A structural risk-neutral model for pricing and hedging power derivatives

FiME Research Centre Monthly Seminar - Paris

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Paris-Dauphine University - Paris Diderot University
EDF R&D - FiME Research Centre

Agenda

Looking for a power spot price model

Applications

- pricing of derivatives on the spot
- asset valuation (strip of hourly fuel spread options)
- hedging
- energy market risk management

Model requirements

- realistic
- robust
- tractable
- consistent

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- **pros** modeling the real available instruments
- **cons** introduction of many parameters to reconstruct hourly futures prices

Modeling spot prices

Exogenous

Equilibrium

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Electricity prices exogeneous dynamics

Deng (00), Benth et al. (03, 07, 09), Burger et al. (04), Kolodnyi (04), Cartea & Figueroa (05), Geman & Roncoroni (06)

Equilibrium model

	Spot	Futures	Options
Pirrong & Jermakyan (00)	×	×	
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pricing and hedging power derivatives...

... using an improved version of A., Campi Nguyen & Touzi (09) Structural Risk-Neutral model

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Initial SRN Model

Variables

n	fuels, $1 \leq i \leq n$
D_t	demand (MW)
C_t^i	capacities (en MW)
S_t^i	fuel prices
h_i	*heat rates* ($h_i S_t^i$ en €/MWh, ↗ en i)

Electricity price (€/MWh)

$$\widehat{P}_t = \sum_{i=1}^{n} h_i S_t^i \mathbf{1}_{\left\{\sum_{k=1}^{i-1} C_t^k \leq D_t \leq \sum_{k=1}^{i} C_t^k\right\}}$$

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Initial SRN model

Pros

- Consistency between electricity prices and fuel prices
- Consistency between electricity prices and demand

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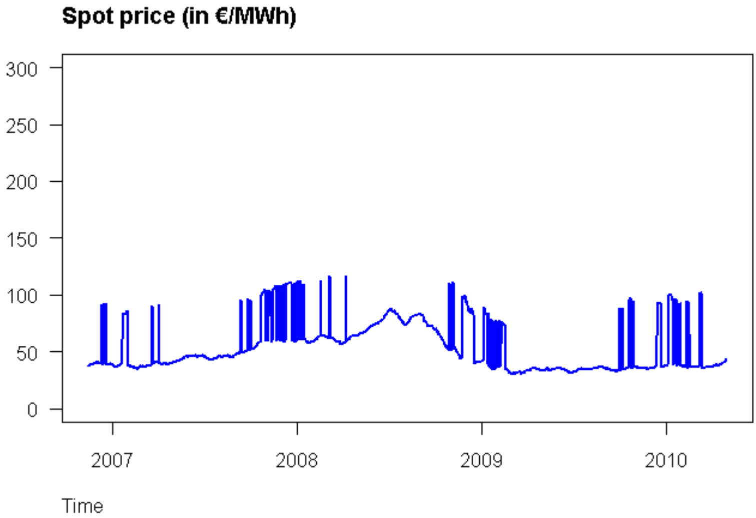
Initial SRN Model - illustration



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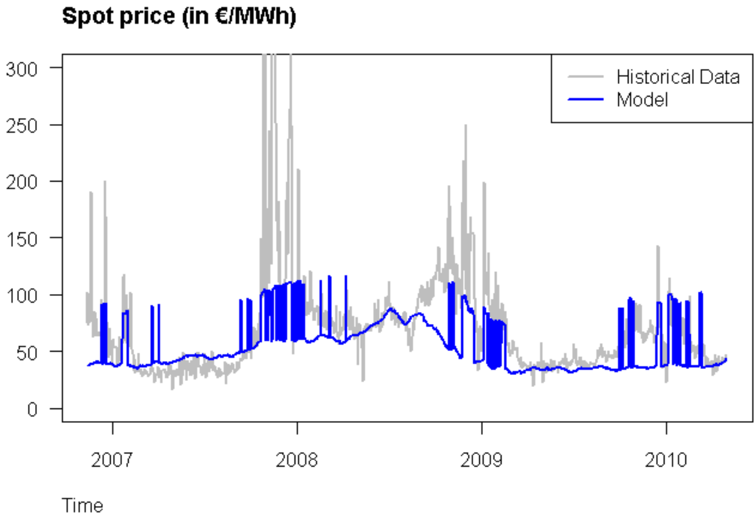

