Dynamic Contracts Under Loss Aversion

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Motivation I

- Framework: Principal-Agent and contracts.
- Hidden action problems: trade-off between incentives and insurance.
  - Reward the agent (employee) most for outcomes that are more likely to arise when he puts in more effort.
  - Punish the agent most for outcomes that are more likely to occur when he shirks.
- Predicted optimal contracts by theory are often very complex.

Motivation II

- May be even more complex in dynamic contexts.
- Why? Richer environment:
  - intertemporal risk-sharing - the agent can self-insure,
  - repeated output observations - more information is revealed,
  - larger set of available actions to the agent.
- Additional result: optimal contract depends on the entire history of outputs.

Motivation III

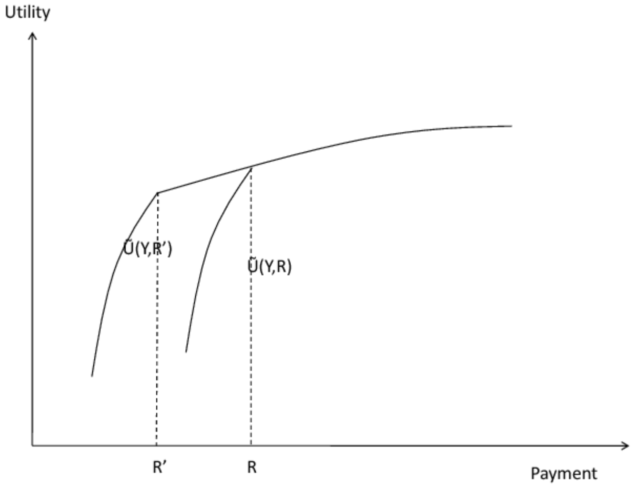
- Real life contracts seem very simple, with little fine tuning between outputs and payments (Bolton and Dewatripont, 2005; Prendergast, 1999)
- In addition, real wages are highly persistent (Dickens at al., 2007).
- Discussion on whether observed contracts are optimally designed.
- There are many reasons why theoretical predictions may be unrealistically complex; e.g., multidimensional incentive problems or career concerns.

Modeling

- In this paper we explore the case of a loss averse agent and dynamic:
- We introduce reference dependent preferences and loss aversion - Kahneman and Tversky (1979) - to the dynamic moral hazard principal-agent model
- Utility has a gain/loss component.
- Losses loom larger than gains.

The agent's utility function

Figure: Utility for different reference points.



Analyses and results I

- We analyze the dynamic optimal contracting problem of Rogerson (1985) with reference dependent preferences and loss aversion.
- We find similarities with the classical model, i.e.
    - non decreasing optimal schemes,
    - memory,
    - consumption smoothing and
    - renegotiation proofness.

Analyses and results II

- But we also find relevant differences:
  - Optimal payment schemes may be insensitive to outcomes in an interval, as De Meza and Webb (2007) proved in a one period setup.
  - There is a positive probability of constant wages over time.
  - Incentives may be postponed until the last period.
  - When allowed to borrow and save, the agent might prefer to consume his full income –status quo bias.

Outline

- The model.
- The optimal payment scheme.
- Intra period and intertemporal properties.
- Two period example.
- Concluding remarks.

The model

- We follow Rogerson (1985).
- Principal-agent relationship lasts $T+1$ periods.
- In each period i the agent chooses an unobservable action $a_i \in \{a_L, a_H\}$ where $a_L < a_H$.
- The outcome in period i is $x_i \in [\underline{x}_i, \bar{x}_i]$ with a differentiable probability distribution function $f^i(x_i|a_i)$.
- MLRP (Monotone likelihood ratio property) holds, i.e., if $f^i_{a_i}(x_i|a_i) = f^i(x_i|a_H) - f^i(x_i|a_L)$, then $f^i_{a_i}(x_i|a_i)/f^i(x_i|a_i)$ is increasing in x_i.
- Let the wage schedule in i be $\omega_i(x_0, x_1, \ldots, x_i)$ and let consumption be c_i.
- The agent has no access to credit markets.
- Full commitment.

