Position of the problem Spot model Pricing & hedging Risk premium vs error model Conclusion

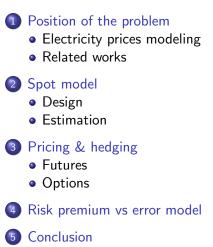
A structural risk-neutral model for pricing and hedging power derivatives FiME Research Centre Monthly Seminar - Paris

René Aïd, Luciano Campi, Nicolas Langrené Paris-Dauphine University - Paris Diderot University EDF R&D - FiME Research Centre



Position of the problem Spot model Pricing & hedging Risk premium vs error model Conclusion

Agenda



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Electricity prices modeling Related works

Looking for a power spot price model

Applications

- pricing of derivatives on the spot
- asset valuation (strip of hourly fuel spread options)
- hedging
- energy market risk management

Model requirements

- realistic
- robust:
- tractable

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Spot model Pricing & hedging Risk premium vs error model Conclusion

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Modeling strategies

Modeling futures prices

pros modeling the real available instruments cons introduction of many parameters to reconstruct hourly futures prices

Modeling spot prices

Exogeneous

O Equilibrium

Spot model Pricing & hedging Risk premium vs error model Conclusion

Electricity prices modeling Related works

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Modeling spot prices Exogeneous prostractability come dependencies Equilibrium Equilibrium Equilibrium Equilibrium

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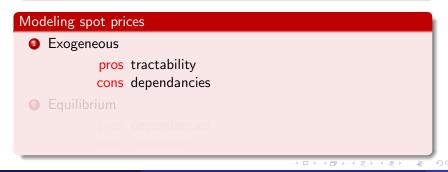
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Related works

Electricity prices exogeneous dynamics

Deng (00), Benth et al. (03, 07, 09), Burger et al. (04), Kolodnyi (04), Cartea & Figueroa (05), Geman & Roncoroni (06)

Equilibrium model

Pirrong & Jermakyan (00) Barlow (02) Kanamura & Ohashi (07)

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	Spot	Futures	Options
Pirrong & Jermakyan (00)			
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Coulon & Howison (09)			
			×

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Coulon & Howison (09)	×	×	
Lyle & Elliot (09)	×	×	×

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This talk



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This talk

Objectives

pricing and hedging power derivatives...

... using an improved version of A., Campi Nguyen & Touzi (09) Structural Risk-Neutral model

Spot Futures Options A., Campi, Nguyen & Touzi (09) × ×

improved SRN model

Spot model Pricing & hedging Risk premium vs error model Conclusion

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Objectives pricing and hedging power derivatives... ... using an improved version of A., Campi Nguyen & Touzi (09) Structural Risk-Neutral model Spot Futures A., Campi, Nguyen & Touzi (09) A., Campi, Nguyen & Touzi (09) mproved SRN model ×

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Spot model Pricing & hedging Risk premium vs error model Conclusion

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Objectives pricing and hedging power derivatives... ... using an improved version of A., Campi Nguyen & Touzi (09) Structural Risk-Neutral model Spot Futures Options A., Campi, Nguyen & Touzi (09) × × improved SRN model × × × ×

Design Estimation

Initial SRN Model

Variables

- fuels, $1 \le i \le n$
- D_t demand (MW)
 - capacities (en MW)
 - fuel prices
 - *heat rates* ($h_i S_t^i$ en €/MWh, \nearrow en i

Electricity price (€/MWh)

$\widehat{P}_t = \sum_{i=1}^n h_i S_t^i \mathbf{1}_{\left\{\sum_{k=1}^{i-1} C_t^k \le D_t \le \sum_{k=1}^{i} C_t^k ight\}}$

Design Estimation

Initial SRN Model

Variables

п	fuels, $1 \leq i \leq n$
	demand (MW)
h_i	<i>heat rates</i> $(h_i S_t^i$ en \in /MWh, \nearrow en i)

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Variables

 $\begin{array}{ll} n & \text{fuels, } 1 \leq i \leq n \\ D_t & \text{demand (MW)} \\ C_t^i & \text{capacities (en MW)} \\ S_t^i & \text{fuel prices} \\ h_i & heat rates (h_i S_t^i \text{ en } \in /\text{MWh}, \nearrow \text{ en } i \end{array}$

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René Aïd, Luciano Campi, Nicolas Langrené

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Position of the problem Spot model

Design

Initial SRN Model

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Spot model

Design

Initial SRN Model

Variables

- n fuels, 1 < i < n
- demand (MW) D_t
- C_t^i S_t^i capacities (en MW)
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- heat rates $(h_i S_t^i \text{ en } \in /MWh, \nearrow \text{ en } i)$ h;

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Spot model

Design

Initial SRN Model

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Electricity price (\in /MWh)

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Design Estimation

Initial SRN model

Pros

• Consistency between electricity prices and fuel prices

• Consistency between electricity prices and demand

Design Estimation

Initial SRN model

Pros

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Cons

Marginal fuel cost is not the spot price.