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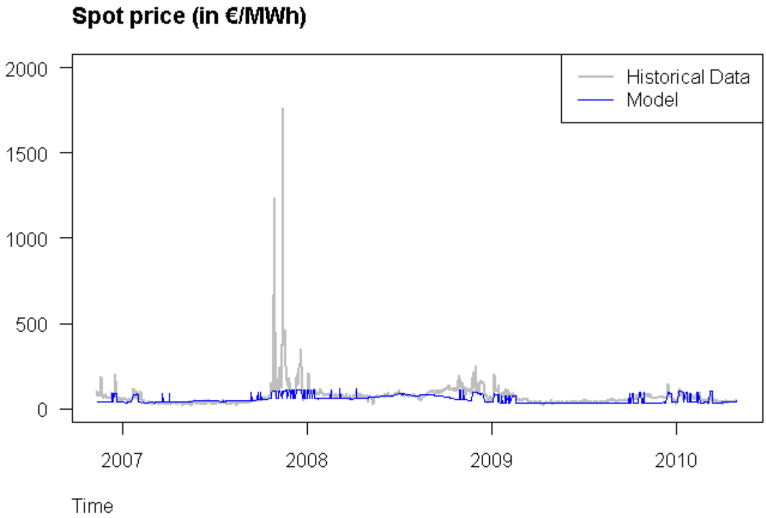

Improved SRN model

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$y_t := \frac{P_t}{\widehat{P}_t}$ **as a (nonlinear) function of** $x_t := \overline{C}_t - D_t$

Improved SRN model - Estimation

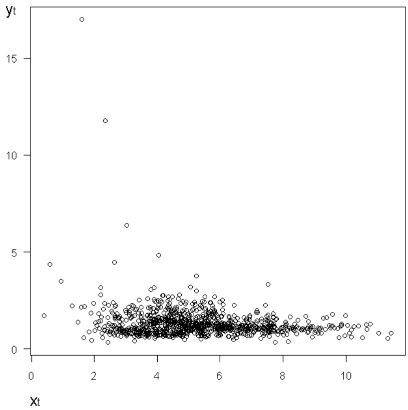


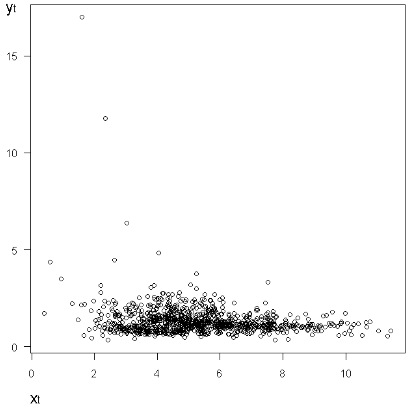
Figure: PowerNext - 19th hours
Nov, 13th 06 to April 30th 10

Observation

- Decreasing relation
- Difficult estimation

Idea

Improved SRN model - Estimation



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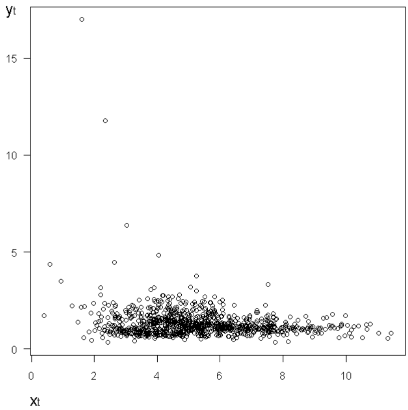

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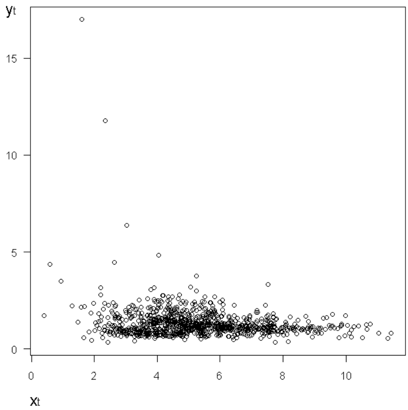

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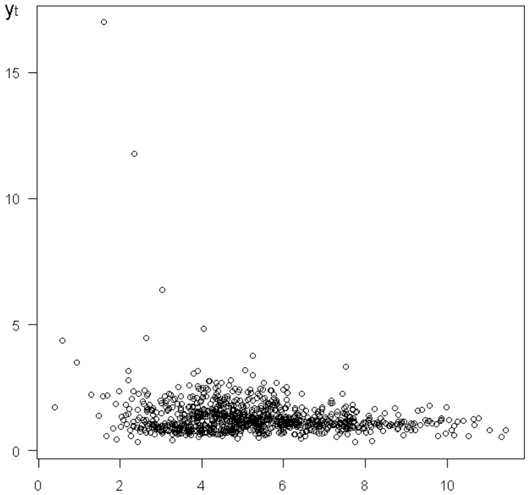

