Structural Model	Improved Structural Model	Hedging and Pricing	Numerics 000	Future work	Conclusion 0

A structural risk-neutral model for pricing and hedging electricity derivatives

#### Nicolas Langrené

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Joint work with René Aïd and Luciano Campi

Clamart, 6th Dec. 2011



Structural Model	Improved Structural Model	Hedging and Pricing	Numerics 000	Future work 00	Conclusion O
Motivatio	n				

#### Modelling power prices

• Joint modelling with fuel prices

### Applications

- Pricing and hedging power derivatives
- $\bullet$  Especially spread options  $\Longrightarrow$  valuation of power plants



A structural risk-neutral model for pricing and hedging electricity derivatives

Structural Model ●00000	Improved Structural Model	Hedging and Pricing	Numerics 000	Future work 00	Conclusion O
Previous n fuels	Structural Mod	el			

# Nariablesnfuels, $1 \le i \le n$ $D_t$ demand (in MW) $C_t^i$ capacities (in MW) $S_t^i$ fuel prices $h_i$ heat rates ( $h_i S_t^i$ in $\in$ /MWh, $\nearrow$ in i)

#### Electricity price (in €/MWh)

# $P_t = \sum_{i=1}^n h_i S_t^i \mathbf{1}_{\left\{\sum_{k=1}^{i-1} C_t^k \le D_t \le \sum_{k=1}^{i} C_t^k ight\}}$

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A structural risk-neutral model for pricing and hedging electricity derivatives

Structural Model ●00000	Improved Structural Model	Hedging and Pricing	Numerics 000	Future work 00	Conclusion O
Previous	Structural Mod	el			

Variables		
n	ו	fuels, $1 \le i \le n$
		demand (in MW)
		capacities (in MW)
		fuel prices
h		heat rates $(h_i S_t^i$ in $\in$ /MWh, $\nearrow$ in $i$ )



Structural Model ●00000	Improved Structural Model	Hedging and Pricing	Numerics 000	Future work 00	Conclusion O
Previous n fuels	Structural Mod	el			

Variables	
n	fuels, $1 \le i \le n$
$D_t$	demand (in MW)
	capacities (in MW)
	fuel prices
	heat rates $(h_i S_t^i \text{ in } \in /MWh, \nearrow \text{ in } i)$



Structural Model	Improved Structural Model	Hedging and Pricing	Numerics	Future work	Conclusion
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Structural Model 0●0000	Improved Structural Model	Hedging and Pricing	Numerics 000	Future work 00	Conclusion O
Pros and	cons				

# +

- Price formation mechanism
- Simple, interpretable
- Observable processes

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Structural Model 0●0000	Improved Structural Model	Hedging and Pricing	Numerics 000	Future work 00	Conclusion O
Pros and	cons				

# +

- Price formation mechanism
- Simple, interpretable
- Observable processes

#### • Marginal cost vs. Market price

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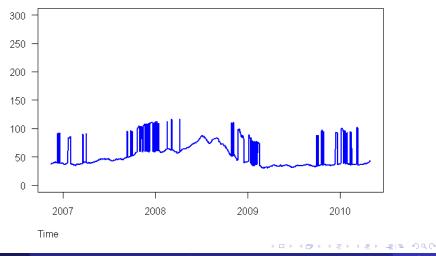
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Structural Model	Improved Structural Model	Hedging and Pricing	Numerics 000	Future work 00	Conclusion O
Illustratio Dataset	n (1/4)				

- French power market
- $\bullet$  Time period : 13  $^{\rm th}$  Nov. 2006 to 30  $^{\rm th}$  Apr. 2010 :  $\sim$  3.5 years
- $\bullet\,$  Focus on  $19^{\rm th}$  hour of each day
- 2 possible marginal fuels : coal or oil
- $\rm CO_2$  price included
- Average prices (in €/MWh) : Coal~47, Elec~74, Oil~102

Structural Model 000●00	Improved Structural Model	Hedging and Pricing	Numerics 000	Future work 00	Conclusion O
Illustratio Prices generate	n $(2/4)$ ed by the model				

