The VaR at risk

Alfred Galichon

(Ecole polytechnique)

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Introduction

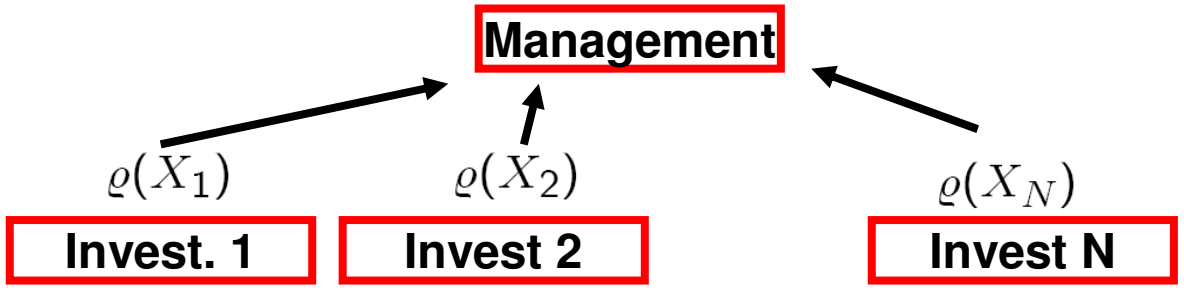
- Risk measures such as Value-at-Risk (VaR) have become standard tools to determine regulatory capital, especially since the Basle II agreements
  - *VaR at level 95% is the amount of capital I need to budget in order to cover my losses with probability 95%.*
  - Simple and intuitive definition…
- However, widely criticized, both among practitioners and academics…
- One serious flaw of VaR identified here is that **regulatory capital is not independent of the structure of the firm.**

Structure of the firm

- Modigliani-Miller (1958): The structure of the capital of the firm is neutral to its **value**.
  - corporate finance theory has ever since tried to dispute this claim
- Deconsolidation is the art of manipulating of the firm's capital structure in order to optimize certain accounting ratios (reduce assets, net debt, etc.)
  - example: Enron's Special Purpose Vehicles (SPVs)
- I will demonstrate a deconsolidation scheme which minimizes the firm's regulatory capital under VaR budgeting rules.

Budgeted capital and aggregation

Holding's manager supervises N business units with contingent losses $X_1, ..., X_N$, where $X_i \in [0, l_M]$ (net losses are accounted positively).
Eg. investments portfolio of a **fund of funds**. True economic risk of the fund is $X = X_1 + ... + X_N$.



Business units: portfolio of (contingent) losses X_i - compute required capital $\varrho(X_i)$ and report request to management. With Basle II, ϱ is the Value-at-Risk:

$$\varrho(X) = VaR_\alpha(X) = F_X^{-1}(\alpha).$$

Manager needs to budget amount $\varrho(X_1) + ... + \varrho(X_N)$.

The holding manager's problem

Manager's problem. Suppose restructuring the firm is free. Then given economic risk X, manager's problem is to **minimize amount of capital to budget**. Thus manager problem's is to determine

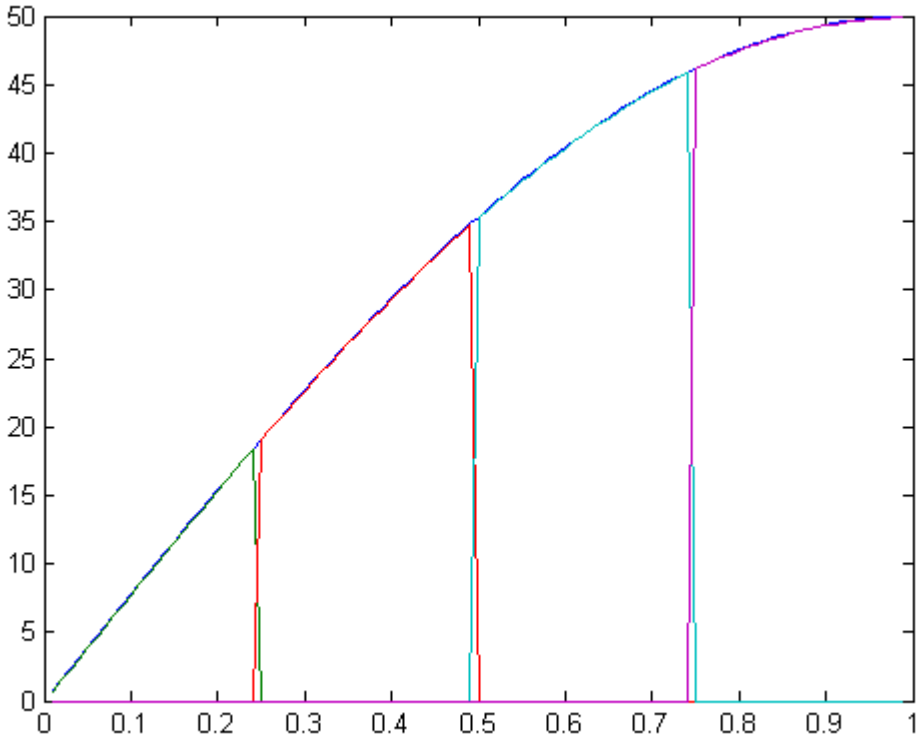
$$\inf_{X_1+...+X_N=X} \varrho(X_1) + ... + \varrho(X_N)$$