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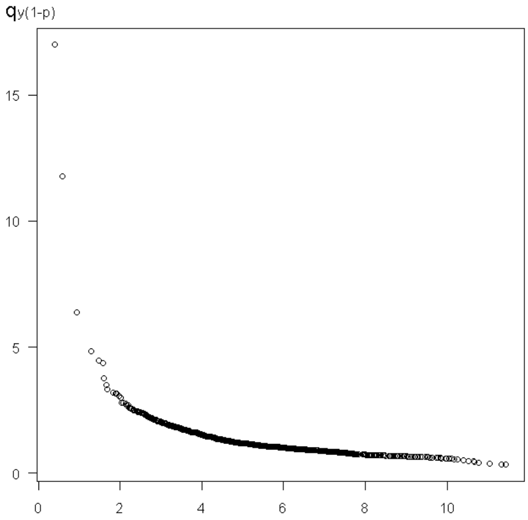

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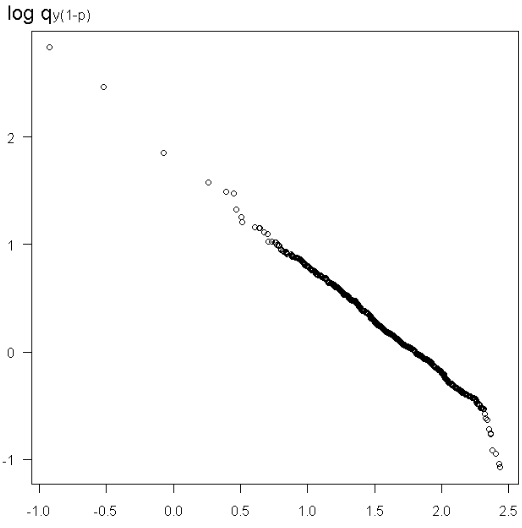

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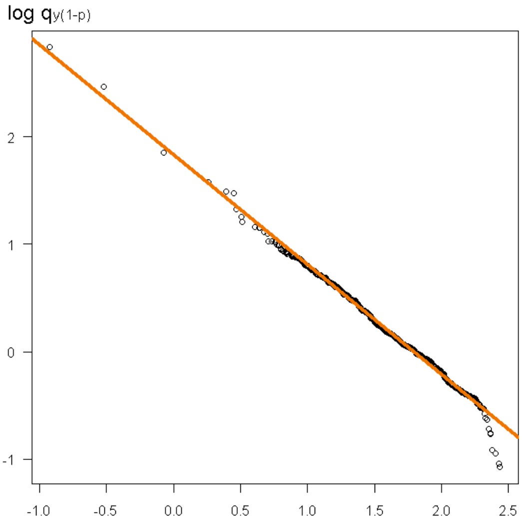

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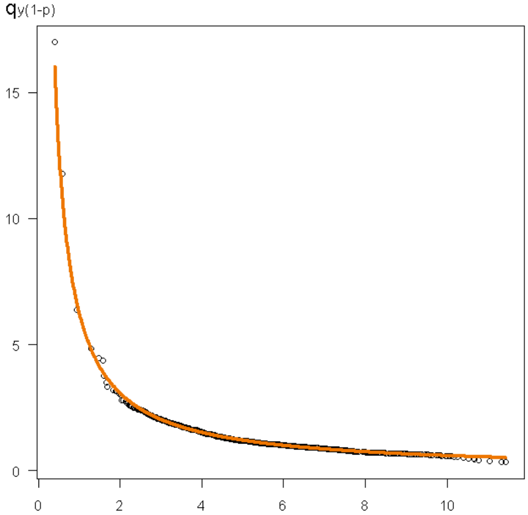

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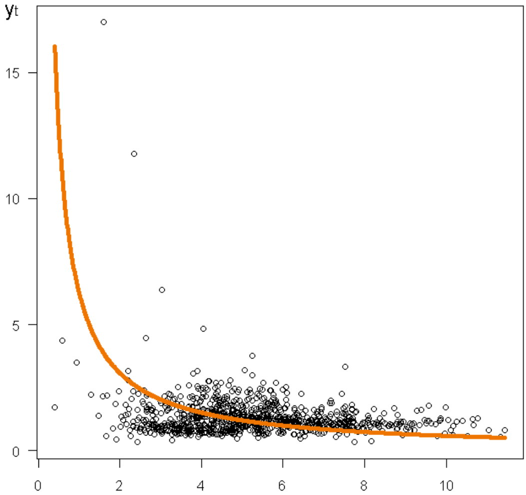


