A structural risk-neutral model for pricing and hedging electricity derivatives

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Joint work with **René Aïd** and **Luciano Campi**

Clamart, 6th Dec. 2011

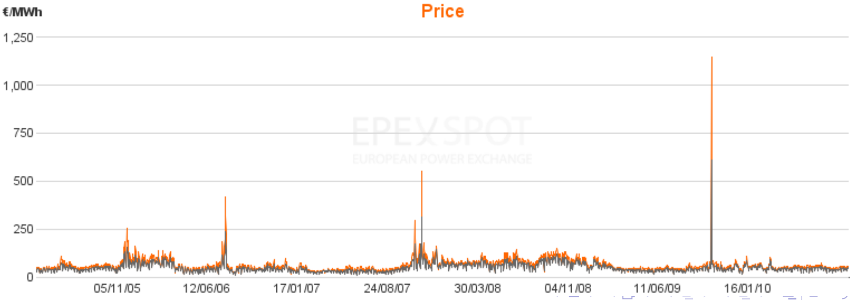
Motivation

Modelling power prices

- Joint modelling with fuel prices

Applications

- Pricing and hedging power derivatives
- Especially spread options $\Longrightarrow$ valuation of power plants



Previous Structural Model
n fuels

Variables

n	fuels, $1 \leq i \leq n$
D_t	demand (in MW)
C_t^i	capacities (in MW)
S_t^i	fuel prices
h_i	heat rates ($h_i S_t^i$ in €/MWh, $\nearrow$ in i)

Electricity price (in €/MWh)

$$P_t = \sum_{i=1}^{n} h_i S_t^i \mathbf{1}_{\left\{\sum_{k=1}^{i-1} C_t^k \leq D_t \leq \sum_{k=1}^{i} C_t^k\right\}}$$

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Pros and cons

+

- Price formation mechanism
- Simple, interpretable
- Observable processes

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- Marginal cost vs. Market price

Illustration (1/4)

Dataset

- French power market
- Time period : 13^{th} Nov. 2006 to 30^{th} Apr. 2010 : $\sim$ 3.5 years
- Focus on 19^{th} hour of each day
- 2 possible marginal fuels : coal or oil
- CO_2 price included
- Average prices (in €/MWh) : Coal$\sim$47, Elec$\sim$74, Oil$\sim$102

Illustration (2/4)

Prices generated by the model

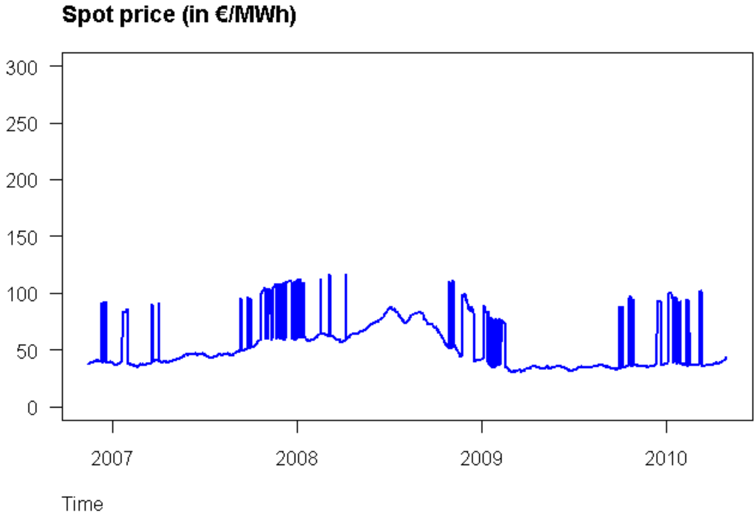
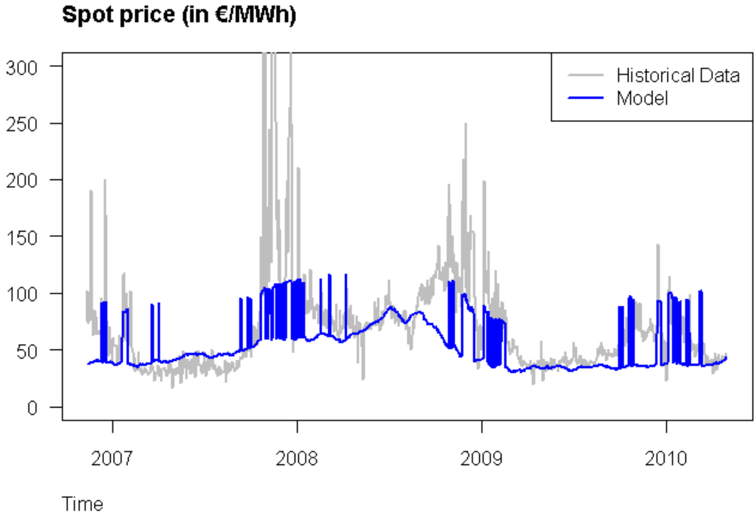


Illustration (3/4)

Comparison to realized prices



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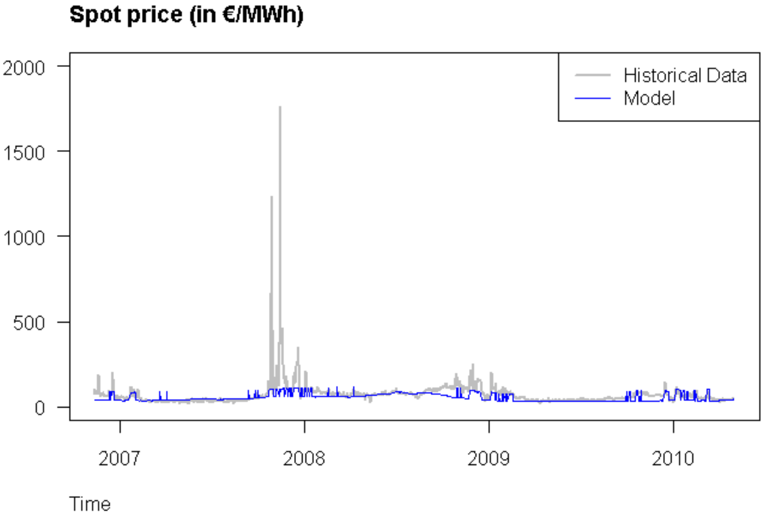