#### Spot price (in €/MWh)



Structural Model 0000●0	Improved Structural Model	Hedging and Pricing	Numerics 000	Future work 00	Conclusion O
Illustratio Comparison to	n (3/4) realized prices				

#### 300 Historical Data Model 250 -200 -150 \_ 100 50 0 2007 2008 2009 2010 Time

#### Spot price (in €/MWh)

-= 1= 990

Structural Model 00000●	Improved Structural Model	Hedging and Pricing	Numerics 000	Future work 00	Conclusion O
Illustratio Comparison to	n (4/4) realized prices				

# 2000 -Historical Data Model 1500 -1000 -500 AMULA 0 2007 2008 2009 2010 Time

#### Spot price (in €/MWh)

Structural Model	Improved Structural Model ●0000	Hedging and Pricing	Numerics 000	Future work 00	Conclusion O
Price spik	ies				

• Marginal cost  $MC_t := \sum_{i=1}^n h_i S_t^i \mathbf{1}_{\left\{\sum_{k=1}^{i-1} C_t^k \le D_t \le \sum_{k=1}^{i} C_t^k\right\}}$ 

Structural Model	Improved Structural Model ●0000	Hedging and Pricing	Numerics 000	Future work 00	Conclusion O
Price spik	kes				

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$$MC_t := \sum_{i=1}^n h_i S_t^i \mathbf{1}_{\left\{\sum_{k=1}^{i-1} C_t^k \le D_t \le \sum_{k=1}^i C_t^k 
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• Idea : price spikes are more likely when the residual capacity  $RC_t := \sum_{k=1}^n C_t^k - D_t$  is small

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Structural Model	Improved Structural Model ●0000	Hedging and Pricing	Numerics 000	Future work 00	Conclusion O
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• Idea : price spikes are more likely when the residual capacity  $RC_t := \sum_{k=1}^n C_t^k - D_t$  is small

$$\hookrightarrow$$
 Write  $\frac{P_t}{MC_t}$  as a function of  $RC_t$ 

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Structural Model	Improved Structural Model	Hedging and Pricing	Numerics 000	Future work 00	Conclusion O
New spot	model				

Estimated relation : 
$$\frac{P_t}{MC_t} = \frac{\gamma}{\left(RC_t\right)^{
u}}$$
 ,  $\gamma = 6.2 \pm 0.06$  ,  $u = 1.0 \pm 0.01$ 

Model
$$P_t = g\left(\sum_{k=1}^n C_t^k - D_t\right) \times \left(\sum_{i=1}^n h_i S_t^i \mathbf{1}_{\left\{\sum_{k=1}^{i-1} C_t^k \le D_t \le \sum_{k=1}^i C_t^k\right\}}\right)$$

Scarcity function

$$g(x) := \min\left(\frac{\gamma}{x^{\nu}}, M\right) \mathbf{1}_{\{x \ge 0\}} + M \mathbf{1}_{\{x \le 0\}}$$

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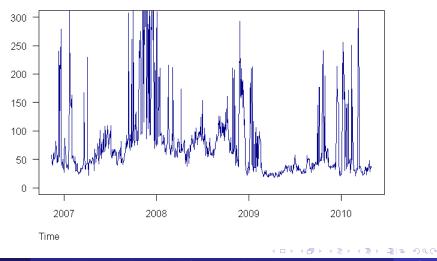
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$$g(x) := \min\left(\frac{\gamma}{x^{\nu}}, M\right) \mathbf{1}_{\{x > 0\}} + M \mathbf{1}_{\{x \le 0\}}$$

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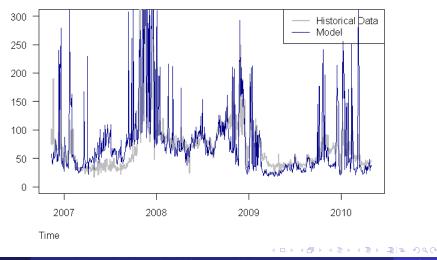
Structural Model	Improved Structural Model	Hedging and Pricing	Numerics 000	Future work 00	Conclusion O
Illustratio Prices generate	N ed by the model				