Improved SRN model - Estimation

Estimated relation : $y_t = \frac{\gamma}{x_t^\nu}$

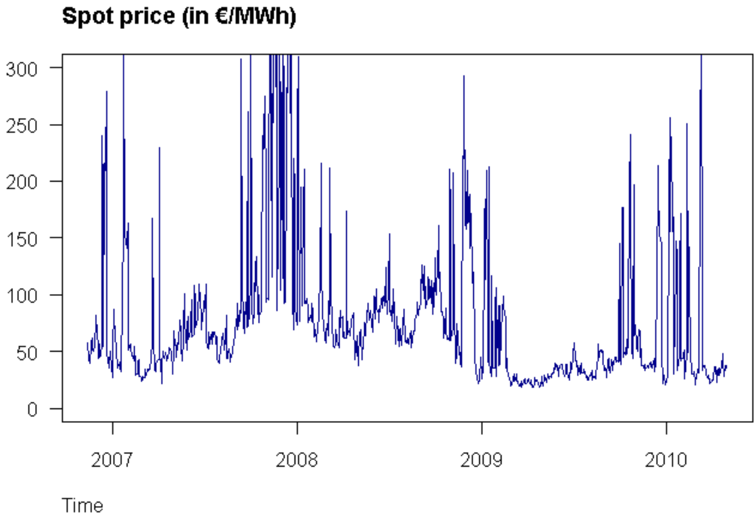
Improved SRN model

$$P_t = g\left(\sum_{k=1}^{n} C_t^k - D_t\right) \times \left(\sum_{i=1}^{n} h_i S_t^i \mathbf{1}_{\left\{\sum_{k=1}^{i-1} C_t^k \leq D_t \leq \sum_{k=1}^{i} C_t^k\right\}}\right)$$

with scarcity function

$$g(x) := \min\left(\frac{\gamma}{x^\nu}, M\right) \mathbf{1}_{\{x>0\}} + M\mathbf{1}_{\{x\leqslant 0\}}$$

Improved SRN model - Back-testing



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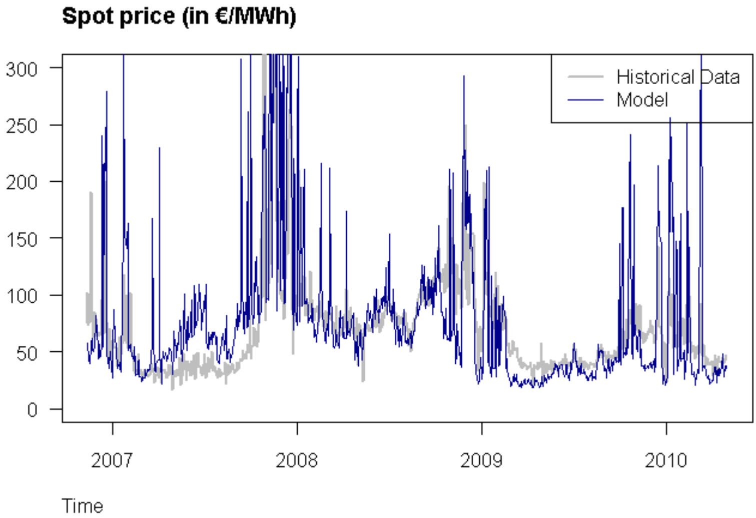

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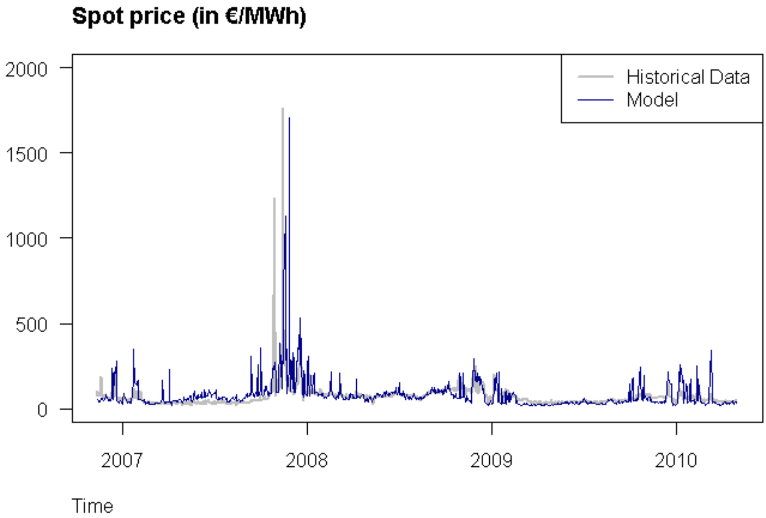