Price spikes

- Marginal cost $MC_t := \sum_{i=1}^{n} h_i S_t^i \mathbf{1}_{\{\sum_{k=1}^{i-1} C_t^k \leq D_t \leq \sum_{k=1}^{i} C_t^k\}}$

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- Idea : price spikes are more likely when the residual capacity $RC_t := \sum_{k=1}^{n} C_t^k - D_t$ is small

$\hookrightarrow$ **Write $\frac{P_t}{MC_t}$ as a function of RC_t**

New spot model

Estimated relation : $\frac{P_t}{MC_t} = \frac{\gamma}{(RC_t)^\nu}$, $\gamma = 6.2 \pm 0.06$, $\nu = 1.0 \pm 0.01$

Model

$$P_t = g\left(\sum_{k=1}^{n} C_t^k - D_t\right) \times \left(\sum_{i=1}^{n} h_i S_t^i \mathbf{1}_{\left\{\sum_{k=1}^{i-1} C_t^k \leq D_t \leq \sum_{k=1}^{i} C_t^k\right\}}\right)$$

Scarcity function

$$g(x) := \min\left(\frac{\gamma}{x^\nu}, M\right) \mathbf{1}_{\{x>0\}} + M \mathbf{1}_{\{x \leqslant 0\}}$$

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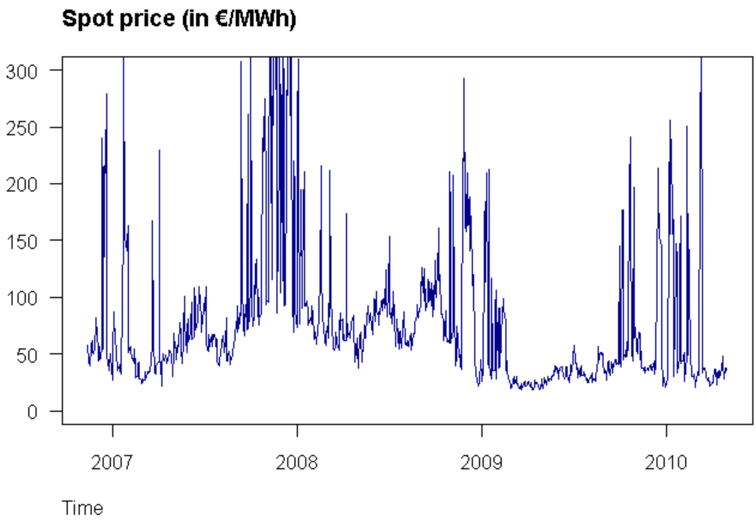

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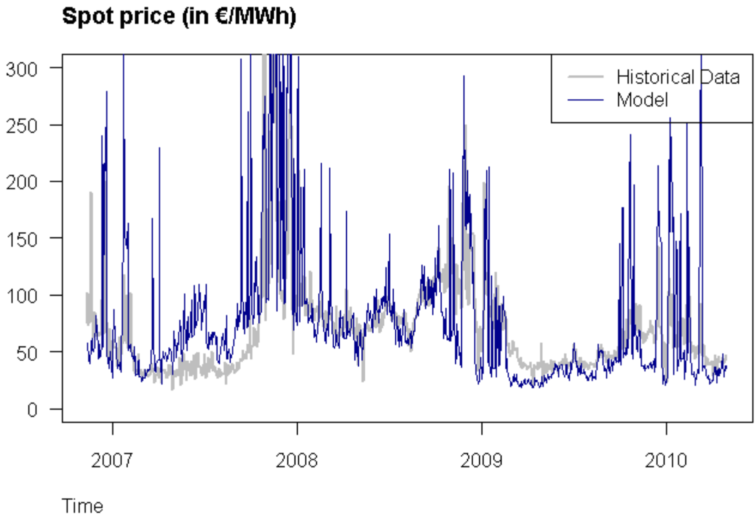

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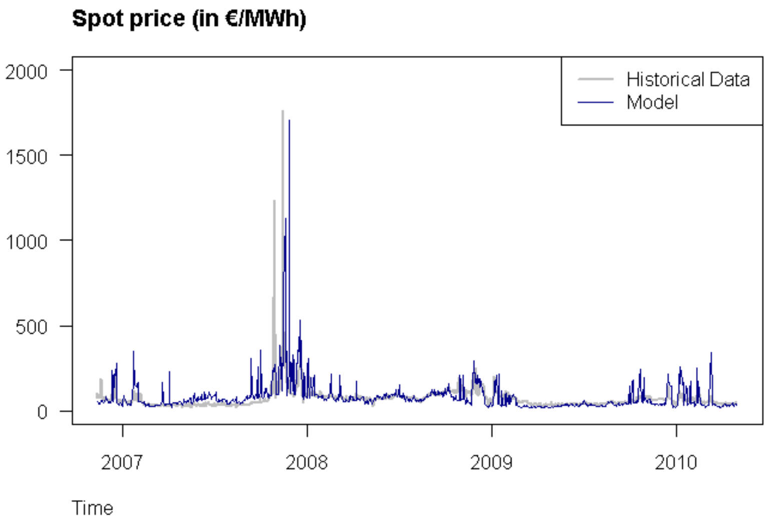

