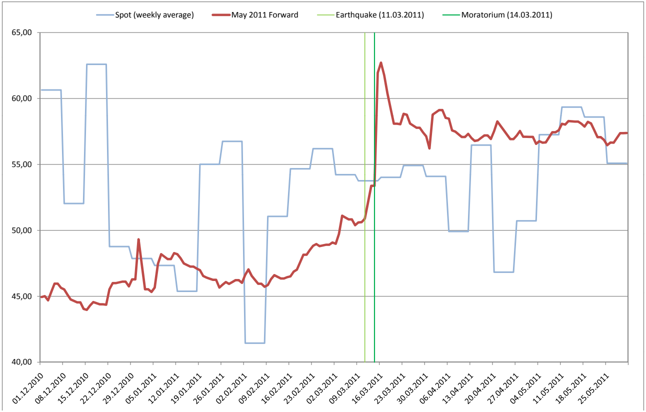

Figure : EEX forward prices delivery May 2011

German Moratorium III

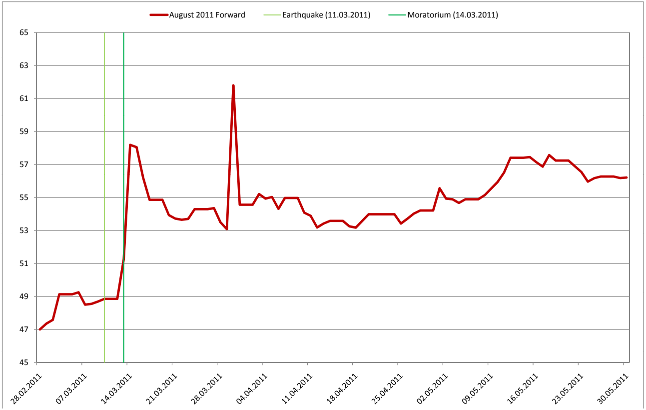

Figure : EEX forward prices delivery August 2011

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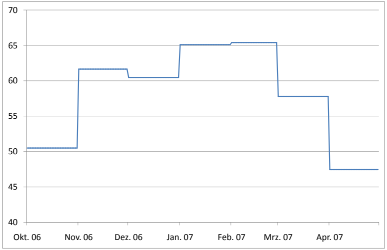
Example: 2008 CO_2 Emission Costs



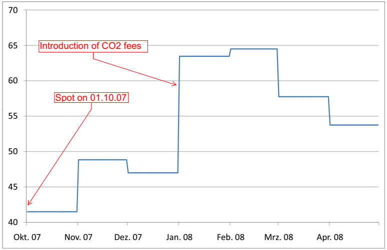


Figure : EEX Forward prices observed on 01/10/06 (left) and 01/10/07 (right)

- Typical winter and bank holidays behaviour in both graphs
- General upward shift in 2008

$\Rightarrow$ 2nd phase of CO_2 certificates

Information Approach

- Future information is incorporated in the forward price
- ... but not necessarily in the spot price due to non-storability
- ... buy-and-hold strategy does not work

Information Approach

- The usual pricing relation between spot and forward:

$$F(t,T) = \mathbb{E}^{\mathbb{Q}}[S_T|\mathcal{F}_t]$$

- Not sufficient: natural filtration $\mathcal{F}_t = \sigma(S_s, s \leq t)$
- Idea: enlarge the filtration!
- ... by information about the spot at some future time T_Υ
- Info could be that spot will be in certain interval...
- ... or the value of a correlated process (temperature)

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Information Approach

Filtrations

- $\mathcal{F}_t$ - the historical filtration
- $\mathcal{H}_t$ - complete information, i.e. $\mathcal{H}_t = \mathcal{F}_t \vee \sigma(S(T_\Upsilon))$
- $\mathcal{G}_t$ - the filtration of all information publicly available to the market

- Hence, we have the relation $\mathcal{F}_t \subseteq \mathcal{G}_t \subseteq \mathcal{H}_t$
- In the following we will consider the observed forward as

$$F(t, T) = \mathbb{E}^{\mathbb{Q}}[S(T)|\mathcal{G}_t]$$

The Information Premium

- Quantify the influence of future information using:

Information Premium

The information premium is defined to be

$$I(t, T) = \mathbb{E}[S_T|\mathcal{G}_t] - \mathbb{E}[S_T|\mathcal{F}_t]$$

i.e. the difference between the prices of the forward under $\mathcal{G}$ and $\mathcal{F}$.

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The Information Premium

Lemma

There is no information on the information premium in $\mathcal{F}$.

Proof:

$$\mathbb{E}[I_{\mathcal{G}}(t,T) \mid \mathcal{F}_t] = \mathbb{E}[\ \mathbb{E}[S(T) \mid \mathcal{G}_t] - \mathbb{E}[S(T) \mid \mathcal{F}_t]\ \mid \mathcal{F}_t] = 0$$

- Result valid for all measures equivalent to $\mathbb{P}$
- Usual method to attain the *market price of risk* is a measure change
- This is not possible for the *Information Premium*

Spot price model and forward price with delivery

Two-factor arithmetic Spot Price

$$S(t) = \Lambda(t) + X(t) + Y(t)$$

$$X(T) = e^{-\alpha(T-t)}X(t) + \sigma \int_t^T e^{\alpha(T-s)}dW(s)$$

$$Y(T) = e^{-\beta(T-t)}Y(t) + \int_t^T e^{\beta(T-s)}dL(s)$$

where $\Lambda(t)$ is deterministic, $W(t)$ a BM, $L(t)$ a Lévy process.