Design Estimation

Initial SRN model

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Initial SRN model

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Marginal fuel cost is not the spot price

Non-convex technical constraints (may lead to negative price Strategic behaviour (Hortaçsu & Puller, RAND J. of Economics 2008)

Design Estimation

Initial SRN model

Pros

- Consistency between electricity prices and fuel prices
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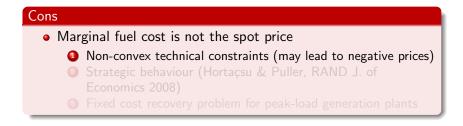
- Marginal fuel cost is not the spot price
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 - Fixed cost recovery problem for peak-load generation plants

Design Estimation

Initial SRN model

Pros

- Consistency between electricity prices and fuel prices
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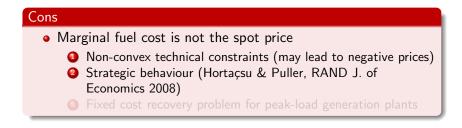


Design Estimation

Initial SRN model

Pros

- Consistency between electricity prices and fuel prices
- Consistency between electricity prices and demand



Design Estimation

Initial SRN model

Pros

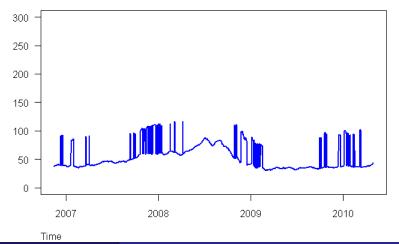
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Marginal fuel cost is not the spot price Non-convex technical constraints (may lead to negative prices) Strategic behaviour (Hortaçsu & Puller, RAND J. of Economics 2008) Fixed cost recovery problem for peak-load generation plants

Design Estimation

Initial SRN Model - illustration

Spot price (in €/MWh)

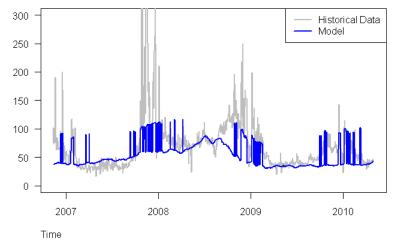


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Design Estimation

Initial SRN Model - illustration

Spot price (in €/MWh)

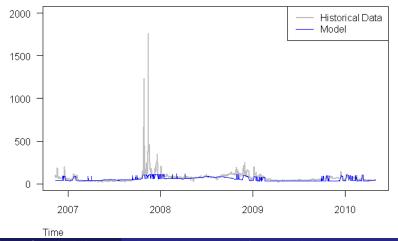


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Design Estimation

Initial SRN Model - illustration

Spot price (in €/MWh)



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Design Estimation

Improved SRN model

• Marginal fuel cost $\widehat{P}_t := \sum_{i=1}^n h_i S_t^i \mathbf{1}_{\{\sum_{k=1}^{i-1} C_t^k \le D_t \le \sum_{k=1}^{i} C_t^k\}}$

Design Estimation

Improved SRN model

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- Available capacity $\overline{C}_t := \sum_{k=1}^n C_t^k$

Design Estimation

Improved SRN model

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- Available capacity $\overline{C}_t := \sum_{k=1}^n C_t^k$
- Price spikes occur when the electric system is under stress, i.e. $\overline{C}_t D_t$ is small

Design Estimation

Improved SRN model

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- Corresponds to peak-load fixed cost problem recovery...