Spot price (in €/MWh)

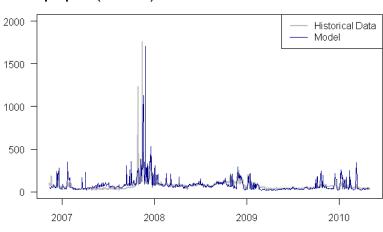


Structural Model	Improved Structural Model	Hedging and Pricing	Numerics 000	Future work 00	Conclusion O
Illustratio Comparison to	N realized prices				

Spot price (in €/MWh)



Structural Model 000000	Improved Structural Model	Hedging and Pricing	Numerics 000	Future work 00	Conclusion O
Illustratio Comparison to	N realized prices				



#### Spot price (in €/MWh)

Time

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Structural Model	Improved Structural Model	Hedging and Pricing ●00000	Numerics 000	Future work 00	Conclusion O
Hedging	criterion				

#### Incomplete market

- Choice of a hedging criterion
- Ex : super-replication, utility indifference, mean-variance,...
- Our choice : Local Risk Minimization

#### Local Risk Minimization

- Explicit formulae
- Split contingent claims between the **hedgeable part** (fuels) and the **non-hedgeable part** (demand, capacities)
- $\bullet$  Pricing : expectated discounted payoff under  $\widehat{\mathbb{Q}}$

Structural Model	Improved Structural Model	Hedging and Pricing ●00000	Numerics 000	Future work 00	Conclusion O
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Structural Model	Improved Structural Model	Hedging and Pricing	Numerics	Future work	Conclusion
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Futures <sub>Example</sub>					

# Pricing

$$F_{t}^{e}(T) = \sum_{i=1}^{n} h_{i}G_{i}^{T}(t, C_{t}, D_{t}) F_{t}^{i}(T)$$

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Structural Model	Improved Structural Model	Hedging and Pricing	Numerics	Future work	Conclusion
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Futures <sub>Example</sub>					

# Pricing

$$F_{t}^{e}(T) = \sum_{i=1}^{n} h_{i}G_{i}^{T}(t, C_{t}, D_{t}) F_{t}^{i}(T)$$

$$G_i^{T}(t,C_t,D_t) = \mathbb{E}_t \left[ g \left( \sum_{k=1}^n C_T^k - D_T \right) \mathbf{1}_{\left\{ \sum_{k=1}^{i-1} C_T^k \le D_T \le \sum_{k=1}^i C_T^k \right\}} \right]$$

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Structural Model	Improved Structural Model	Hedging and Pricing 00●000	Numerics 000	Future work 00	Conclusion O
Modelling	the inputs				

# $D_t, C_t^i, i=1\ldots n$

## deterministic function + Ornstein-Uhlenbeck

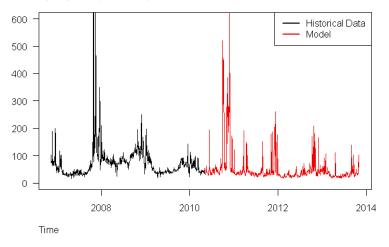
# $S_t^i$ , $i = 1 \dots n$

spread 
$$Y_t^i := h_i S_t^i - h_{i-1} S_t^{i-1}$$
  
 $Y_t^i$  : geometric Brownian motion

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Structural Model	Improved Structural Model	Hedging and Pricing	Numerics 000	Future work 00	Conclusion O
Spot Traj	ectories $(1/3)$				

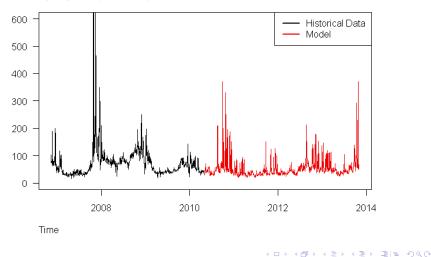
Spot price (in €/MWh)