- The forward price with delivery in $[T_1, T_2]$ is then given by

$$F(t, T_1, T_2) = \frac{1}{T_2 - T_1} \mathbb{E}\left[\int_{T_1}^{T_2} S(u)du \mid \mathcal{F}_t\right]$$

Enlargement of filtrations

- We will demonstrate how to calculate the information premium in this model
- We will enlarge the historical filtration of Lévy process L_t...
- ... with future information about the value L_{T_Υ}
- *Grossissements de filtrations*:
  - Developed by French Mathematicians (Jeulin, Yor) in the 1980s
  - First theorem by Itō in 1976

Itō's theorem for Lévy processes and additional incomplete information

Theorem

Let L_t be a Lévy process and $\mathcal{G}_t \subseteq \mathcal{H}_t = \mathcal{F}_t \vee \sigma(L_{T_\Upsilon})$. Then

1. *L is still a semimartingale with respect to $\mathcal{G}_t$*
2. *if $\mathbb{E}[|L_t|] < \infty$ then*

$$\xi(t) = L_t - \int_0^{t \wedge T_\Upsilon} \frac{\mathbb{E}[L_{T_\Upsilon} - L_s | \mathcal{G}_s]}{T_\Upsilon - s} \, ds$$

is a $\mathcal{G}_t$-martingale.

Future Lévy information

- The info premium is

$$I_{\mathcal{G}}(t, T_1, T_2; T_\Upsilon) = F_{\mathcal{G}}(t, T_1, T_2) - F_{\mathcal{F}}(t, T_1, T_2)$$

- Brownian motion terms as well as X_t and Y_t terms cancel (both filtrations coincide), thus

$$I_{\mathcal{G}}(t, T_1, T_2; T_\Upsilon) = \frac{1}{T_2 - T_1} \mathbb{E}\left[\int_{T_1}^{T_2} \int_t^u e^{-\beta(u-s)} dL(s) du | \mathcal{G}_t \right] - \frac{1}{T_2 - T_1} \hat{\beta}(t, T_1, T_2) \phi'(0)$$

- where $\hat{\beta}$ is some deterministic function (> 0) and ϕ is the log-moment-generating function of L_1

Future Lévy information

- We now apply Itō's theorem

(remember $\xi(t) = L(t) - \int_0^t \frac{\mathbb{E}[L(T_\Upsilon) - L(s)|\mathcal{G}_s]}{T_\Upsilon - s} ds$ is a $\mathcal{G}$-martingale)

$$
\begin{aligned}
&\mathbb{E}\left[\int_{T_1}^{T_2}\int_t^u e^{-\beta(u-s)} dL(s) du | \mathcal{G}_t\right] \\
&\quad = \mathbb{E}\left[\int_{T_1}^{T_2}\int_t^u e^{-\beta(u-s)} \frac{\mathbb{E}[L_{T_\Upsilon} - L_s|\mathcal{G}_s]}{T_\Upsilon - s} ds\, du | \mathcal{G}_t\right] \\
&\quad = \dots \\
&\quad = \frac{\mathbb{E}[L_{T_\Upsilon} - L_t|\mathcal{G}_t]}{T_\Upsilon - t} \hat{\beta}(t, T_1, T_2)
\end{aligned}
$$

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Future Lévy information

- Collecting terms yields

$$
\begin{aligned}
& I_{\mathcal{G}}(t, T_1, T_2; T_\Upsilon) \\
& = \frac{1}{T_2 - T_1} \hat{\beta}(t, T_1, T_2) \left(\frac{\mathbb{E}[L_{T_\Upsilon} - L_t | \mathcal{G}_t]}{T_\Upsilon - t} - \phi'(0) \right) \\
& = \frac{1}{T_2 - T_1} \frac{\hat{\beta}(t, T_1, T_2)}{T_\Upsilon - t} \left(\mathbb{E}[L_{T_\Upsilon} | \mathcal{G}_t] - \mathbb{E}[L_{T_\Upsilon} | \mathcal{F}_t] \right)
\end{aligned}
$$

- Sign of the premium depends on $\mathbb{E}[L_{T_\Upsilon} | \mathcal{G}_t] - \mathbb{E}[L_{T_\Upsilon} | \mathcal{F}_t]$
- ... which matches the intuition:
  - i.e. CO_2 certificates: positive premium $\Rightarrow$ $\mathbb{E}[L_{T_\Upsilon} | \mathcal{G}_t] > \mathbb{E}[L_{T_\Upsilon} | \mathcal{F}_t]$

Agenda

Explaining the Spot-Forward Relationships

Information Approach

Empirical Study

- CO_2 Emission Certificates Introduction
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Options on Electricity Futures and Additional Information

Showing the existence of the info premium

- Agenda:
  1. Calibrate the spot model to observed data (EEX)
  2. Calculate expectations under $\mathbb{P}$
  3. Conduct for each class of month-forwards a constant distance-minimising change of measure (ls-sense)
  4. Calculate expectation under $\mathbb{Q}$
  5. Assume observed forward price $\hat{F}(t, T_1, T_2)$ is $F_{\mathcal{G}}^{\mathbb{Q}}(t, T_1, T_2)$
  6. For the life-time of different forwards calculate

$$\hat{I}_{\mathcal{G}}^{\mathbb{Q}}(t, T_1, T_2) = \hat{F}(t, T_1, T_2) - F_{\mathcal{F}}^{\mathbb{Q}}(t, T_1, T_2)$$

