Validation of hedging models for energy markets

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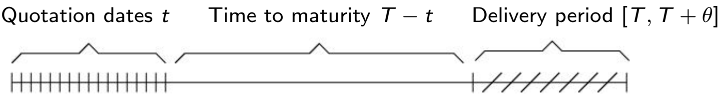
1

Hedging with futures

Hedging with futures

• Let us consider, F_t, the price quoted at time t for the delivery of one MWh on the period $[T_F, T_F + \theta]$, with $T_F \geq t$.



• Let V_t denote the value of a self-financed portfolio with position Φ_t at time t on the future F_t. Recall that entering in a futures contract is free, hence

$$V_{t+\Delta t} = \varphi_t (F_{t+\Delta t} - F_t) + e^{r\Delta t} V_t \ , \quad \text{where} \tag{1}$$

- r is the interest rate ;
- V_0 is the initial value of the portfolio and should correspond to the initial price of the corresponding option for a hedging portfolio ;

Hedging tests description

• 6 Hedging strategies: Implicit-Delta, SABR-Delta, SABR-LRM, Heston and VO-NIG are implemented daily on EEX futures prices with the same initial value V_0 observed on the options market.

• Calibrating and fitting models

- BS, SABR, Heston and the Local Volatility model are calibrated daily on EEX options prices.
- The Lévy model is fitted at the beginning of the hedging period on EEX futures prices of the hedging period.

• The *hedging error* comparing the hedging portfolio value V_T with the option payoff at maturity is computed

$$\varepsilon_T = V_T - (F_T - K)+ \ .$$

• 12×21 Call options are considered : on 12 months (Jan-08, ..., Dec-08) with for each month 21 strikes.
=> The set of (quasi-independent) obversations $\varepsilon_T^{Month,K}$ is too small to produce precise statistics.

2

Black-Scholes framework

Black model for futures prices

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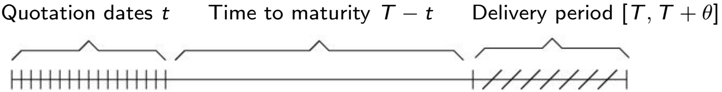


• The Black-Scholes model assumes that the price process $(F_t)_{t \geq 0}$ is such that

$$dF_t = F_t \left(\mu dt + \sigma dW_t \right) \ , \quad \text{where} \tag{2}$$

- W is the Standard Brownian motion on $\mathbb{R}$;
- σ is the constant volatility ;
- μ the drift.

BS formula for pricing

• Under the BS assumption, the value of the European call, with underlying $(F_t)_{t\geq 0}$, maturity $T \leq T_F$ and strike K is

$$C(F,t;K,T,r,\sigma) = e^{-r(T-t)}\tilde{\mathbb{E}}[(F_T - K)^+ \mid F_t = F] \ , \qquad (3)$$

the expectation of the pay-off under the martingale measure $\tilde{\mathbb{P}}$ for which (F_t) is a martingale.

• The log-normal assumption yields the BS formula :

$$C = C^{BS}(F,t;K,T,r,\sigma) = e^{-r(T-t)}\left(F\mathcal{N}(d_1) - K\mathcal{N}(d_2)\right) \ , \quad (4)$$

where $\mathcal{N}$ denotes the distribution function of $\mathcal{N}(0,1)$ and

$$d_1 = \frac{\log(F/K)}{\sigma\sqrt{T-t}} + \frac{\sigma\sqrt{T-t}}{2} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T-t} \ . \qquad (5)$$

- By Ito's Formula

$$e^{-rT}(F_T - K)^+ = C_0 + \int_0^T e^{-rt}\left[\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 F^2 \frac{\partial^2 C}{\partial F^2} - rC\right] dt + \int_0^T e^{-rt}\frac{\partial C}{\partial F} dF_t \ . \tag{6}$$

- As (F_t) and $e^{-rt}C(F, t; K, T, r, \sigma)$ are martingales under $\tilde{\mathbb{P}}$ then

$$\begin{cases} \frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 F^2 \frac{\partial^2 C}{\partial F^2} - rC = 0 & \textit{Pricing} \ , \\ (F_T - K)^+ = C_0 + \int_0^T e^{r(T-t)}\Delta_t^{BS} d\tilde{F}_t & \textit{Hedging} \ . \end{cases}$$

with

$$\Delta_t^{BS} = \left.\frac{\partial C^{BS}}{\partial F}\right|_{F=F_t} = \mathcal{N}(d_1(F_t, t; K, T, \sigma)) \ . \tag{7}$$