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Structural Model	Improved Structural Model	Hedging and Pricing 0000●0	Numerics 000	Future work 00	Conclusion O
Spot Traj	ectories (2/3)				

Spot price (in €/MWh)

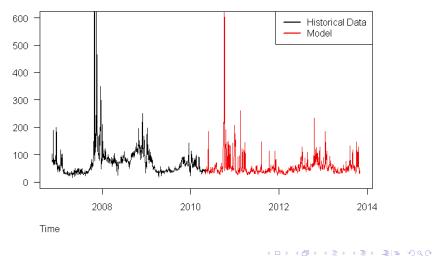


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Structural Model	Improved Structural Model	Hedging and Pricing	Numerics	Future work	Conclusion
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Spot Traj	ectories (3/3)				

#### Spot price (in €/MWh)



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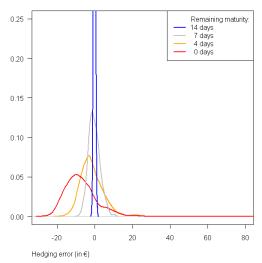
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Structural Model 000000	Improved Structural Model	Hedging and Pricing	Numerics ●00	Future work 00	Conclusion 0
	merical test of Power Futures (1/2	?)			

- Hedging a (fictitious) power futures with a delivery period of 1 hour
- Using a (daily rebalanced) portfolio of futures on fuels

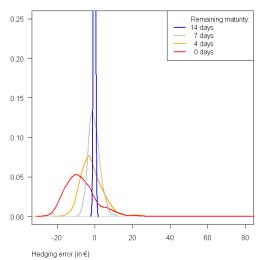
Structural Model	Improved Structural Model	Hedging and Pricing	Numerics 0●0	Future work 00	Conclusion O
Result Partial hedging	g of Power Futures (2/2	2)			

#### Distribution of hedging error: Time evolution



Structural Model 000000	Improved Structural Model	Hedging and Pricing	Numerics ○●○	Future work 00	Conclusion O
Result Partial hedging	g of Power Futures (2/2	2)			

#### Distribution of hedging error: Time evolution



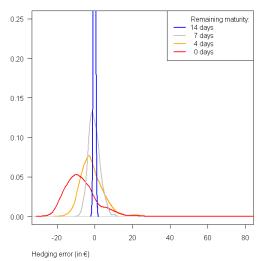
• Far from maturity : perfect hedging, basket of fuels

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Structural Model 000000	Improved Structural Model	Hedging and Pricing	Numerics ○●○	Future work 00	Conclusion O
Result Partial hedgin	g of Power Futures (2/2	2)			

Distribution of hedging error: Time evolution



• Far from maturity : perfect hedging, basket of fuels

• Close to maturity : inefficient partial hedging

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Structural Model	Improved Structural Model	Hedging and Pricing	Numerics	Future work 00	k Conclusion O
Other exa	amples				

- Spread options, options on futures,...
- Semi-explicit pricing : numerical integration
- Partial hedging by means of fuel futures and power futures

Structural Model	Improved Structural Model	Hedging and Pricing	Numerics 000	Future work 00	Conclusion O	
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Structural Model	Improved Structural Model	Hedging and Pricing	Numerics 000	Future work 00	Conclusion O
Other exa	amples				

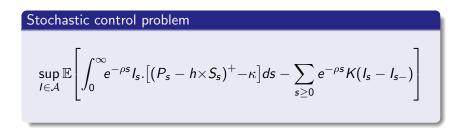
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Structural Model	Improved Structural Model	Hedging and Pricing	Numerics 000	Future work ●0	Conclusion O
Investmer	nts in power pla	nts			

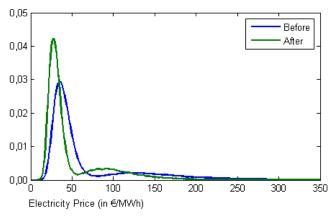
#### Spot model $\Longrightarrow$ Spread options $\Longrightarrow$ Power plants $\Longrightarrow$ Investments