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Specific Two-Factor Model

We use

$$S(t) = \Lambda(t) + X(t) + Y(t), \tag{3}$$

- with $\Lambda(t)$ a deterministic function to capture seasonal influences,
- $X(t)$ a standard *Ornstein-Uhlenbeck processes*

$$dX(t) = -\alpha X(t)dt + \sigma dW(t)$$

where $\alpha \in \mathbb{R}$ is the mean reversion parameter, $\sigma > 0$ the volatility,

Specific Two-Factor Model

$$dY(t) = -\beta Y(t)dt + dL(t)$$

- $\beta \in \mathbb{R}$ is the mean reversion parameter
- Here, we use

$$L_t = \sum_{i=1}^{N_t} D_i$$

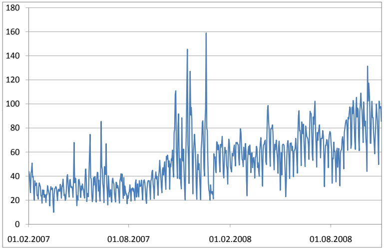
  where N_t is a Poisson process with intensity $\lambda > 0$ and D_i are the i.i.d jump sizes
- double-exponentially distributed with density

$$f_D(x) = p\eta_1 e^{-\eta_1 x} \mathbb{1}_{x \geq 0} + q\eta_2 e^{-\eta_2 |x|} \mathbb{1}_{x \leq 0}$$

  where $p + q = 1$ and $\eta_1, \eta_2 \geq 0$.

Spot Calibrating

- EEX spot from 01/02/2007 to 30/10/2008
- Includes CO_2-date 01/01/08 as midpoint



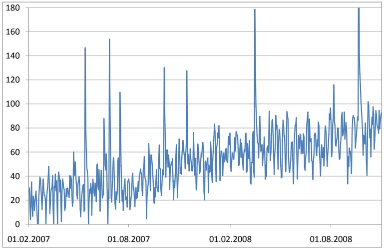


Figure : Spot and Simulation for data set

Estimation Results

- Fitted parameter values are

Parameter	α	σ	β	λ	p	q	η_1	η_2
Value	0.538	11.108	0.786	0.034	0.955	0.045	0.019	0.027

- The change of measure parameters are positive for the first three months and negative for more distant delivery periods.

Forward	1 m	2 m	3 m	4 m	5 m	6 m
θ_W	0.164	0.734	0.153	-0.593	-1.893	-3.199

Expectations and change of measure

- Prices under $\mathbb{P}$ and $\mathbb{Q}$ and observed January 2008 forward

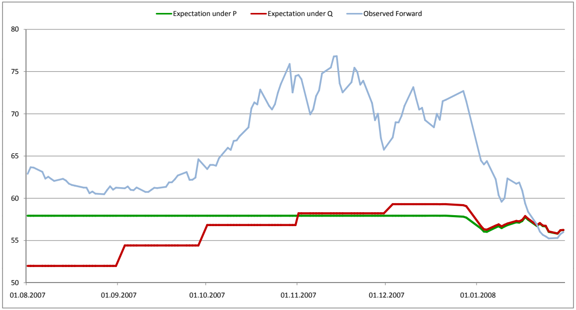


Figure : Observed, $\mathbb{E}^{\mathbb{P}}$ and $\mathbb{E}^{\mathbb{Q}}$ Prices

The information premium?

- The residual $\hat{I}^{\mathbb{Q}}_{\mathcal{G}}(t, T_1, T_2) = \hat{F}(t, T_1, T_2) - F^{\mathbb{Q}}_{\mathcal{F}}(t, T_1, T_2)$
  - is positive approx. between 5 and 20 €
  - converges in the delivery period

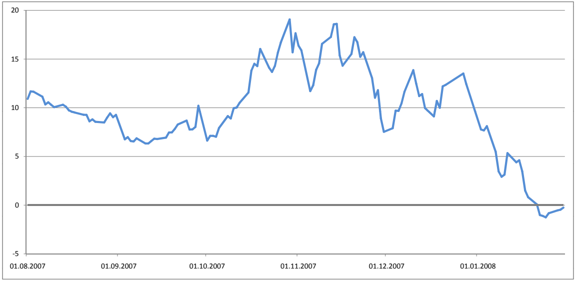


Figure : The residual $\hat{I}^{\mathbb{Q}}_{\mathcal{G}}(t, T_1, T_2)$ for the January 2008 forward

Showing the existence of the info premium

- $\hat{I}^{\mathbb{Q}}_{\mathcal{G}}(t, T_1, T_2)$ is our best guess for $I(t, T_1, T_2)$!
- We need to show that:
  1. $\hat{I}^{\mathbb{Q}}_{\mathcal{G}} \neq 0$
  2. $\hat{I}^{\mathbb{Q}}_{\mathcal{G}}$ is not $\mathcal{F}_t$-measurable, i.e. $\mathbb{E}[\hat{I}^{\mathbb{Q}}_{\mathcal{G}}|\mathcal{F}_t] = 0$
- We will consider Nov07, Jan08, Mar08 and Aug08 contracts (lifetime before, during and after 01/01/08)

$\hat{l}_{\mathcal{G}}^{\mathbb{Q}}$ non-zero?

