A structural risk-neutral model of electricity prices

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R. Aïd, L. Campi, N. Touzi, A. Nguyen Huu

Laboratoire FiME (Paris-Dauphine, CREST, EDF R&D)

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Introduction I : Electricity market

- Electricity is not storable, thus buy-and-hold strategies on the spot are just impossible : $F_t(T) \neq P_t e^{r(T-t)}$
- Electricity is created via transformation of storable commodities
- Delivery periods forward contracts: next day, week or month ; quarterly ; yearly
- European options on forward (quarterly, yearly)

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Introduction II : Toy model

Imagine an fictitious economy where electricity is produced only out of coal, so that electricity spot price $P_t = c_c S_t^c$, then one can at time t:

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Under NA assumption we have the following relation:

$$F_0^e(T) = c_c F_0^c(T)$$

The Model

- **Randomness :** $(W^0, W) = (W^0, W^1, \ldots, W^n)$ Wiener process defined on a given $(\Omega, \mathcal{F}, \mathbb{P})$. $\mathcal{F}_t^0$ and $\mathcal{F}_t^W$ are their natural filtrations.
- Riskless asset $S_t^0 = S_0 \exp rt$, $r, t \geq 0$.
- **Commodities market:** $n \geq 1$ commodities (coal, gas, ...) whose prices S^i to produce 1 MWh of electricity follows

$$dS_t^i = S_t^i(\mu_t^i dt + \sum_j \sigma_t^{ij} dW_t^j), \quad t \geq 0.$$

For simplicity, assume that convenience yields $y^i = 0$ for all $i = 1, \ldots, n$.
- **Electricity demand:** $D = (D_t)_{t \geq 0}$ $\mathcal{F}_t^0$-adapted, (positive) process; notice that D is independent of each S^i.

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Electricity spot price P_t when all technologies are available :

- Order commodities prices $S_t^{(1)}(\omega) \leq \ldots \leq S_t^{(n)}(\omega)$ from the cheapest to the most expensive, giving an $\mathcal{F}_t^W$-adapted *random* permutation $\pi_t(\omega)$ of $\{1, \ldots, n\}$
- $\Delta^i > 0$ denotes the maximal capacity of i-th commodity for electricity at every instant, a constant known to the producer
- Look at the demand D_t:

$$D_t \in I_k^{\pi_t} := \left[\sum_{i=1}^{k-1} \Delta^{\pi_t(i)}, \sum_{i=1}^{k} \Delta^{\pi_t(i)}\right) \Rightarrow P_t = S_t^{(k)}$$

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Technologies failures I : The case of two commodities

- If $n = 2$ we have $S_t^1 \leq S_t^2$ or $S_t^2 \leq S_t^1$, let's consider the first case $\pi_t = \{1, 2\}$
- Introduce two r.v.'s Δ_t^i, $i = 1, 2$ such that
  - $\Delta_t^i, i = 1, 2$ are independent and have their own natural filtration $\mathcal{F}_t^\Delta$
  - $\Delta_t^i = M_i$ when technology i is fully available.
  - $\Delta_t^i = m_i$ when technology is partially available.
- Four cases may happen at each time t
  1. $\Delta_t^1 = M_1, \Delta_t^2 = M_2$
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- To sum up: $P_t = S_t^1 \mathbf{1}_{[0, \Delta_t^1)}(D_t) + S_t^2 \mathbf{1}_{[\Delta_t^1, \Delta_t^1 + \Delta_t^2)}(D_t)$

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No-arbitrage assumption on commodities market

Let $T > 0$. There exists $\mathbb{Q} \sim \mathbb{P}$ on $\mathcal{F}_T^W$ such that :

1. each $\tilde{S}^i / S^0$ is a $\mathbb{Q}$-martingale w.r.t. $\mathcal{F}^W$
2. the laws of W^0 and Δ_t^i for all i do not change
3. filtrations $(\mathcal{F}_t^0), (\mathcal{F}_t^W), (\mathcal{F}_t^\Delta)$ are $\mathbb{Q}$-independent

Remarks

1. Property 3 above is satisfied if W^0, W and Δ^i are constructed on the canonical product space and the change of measure affects only the factor where W is defined. Such a $\mathbb{Q}$ is usually called "minimal martingale measure".
2. Since D is not tradable, this market is not complete. We choose $\mathbb{Q}$ as the pricing measure.
3. Notation: $\mathcal{F}_t = \mathcal{F}_t^0 \vee \mathcal{F}_t^W \vee \mathcal{F}_t^\Delta$ is the market filtration.