Design Estimation

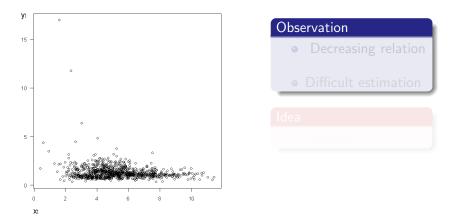
Improved SRN model

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- Corresponds to peak-load fixed cost problem recovery...

$$y_t := \frac{P_t}{\widehat{P}_t}$$
 as a (nonlinear) function of $x_t := \overline{C}_t - D_t$

Design Estimation

Improved SRN model - Estimation



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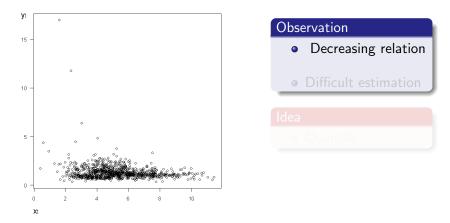
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Figure: PowerNext - 19th hours Nov, 13th 06 to April 30th 10

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Design Estimation

Improved SRN model - Estimation



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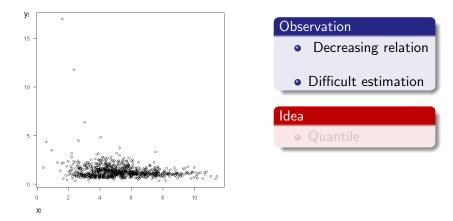
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Design Estimation

Improved SRN model - Estimation



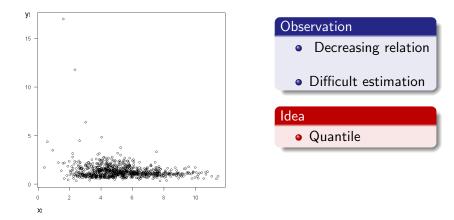
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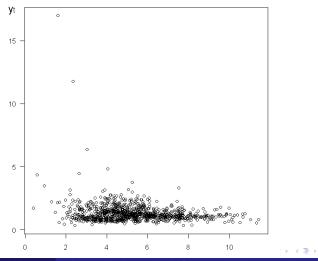
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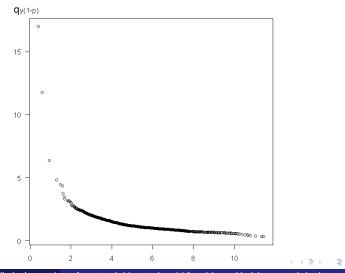
Improved SRN model - Estimation



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Design Estimation

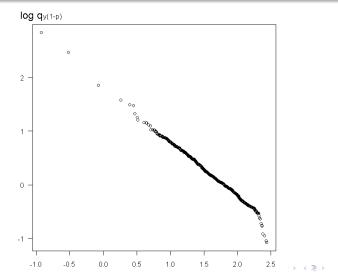
Improved SRN model - Estimation



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Design Estimation

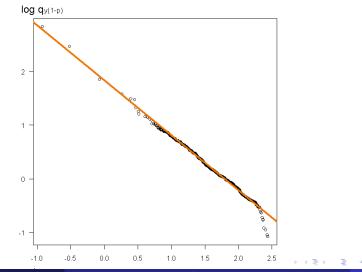
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Design Estimation

Improved SRN model - Estimation

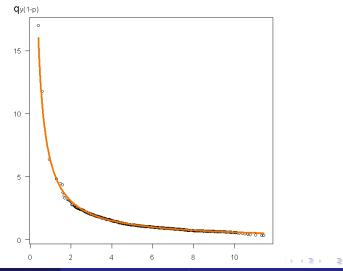


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A structural risk-neutral model for pricing and hedging power derivatives 17 / 43

Design Estimation

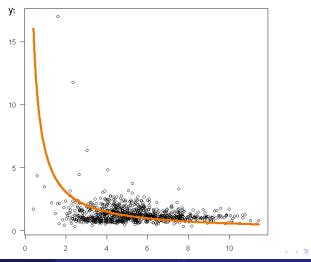
Improved SRN model - Estimation



René Aïd, Luciano Campi, Nicolas Langrené

Design Estimation

Improved SRN model - Estimation



René Aïd, Luciano Campi, Nicolas Langrené 🛛 A s

Design Estimation

Improved SRN model - Estimation

Estimated relation :
$$y_t = \frac{\gamma}{x_t^{\nu}}$$

Improved SRN model

$$P_t = g\left(\sum_{k=1}^n C_t^k - D_t\right) \times \left(\sum_{i=1}^n h_i S_t^i \mathbf{1}_{\left\{\sum_{k=1}^{i-1} C_t^k \le D_t \le \sum_{k=1}^i C_t^k\right\}}\right)$$