Implicit volatility

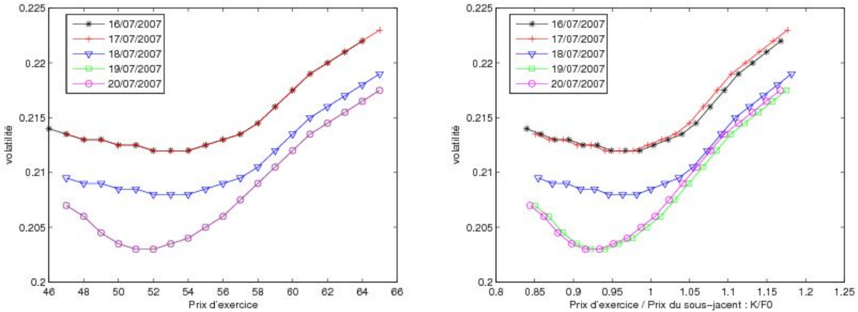
• As C^{BS} is strictly increasing with $\sigma > 0$, one can define the concept of implicit volatility associated with a call price C at time t for an underlying F by inversion of the BS formula :

$$C^{BS}(F, t; K, T, r, \sigma_{imp}) = C \ . \tag{8}$$

- A single number to compare option prices corresponding to different strikes and maturities.
- A simple hedging strategy consisting in injecting the implicit volatility in the Delta formula : *Implicit Delta*
- Unfortunately, the model is intrinsicly incoherent => difficult to apply for exotic options.

Example of volatility smile on electricity market

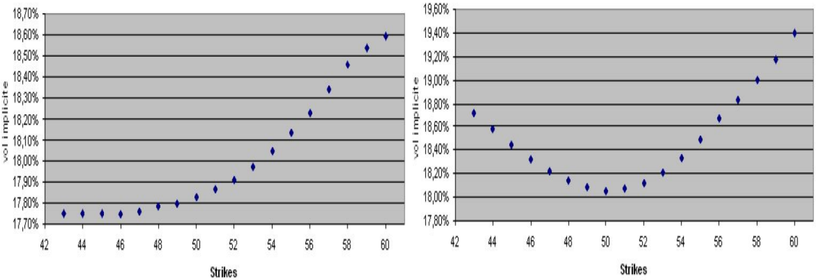
Implicit volatility on the futures Germany 2008 (ICAP data) :



$=>$ Necessity to introduce a new model to price exotic options involving multiple strikes and maturities.

Example of volatility smile on electricity market

Implicit volatility on EEX market :



=> Necessity to introduce a new model to price exotic options involving multiple strikes and maturities.

3

The local volatility model

Local volatility model

- Dupire (1994) proposed the local volatility model :

$$dF_t = F_t \left(\mu(t, F_t)dt + \sigma(F_t, t)dW_t \right) \ . \tag{9}$$

- Explains the "smile" in a coherent model:
Dupire showed that for any pricing rule without arbitrage *there exists a unique local volatility function* inducing the same call prices.

- Complete market model which provides in theory a perfect hedging strategy that does not involve options which are not liquid in energy markets.

Calibration of Local volatility

• Dupire's model yields call prices $C(F, t; K, T, r, \sigma)$ solutions of Dupire's PDE for a given initial value F of the underlying future price:

$$\begin{cases} \dfrac{\partial C}{\partial T} = \dfrac{1}{2}\sigma^2(K,T)K^2\dfrac{\partial^2 C}{\partial K^2} - rC \ , & \text{for all } K > 0 \ , \ 0 < T \leq T_l \\ C(K, T=0) = (F-K)^+ \ , & \text{for all } K > 0 \ . \end{cases}$$

=> Solving this PDE gives straightforwardly $C(K_j, T_j)$ for a grid $(K_j, T_j)_{j\in\mathcal{G}}$.

• Calibration consists in solving an inverse problem : Looking for the local volatility function $(F, t) \mapsto \sigma(F, t)$ inducing option prices (or implicit volatility points) as close as possible to observations $\left(C_i^*\right)_{i\in\mathcal{I}}$ (or $\left(\sigma^i_{imp}\right)_{i\in\mathcal{I}}$) for a given set of strike and maturities $(K_i, T_i)_{i\in\mathcal{I}}$.