Hedging criterion

Incomplete market

- Choice of a hedging criterion
- Ex : super-replication, utility indifference, mean-variance,. . .
- Our choice : Local Risk Minimization

Local Risk Minimization

- Explicit formulae
- Split contingent claims between the **hedgeable part** (fuels) and the **non-hedgeable part** (demand, capacities)
- Pricing : expectated discounted payoff under $\widehat{\mathbb{Q}}$

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Futures

Example

Pricing

$$F_t^e(T) = \sum_{i=1}^{n} h_i G_i^T(t, C_t, D_t) F_t^i(T)$$

Futures
Example

Pricing

$$F_t^e(T) = \sum_{i=1}^{n} h_i G_i^T(t, C_t, D_t)\, F_t^i(T)$$

$$G_i^T(t, C_t, D_t) = \mathbb{E}_t\left[g\left(\sum_{k=1}^{n} C_T^k - D_T \right) \mathbf{1}_{\left\{\sum_{k=1}^{i-1} C_T^k \le D_T \le \sum_{k=1}^{i} C_T^k\right\}} \right]$$

Modelling the inputs

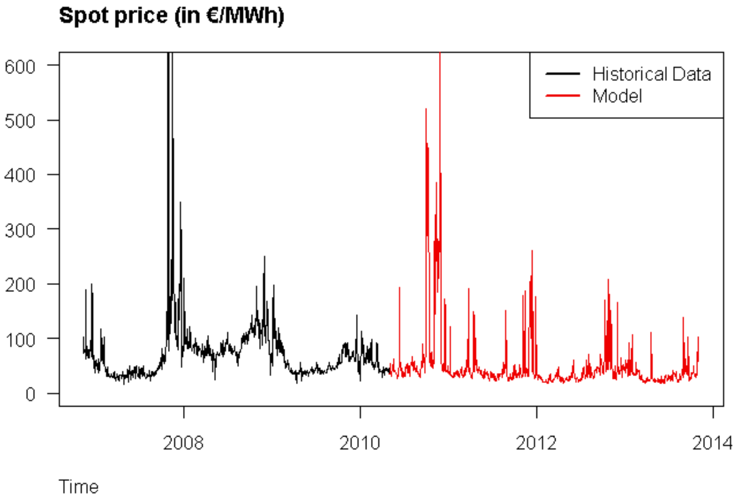
D_t, C_t^i, $i = 1 \dots n$

deterministic function + Ornstein-Uhlenbeck

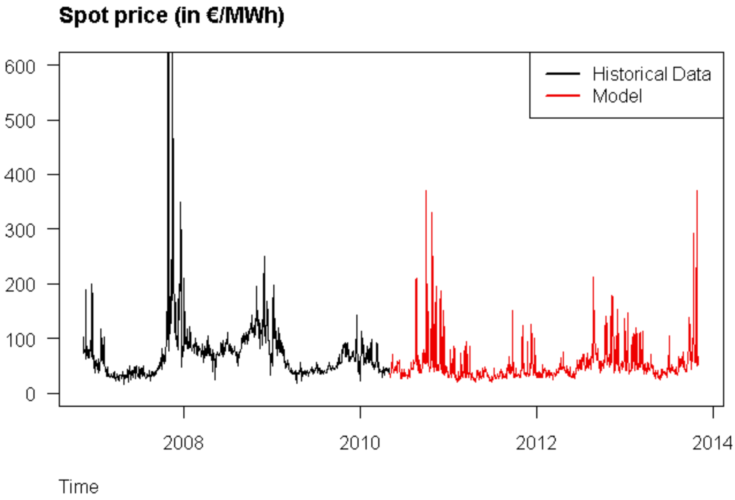
S_t^i, $i = 1 \dots n$

spread $Y_t^i := h_i S_t^i - h_{i-1} S_t^{i-1}$
Y_t^i : geometric Brownian motion

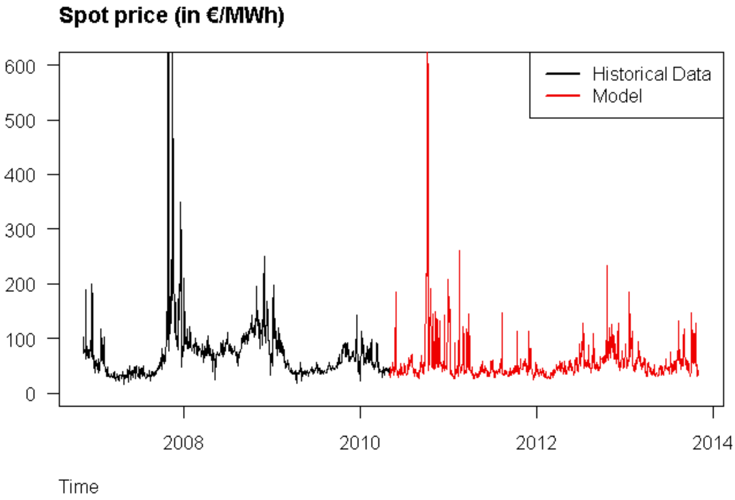
Spot Trajectories (1/3)



Spot Trajectories (2/3)



Spot Trajectories (3/3)



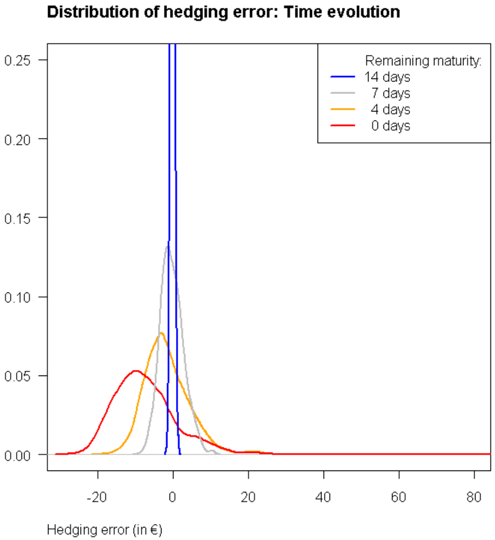
Simple numerical test

Partial hedging of Power Futures (1/2)