Pricing & hedging

Pricing

- incomplete market
- need for a hedging criterion
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Futures

Futures prices $F_t^e(T) = \mathbb{E}_t^{\widehat{\mathbb{Q}}}\left[e^{-r(T-t)}P_T\right]$

$$F_t^e(T) = \sum_{i=1}^{n} h_i G_i^T(t, C_t, D_t)\, F_t^i(T)$$

with :

$$G_i^T(t,C_t,D_t) = \mathbb{E}_t\left[g\left(\sum_{k=1}^{n} C_T^k - D_T\right)\mathbf{1}_{\left\{\sum_{k=1}^{i-1} C_T^k \le D_T \le \sum_{k=1}^{i} C_T^k\right\}}\right]$$

Futures prices - hedging

Demand & capacities

$$dD_t = a\left(t, D_t\right) dt + b\left(t, D_t\right) dW_t^D$$
$$dC_t^i = \alpha_i\left(t, C_t^i\right) dt + \beta_i\left(t, C_t^i\right) dW_t^{C,i}$$

Futures price dynamics

$$dF_t^e(T) = \sum_{i=1}^n h_i \left[G_i^T(t, C_t, D_t) dF_t^i(T) + F_t^i(T) dG_i^T(t, C_t, D_t)\right]$$

$$dG_i^T(t, C_t, D_t) = \sum_{k=1}^n \frac{\partial G_i^T}{\partial c_k}(t, C_t, D_t)\beta_k(t, C_t^k) dW_t^{C,k} + \frac{\partial G_i^T}{\partial z}(t, C_t, D_t) b(t, D_t) dW_t^D$$

Futures prices - hedging

- To go further, need to choose dynamics for demand and capacities

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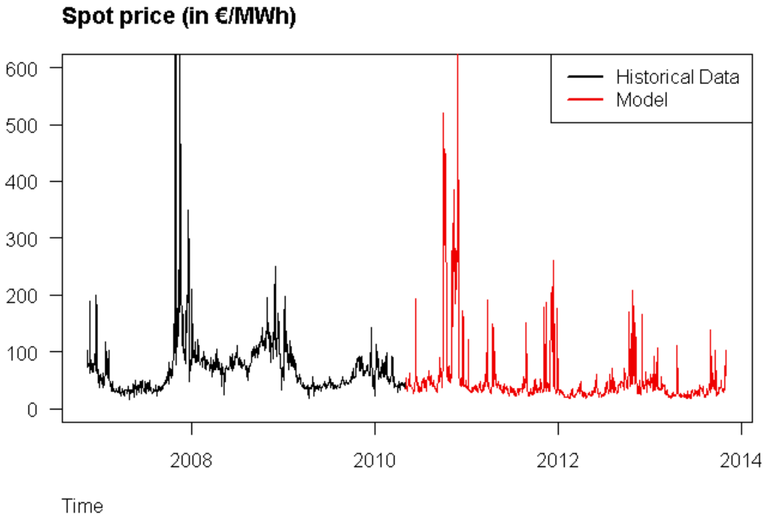
- ... for which efficient numerical algorithms are provided in A., Campi & Langrené (10).

Futures prices - hedging : spot simulations



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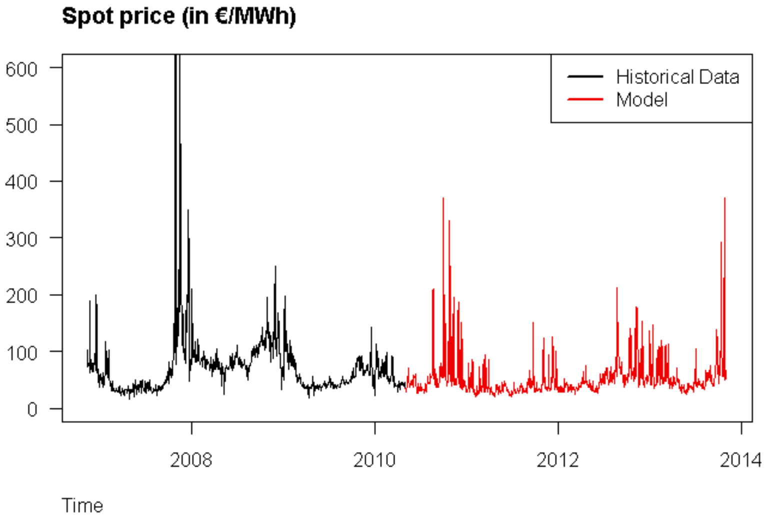