Calibration algorithm (Cont and BenHamida 2005)

- Calibration problem:

$$\inf_{\theta \in E} G(\theta) \quad \text{where} \quad G(\theta) = \sum_{i=1}^{I} |C^{\theta}(t, F_t, T_i, K_i) - C_i^*|^2 \omega_i \ . \quad (10)$$

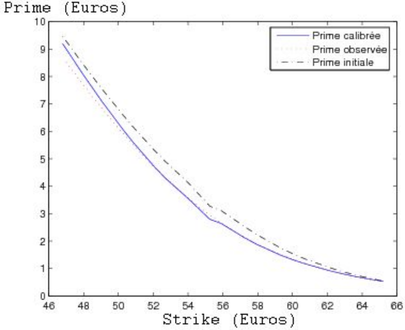
- Parametrization of the local volatility by Splins $(\phi_i)_{i=1,\cdots,n}$:

$$\sigma^{\theta}(x) = \sum_{i=1}^{n} \theta_i \phi_i(x) \ , \quad \text{for all } x \in \mathbb{R} \ . \qquad (11)$$

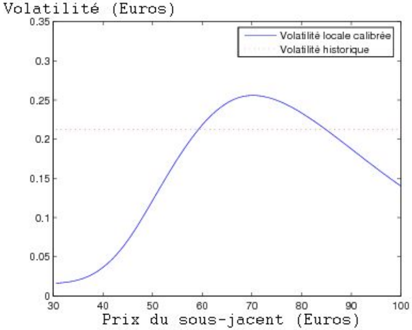
- Particle system $(\theta^1, \cdots, \theta^N)$ converging to glocal minima of G:
  - **Initialisation**: simulate N iid random variables $(\theta^{1,0}, \cdots, \theta^{N,0})$ representing N possible values of parameter θ.
  - **Mutation**: each particle $\theta^{k,i}$ evolves independently according to a transition kernel Q_k which yields a new particle system $(\tilde{\theta}^{k+1,1}, \cdots, \tilde{\theta}^{k+1,N})$.
  - **Selection**: each particle is selected according to the value of the cost function G. Particles with high value of G are killed whereas particles with small value of G are multiplied.

Example of calibration on electricity market

Local volatility calibration on the futures Germany 2008 (ICAP data) :



Option prices for different strikes



Local volatility function

Local volatility and hedging

• If $(F_t)_{t\geq 0}$ follows the local volatility model (14). By definition of implicit volatility (8), we have :

$$C^{BS}(F,t;K,T,r,\sigma^{BS}_{imp}) = C(F,t;K,T,r,\sigma)\ . \qquad (12)$$

For fixed K, T and r, σ^{BS}_{imp} is a function of the initial price F.
• The sensitivity of C w.r.t. the underlying F is :

$$\frac{\partial C}{\partial F} = \frac{\partial C^{BS}}{\partial F} + \frac{\partial C^{BS}}{\partial \sigma^{BS}_{imp}} \frac{\partial \sigma^{BS}_{imp}}{\partial F}\ . \qquad (13)$$

=> Implicit Δ hedging induces a risk proportionnal to the Vega.
=> To implement an efficient hedging strategy one has to either

- make assumptions on the "skew" $\frac{\partial \sigma^{BS}_{imp}}{\partial F}$, which is in general difficult to handle;
- use Hagan & Woodward formula (1999): $\exists$ a deterministic function $\mathcal{G}^{\mathrm{HW}}$ s.t.

$$\sigma^{BS}_{imp} \approx \mathcal{G}^{\mathrm{HW}}(F,K,T,\sigma,\sigma',\sigma'')\ .$$

Local volatility model: Pros and Cons

- Dupire (1994) proposed the local volatility model :

$$dF_t = F_t \left(\mu(t, F_t)dt + \sigma(F_t, t)dW_t \right) \ . \tag{14}$$

Pros

- Explains the "smile" in a coherent model
- Complete market model with hedging strategies that do not involve options which are not liquid in energy markets

Cons (...to check on electricity market)

- The dynamics of the volatility surface predicted by the model is often wrong
- The model proposes a Delta hedging strategy which in theory is perfect but in practice there is a vega risk which is not taken into account in the pricing

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Stochastic volatility models (Etchepare and Tankov)

• The futures price process is supposed to follow under the risk neutral probability, the SDE

$$\begin{cases} \dfrac{dF_t}{F_t} = \sqrt{v_t}dW_t \ , \quad \text{with} \\ dv_t = k(\theta - v_t)dt + \xi\rho\sqrt{v_t}dW_t + \xi\sqrt{1-\rho^2}\sqrt{v_t}dW_t' \ , \end{cases} \tag{15}$$

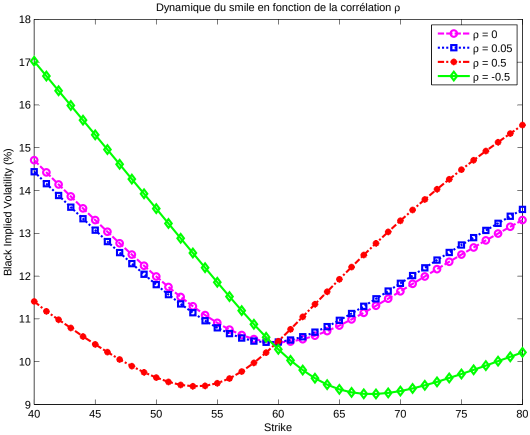
where W and W' are two independant Brownian Motions.