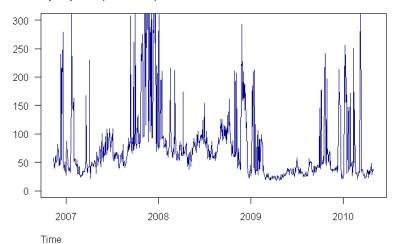
with scarcity function

$$g(x) := \min\left(\frac{\gamma}{x^{\nu}}, M\right) \mathbf{1}_{\{x \ge 0\}} + M \mathbf{1}_{\{x \le 0\}}$$

Design Estimation

Improved SRN model - Back-testing

Spot price (in €/MWh)

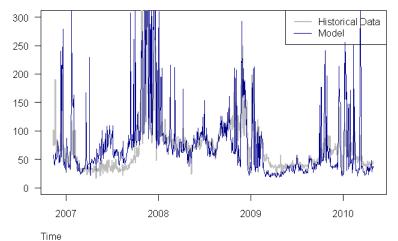


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Design Estimation

Improved SRN model - Back-testing

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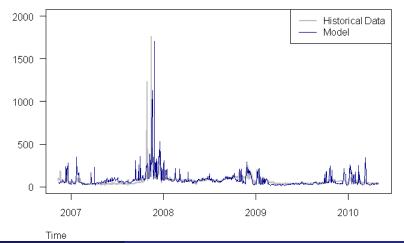


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Design Estimation

Improved SRN model - Backtesting

Spot price (in €/MWh)



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Futures Options

Pricing & hedging

Pricing

- incomplete market
- need for a hedging criterion
- Super-replication, utility indifference or mean-variance
- our choice : Local Risk Minimization

.ocal Risk Minimization (Pham (00), Schweizer (01))

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- allows to decompose contingent claim between hedgeable part: (fuels) and non-hedgeable part (demand, capacities)

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Pricing & hedging Risk premium vs error model

Futures

Futures

Futures prices
$$F_t^e(T) = \mathbb{E}_t^{\mathbb{Q}} \left[e^{-r(T-t)} P_T \right]$$

$$F_t^e(T) = \sum_{i=1}^n h_i G_i^T(t, C_t, D_t) F_t^i(T)$$
with :

with

$$G_i^{\mathsf{T}}(t,C_t,D_t) = \mathbb{E}_t \left[g \left(\sum_{k=1}^n C_{\mathsf{T}}^k - D_{\mathsf{T}} \right) \mathbf{1}_{\left\{ \sum_{k=1}^{i-1} C_{\mathsf{T}}^k \le D_{\mathsf{T}} \le \sum_{k=1}^i C_{\mathsf{T}}^k \right\}} \right]$$

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Futures Options

Futures prices - hedging

Demand & capacities

$$dD_{t} = a(t, D_{t}) dt + b(t, D_{t}) dW_{t}^{D}$$
$$dC_{t}^{i} = \alpha_{i} (t, C_{t}^{i}) dt + \beta_{i} (t, C_{t}^{i}) dW_{t}^{C,i}$$

Futures price dynamics

$$dF_t^e(T) = \sum_{i=1}^n h_i \left[G_i^T(t, C_t, D_t) dF_t^i(T) + F_t^i(T) dG_i^T(t, C_t, D_t) \right]$$

$$dG_{i}^{T}(t, C_{t}, D_{t}) = \sum_{k=1}^{n} \frac{\partial G_{i}^{T}}{\partial c_{k}}(t, C_{t}, D_{t})\beta_{k}(t, C_{t}^{k})dW_{t}^{C, k}$$
$$+ \frac{\partial G_{i}^{T}}{\partial z}(t, C_{t}, D_{t})b(t, D_{t})dW_{t}^{D}$$

Futures Options

Futures prices - hedging

• To go further, need to choose dynamics for demand and capacities

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Futures Options

Futures prices - hedging

- To go further, need to choose dynamics for demand and capacities
- deterministic part for seasonality + Ornstein-Uhlenbeck

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Futures Options

Futures prices - hedging

- To go further, need to choose dynamics for demand and capacities
- deterministic part for seasonality + Ornstein-Uhlenbeck
- G_i^T explicite as function of *extended incomplete Goodwin-Staton integral* :

$$\widetilde{\mathcal{G}}(x,y;\nu) = \int_{x}^{\infty} \frac{1}{\left(y+z\right)^{\nu}} e^{-z^{2}} dz$$

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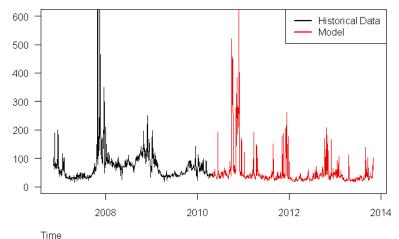
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• ... for which efficient numerical algorithms are provided in A., Campi & Langrené (10).

