Mean field games and technology switch modeling

Aimé Lachapelle

CEREMADE

Conférence FiME, 29 juin 2010

UNIVERSITE PARIS DAUPHINE

The player's individual problem

- Space of states: Ω, time period $[0, T]$, initial distribution of agents is given: m_0
- Controlled evolution of a player starting at x: $dX_t^x = \alpha_t dt + \sigma dW_t$
- α =control, W_t = standard Brownian motion

Individual problem of a player starting at x:

$$\inf_{\alpha} \mathbb{E}\left[\int_0^T L(X_t^x, \alpha_t) + V[m_t](X_t^x)dt + g[m_T](X_T^x)\right]$$

- Key Point: **the criteria depends on the mean field m_t, i.e. the distribution of agents at time t**
- From here one can get the MFG system (HJB and Fokker-Planck PDEs)

Optimization setting of MFG

- **Particular case:** $\exists \Phi, \Psi$ **s.t.** $V = \Phi'$ **and** $g = \Psi'$

Optimal control of Fokker-Planck:

$$\begin{cases} \inf_{\alpha} J(\alpha) := \int_0^T \Big(\int_\Omega L(x,\alpha) m(t,x) dx + \Phi(m_t) \Big) dt + \Psi(m_T) \\ \partial_t m - \frac{\sigma^2}{2} \Delta m + \mathrm{div}(\alpha m) = 0 \ , \ m(0,.) = m_0(.). \end{cases}$$

- The critical points verify the system (for $H = L^*$) :

(MFG) system

$$\partial_t m - \frac{\sigma^2}{2} \Delta m + \mathrm{div}(m \nabla_p H(., \nabla v)) = 0 \ , \ m(0,.) = m_0,$$

$$\partial_t v + \frac{\sigma^2}{2} \Delta v + H(x, \nabla v) = V(m) \ , \ v(T,.) = g(m_T).$$

- we consider a case where Φ is **concave (non-uniqueness)**

The model: agents & costs

with J. Salomon and G. Turinici (M3AS, 2010)

- **Stylized** model
- Arbitrage between insulation and heating. Any player has an insulation level $x \in [0,1]$ ($x = 0$: no insulation, $x = 1$: maximal insulation)
- **Controlled** process : $dX_t^x = \alpha_t dt + \sigma dW_t + dN_t(X_t)$, $\alpha \to$ insulation effort. Diffusion process with values in $[0,1]$.
- Insulation acquisition cost : $L(x,\alpha) = \frac{|\alpha|^2}{2}$
- Aggregate state cost (**concave with respect to m**) :

$$\Phi(m)(t) := \int_0^1 \left(p(t)(1 - 0,8x) + \frac{c_0 x}{c_1 + c_2 m(t,x)} \right) m(t,x)dx$$

The model: costs and global problem

- *Heating cost*: $p(t)(1-0,8x)$, where $p(t)$ is the unit price of energy
- *Insulation cost*: $\frac{c_0 x}{c_1+c_2 m(t,x)}$, increasing in x and decreasing in m: **scale effect and positive externalities**. The agents should do the same choice (attraction).

Minimization problem :

$$J(\alpha) := \int_0^T \left[\int_0^1 \frac{\alpha(t,x)^2}{2} m(t,x)dx + \Phi(m)(t)\right] dt$$

+ Fokker-Planck PDE with Neumann boundary conditions

Numerical method: discretization and monotonic algorithm

- **Monotonic algorithms** (have their roots in quantum control)
- $J(\alpha) := \int_0^T \left[\int_0^1 \frac{\alpha(t,x)^2}{2} m(t,x)dx + \Phi(m)(t)\right] dt$

Concavity inequality leads to:

$$J(\alpha') - J(\alpha) \leq \int_0^T \Delta(\alpha', \alpha; t, m', m) \cdot \Big(\alpha' - \alpha\Big) dt$$

- The equation $\Delta(\alpha', \alpha; t, m', m) = -\theta(\alpha' - \alpha)$ has a solution ($\Delta(.)$ is of course explicit). This strategy gives the **monotonicity**.
- Finite differences : **Godunov scheme to preserve** $m \geq 0$
- More details in my PhD dissertation

Test (1)

- Initial density is concentrated around 0.1 (agents consume energy)
- The unit price of energy $p(t)$ is **time-dependent**, it reaches a peak before decreasing to its low level

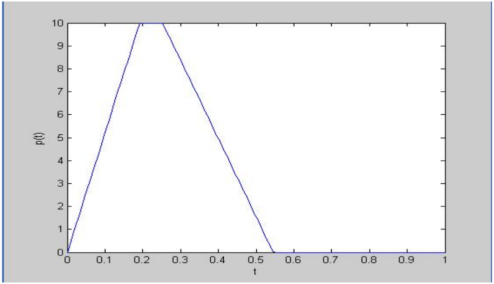
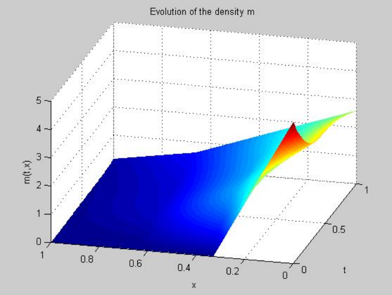
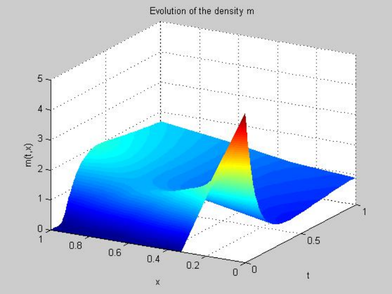


Figure: Question: In such a situation, can we find two MF equilibria, the first one being related to the expectation of a higher insulation level, the second to the expectation of heating ?

Test (2)



(a) solution with energy consumption (no switch)



(b) solution involving insulation (switch)

Test (3) : interpretations

- A possible technology switch/transition
- Olson's Paradox : *The logic of collective action, Harvard UP, 1971*
- Although self-interests could lead to a better (common interest, consensual) situation, they can carry on another equilibrium (switch or not to clean technologies, value of an equilibrium)
- Group behavior, **free rider** phenomenon
- Non-uniqueness: low cost **incentive policies**