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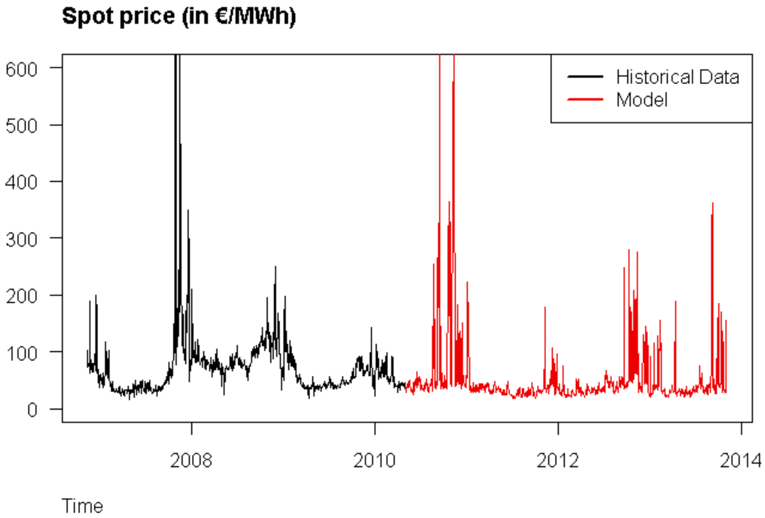

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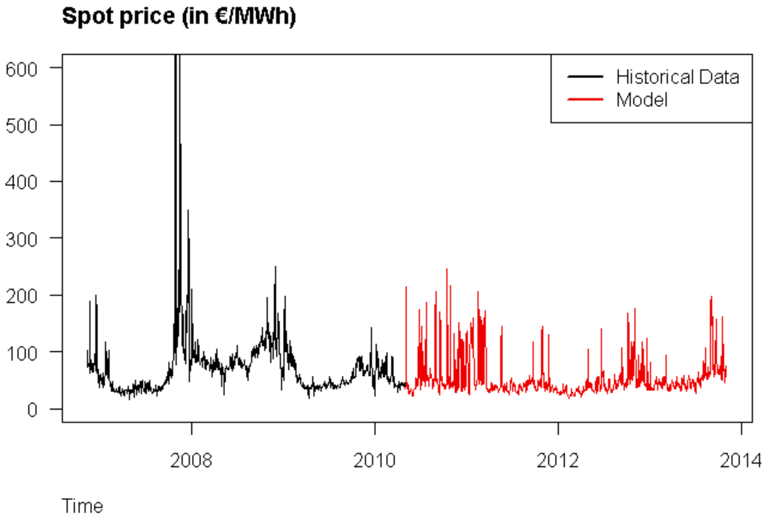

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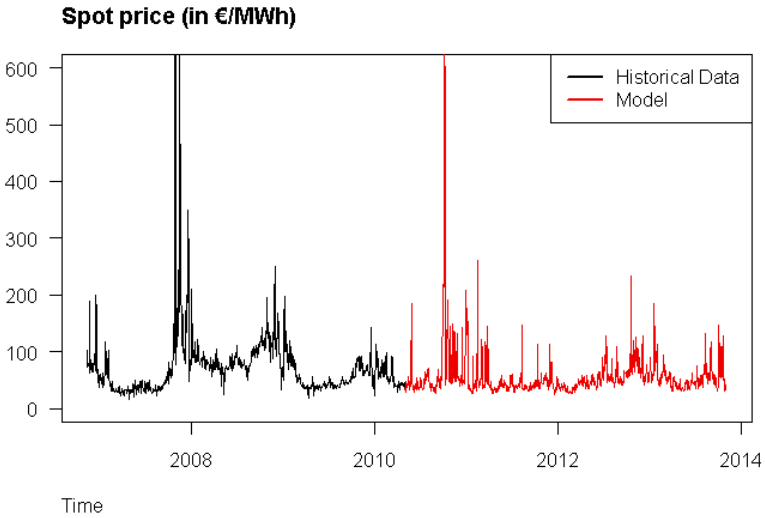

Futures prices - hedging

Numerical test

- Hedging an electricity futures with a delivery period of 1 hour
- with a daily rebalanced basket of futures contracts on fuels

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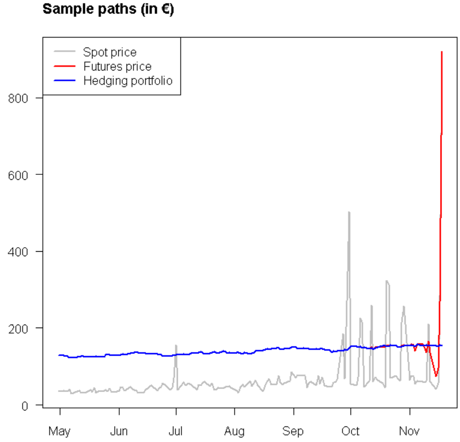