- Visual impression for all four series does not hint to white-noise
- Ljung-Box test for white noise for $\hat{l}_{\mathcal{G}}^{\mathbb{Q}}(t, T_1, T_2)$ rejected at all levels

	Nov 2007	Jan 2008	Mar 2008	Aug 2008
Ljung-Box	867.68	738.84	1606.87	797.01
χ^2 (95%)	36.06	35.73	35.40	35.84

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How do we show non-measurability?

- We want to show $\mathbb{E}[\hat{I}^{\mathbb{Q}}_{\mathcal{G}}|\mathcal{F}_t] = 0$
  - Consider Hilbert space $L^2(\mathcal{F}, \mathbb{Q})$
  - Try to express $\hat{I}^{\mathbb{Q}}_{\mathcal{G}}$ in terms of a countable basis of the spot...
  - ... by means of regression from S onto $\hat{I}^{\mathbb{Q}}_{\mathcal{G}}$
  - Non-measurability $\Rightarrow$ Bad regression results!
- For now, let $\mathcal{B} = \{x^i : i \in \mathcal{I}\}$ the polynomial basis
- To avoid spurious regression (Granger/Newbold) we use (stationary) first differences

Regression results

- Regression: $\triangle \hat{I}^{\mathbb{Q}}_{\mathcal{G}}(t, T_1, T_2) = \sum_{i=1}^{N} c_i \triangle S^i_t + \epsilon(t)$

Regression results for $N = 10$

	Nov 07	Jan 08	Mar 08	Aug 08
R^2	0.14	0.07	0.03	0.07
$F - stat$	1.47	0.65	0.35	0.75

- F-value for 95% is 1.88, thus we cannot reject $c_1 = \ldots = c_N = 0$
- Increasing N does not alter the results
- Contracts living on 01/01/08 show more extreme results!

- We conclude that $\hat{I}^{\mathbb{Q}}_{\mathcal{G}}(t, T_1, T_2)$ is not $\mathcal{F}_t$-measurable!

Discussion

Size of $\hat{I}^{\mathbb{Q}}_{\mathcal{G}}$ for Jan08:

- 2007: CO_2 price practically zero
- 2008: around €22
- assume $0.7 tCO_2/MWh$ efficiency rate
- $\Rightarrow$ info premium should be around $0.7\cdot$ €22 $\approx$ €15
- $\Rightarrow$ which $\hat{I}^{\mathbb{Q}}_{\mathcal{G}}$ is!

Spot Calibrating – Data

- EEX spot from 01/09/2009 to 15/08/2011
- critical dates are 11/03/2011 (earthquake), 14/03/2011 (Moratorium), 31/05/2011 (final decision to close old plants) and 14/06/2011 (end of Moratorium).

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Spot Calibrating – Data

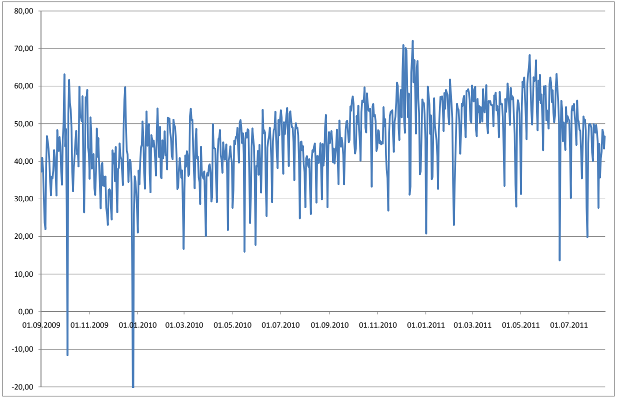


Figure : EEX spot price from 01/09/2009 until 15/08/2011

Estimation Results

- Fitted parameter values are

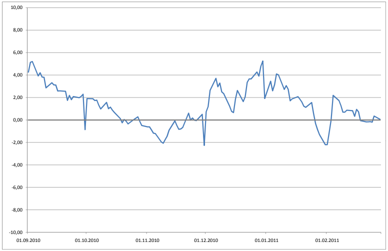
Parameter	α	σ	β	λ	p	q	η_1	η_2
Value	0.499	6.01	0.864	0.027	0.105	0.895	0.046	0.033

- The change of measure parameters for different months and quarters.

Forward	1 m	2 m	3 m	4 m	5 m	6 m
θ_W	0.210	0.624	0.650	0.614	0.512	0.363

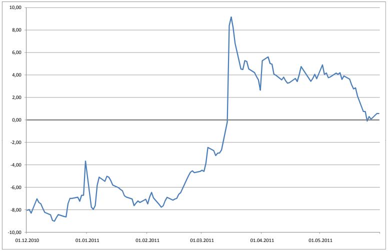
The Info Premium $\hat{I}^{\mathbb{Q}}_{\mathcal{G}}(t, T_1, T_2)$

We will consider Feb11, May11, July11 contracts (lifetime before, during and after the date of the moratorium)

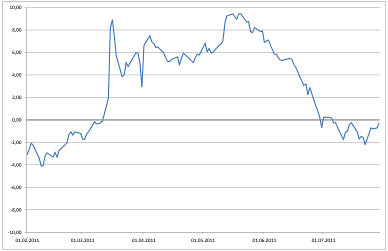


(a) February 2011: $\hat{I}^{\mathbb{Q}}_{\mathcal{G}}$

The Info Premium $\hat{I}^{\mathbb{Q}}_{\mathcal{G}}(t, T_1, T_2)$



(b) May 2011: $\hat{I}^{\mathbb{Q}}_{\mathcal{G}}$



(c) July 2011: $\hat{I}^{\mathbb{Q}}_{\mathcal{G}}$

$\hat{I}_{\mathcal{G}}^{\mathbb{Q}}$ non-zero?