The agent's utility function

$$\tilde{U}(c_i, R_i) - \psi_i(a_i) \tag{1}$$

- R_i is the reference point in period i.
- $\psi_i(\cdot)$ is an increasing and convex cost function.
- $\tilde{U}$ is continuous.
- We assume loss aversion around the reference point

$$\lim_{t \to 0^+} \frac{\tilde{U}_i(R+t, R) - \tilde{U}_i(R, R)}{t} < \lim_{t \to 0^+} \frac{\tilde{U}_i(R-t, R) - \tilde{U}_i(R, R)}{-t}$$

The agent's utility function

For $\ell_0 > 0$, an exogenous reference level R_0 and a smooth, concave and strictly increasing function $U(\cdot)$, without loss of generality, the period 0 utility, $\tilde{U}_0$ can be written as,

$$\tilde{U}_0(c_0, R_0) = U(c_0) - \ell_0 \theta(c_0, R_0) \left(U(R_0) - U(c_0) \right)$$

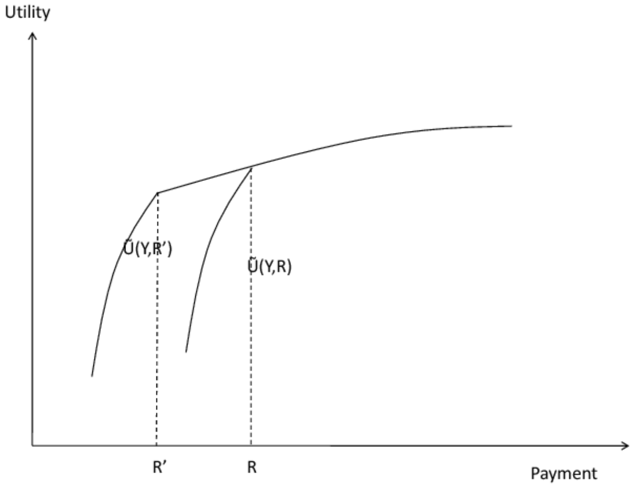
where

$$\theta(c, R) = \begin{cases} 1 & \text{if } c < R \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

- Utility is non-increasing on the reference.
- $\tilde{U}_0$ is non-differentiable at the reference point.

The agent's utility function

Figure: Utility for different reference points.



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where

$$\theta(c, R) = \begin{cases} 1 & \text{if } c < R \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

- Utility is non-increasing on the reference.
- $\tilde{U}_0$ is non-differentiable at the reference point.

The agent's utility function

- We assume that for periods $i > 0$, the $R_i{=}c_{i-1}$.
- Same as in Bowman et al. (1999) and Munro and Sugden (2003).

$$\tilde{U}_{i+1}(c_{i+1}, c_i) = U(c_{i+1}) - \ell_{i+1}\theta(c_{i+1}, c_i)\left(U(c_i) - U(c_{i+1})\right)$$

The principal's utility function

- We assume a risk neutral principal
- His period i payoff is $x_i - \omega_i(x_i)$.
- Discount factor δ.

The principal's problem

$$\max_{(\omega_i(\cdot))_i,(a_i)_i} \sum_{i=0}^{T} \delta^i \mathbb{E}\left(x_i - \omega_i(x_0, x_1, \ldots, x_i) | a_0, a_1, \ldots, a_i\right)$$

subject to

$$\sum_{i=0}^{T} \delta^i \left(\mathbb{E}\left(\tilde{U}_i(\omega_i(x_0, x_1, \ldots, x_i), c_{i-1}) | a_0, a_1, \ldots, a_i\right) - \psi_i(a_i)\right) \geq U^* \tag{PC}$$

$$a = (a_0, a_1(x_1), \ldots a_T(x_0, x_1, \ldots, x_T)) \in$$
$$\text{argmax}_a \sum_{i=0}^{T} \delta^i \left(\mathbb{E}\left(\tilde{U}_i(\omega_i(x_0, x_1, \ldots, x_i), c_{i-1}) | a_0, a_1, \ldots, a_i\right) - \psi(a_i)\right) \tag{IC}$$

Optimal scheme

For all $i < T$

$$\frac{1}{U'(\omega_i(x_0, x_1, \dots, x_i))} = (1 + k_i(x_0, x_1, \dots, x_i)\ell_i)\left(\lambda_i + \mu_i \frac{f^i_{a_i}(x_i|a_i)}{f^i(x_i|a_i)}\right) +$$

$$-\delta\ell_{i+1}\int_{\omega_{i+1}\le\omega_i} k_{i+1}(x_0, x_1, \dots, x_{i+1})(\lambda_{i+1} + \mu_{i+1}\frac{f^{i+1}_{a_{i+1}}(x_{i+1}|a_{i+1})}{f^{i+1}(x_{i+1}|a_{i+1})})f^{i+1}(x_{i+1}|a_{i+1})dx_{i+1}.$$

$$k_i(x_0, x_1, \dots, x_i) \in \begin{cases} \{1\} & \text{if } \omega_i(x_0, x_1, \dots, x_i) < R_i \\ [0,1] & \text{if } \omega_i(x_0, x_1, \dots, x_i) = R_i \\ \{0\} & \text{otherwise} \end{cases}$$

- $\lambda_i = \lambda + \sum_{k=0}^{i-1} \mu_k \frac{f^k_{a_k}(x_k|a_k)}{f^k(x_k|a_k)}$, with λ a multiplier associated to (PC) and $\mu_i = \mu_i(x_0, \dots, x_{i-1})$ the multipliers associated to the incentive compatibility constraints.