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Structural Model 000000	Improved Structural Model	Hedging and Pricing	Numerics 000	Future work ○●	Conclusion O
Feedback	effect				

Before : price density at time T = 1 year After : add 2GW of new peak-load assets



Price densities

-

Structural Model 000000	Improved Structural Model	Hedging and Pricing	Numerics 000	Future work 00	Conclusion •
Conclusio	n				

#### • New spot model : marginal cost + scarcity effect

- Derivatives pricing : futures, spread options, options on futures,...
- Partial hedging using fuel futures and power futures
- Still extensions and improvements
- But interesting preliminary results

Structural Model Improved Structural Model		Hedging and Pricing	Numerics 000	Future work 00	Conclusion •	
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Structural Model Improved Structural Model		Hedging and Pricing	Numerics 000	Future work 00	Conclusion •
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Structural Model Improved Structural Model		Hedging and Pricing	Numerics 000	Future work 00	Conclusion •
Conclusio	n				

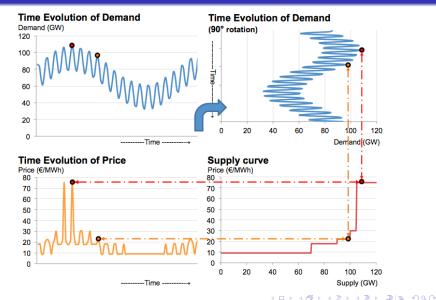
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Qu	estio	ns						

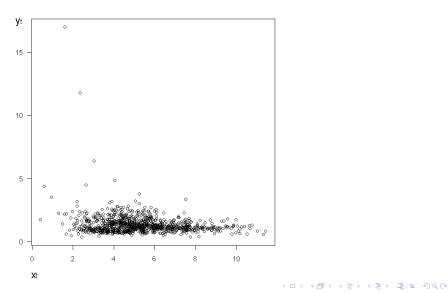




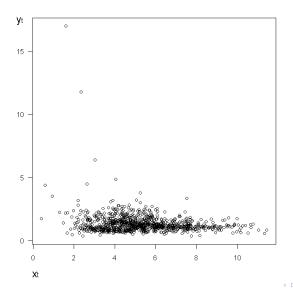
# Time evolution of marginal costs



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Est	timati	ion $(1/8)$	)					





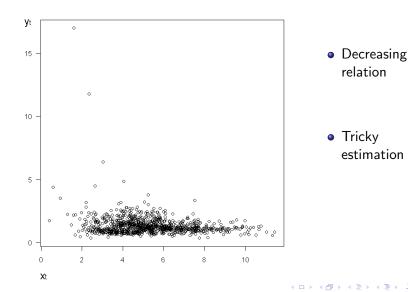




A structural risk-neutral model for pricing and hedging electricity derivatives

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	MC o	Scarcity 0●000000	Comparison 00	Inputs 000	Hedging 000	Spread 00	Premium 00	
Est	timati	ion (2/8)	)					

#### Quantiles

$$\mathbb{P}\left(X\leqslant q_X(p)
ight)=p$$
  
 $\mathbb{P}\left(Y\leqslant q_Y(p)
ight)=p$ 

	MC o	Scarcity 0●000000	Comparison 00	Inputs 000	Hedging 000	Spread 00	Premium 00	
Est	timati	ion (2/8)	)					

#### Quantiles

$$\mathbb{P}\left(X\leqslant q_X(p)
ight)=p$$
  
 $\mathbb{P}\left(Y\leqslant q_Y(p)
ight)=p$ 

#### Idea

If 
$$Y = h(X)$$
,  $h \searrow$   
Then  $q_Y(1-p) = h(q_X(p))$ ,  $0 \le p \le 1$ 

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	MC o	Scarcity 0●000000	Comparison 00	Inputs 000	Hedging 000	Spread 00	Premium 00	
Est	timati	ion (2/8)	)					

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$$\mathbb{P}\left(X\leqslant q_X(p)
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ight)=p$ 