- Hedging a (fictitious) power futures with a delivery period of 1 hour
- Using a (daily rebalanced) portfolio of futures on fuels

Result

Partial hedging of Power Futures (2/2)



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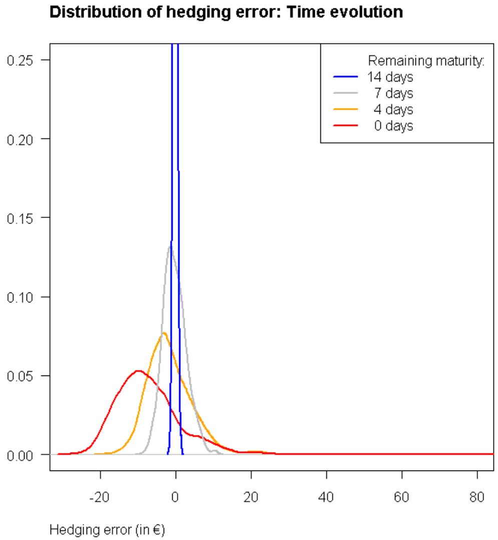

- Far from maturity : perfect hedging, basket of fuels

Result

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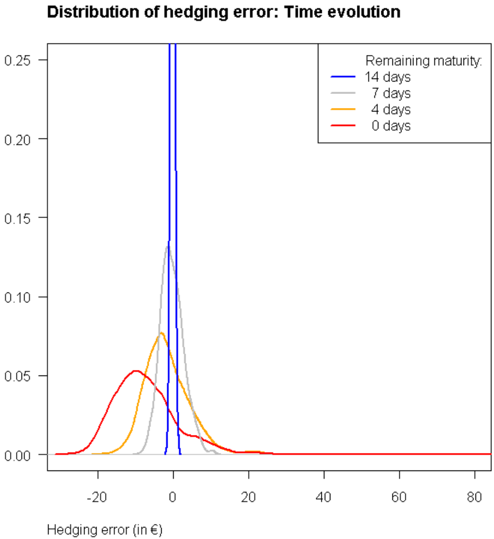

- Far from maturity : perfect hedging, basket of fuels
- Close to maturity : inefficient partial hedging

Other examples

- Spread options, options on futures,. . .
- Semi-explicit pricing : numerical integration
- Partial hedging by means of fuel futures and power futures

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Investments in power plants

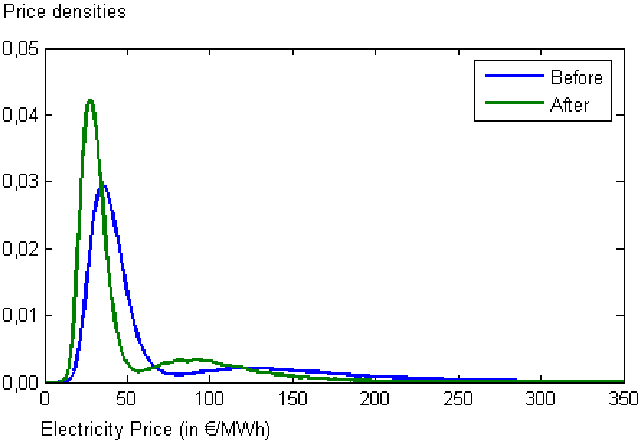
Spot model $\Longrightarrow$ Spread options $\Longrightarrow$ Power plants $\Longrightarrow$ Investments

Stochastic control problem

$$\sup_{I\in\mathcal{A}} \mathbb{E}\left[\int_0^\infty e^{-\rho s} I_s.\left[(P_s - h\times S_s)^+ - \kappa\right] ds - \sum_{s\geq 0} e^{-\rho s} K(I_s - I_{s-})\right]$$

Feedback effect

Before : price density at time $T = 1$ year
After : add 2GW of new peak-load assets



Conclusion

- New spot model : marginal cost + scarcity effect
- Derivatives pricing : futures, spread options, options on futures,. . .
- Partial hedging using fuel futures and power futures

- Still extensions and improvements
- But interesting preliminary results

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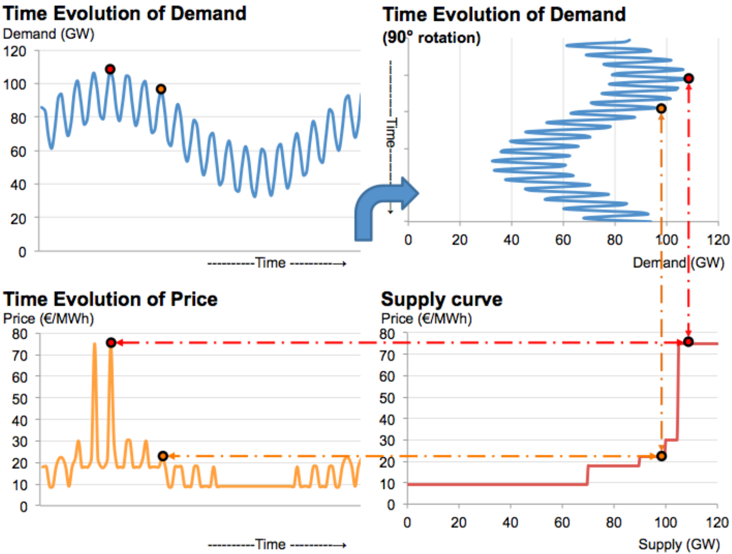
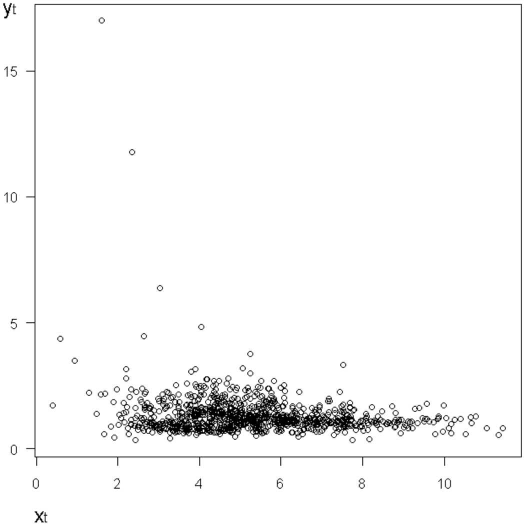
Questions