where the minimization problem is over all choices of $N \geq 1$ and $X_1, ..., X_N$ such that $X_1 + ... + X_N = X$.

Result. *When* $\varrho(.) = VaR_\alpha(.)$, *the solution of this problem is zero.*

In other words, one can divide a firm in a number of business unit which **all report zero VaR**.

Loopholes in the VaR



1. Total loss distribution: probability of X not exceeding value x.
2. Holding's risk (VaR at different levels).
3. **Subsidiaries' risk (X1 to X4).**

- Each subsidiary has a probability equal to 25% of encountering nonzero losses. Hence their VaR at level 75% is zero.
- However, the VaR at level 75% of the holding is 46 MEurs!

Loopholes in the VaR

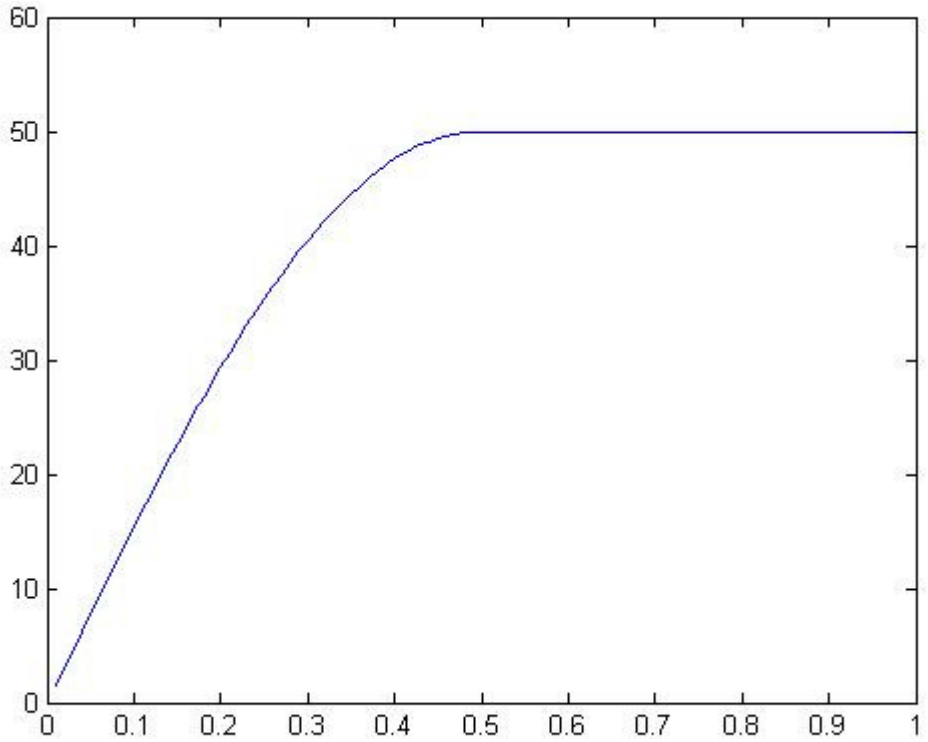
- X_i is approximatively the payoff of **digital options** on the company's payoff X, which is worth
  - **zero** below strike x_i on X, and
  - **zero** above strike x_{i+1} ($x_{i+1} > x_i$) on X
    - In the previous example, x_i = 18 MEur and x_3 = 35M Eur
- If the strikes x_i and x_{i+1} are close enough, the value-at-risk associated to X_i is zero, as the probability that X gets realized between x_i and x_{i+1} is small enough.
- Therefore the structure of the firm is not neutral to the amount of budgeted capital.