- $v_t > 0$ when $2\kappa^*\theta^* \geq \xi^2$.
- v_0, ρ and ξ determine resp. the level, the slope and the convexity of the smile for a fixed maturity.
- κ^* and θ^* determine the implicit volatility term structure (fixed in ou case).

• If the price is supposed to be without arbitrage then

$$dC_t = \left(\frac{\partial C}{\partial F}F_t\sqrt{v_t} + \frac{\partial C}{\partial v}\xi\sqrt{v_t}\right) dW_t + \frac{\partial C}{\partial v}\xi\sqrt{1-\rho^2}\sqrt{v_t}dW_t' \ . \tag{16}$$

Impact of the correlation ρ on the BS implicit volatility produced by Heston Model



- Minimizing the global quadratic risk (=local quadratic risk) under the risk neutral measure i.e. in discrete time solving

$$\min_{\phi_k} \mathbb{E}[(e^{r\Delta t} C_k + \phi_k \Delta F_k - C_{k+1})^2 \,|\, \mathcal{F}_k] \;, \quad \text{for } k = 0, \cdots, n-1,$$

yields the following hedging strategy:

$$\delta_t^{LRM} = \frac{d\langle C, F\rangle_t}{d\langle F, F\rangle_t} = \frac{\partial C}{\partial F} + \frac{\rho\xi}{F_t} \frac{\partial C}{\partial v} \;. \tag{17}$$

  - The risk generated by W is hedged but not W'
  - δ_t^{LRM} coincides with the variance optimal strategy when we are under the martingale probability
  - Estimating δ_t^{LRM} requires to estimate precisely the instantaneous volatility v_t.
  - $\partial C / \partial F$ and $\partial C / \partial v$ are computed via closed formula for the call price.

SABR Model (Hagan, Kumar, Lesniewski & Woodward (2002))

- The price process is supposed to follow the SDE

$$dF_t = \alpha_t F_t^{\beta} dW_t \ , \quad \frac{d\alpha_t}{\alpha_t} = \epsilon \rho dW_t + \epsilon \sqrt{1-\rho^2} dW_t' \ , \qquad (18)$$

  - α_0, ρ and ϵ determine resp. the level, the slope and the convexity of the smile for a fixed maturity.
  - There is no mean-reverting in the volatility process.
  - The model is able to represent precisely the volatility smile for a fixed maturity.

Hedging with SABR model

- There exists a deterministic function $\mathcal{H}^{\text{Hagan}}$ s.t.

$$\widehat{\sigma}_{BS} \approx \mathcal{H}^{\text{Hagan}}(F, K, T, \alpha, \nu, \beta, \rho).$$

- The hedging ratio is given by

$$\delta_t^{\text{LRM}} := \Delta_{\text{opt}} = \frac{\partial \widehat{C}_{BS}}{\partial F} + \frac{\partial \widehat{C}_{BS}}{\partial \widehat{\sigma}_{BS}} \left(\frac{\partial \widehat{\sigma}_{BS}}{\partial F} + \frac{\partial \widehat{\sigma}_{BS}}{\partial \alpha} \frac{\rho \nu}{F^{\beta}} \right).$$

- An alternative approach proposed by Hagan *et al.* is to consider the volatility α as a parameter and not as a state variable:

$$\delta_t^{\text{Hagan}} := \frac{\partial \widehat{C}_{BS}(F, \widehat{\sigma}_{BS}(F, \alpha))}{\partial F} = \frac{\partial \widehat{C}_{BS}}{\partial F} + \frac{\partial \widehat{C}_{BS}}{\partial \widehat{\sigma}_{BS}} \frac{\partial \widehat{\sigma}_{BS}}{\partial F}.$$

eDF R&D

5

The Exponential NIG Lévy model

Properties of the Normal Inverse Gaussian distribution

- Mean variance mixture: $\alpha > 0\,,\ 0 \leq |\beta| < \alpha\,,\ \delta > 0\,,\ \mu \in \mathbb{R}$

$$X = \mu + \beta Y + \sqrt{Y} N\,, \quad \text{where} \quad N \sim \mathcal{N}(0,1)\,, \quad \perp \quad Y \sim \mathrm{IG}(\delta, \gamma)\,,$$

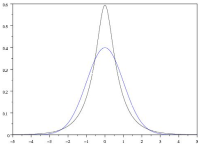
with $\gamma = \sqrt{\alpha^2 - \beta^2}$.