Time evolution of marginal costs



Estimation (1/8)



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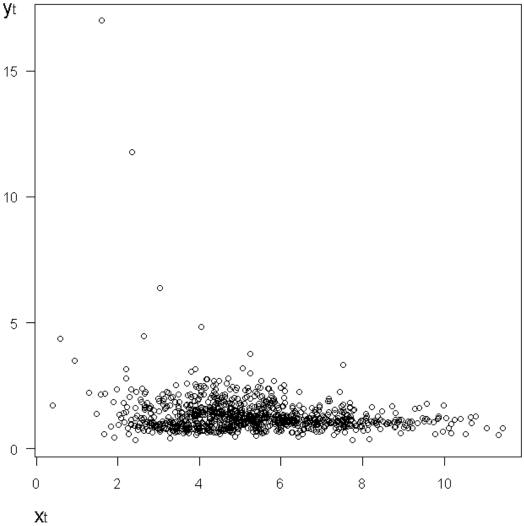

- Decreasing relation

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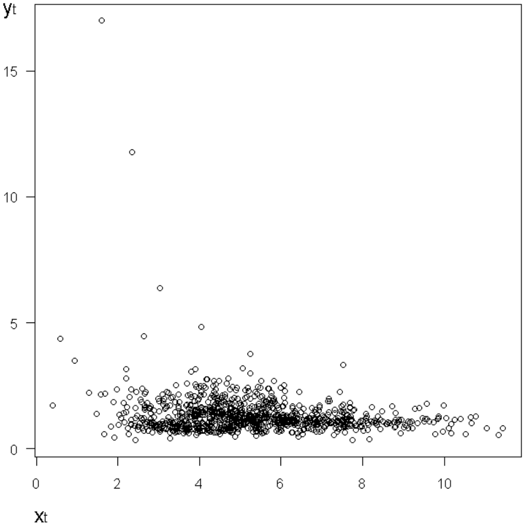

- Decreasing relation
- Tricky estimation

Estimation (2/8)

Quantiles

$$\mathbb{P}\left(X \leqslant q_X(p)\right) = p$$
$$\mathbb{P}\left(Y \leqslant q_Y(p)\right) = p$$

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Idea

If $Y = h(X)$, $h \searrow$

Then $q_Y(1-p) = h(q_X(p))$, $0 \leqslant p \leqslant 1$

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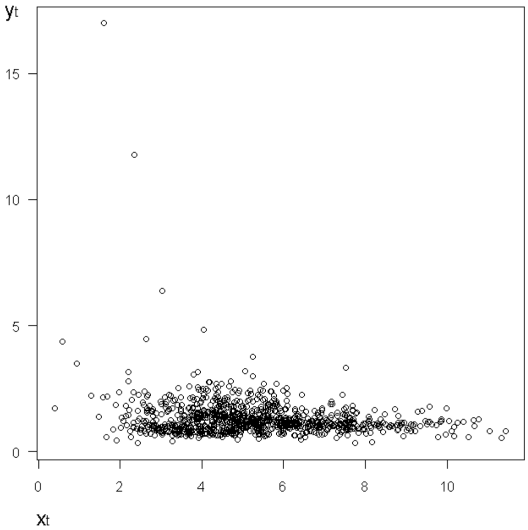
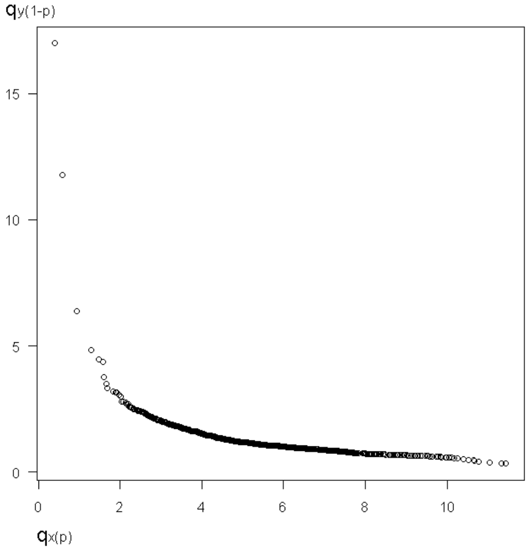
$\Rightarrow$ Estimation on the quantiles

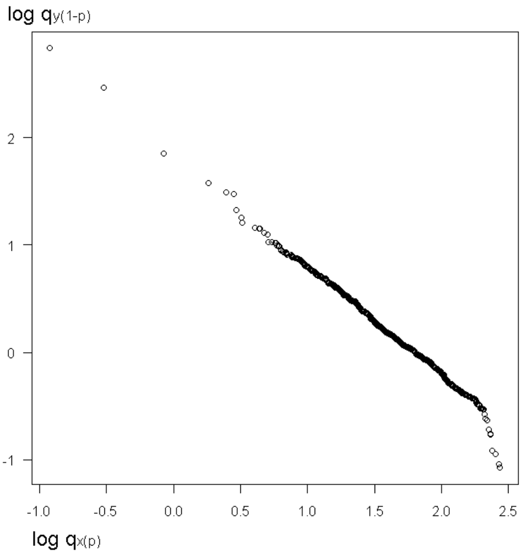
Estimation (3/8)