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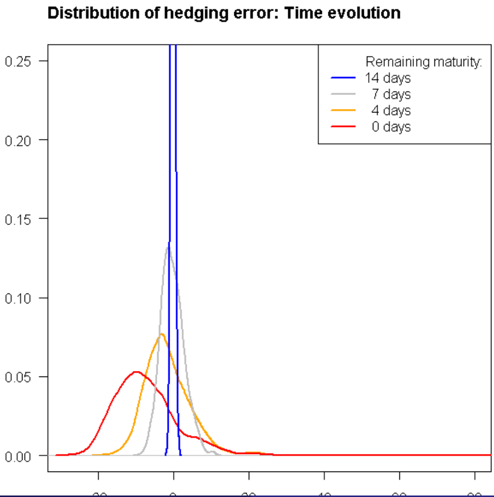
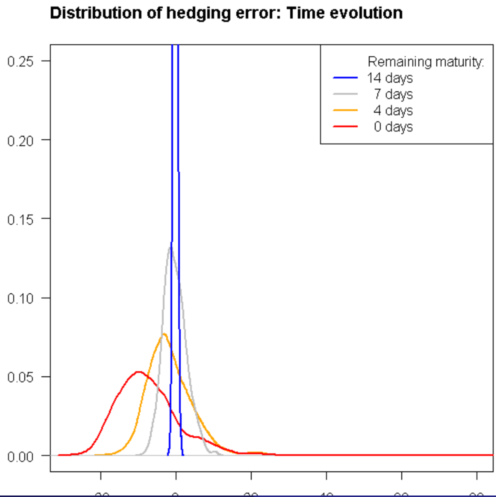


Remarks

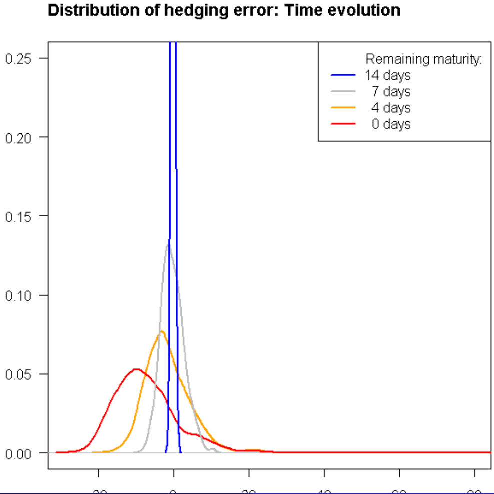
Futures prices - hedging



Remarks

- Positive values are losses

Futures prices - hedging



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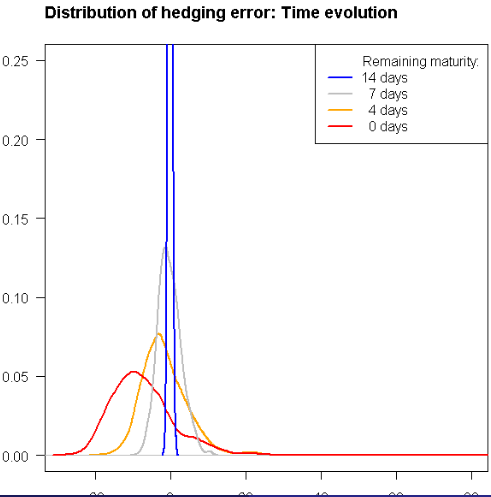

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- Close to maturity : inefficient hedge

Spread options (do not panic)

Spread option with a 2 fuel model

The price π_0 at time $t = 0$ of a call spread option with pay-off $H = (P_T - h_1 S_T^1 - K)^+$ is given by :

$$\pi_0 = \int_{\mathbb{R}^2} f_{C_T^1 - D_T}(z) f_{C_T^2}(c) \left\{ \phi_1(c,z) \mathbf{1}_{\{z>0\}} + \phi_2(c,z) \mathbf{1}_{\{z\leq 0\}} \right\} dcdz,$$

$$\phi_1 = (g-1) BS_0(\sigma_1, K) \mathbf{1}_{\{g>1\}}$$

$$\phi_2 = g \int_0^\infty \hat{f}_{Y_T^1}(y) BS_0 \left(\sigma_2, \frac{K + (1-g)y}{g} \right) \left(\mathbf{1}_{\{g\leq 1\}} + \mathbf{1}_{\{g>1\}} \mathbf{1}_{\{y < \frac{K}{g-1}\}} \right) dy$$

$$+ \left(g Y_0^2 \mathcal{N} \left(\frac{\left(r - \frac{\sigma_1^2}{2}\right) T - \ln\left(\frac{K}{(g-1) Y_0^1} \right)}{\sigma_1 \sqrt{T}} \right) + (g-1) BS_0 \left(\sigma_1, \frac{K}{g-1} \right) \right) \mathbf{1}_{\{g>1\}}$$

with $g := g(c+z)$.