- Density $f_{NIG}(x) = \frac{\alpha}{\pi} \exp\left(\delta\gamma + \beta(x-\mu)\right) \frac{K_1\left(\alpha\delta\sqrt{1+(x-\mu)^2/\delta^2}\right)}{\sqrt{1+(x-\mu)^2/\delta^2}}$

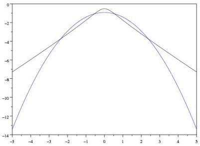
where K_1 the Bessel function of the third type.

- Mean and variance $\mathbb{E}X = \mu + \dfrac{\delta\beta}{\gamma}\,, \quad \mathrm{Var}X = \dfrac{\delta\alpha^2}{\gamma^3}\,.$

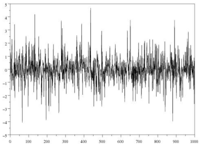
▶ Comparison of the Gaussian (blue) and NIG (black) densities



Densities



Log–densities



NIG residuals

VO strategy for the Lévy model (Hubalek *et al.* 2006)

If the payoff function can be written as a Laplace transform i.e.

$$\Phi(s) = \int_{\mathbb{C}} S^z \Pi(dz) \;, \quad \text{where } \Pi \text{ is a finite complex measure} \;, \quad \text{then}$$

the variance-optimal capital and hedging strategy (V_0, φ) are s.t.

$$V_0 = H_0 \quad \text{and} \quad \varphi_n = \xi_n + \frac{\lambda}{S_{n-1}}\left(H_{n-1} - V_0 - \sum_{k=0}^{n-1} \varphi_k \Delta S_k\right) ,$$

where (H_n, ξ_n) defines the FS decomposition of the payoff:

$$H_n := \int_{\mathbb{C}} S_n^z h(z)^{N-n} \Pi(dz) \;, \quad \text{and} \quad \xi_n := \int_{\mathbb{C}} S_{n-1}^{z-1} g(z) h(z)^{N-n} \Pi(dz) \;,$$

$$\text{with} \quad g(z) := \frac{m(z+1) - m(1)m(z)}{m(2) - m(1)^2} \;, \quad \text{and} \quad h(z) := m(z) - (m(1) - 1)g(z) \;,$$

where m is the moment generating function of $X_1 = \log(\frac{S_1}{S_0})$, and

$$\lambda := \frac{m(1) - 1}{m(2) - 2m(1) + 1} \;.$$

6

Hedging tests on real data

Hedging tests description

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- Calibrating and fitting models
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=> The set of (quasi-independent) obversations $\varepsilon_T^{Month,K}$ is too small to produce precise statistics.

Global results for options of Month-Ahead-Futures 2008

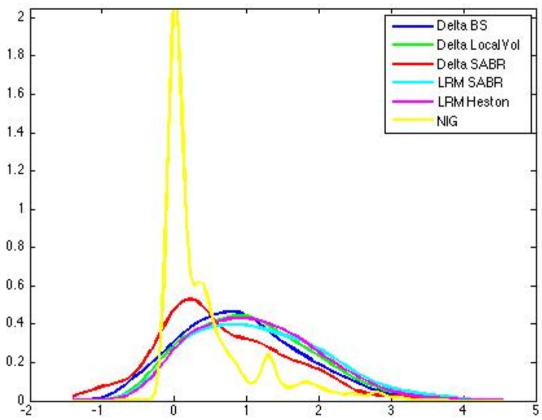
Average initial and terminal option values : $C_0 \approx 5.50$ Euros and $C_T \approx 9.50$ Euros

Indicators Hedging	BS	Vol Loc	SABR Δ	SABR LRM	Heston	NIG
Mean μ	0,92	1,06	0,67	1,18	1,12	0,43
Std σ	0,83	0,82	0,89	0,87	0,81	0,65
Skewness	0,54	0,53	0,61	0,50	0,47	1,89
Kurtosis	0,08	-0,08	0,37	-0,30	-0,26	3,25
Q-0,95	2,43	2,57	2,14	2,64	2,58	1,91
Q-0,99	3,19	3,24	3,22	3,56	3,17	2,81
Q-0,05	-0,40	-0,19	-0,69	-0,02	-0,02	-0,06
Q-0,01	-0,50	-0,23	-1,08	-0,28	-0,29	-0,13

$$\text{Skewness} := \frac{\mathbb{E}(\varepsilon_T - \mu)^3}{\sigma^3} \quad \text{and} \quad \text{Kurtosis} := \frac{\mathbb{E}(\varepsilon_T - \mu)^4}{\sigma^4} - 3.$$

Global results for options of Month-Futures 2008

Empirical densities of hedging errors associated with each model.



Global results for Months-Quarter and Years 2008.

• Over-hedging in average for a call seller (the std of the mean estimator computed on 12 contracts is 0,3).

• The NIG model shows significantly different results from other models: smaller mean and standard deviation.
=> This can be explained either by the hedging model or by the data used to fit the parameters (historical vs. implicit).