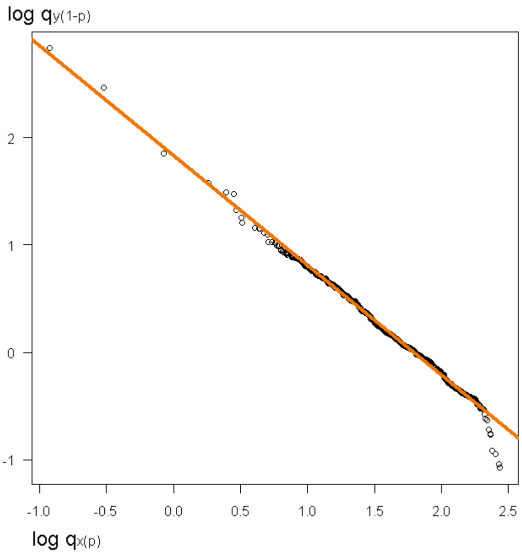
Estimation (4/8)



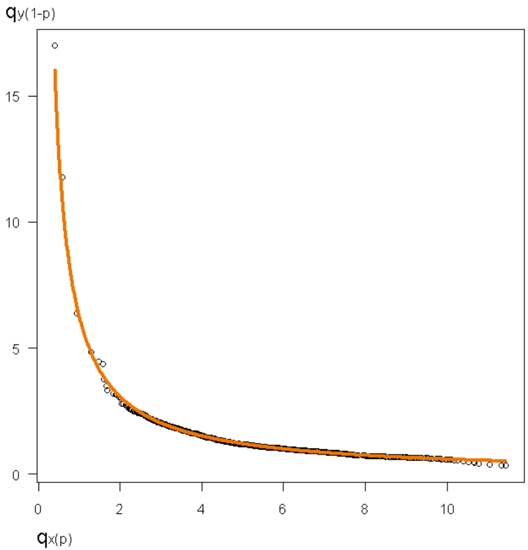
Estimation (5/8)



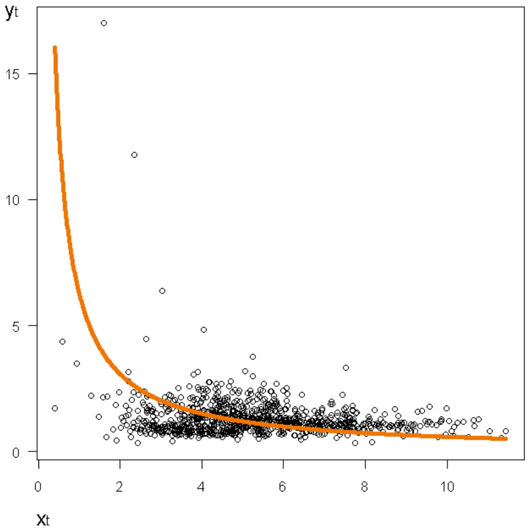
Estimation (6/8)



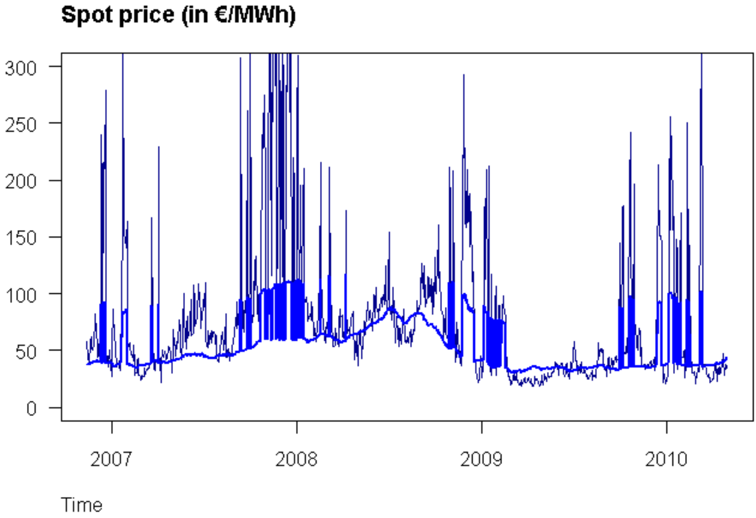
Estimation (7/8)



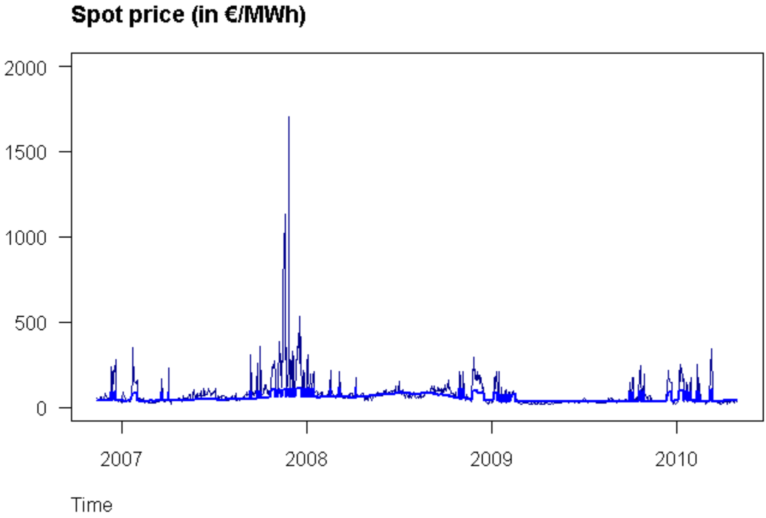
Estimation (8/8)