- Visual impression does no suggest white noise
- Ljung-Box test for white noise for $\hat{I}_{\mathcal{G}}^{\mathbb{Q}}(t, T_1, T_2)$ rejected at all levels

	Feb 2011	May 2011	Jul 2011
Ljung-Box	522.97	1645.87	1094.83
χ^2 (95%)	36.06	35.95	35.62

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Regression results

Regression Equation

$$\triangle \hat{I}_{\mathcal{G}}^{\mathbb{Q}}(t, T_1, T_2) = \sum_{i=1}^{N} \alpha_i \triangle S^i(t) + \triangle \epsilon(t)$$

Regression results for $N = 10$

	Feb 11	May 11	Jul 11
R^2	0.14	0.06	0.09
F-statistic	1.96	0.69	1.09

Table : R^2s and F-statistics of the regression from $\triangle S$ onto $\triangle \hat{I}_{\mathcal{G}}^{\mathbb{Q}}$

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Regression results

- F-value for 95% is 1.88, thus the hypothesis of value zero coefficients can not be rejected for the May and July forwards, although for the February contract we can only use the 90% level.
- Values and t-statistics of the individual coefficients confirm zero coefficients
- We conclude that $\hat{I}^{\mathbb{Q}}_{\mathcal{G}}(t, T_1, T_2)$ is not $\mathcal{F}_t$-measurable!

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Discussion

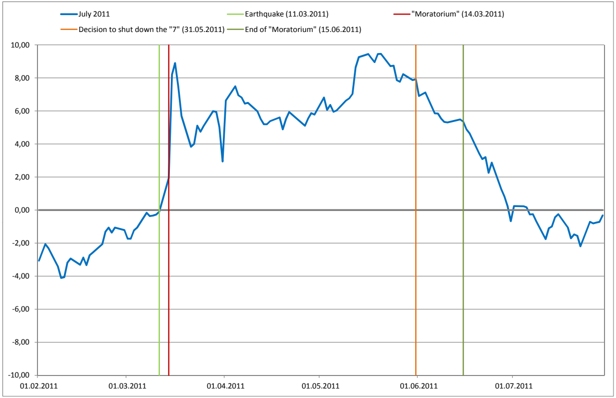

Figure : The information premium for the July 2011 forward contract

Non-Markovian Structure

- Regression equation

$$\triangle \hat{I}_{\mathcal{G}}^{\mathbb{Q}}(t, T_1, T_2) = a_1 MA_l(S_t) + \sum_{i=2}^{N+1} a_i \triangle S^i(t) + \epsilon(t)$$

  where l denotes the length of the moving average; we tried lengths $l = 2, 4, 7, 10, 30$.
- For both data sets we found the general result was not altered and the moving average's coefficient was significantly zero.

Other Assets and Commodities

- Regression equation (e.g. DAX)

$$\triangle \hat{I}^{\mathbb{Q}}_{\mathcal{G}}(t, T_1, T_2) = \sum_{i=1}^{N_1} a_i \triangle DAX^i(t) + \sum_{j=N_1+1}^{N_2} a_j \triangle S^j(t) + \epsilon(t)$$

- We used Brent oil spot (EUETS data set), EEX gas (Moratorium data set) and the DAX stock index (EUETS data set). None of these add to the quality or the significance of the regressions (less than $5\% R^2$, mostly insignificant coefficients).

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DAX and Moratorium

Forward	Regression	R^2	F-stat
Feb 2011	$N_1 = 1, N_2 = 1$	0.007	2.48
Feb 2011	$N_1 = 5, N_2 = 15$	0.25	2.38
May 2011	$N_1 = 1, N_2 = 1$	0.08	10.52
May 2011	$N_1 = 5, N_2 = 15$	0.62	11.97

Table : R^2s and F-statistics for regressions including the DAX index for February and May 2011 forwards

DAX and Moratorium

- The earthquake occurred on a Friday whereas the Moratorium and its consequent rise in forward prices was the Monday and Tuesday thereafter.
- The price of the May 2011 forward was 50.88 Euro on Friday, it jumped to 61.95 on Tuesday and settled to a level around 58.00 Euro by the end of that week.
- Exactly the opposite took place on the stock exchange. On Friday, the DAX was at 6981 points, when the stock exchange reopened on Monday the DAX fell by more than 400 (i.e. 5.7%) to 6513 points.