Futures Options

Futures prices - hedging : spot simulations

Spot price (in €/MWh)

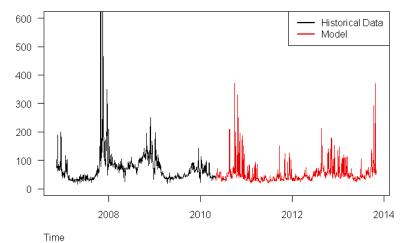


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Futures Options

Futures prices - hedging : spot simulations

Spot price (in €/MWh)

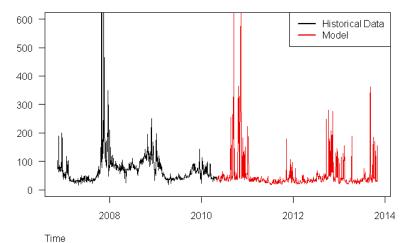


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Futures Options

Futures prices - hedging : spot simulations

Spot price (in €/MWh)

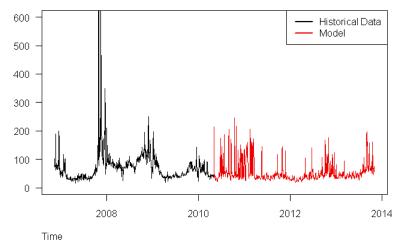


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Futures Options

Futures prices - hedging : spot simulations

Spot price (in €/MWh)

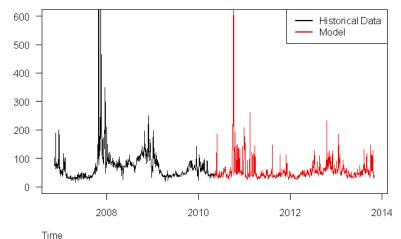


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Futures Options

Futures prices - hedging : spot simulations

Spot price (in €/MWh)



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Futures Options

Futures prices - hedging

Numerical test

- Hedging an electricity futures with a delivery period of 1 hour
- with a daily rebalanced basket of futures contracts on fuels

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Futures Options

Futures prices - hedging

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Futures Options

Futures prices - hedging

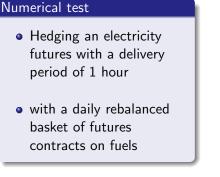
Numerical test

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Futures Options

Futures prices - hedging



Sample paths (in €)

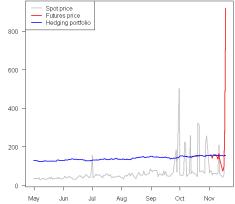
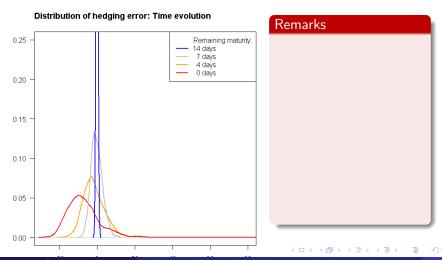


Image: A matrix

Futures Options

Futures prices - hedging

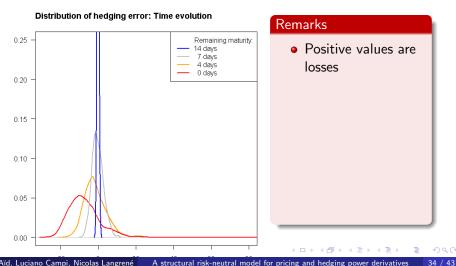


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Pricing & hedging

Futures

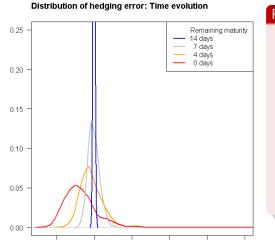
Futures prices - hedging



René Aïd, Luciano Campi, Nicolas Langrené A structural risk-neutral model for pricing and hedging power derivatives Position of the problem Pricing & hedging

Futures

Futures prices - hedging



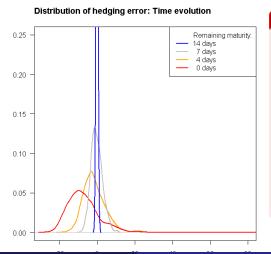
Remarks

- Positive values are losses
- Far from maturity : perfect hedge; electricity futures is equivalent to a basket of fuels