Comparison of the two models (1/2)



Comparison of the two models (2/2)



C_t and D_t

Building blocks

$$D_t = f_D(t) + Z_D(t)$$
$$C_t^i = f_i(t) + Z_i(t)$$

Stochastic part

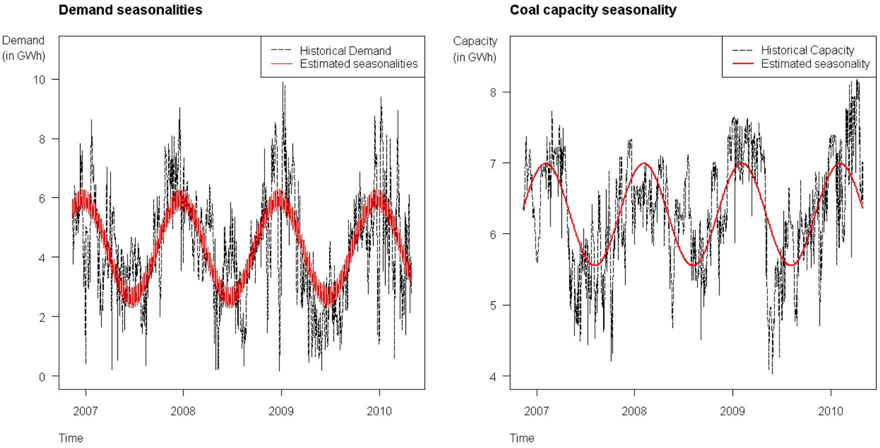
$$dZ_D(t) = -\alpha_D Z_D(t)\,dt + b\,dW_t^D$$
$$dZ_i(t) = -\alpha_i Z_i(t)\,dt + \beta_i\,dW_t^i$$

Deterministic part

$$f_D(t) = d_1 + d_2 \cos\left(2\pi \frac{t-d_3}{l_1}\right) + d_4 \cos\left(2\pi \frac{t-d_5}{l_2}\right)$$
$$f_i(t) = c_1^i + c_2^i \cos\left(2\pi \frac{t-c_3^i}{l_1}\right) + f_i^{\mathrm{evo}}(t)$$

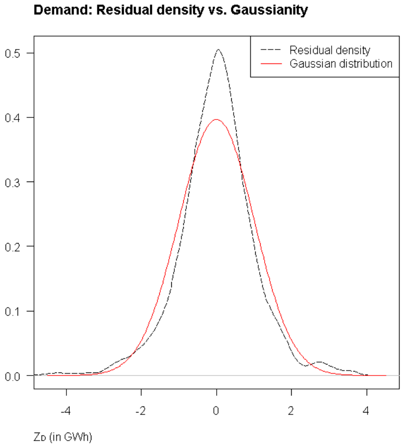
C_t and D_t

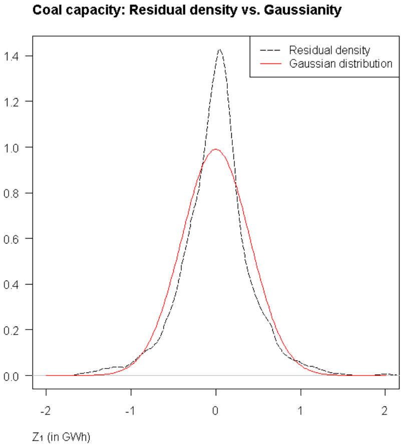
Historical data



C_t and D_t

Residuals





Hedging strategy

Payoff H, price $V_t^H := \widehat{\mathbb{E}}[H|\mathcal{F}_t] = \phi(t, F_t(T^*), C_t, D_t)$

Hedging strategy

$$\xi_t^e = \frac{1}{||\theta_t^C, \theta_t^D||^2} \left\{ \sum_{i=1}^{n} \theta_t^{C,i} \frac{\partial \phi}{\partial c_i} \beta_i + \theta_t^D \frac{\partial \phi}{\partial z} b \right\}$$

$$\xi_t^i = \frac{\partial \phi}{\partial y_i} + \frac{h_i G_i^{T^*}(t, C_t, D_t)}{\|\theta_t^C, \theta_t^D\|^2} \left\{ \sum_{i=1}^{n} \theta_t^{C,i} \frac{\partial \phi}{\partial c_i} \beta_i + \theta_t^D \frac{\partial \phi}{\partial z} \right\}$$

where $dF_t^e := \theta_t^S.d\widehat{W}_t + \theta_t^C.dW_t^C + \theta_t^D dW_t^D$

Dynamics of power futures

$$
\begin{aligned}
dF_t^e &= \theta_t^S.d\widehat{W}_t + \theta_t^C.dW_t^C + \theta_t^D dW_t^D \\
&= e^{r(T^*-t)} \sum_{i=1}^{n} \left(\sum_{k=i}^{n} G_k^{T^*}(t, C_t, D_t) \right) \sigma_i Y_t^i d\widehat{W}_t^i \\
&\quad + \sum_{i=1}^{n} h_i F_t^i(T^*) \frac{\partial G_i^{T^*}}{\partial d}(t, C_t, D_t) b(t, D_t) dW_t^D \\
&\quad + \sum_{i=1}^{n} h_i F_t^i(T^*) \sum_{k=1}^{n} \frac{\partial G_i^{T^*}}{\partial c_k}(t, C_t, D_t) \beta_k(t, C_t^k) dW_t^{C,k}
\end{aligned}
$$

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and its partial derivatives

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$$\Gamma(\alpha, x) = \text{incomplete Gamma function} = \int_x^{\infty} t^{\alpha-1} e^{-t} dt$$