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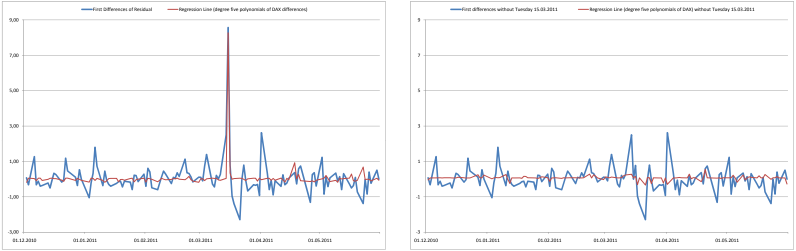
DAX and Moratorium



(a) Residuals vs regression line including 15.03.2011

(b) Residuals vs regression line not including 15.03.2011

Figure : May 2011 residuals vs regression line with respect to DAX polynomials of degree five

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German Moratorium - Spot

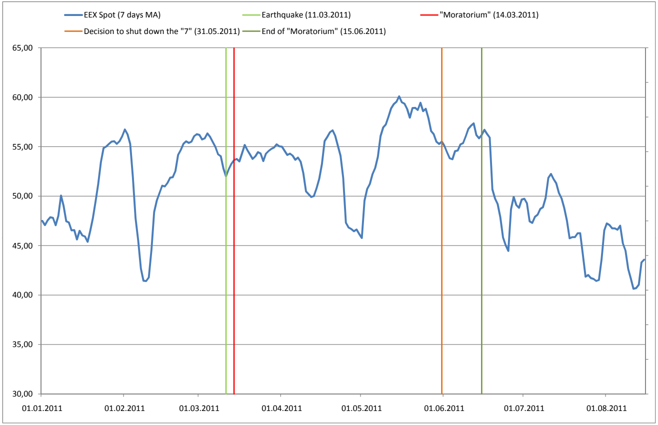

Figure : EEX day-ahead baseload (spot) prices 2010/2011

- No significant impact on spot prices

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German Moratorium - Forward

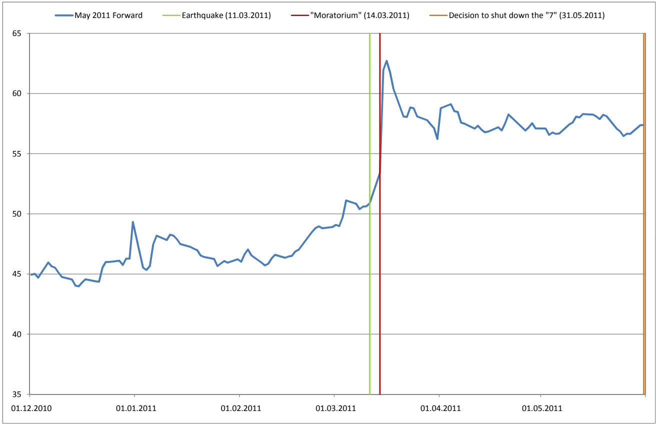

Figure : EEX May 2011 forward price

- Permanent increase from 14/03/2011

Plants offline

- Brunsbüttel and Krümel offline anyway (800 MW + 1400 MW)
- Biblis B in regular revision (another 1300 MW)

- This leaves around 4000 MW that were switched off immediately
- Still, it seems that there was no change in price setting technology!?

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Renewables

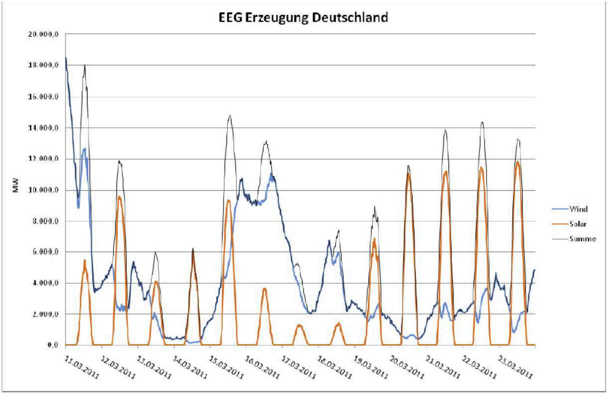

Figure : Wind and solar (report from BNA for BMWi)

- Some nuclear energy replaced by wind first, then solar...

Import/Export

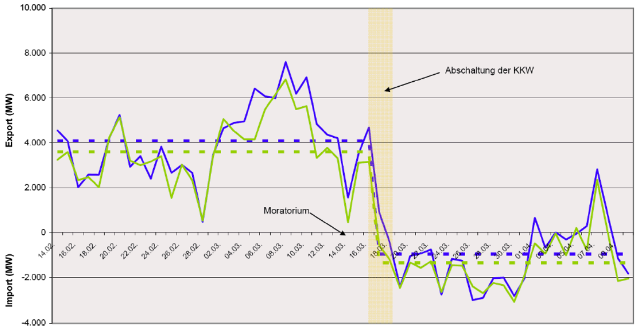

Figure : Cross-border trading (report from BNA for BMWi)

- ... and some brought in from France

Agenda

Explaining the Spot-Forward Relationships

Information Approach

Empirical Study

Options on Electricity Futures and Additional Information
- Mathematical Framework
- Vanilla Call Option
- Discussion
- Stylised Example

Spot-model and Forward Prices

- To illustrate, we choose the easiest spot model:

$$dX_t = -\alpha X_t dt + \sigma dW_t$$
$$X_T = e^{-\alpha(T-t)} X_t + \sigma \int_t^T e^{-\alpha(T-s)} dW_s$$

- The traditional forward price with delivery in $[T_1, T_2]$ is

$$F_{\mathcal{F}}^{\mathbb{P}}(t, T_1, T_2) = \tfrac{1}{T_2 - T_1} \bar{\alpha}(t, T_1, T_2) X_t$$
$$dF_{\mathcal{F}}^{\mathbb{P}}(t, T_1, T_2) = \tfrac{1}{T_2 - T_1} \sigma \bar{\alpha}(t, T_1, T_2) dW_t$$

- Thus, our pricing measure satisfies $\mathbb{Q} = \mathbb{P}$ and we are in a Bachelier-world!