Optimal scheme II

- First order conditions with subgradients set - convex analysis.
- Classical case if $\ell_i = 0\ \forall i$; loss aversion if $\ell_i > 0$ for some i.
- Payments today affect future references.

Optimal scheme

For $i = T$

$$\frac{1}{U'(\omega_T(x_0, x_1, \ldots, x_T))} = (1 + k_T(x_0, x_1, \ldots, x_T)\ell_T)\left(\lambda_T + \mu_T \frac{f^T_{a_T}(x_T|a_i)}{f^i(x_i|a_i)}\right) \tag{5}$$

Optimal scheme

The scheme balances different effects:

- Payments over the reference provide relatively low marginal utility.
- But payments under the reference strain the PC.
- In addition, a lower payment today reduces tomorrow's reference, increasing the agent's utility in the loss area.

Optimal scheme

Thus,

- A payment that gives the reference for an outcome, might pay the reference for close outcomes as well.
- We observe flat segments at the reference, that may extend for the whole support of outcomes.
- Except for period T: incentives may be deferred to the last period.

Possible payment schemes

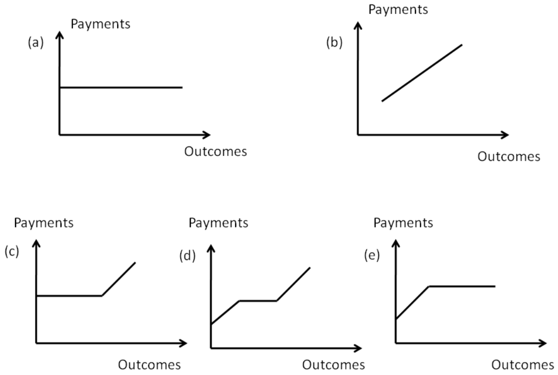


Figure: Schematic representation of monotonicity of contracts

(a) possible in $\{0, \ldots, T-1\}$, (b) possible in period 0, and (c), (d), (e) in every period.

Shape of the optimal contract

- schemes that are more realistic
- examples: options or "tenure track"

Dependence on outcomes' history

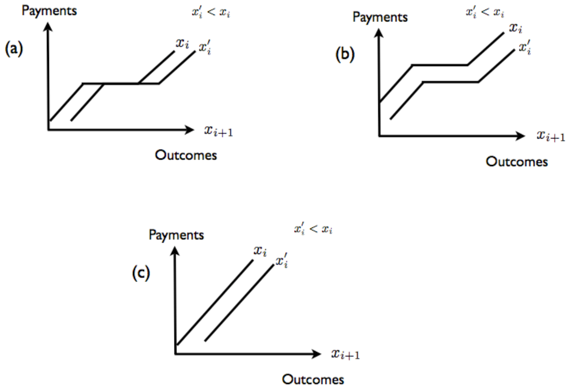


Figure: Dependence across periods

- Classical case: a higher payment in one period leads to higher payments in all subsequent periods.
- Same here, but overlaps are possible.

Consumption smoothing

- In Rogerson (1985) the inverse of the marginal utility of consumption equals the conditional expected value of the inverse of marginal utility.
- Here this equality might not hold.
- The principal takes into account the cost of changing future references.

Relationship between consecutive periods

$$\frac{1}{U'(\omega_{i-1}(x_{i-1}))(1+k_{i-1}(x_{i-1})\ell_{i-1})} = \int \frac{1}{U'(\omega_i(x_i))(1+k_i(x_i)\ell_i)} f^i(x_i|a_i)dx_i + c(x_{i-1})$$

where

$$\begin{aligned} c(x_{i-1}) &= -\frac{\ell_i \delta}{1+k_{i-1}(x_{i-1})\ell_{i-1}} \int k_i(x_i) \left(\lambda_i + \mu_i \frac{f^i_{a_i}(x_i|a_i)}{f^i(x_i|a_i)} \right) f^i(x_i|a_i)dx_i + \\ & \ell_{i+1}\delta \int \int \frac{k_{i+1}(x_{i+1})}{1+k_i(x_i)\ell_i} \left(\lambda_{i+1} + \mu_{i+1} \frac{f^{i+1}_{a_{i+1}}(x_{i+1}|a_{i+1})}{f^{i+1}(x_{i+1}|a_{i+1})} \right) f^{i+1}(x_{i+1}|a_{i+1}) f^i(x_i|a_i) dx_{i+1} dx_i \end{aligned}$$

Status quo bias

- In Rogerson (1985) the agent is not fully insured and is left with the desire to save.
- We find that if allowed to save or borrow, the agent might prefer to consume his allocation.
- Marginal utility of saving not equal to - (marginal utility of borrowing).
- Infimum interest rate that motivates savings > supremum interest rate willing to take a loan.