Spread option

Payoff $H = (P_T - h_1 S_T^1 - K)^+$

Pricing

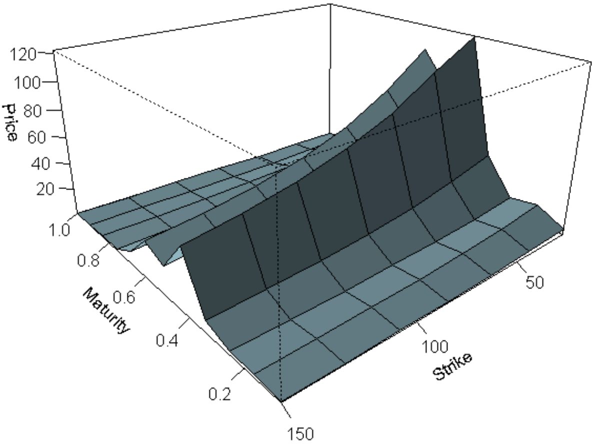
$$\pi_0 = \int_{\mathbb{R}^2} f_{C_T^1 - D_T}(z) f_{C_T^2}(c) \left\{ \phi_1(c,z)\mathbf{1}_{\{z>0\}} + \phi_2(c,z)\mathbf{1}_{\{z\leq 0\}} \right\} dcdz,$$

$$\phi_1 = (g-1)BS_0\left(\sigma_1, \frac{K}{g-1}\right)\mathbf{1}_{\{g>1\}} \qquad g = g(c+z)$$

$$\phi_2 = g\int_0^\infty \hat{f}_{Y_T^1}(y)BS_0\left(\sigma_2, \frac{K+(1-g)y}{g}\right)\left(\mathbf{1}_{\{g\leq 1\}} + \mathbf{1}_{\{g>1\}}\mathbf{1}_{\{y<\frac{K}{g-1}\}}\right)dy$$

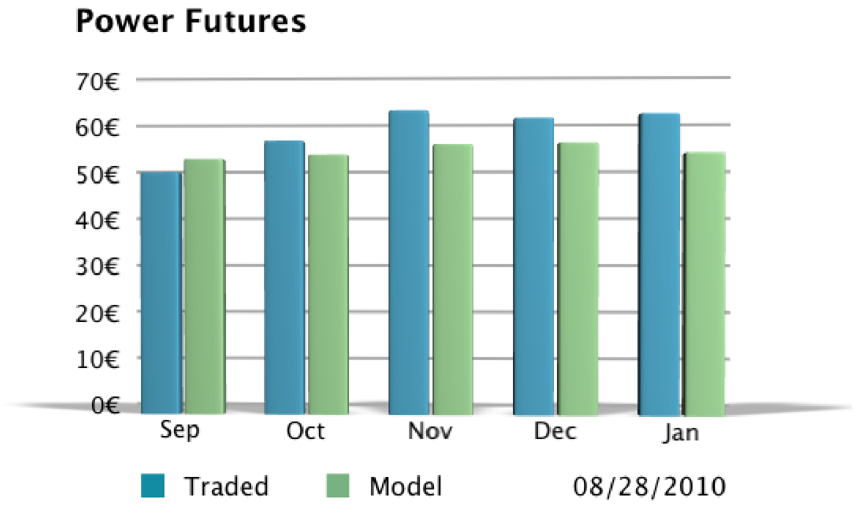
$$+\left(gY_0^2\mathcal{N}\left(\frac{\left(r-\frac{\sigma_1^2}{2}\right)T - \ln\left(\frac{K}{(g-1)Y_0^1}\right)}{\sigma_1\sqrt{T}}\right) + (g-1)\,BS_0\left(\sigma_1, \frac{K}{g-1}\right)\right)\mathbf{1}_{\{g>1\}}$$

Spread option
Numerical example



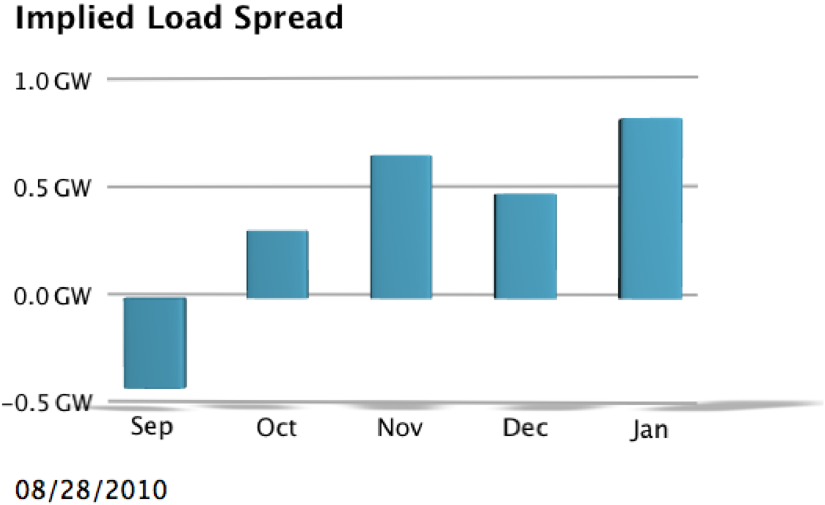
Market vs. model (1/2)

Risk premium vs. model error



Market vs. model (2/2)

Risk premium vs. model error



Extensions

Done

- Extend the numerical application to every single hour of the day (3 marginal fuels)
- Extension to payoffs with delivery periods $[T_1, T_2]$
- Compute the residual market price of risk

To do

- Better models for fuels (spot and futures) : convenience yield, interest rates, cointegration. . .
- Better volatility models
- Compare different hedging criteria
- Application to investment opportunities