Electricity forward prices : The main formula

Proposition

Under previous assumptions and if $S^i_T \in L^1(\mathbb{Q}_T)$, $1 \leq i \leq n$ *: for all* $t \in [0, T]$

$$F_t(T) = \sum_{i=1}^{n} \sum_{\pi \in \Pi_n} c_{\pi(i)} F_t^{\pi(i)}(T) \mathbb{Q}[D_T \in I_i^\pi(T) | \mathcal{F}_t^0] \times \mathbb{Q}^{\pi(i)}[\pi_T = \pi | \mathcal{F}_t^W]$$

where :

- Π_n *is the set of all permutations of* $\{1, \ldots, n\}$
- $F^i_t(T)$ *is forw. price of* 1 *unit of commodity* i, *maturity* T
- $d\mathbb{Q}^{\pi(i)}/d\mathbb{Q} = S_T^{\pi(i)}/\mathbb{E}^{\mathbb{Q}}[S_T^{\pi(i)}]$ *on* $\mathcal{F}_T^W$

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The constant coefficients model : more explicit formulae

- Commodities prices S^i follow n-dim Black-Scholes model : volatilities $\sigma^{ij} > 0$ and interest rate $r > 0$ constant.
- Production capacity Δ_t^i are independent componed Poisson processes with two values $(M_i > m_i)$
- Demand of electricity : D follows a OU process

$$dD_t = a(b(t) - D_t)dt + \delta dW_t^0, \quad D_0 > 0$$

with $a, b(t), \delta > 0$. $b(t)$ stands for seasonnality in Demand.

- Under these assumptions probabilities $\mathbb{Q}[D_T \in I_k^\pi(T)|\mathcal{F}_t^0]$ and $\mathbb{Q}^{\pi(i)}[\pi_T = \pi|\mathcal{F}_t^W]$ can be *computed explicitly as functions of the parameters.*
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Back to reality I : what do we know?

We focus on French market, and two technologies :

- Gas plants (gas and CO2 prices)
- Fuel combustion turbines (fuel and CO2 prices)

Several approximations :

- We select midday hourly prices on peakload to ensure the only use of these technologies (this implies knowledge on demand)
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Back to reality II : how do we proceed?

We can calibrate the model:

- On spot prices to price forward assets
- On forward prices to adjust meaningful parameters

The data we calibrate on are

- S^i or $F^i(T)$ for technologies costs (spot and forward)
- P_t or $F_t(T)$ for electricity spot and forward prices
- R^i_t residual demand for i-th technology given by :

$$R^i_t = \min\left\{\Delta^i_t, \left(D_t - \sum_{k=1}^{i-1}\Delta^k_t\right)^+\right\}$$

where D_t stands for total demand (sum of residuals).

Consequence : R^1 and R^2 allows for straight spot price fitting.

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Spot price historical fitting

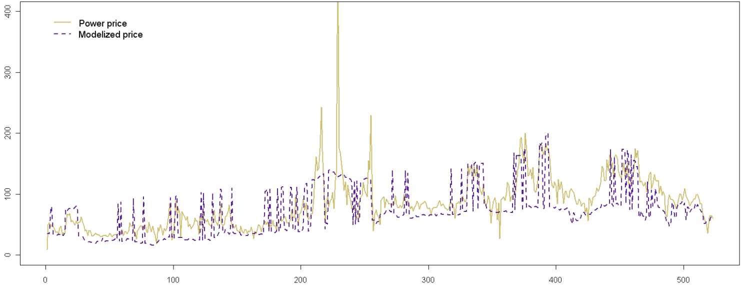
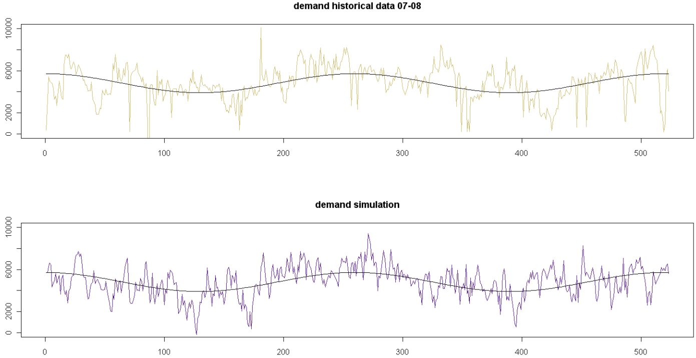


Figure: Spot price fitting with technologies spot prices and residual demand. Daily data from January 2007 to December 2008.

Estimation method

- Demand process is estimated via MLE.