What went wrong in the previous example?

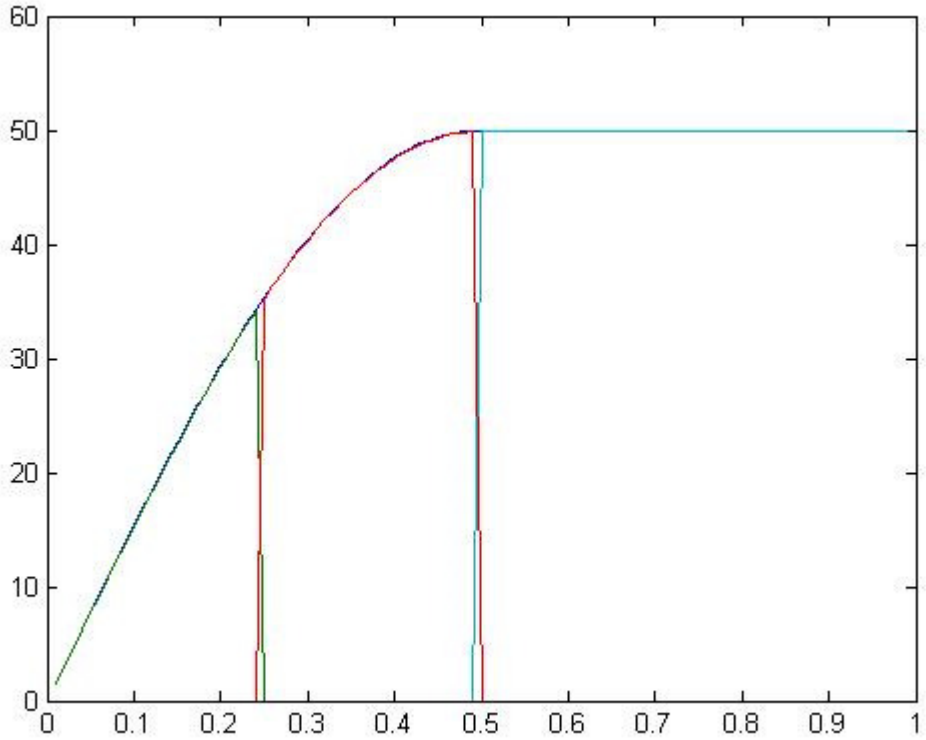
- By dividing their risk into several subsidiaries, the holding manager has managed to decrease his regulatory capital from 46 M Eur to 0…
- The Capital Budgeting rule should satisfy the property that given any possible way to split up the risk, there is no decrease in the amount of budgeted capital for doing to.
- This property is called **subadditivity**, which VaR notoriously fails to satisfy.

What about disasters?

- Suppose company has probability 50% of incurring exactly its maximal possible loss (50 MEur)…



What about disasters?



X_1 has 0 $VaR_{75\%}$

X_2 has 0 $VaR_{75\%}$

X_3 has $VaR_{75\%}$ = 50MEur!!!

Company needs to budget 50MEur, as before deconsolidation. Previous scheme no longer works.

What about disasters?

- Can we still reduce the VaR?
- YES WE CAN!
  - Recall that X_3 has a probability 50% of being zero. In place of X_3, create two companies X_{3A} and X_{3B} such that:
    - X_{3A} = X if Obama gets elected, zero otherwise
    - X_{3B} = X if McCain gets elected, zero otherwise
    We do have $X_3 = X_{3A} + X_{3B}$ (either Obama or McCain is elected).
  - Assuming Obama and McCain have the same probability of being elected, X_{3A} and X_{3B} both have a probability 75% of being zero. Hence, both associated VaRs are zero.
- Hence, we have again managed to reduce the amount of budgeted capital to zero.

Conclusion

- VaR values do not sum up lines by lines
- The amount of capital budgeted under VaR is dependent of the structure of the firm
  - room for regulatory arbitrage: “The VaR is at risk!”
- Imposing the independence of amount of capital budgeted with respect to the structure of the firm leads to new characterizations of alternative risk measures
  - Cf. Ekeland, Galichon and Henry (2009)
  - These measures encompass well-known alternatives to VaR such as TailVaR.

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Thank you !

alfred.galichon@polytechnique.edu