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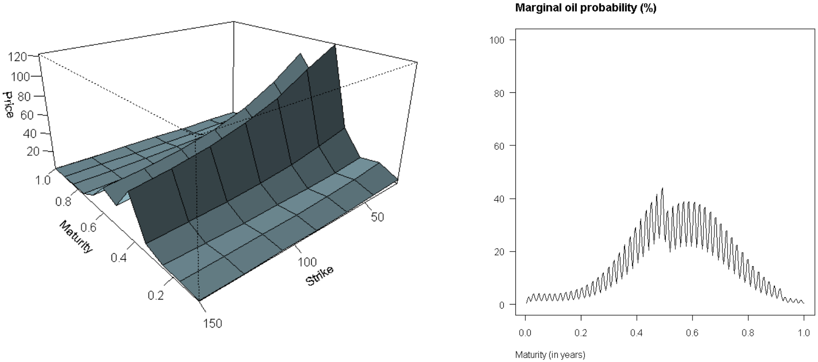

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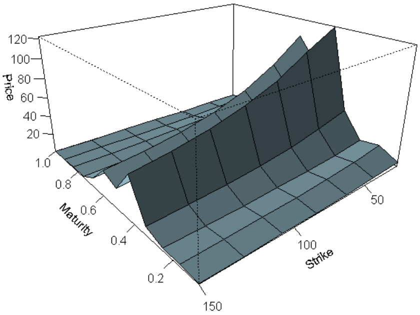
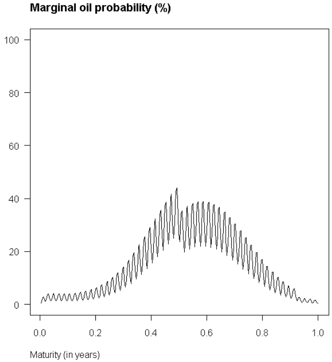
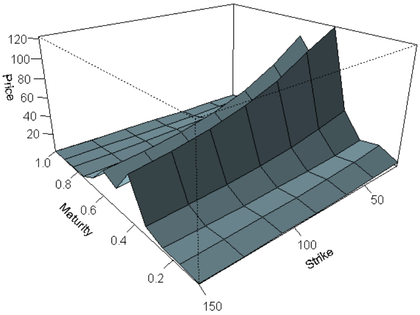
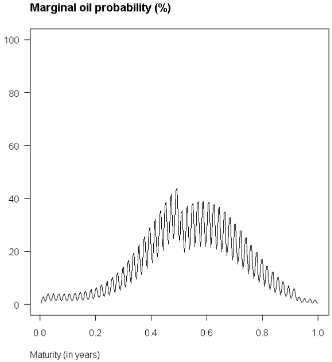




- seasonality pattern

Spread options





- seasonality pattern
- information on planned outages

Risk premium vs error model

Comparison between real quoted electricity futures and estimated price using the spot model

Risk premium

$$F_t^e(T,\theta) - \widehat{F}_t^e(T,\theta)$$

with estimated electricity futures price

$$\widehat{F}_t^e(T,\theta) = \int_0^\theta F_t^e(T+\theta)\,d\theta$$

with :

$$F_t^e(T) = \sum_{i=1}^n h_i G_i^T(t, C_t, D_t)\, F_t^i(T)$$

Risk premium vs error model

Estimation done on August, 28th, 2010 for baseload month electricity futures on PowerNext.

	SEPT10	OCT10	NOV10	DEC10	JAN11
Quoted	49.5	55.69	62.	60.45	61.36
Estimation	52.2	53.1	55.2	55.5	53.4
Premium	-2.7	2.59	6.8	4.95	7.96
Relative error (%)	-5.5	4.7	11	8.2	13
Implied excess demand (GW)	-0.4	0.3	0.63	0.46	0.79

Is there a way to make a distinction between risk premium and error model ?

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- allows futures and derivatives pricing and hedging
- nevertheless, only fuels dependancies can he hedged...
- ... and present work only treated hourly futures

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