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Forward Price with Additional Information

- Theory of enlargement of filtrations helps us to find a process $\mu_t^{\mathcal{G}}$ such that

$$\xi_t = W_t - \int_0^t \mu_s^{\mathcal{G}} ds$$

is a $\mathcal{G}$-Brownian motion

- We then find the $\mathcal{G}$-dynamics of the forward

$$dF_{\mathcal{G}}^{\mathbb{P}}(t, T_1, T_2) = \frac{1}{T_2 - T_1} \sigma\bar{\alpha}(t, T_1, T_2)\, d\left(\xi_t + \int_0^t \mu_s^{\mathcal{G}} ds\right)$$
$$= \frac{1}{T_2 - T_1} \left(\sigma\bar{\alpha}(t, T_1, T_2) d\xi_t + \sigma\bar{\alpha}(t, T_1, T_2)\mu_t^{\mathcal{G}} dt\right)$$

i.e. with drift term

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Changing the Measure 1/2

- We define processes

$$M_t = \int_0^t (-\mu_s^{\mathcal{G}}) d\xi_s \quad , \quad N_t = 1 + \int_0^t N_s dM_s$$

- Now, we define a new measure $\tilde{\mathbb{P}}$ by $\frac{d\tilde{\mathbb{P}}}{d\mathbb{P}}|_{\mathcal{G}_t} = N_t$
- Girsanov-Meyer gives the $\tilde{\mathbb{P}}$-decomposition of the $(\mathcal{G}, \mathbb{P})$-BM ξ_t:

$$\begin{aligned}\xi_t &= \left(\xi_t - \int_0^t \frac{1}{N_s} d < N, \xi >_s\right) + \int_0^t \frac{1}{N_s} d < N, \xi >_s \\ &= W_t - \int_0^t \mu_s^{\mathcal{G}} ds\end{aligned}$$

Changing the Measure 2/2

- It turns out the original $(\mathcal{F}, \mathbb{P})$-BM W_t is also a $(\mathcal{G}, \tilde{\mathbb{P}})$-BM and the forward dynamics are

$$dF_{\mathcal{G}}^{\tilde{\mathbb{P}}}(t, T_1, T_2) = \frac{1}{T_2 - T_1}\, \sigma\bar{\alpha}(t, T_1, T_2) dW_t^{\mathcal{G}, \tilde{\mathbb{P}}}$$

- Hence, the pricing measure with additional information is $\tilde{\mathbb{P}}$!
- Note: $\mu_s^{\mathcal{G}}$ is not $\mathcal{F}$-measurable and thus the measure $\tilde{\mathbb{P}}$ is only available to the trader taking future information into account
- This connection between enlargement of filtration and changing measure was discovered by Protter in 1989

Vanilla Call Option

- We can integrate

$$F_{\mathcal{G}}^{\tilde{\mathbb{P}}}(T, T_1, T_2) = F_{\mathcal{G}}^{\tilde{\mathbb{P}}}(t, T_1, T_2) + \frac{1}{T_2 - T_1} \sigma \int_t^T \bar{\alpha}(t, T_1, T_2) dW_s^{\mathcal{G}, \tilde{\mathbb{P}}}$$

- Thus, we are, once again, in a Bachelier-world!
- And the option price with future information is

$$C_{\mathcal{G}}(t, T, F_{\mathcal{G}}, K) = (F_{\mathcal{G}}^{\tilde{\mathbb{P}}}(t, T_1, T_2) - K)\Phi(d_1^{\mathcal{G}}) + \Sigma\phi(d_1^{\mathcal{G}})$$

- Here

$$d_1^{\mathcal{G}} = \frac{F_{\mathcal{G}}^{\tilde{\mathbb{P}}}(t, T_1, T_2) - K}{\Sigma(t, T, T_1, T_2)}$$

$$\Sigma^2(t, T, T_1, T_2) = Var(F_{\mathcal{G}}^{\tilde{\mathbb{P}}}(T, T_1, T_2) \mid \mathcal{G}_t)$$

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Insider Trading, Assets and Market Players 1/2

- The insider-literature finds that honest traders and insiders assign the same value to options!
  - Mathematics: ... dynamics are the same under pricing measures
  - Intuition: ... both have access to a replicating strategy
- Our calculations seem to propose the same for electricity!?
- But stocks are storable and buy-and-hold arguments work
- Intuitive argument not applicable for our underlying
- The electricity spot is not a classical asset!
- Forwards under $\mathcal{F}$ and $\mathcal{G}$ belong to two different pricing models

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Insider Trading, Assets and Market Players 2/2