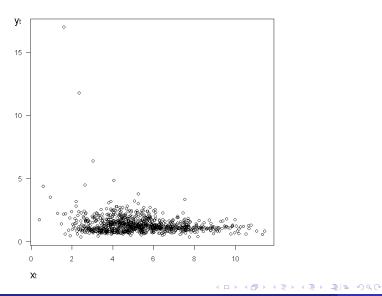
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#### $\Rightarrow$ Estimation on the quantiles

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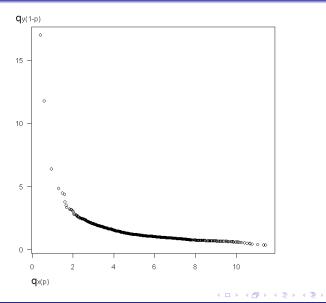
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Es	timati	on (3/8	)					



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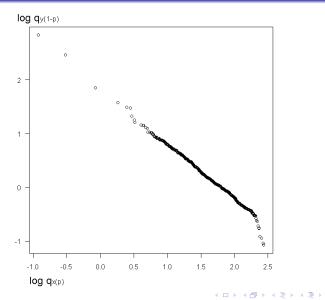


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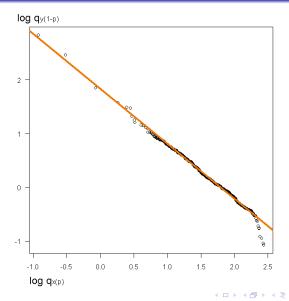
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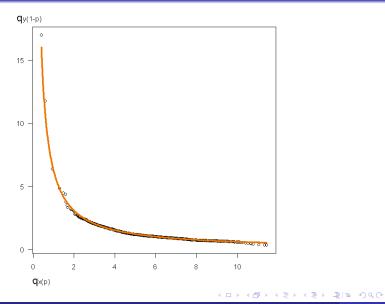


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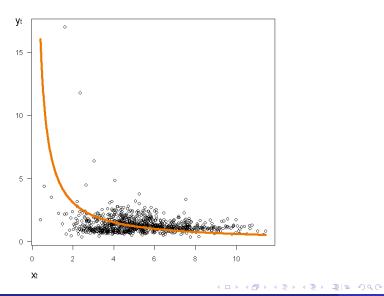
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Es	timati	ion (7/8)	)					



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Es	timati	ion (8/8)	)					

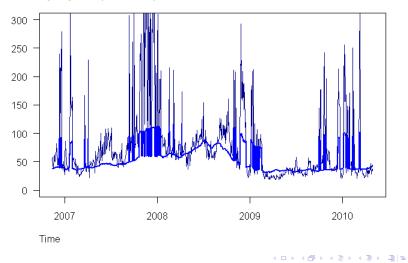


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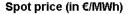
A structural risk-neutral model for pricing and hedging electricity derivatives

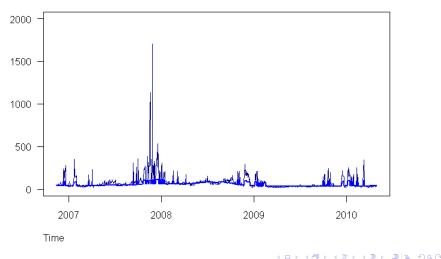


Spot price (in €/MWh)









	MC o	Scarcity 00000000	Comparison 00	Inputs ●00	Hedging 000	Spread 00	Premium 00	
$C_t$	and	$D_t$						

#### Building blocks

$$D_{t} = f_{D}(t) + Z_{D}(t)$$
$$C_{t}^{i} = f_{i}(t) + Z_{i}(t)$$

#### Stochastic part

$$dZ_{D}(t) = -\alpha_{D}Z_{D}(t) dt + bdW_{t}^{D}$$
$$dZ_{i}(t) = -\alpha_{i}Z_{i}(t) dt + \beta_{i}dW_{t}^{i}$$