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- Capacity process :

$$d\Delta_t^1 = (m_1 - M_1)\mathbf{1}_{(\Delta_t^1 = M_1)} dN_t^{1,d} + (M_1 - m_1)\mathbf{1}_{(\Delta_t^1 = m_1)} dN_t^{1,u}$$

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$\Delta_0^1 = M_1$. We estimate intensities $\lambda^{1,d}$ and $\lambda^{1,u}$.
Two estimation problems :

1. The observed process is not pure jump ($\Delta_t^1 \neq m_1$ or M_1)
2. Data is truncated (we observe Δ_t^1 knowing $D_t > \Delta_t^1$)

Estimation method

- Demand process is estimated via MLE.
- Capacity process :

$$d\Delta_t^1 = (m_1 - M_1)\mathbf{1}_{(\Delta_t^1 = M_1)} dN_t^{1,d} + (M_1 - m_1)\mathbf{1}_{(\Delta_t^1 = m_1)} dN_t^{1,u}$$

$\Delta_0^1 = M_1$. We estimate intensities $\lambda^{1,d}$ and $\lambda^{1,u}$.
Two estimation problems :

1. The observed process is not pure jump ($\Delta_t^1 \neq m_1$ or M_1)
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Goal : we compute $\mathbb{Q}[D_T > \Delta_T^1 | \mathcal{F}_t^0]$ and $\mathbb{Q}[\Delta_T^1 = M_i | \mathcal{F}_t^\Delta]$

Forwards pricing

Forwards have a delivery period (1 month, 1 quarter,..) $\Rightarrow$ the expression must be modified

$$F_t(T_1, T_2) = \frac{1}{T_2 - T_1} \sum_{i=1}^{n} \sum_{\pi \in \Pi_n} \int_{T_1}^{T_2} F_t^{\pi(i)}(T) \times \mathbb{Q}_T^{\pi(i)}[\pi_T = \pi | \mathcal{F}_t^W] \mathbb{Q}[D_T \in I_i^\pi(T) | \mathcal{F}_t^{0,\Delta}] dT$$

Approximations again :

- $F_t^i(T)$ are replaced by prices on delivery period $F_t^i(T_1, T_2)$
- Integral is numerically computed

Forwards prices results

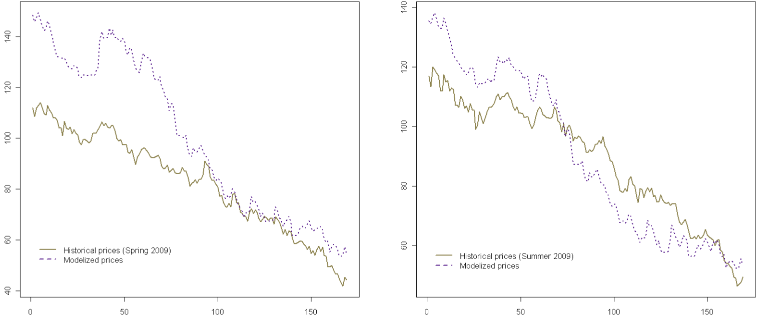

Figure: Historical forward prices (2QAH and 3QAH) and modelized forward prices.

Forwards prices results

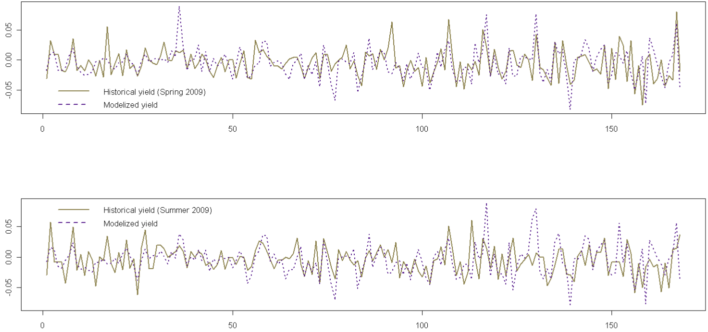

Figure: Historical forward yields (2QAH and 3QAH) and modelized forward yields. Correlation with historical yield : 0.4178213 (2QAH) and 0.4103652 (3QAH)

The other side : calibration on forwards

Calibration issues :

- Which parameters to calibrate?
  - Demand process is supposed to be known
  - Capacity thresholds are the same

  $\Rightarrow$ Intensity of jump or probability of failure $\left(\frac{\lambda^{i,d}}{\lambda^{i,d}+\lambda^{i,u}}\right)$
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Example : we find $P(\Delta_T = M_1, T \in [T_1, T_2)|\mathcal{F}_t) = 0.865$ for Summer 2009 forward.

Back to reality III : what next?

What is achieved?

- The simple model allows for basic cross hedging
- The coherence even with several approximations
- The demand adds significative information to the link between prices
- The residual demand is a fundamental data for fitting

What next?

- Refine capacity evolution
- Look for other potentially useful data
- Add commodities (and improve their models)
- Extend to option pricing
- Analyse the risk premium $F_t(T) - P_t$

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Thank you for your attention!