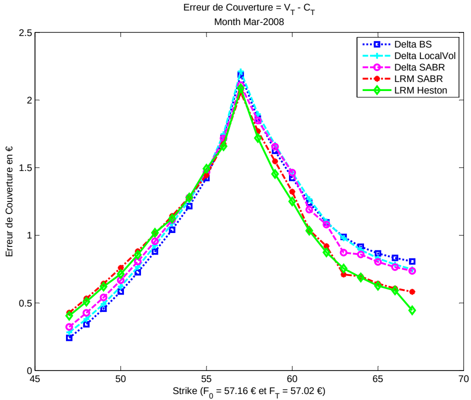
• Other hedging models show quasi-similar performances in terms of bias and variance

Strike Impact

- Over-hedging is maximal for strikes which are ATM at maturity.

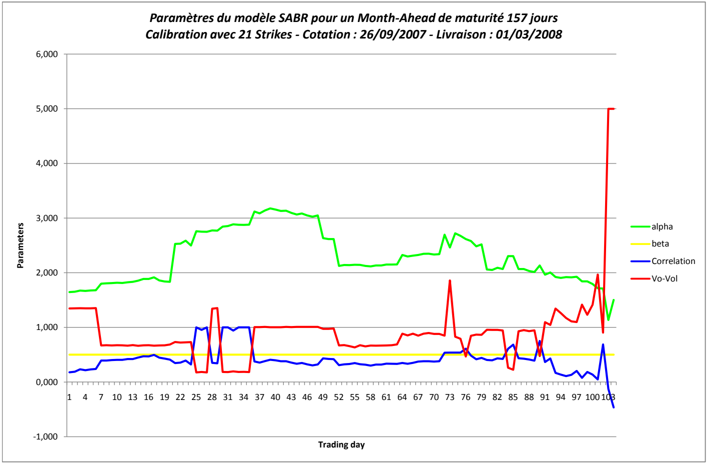
$$\varepsilon_T^{F,\Phi,\delta} = V_T - \Phi(F_T) = V_0 + \int_0^T \delta_u dF_u - (F_T - K)_+ \ .$$

  - For $K < F_T$, $V_0(K)$ decreases at a smaller rate ($\mathcal{N}(d_2) \leq 1$) than $(F_T - K)$.
    For $K > F_T$, $(F_T - K)_+ = 0$ and $V_0(K)$ still decreases.
  - The impact of the strategy δ seems to be of second order.



Static calibration vs dynamic calibration

- Parameters evolution during the hedging period



Static calibration for options on Monthly Futures 2008

Indicators	BS	Vol Loc	SABR Δ	SABR LRM	Heston
Mean μ	0.69	0.77	0.50	0.81	0.80
STD σ	0.81	0.82	0.88	0.81	0.81
Skewness (Sk)	0.91	0.73	0.90	0.62	0.71
Kurtosis (Ku)	1.25	0.87	1.56	1.04	0.97
Quantile-0,95	2.33	2.36	2.28	2.32	2.33
Quantile-0,99	3.27	3.26	3.36	3.29	3.27
Quantile-0,05	-0.37	-0.34	-0.63	-0.25	-0.27
Quantile-0,01	-0.92	-0.92	-1.46	-1.11	-0.92

Hedging portfolio with static calibration daily rebalanced:

- Small decrease of the mean P&L $\approx -20\%$ for the three first hedging models $\approx -10\%$ for others.
- Same standard deviation.
- Noticeable increase of extreme P&L.

Weekly rebalanced hedging

Indicators	BS	Vol Loc	SABR Δ	SABR LRM	Heston
Mean μ	0.40	0.54	0.12	0.60	0.55
STD σ	1.09	1.06	1.22	1.19	1.15
Skewness (Sk)	0.61	0.46	0.32	0.12	0.14
Kurtosis (Ku)	0.35	0.10	0.63	-0.35	-0.07
Quantile-0,95	2.54	2.52	2.53	2.70	2.60
Quantile-0,99	3.45	3.40	3.25	3.44	3.44
Quantile-0,05	-1.35	-1.12	-2.18	-1.26	-1.30
Quantile-0,01	-1.72	-1.63	-2.7	-1.86	-1.97

Hedging portfolio with dynamic calibration weekly rebalanced:

- Decrease of the mean P&L $\approx -50\%$.
- Increase of the standard deviation $\approx +30\%$.
- Significant increase of extreme P&L.