Two period example

Distributions of outcomes $x_i \in [0,1]$ in periods $i \in \{1,2\}$ for actions $a_j \in \{a_L, a_H\}$ is triangular:

$$f^i(x_i|a_j) = \begin{cases} \frac{2x_i}{a_j} & x_i \leq a_j \\ \frac{2(1-x_i)}{1-a_j} & x_i > a_j \end{cases}$$

and $U(Y) = \sqrt{Y}$. Thus,

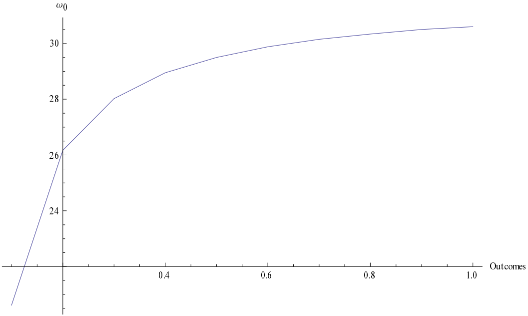
$$\widetilde{U}_i(Y_i, R_i) = \sqrt{Y_i} - \theta(Y_i, R_i)\ell_i(\sqrt{R_i} - \sqrt{Y_i})$$

To solve

- We assume $\ell_0 = 1,\ \ell_1 = 1,\ a_H = 1,\ a_L = 0.1$
- First period computed using fixed point algorithm computed for only some values of x_0. $x_0 \in \{0, 0.1, 0.2, ...0.9, 1\}$

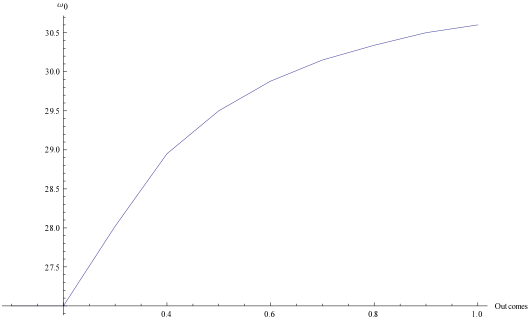
First period scheme

For $1/U'(R_0) < 20.61$ we obtain



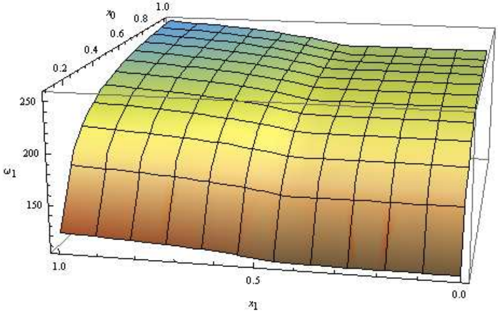
First period scheme

For $1/U'(R_0) = 27$



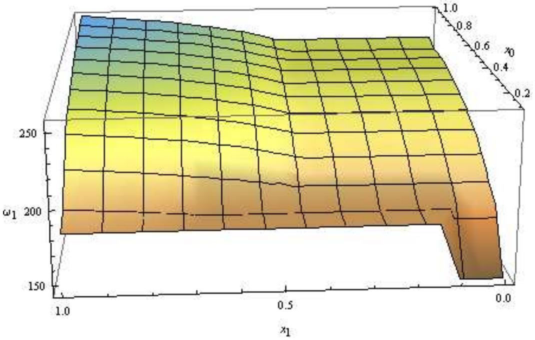
Second period payment schemes

If the references for the first period satisfy $1/U'(R_0) < 20.61$ then schemes for second period are:



Schemes for second period

If the reference for the first period is such that $1/U'(R_0) = 27$ then, second period scheme is:



Concluding remarks

- Many of the properties of the canonical moral hazard dynamic contracts model hold when loss aversion is introduced.
- But it also predicts new features:
  - Flats in the schedules.
  - Persistent wages.
  - Incentives that are deferred into the future.
- More realistic wage schedules in any given period and over time.