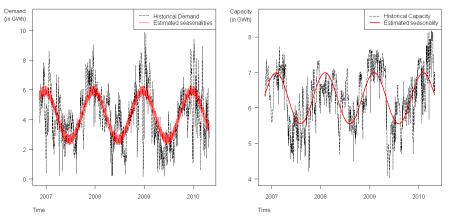
#### Deterministic part

$$f_{D}(t) = d_{1} + d_{2} \cos\left(2\pi \frac{t - d_{3}}{l_{1}}\right) + d_{4} \cos\left(2\pi \frac{t - d_{5}}{l_{2}}\right)$$
$$f_{i}(t) = c_{1}^{i} + c_{2}^{i} \cos\left(2\pi \frac{t - c_{3}^{i}}{l_{1}}\right) + f_{i}^{evo}(t)$$

		Scarcity 00000000	Comparison 00	Inputs 0●0	Hedging 000	Spread 00	Premium 00	
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#### Demand seasonalities

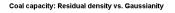




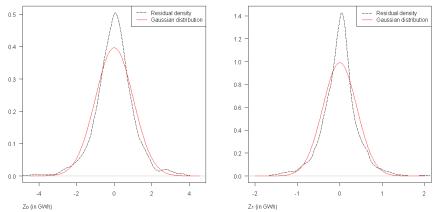
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Demand: Residual density vs. Gaussianity



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	MC o	Scarcity 00000000	Comparison 00	Inputs 000	Hedging ●00	Spread 00	Premium 00	
He	dging	strateg	y					

Payoff H, price 
$$V_t^H := \widehat{\mathbb{E}}[H|\mathcal{F}_t] = \phi(t, \mathcal{F}_t(T^*), \mathcal{C}_t, D_t)$$

# Hedging strategy

$$\begin{aligned} \xi_t^e &= \frac{1}{||\theta_t^C, \theta_t^D||^2} \left\{ \sum_{i=1}^n \theta_t^{C,i} \frac{\partial \phi}{\partial c_i} \beta_i + \theta_t^D \frac{\partial \phi}{\partial z} b \right\} \\ \xi_t^i &= \frac{\partial \phi}{\partial y_i} + \frac{h_i G_i^{T^*}(t, C_t, D_t)}{||\theta_t^C, \theta_t^D||^2} \left\{ \sum_{i=1}^n \theta_t^{C,i} \frac{\partial \phi}{\partial c_i} \beta_i + \theta_t^D \frac{\partial \phi}{\partial z} \right\} \end{aligned}$$

where 
$$dF_t^e := \theta_t^S . d\widehat{W}_t + \theta_t^C . dW_t^C + \theta_t^D dW_t^D$$

	MC	Scarcity	Comparison	Inputs	Hedging	Spread	Premium	Extension
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$$dF_t^e = \theta_t^S \cdot d\widehat{W}_t + \theta_t^C \cdot dW_t^C + \theta_t^D dW_t^D$$
  
=  $e^{r(T^*-t)} \sum_{i=1}^n \left( \sum_{k=i}^n G_k^{T^*}(t, C_t, D_t) \right) \sigma_i Y_t^i d\widehat{W}_t^i$   
+  $\sum_{i=1}^n h_i F_t^i(T^*) \frac{\partial G_i^{T^*}}{\partial d}(t, C_t, D_t) b(t, D_t) dW_t^D$   
+  $\sum_{i=1}^n h_i F_t^i(T^*) \sum_{k=1}^n \frac{\partial G_i^{T^*}}{\partial c_k}(t, C_t, D_t) \beta_k(t, C_t^k) dW_t^{C,k}$ 

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A structural risk-neutral model for pricing and hedging electricity derivatives

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• Explicit, as a function of the *extended incomplete Goodwin-Staton integral* :

$$\widetilde{\mathcal{G}}(x,y;\nu) = \int_{x}^{\infty} \frac{1}{\left(y+z\right)^{\nu}} e^{-z^{2}} dz$$

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$$\widetilde{\mathcal{G}}(x,y;\nu) = \int_{x}^{\infty} \frac{1}{(y+z)^{\nu}} e^{-z^{2}} dz$$