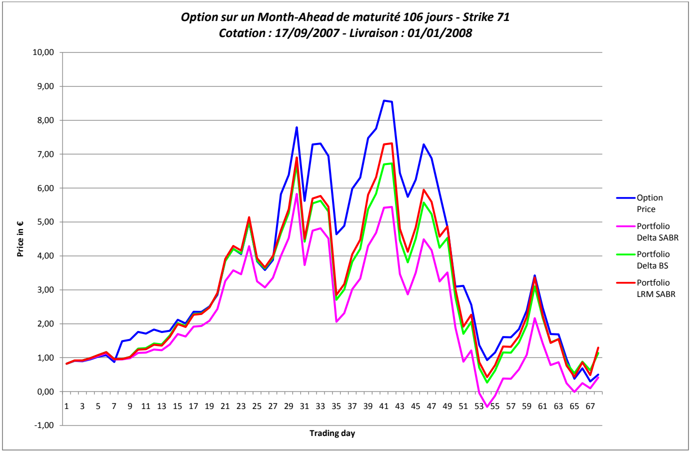
Transaction costs impact

- Adding the transaction costs of 5 cents/MWh induces an average cost of:
  - Dynamic calibration daily rebalanced
    cost $\approx$ 0.12 Euros.
  - Static calibration daily rebalanced
    cost $\approx$ 0.12 Euros.
  - Dynamic calibration weekly rebalanced
    cost $\approx$ 0.08 Euros.

=> Transaction costs explain only 10% of the over-hedging.

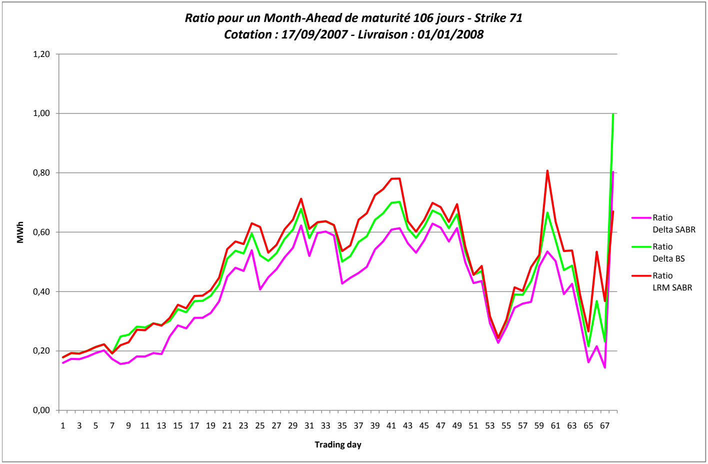
Option and hedging portfolio values path

Prices evolution for the call on Jan-08 and hedging portfolio values using SABR δ_t^{LRM}, SABR $\delta_t^{\mathrm{Hagan}}$ and Δ^{BS} strategies.



Hedging ratio path

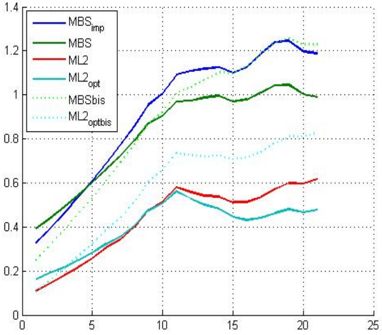
Hedging ratios for the call on Jan-08: SABR δ_t^{LRM}, SABR δ_t^{Hagan} and Δ^{BS}



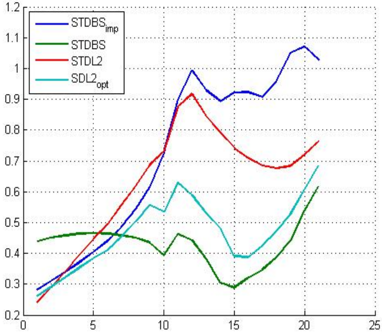
Hedging error computed on real data

Hedging error of Implicit Δ^{BS}, Historic Δ^{BS} and VO-NIG strategies implemented on real data:

- for Implicit Δ^{BS}, calibration is done once at the beginning of the hedging period, on option prices of the day;
- for Historic Δ^{BS} and VO-NIG, parameters are fitted on historical log-returns of the underlying futures price during the hedging period;
- Options are sold at the first day of quotation.



Mean

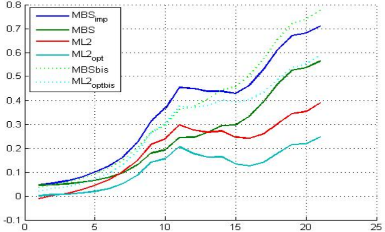


Standard Deviation

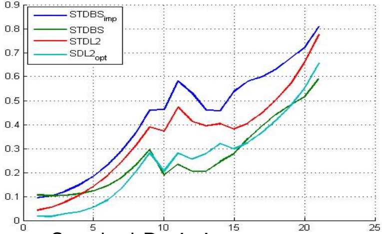
Hedging error computed on real data

Hedging error of Implicit Δ^{BS}, Historic Δ^{BS} and VO-NIG strategies implemented on real data:

- for Implicit Δ^{BS}, calibration is done once at the beginning of the hedging period, on option prices of the day;
- for Historic Δ^{BS} and VO-NIG, parameters are fitted on historical log-returns of the underlying futures price during the hedging period;
- Options are sold 20 trading days before delivery, to insure *independence* of hedging portfolio with different underlying (month) that are implementd on different periods of time.