- Thus, different derivative prices make sense again!
- Do both models (types of traders?) coexist on the market? How are market prices amalgamated?
- Summarising: the initial forward price plays an important role:

$$C_{\mathcal{G}}(t, T, F_{\mathcal{G}}, K) = (F_{\mathcal{G}}^{\tilde{\mathbb{P}}}(t, T_1, T_2) - K)\Phi(d_1^{\mathcal{G}}) + \Sigma\phi(d_1^{\mathcal{G}})$$

- Conclusion: Pricing options with future information...
  - ... reduces to calculating the forward price under $\mathcal{G}$
  - ... we can then use standard approaches (for example Bachelier for standard OU-process)

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Stylised Example

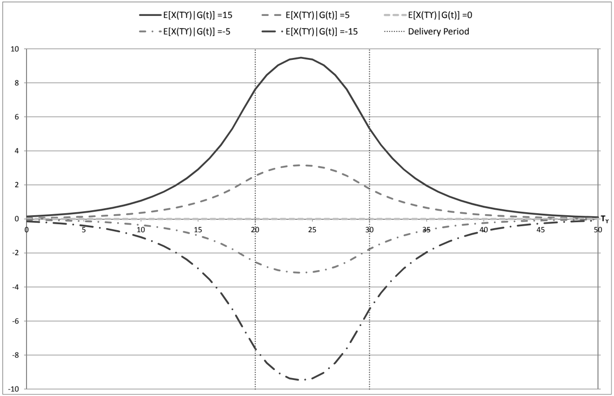

Figure : The information premium over T_Υ for different values of $\mathbb{E}[X_{T_\Upsilon}|\mathcal{G}_t]$. Other parameters are: $t = 0, T_1 = 20, T_2 = 30, X_0 = 0, \alpha = 0.2$ and $\sigma = 3.0$

Stylised Example

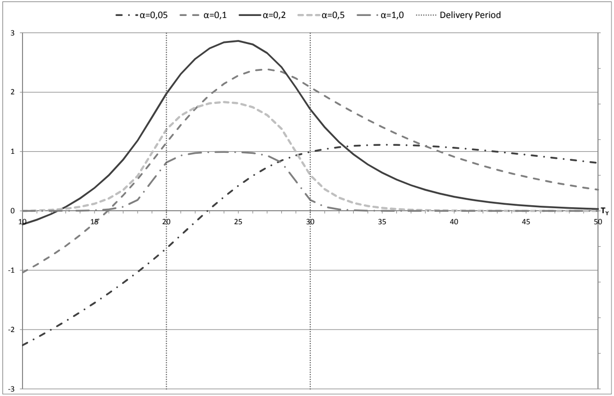

Figure : The information premium over T_Υ for different values of α. Other parameters are:
$t = 10$, $T_1 = 20$, $T_2 = 30$, $X_{10} = 10$, $\mathbb{E}[X_{T_\Upsilon} | \mathcal{G}_{10}] = 5$ and $\sigma = 3.0$

Stylised Example

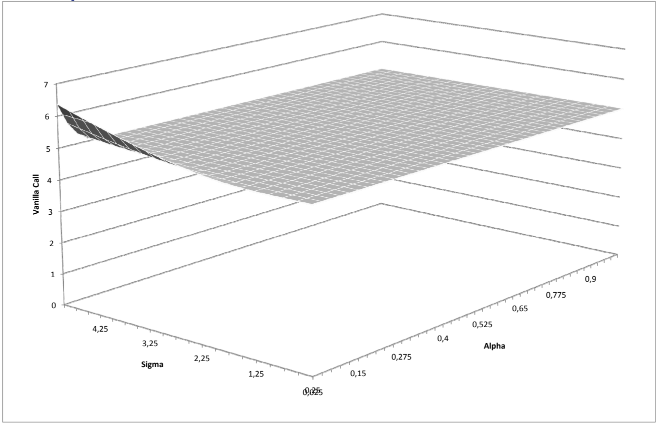

Figure : Vanilla Call Price under $\mathcal{F}$. For different α and σ. Other parameters are: $t = 10$, $T_1 = 20$, $T_2 = 30$, $X_{10} = 0$, $\mu = 30$ and $K = 25$.

Stylised Example

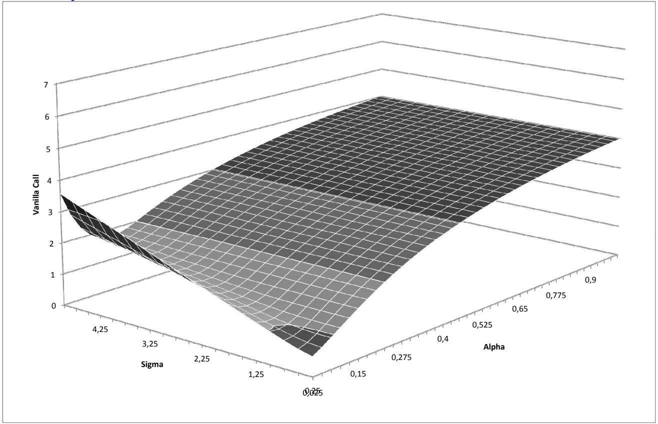

Figure : Vanilla Call Price under $\mathcal{G}$. For different α and σ. Other parameters are: $t = 10$, $T_1 = 20$, $T_2 = 30$, $X_{10} = 0, \mathbb{E}[X_{25}|\mathcal{G}_{10}] = -5, \mu = 30$ and $K = 25$.

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- Thank you for your attention...