• Numerically :

$$\widetilde{\mathcal{G}}(x,y;\nu) = \frac{1}{2}e^{-y^2}\sum_{n=0}^{\infty}\Gamma\left(\frac{1-\nu}{2} + \frac{n}{2},(x+y)^2\right)\frac{(2y)^n}{n!}$$



• Explicit, as a function of the *extended incomplete Goodwin-Staton integral* :

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 $\Gamma(\alpha, x) = \text{incomplete Gamma function} = \int_x^\infty t^{\alpha-1} e^{-t} dt$ 

	MC o	Scarcity 00000000	Comparison 00	Inputs 000	Hedging 000	Spread ●0	Premium 00	
Spi	read o	option						

Payoff 
$$H = (P_T - h_1 S_T^1 - K)^+$$

## Pricing

$$\pi_{0} = \int_{\mathbb{R}^{2}} f_{C_{T}^{1} - D_{T}}(z) f_{C_{T}^{2}}(c) \left\{ \phi_{1}(c, z) \mathbf{1}_{\{z > 0\}} + \phi_{2}(c, z) \mathbf{1}_{\{z \le 0\}} \right\} dcdz,$$

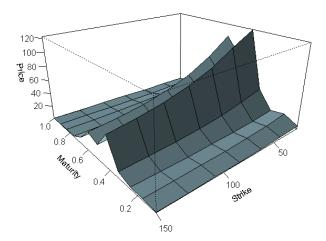
$$\begin{split} \phi_1 &= (g-1)BS_0\left(\sigma_1, \frac{K}{g-1}\right) \mathbf{1}_{\{g>1\}} \qquad g = g\left(c+z\right) \\ \phi_2 &= g\int_0^\infty \hat{f}_{Y_T^1}(y)BS_0\left(\sigma_2, \frac{K+(1-g)y}{g}\right) \left(\mathbf{1}_{\{g\leq1\}} + \mathbf{1}_{\{g>1\}}\mathbf{1}_{\{y<\frac{K}{g-1}\}}\right) dy \\ &+ \left(gY_0^2 \mathcal{N}\left(\frac{\left(r - \frac{\sigma_1^2}{2}\right)T - \ln\left(\frac{K}{(g-1)Y_0^1}\right)}{\sigma_1\sqrt{T}}\right) + (g-1)BS_0\left(\sigma_1, \frac{K}{g-1}\right)\right) \mathbf{1}_{\{g>1\}} \end{split}$$

A structural risk-neutral model for pricing and hedging electricity derivatives

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	MC	Scarcity	Comparison	Inputs	Hedging	Spread	Premium	Extension
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Spread option Numerical example								



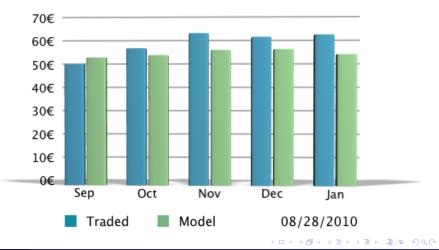
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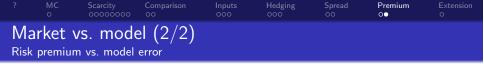
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		Scarcity 00000000	Comparison 00	Inputs 000	Hedging 000	Spread 00	Premium ●0	
Market vs. model $(1/2)$ Risk premium vs. model error								

### **Power Futures**



A structural risk-neutral model for pricing and hedging electricity derivatives



# Implied Load Spread 1.0 GW 0.5 GW 0.0 GW -0.5 GW

Oct

#### 08/28/2010

Sep

Nov

Dec

Jan

	MC o	Scarcity 00000000	Comparison	Inputs 000	Hedging 000	Spread 00	Premium 00	Extension ●
Extensions								

#### Done

- Extend the numerical application to every single hour of the day (3 marginal fuels)
- Extension to payoffs with delivery periods  $[T_1, T_2]$
- Compute the residual market price of risk

#### To do

- Better models for fuels (spot and futures) : convenience yield, interest rates, cointegration...
- Better volatility models
- Compare different hedging criteria
- Application to investment opportunities