Mean



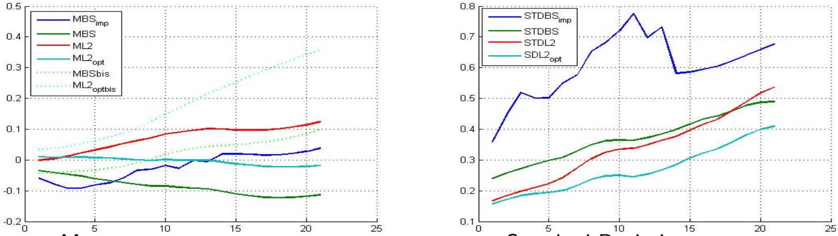
Standard Deviation

- The mean P&L is significantly positive for all models;
- Using historical volatility to hedge and implicit volatility to price keeps the same mean P&L and decreases the P&L STD.
  => the gap between implicit and historical volatility explains the small P&L STD of VO-NIG observed on real data.

Hedging error computed on simulated Gaussian data

Hedging error of Implicit Δ^{BS}, Historic Δ^{BS} and VO-NIG strategies implemented on data simulated according to a BS model (with paraters estimated on real data):

- for Implicit Δ^{BS}, calibration is done once at the beginning of the hedging period, on option prices of the day;
- for Historic Δ^{BS} and VO-NIG, parameters are fitted on historical log-returns of the underlying futures price during the hedging period;
- The hedging period corresponds to the 20 trading days before delivery.



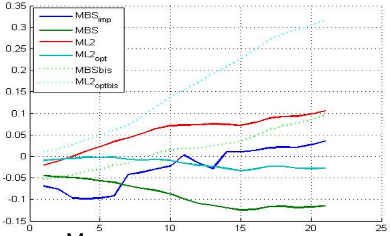
Mean — Standard Deviation

- The mean P&L is close to zero;
  => most part of the mean P&L observed on real data is explained by the fact that futures log-returns don't follow the BS model.
- The P&L STD is close to the P&L STD observed on real data;
- With simulated data, VO startegy is noticeably more performant than BS approach.

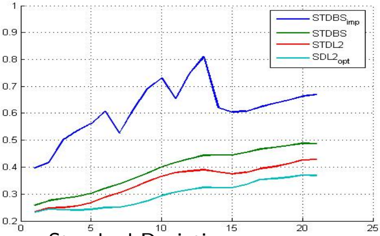
Hedging error computed on simulated NIG data

Hedging error of Implicit Δ^{BS}, Historic Δ^{BS} and VO-NIG strategies implemented on data simulated according to a Lévy NIG model (with paraters estimated on real data):

- for Implicit Δ^{BS}, calibration is done once at the beginning of the hedging period, on option prices of the day;
- for Historic Δ^{BS} and VO-NIG, parameters are fitted on historical log-returns of the underlying futures price during the hedging period;
- The hedging period correspond to the 20 trading days before delivery.



Mean



Standard Deviation

- The mean P&L is close to zero;
  => most part of the mean observed on real data would be explained by the fact that futures log-returns are probably not independent and stationnary.
- The P&L STD is close to the P&L STD observed on real data;
- With simulated data, VO NIG is noticeably more performant than BS approach.

7

Conclusion

Conclusion

- The number of *quasi-independent* observation is too small to formulate definitive conclusions.

- All models show similar hedging performances: the choice of the parameters seems to have a more crucial impact that the choice pf the model.
=> One should probably use different volatilities for pricing (implicit volatility) and hedging (historical volatility).

- Call options are in average over-hedged by all models for a seller.
  - $\approx$ 10% of over-pricing can be explained by an over-evaluation of the volatility on the option market; this over-evaluation of volatility can be viewed as a risk premium to hedge the P&L fluctations or simply to hedge transaction costs which are of the same order;
  - The major part of the average P&L is not explained and can be due to the non-stationnarity or non-independence of futures log-returns.

- Static calibration, transaction costs seem to have a relatively small impact on hedging performances contrarily to the rebalancing frequency.

- When considering options on Monthly futures, non Gaussianity does not invalidate the performances of BS approach when compared to other approaches.

- The presence of a drift has a noticeable impact on the hedging error.

- Non stationarity or dependence of observed log-returns seems to have a significant impact on the hedging errors.