René Aïd, Luciano Campi, Nicolas Langrené

Futures Options

Futures prices - hedging



Remarks

- Positive values are losses
- Far from maturity : perfect hedge; electricity futures is equivalent to a basket of fuels
- Close to maturity : inefficient hedge

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A structural risk-neutral model for pricing and hedging power derivatives 34 / 43

Futures Options

Spread options (do not panic)

Spread option with a 2 fuel model

The price π_0 at time t = 0 of a call spread option with pay-off $H = (P_T - h_1 S_T^1 - K)^+$ is given by :

$$\pi_{0} = \int_{\mathbb{R}^{2}} f_{C_{T}^{1} - D_{T}}(z) f_{C_{T}^{2}}(c) \left\{ \phi_{1}(c, z) \mathbf{1}_{\{z > 0\}} + \phi_{2}(c, z) \mathbf{1}_{\{z \le 0\}} \right\} dcdz$$

$$\phi_1 = (g-1)BS_0(\sigma_1, K)\mathbf{1}_{\{g>1\}}$$

$$\phi_2 = g \int_0^\infty \hat{f}_{Y_T^1}(y) BS_0\left(\sigma_2, \frac{K + (1 - g)y}{g}\right) \left(\mathbf{1}_{\{g \le 1\}} + \mathbf{1}_{\{g > 1\}} \mathbf{1}_{\{y < \frac{K}{g - 1}\}}\right) dy$$

$$+ \left(gY_0^2 \mathcal{N}\left(\frac{\left(r - \frac{\sigma_1^2}{2}\right) \mathcal{T} - \ln\left(\frac{\kappa}{(g-1)Y_0^1}\right)}{\sigma_1 \sqrt{\mathcal{T}}}\right) + (g-1) BS_0\left(\sigma_1, \frac{\kappa}{g-1}\right)\right) \mathbf{1}_{\{g>1\}}$$

with
$$g := g(c + z)$$
.

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with g := g(c + z).

Futures Options

Spread options

• semi-explicit formula : numerical integration

- partial hedging with futures on fuels and electricity
- applied on European dark spread call option with a period of delivery of 1 hour

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Futures Options

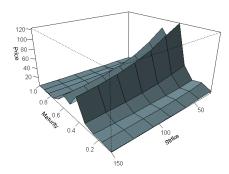
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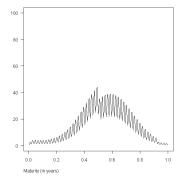
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Futures Options

Spread options



Marginal oil probability (%)

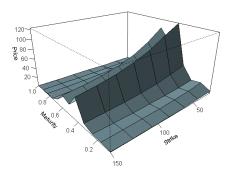


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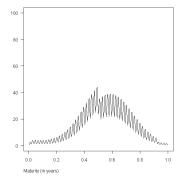
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Futures Options

Spread options



Marginal oil probability (%)

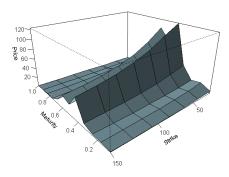


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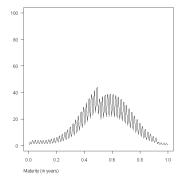
• seasonality pattern

Futures Options

Spread options



Marginal oil probability (%)



- seasonality pattern
- information on planned outages

Risk premium vs error model

Comparison between real quoted electricity futures and estimated price using the spot model

Risk premium

$$F_t^e(T, \theta) - \widehat{F}_t^e(T, \theta)$$

with estimated electricity futures price

$$\widehat{F}_{t}^{e}(T,\theta) = \int_{0}^{\theta} F_{t}^{e}(T+\theta) \, d\theta$$

with :

$$F_{t}^{e}(T) = \sum_{i=1}^{n} h_{i}G_{i}^{T}(t, C_{t}, D_{t}) F_{t}^{i}(T)$$

Risk premium vs error model

Estimation done on August, 28th, 2010 for baseload month electricity futures on PowerNext.

,	SEPT10	OCT10	NOV10	DEC10	JAN11
Quoted	49.5	55.69	62.	60.45	61.36
Estimation	52.2	53.1	55.2	55.5	53.4
Premium	-2.7	2.59	6.8	4.95	7.96
Relative error (%)	-5.5	4.7	11	8.2	13
Implied excess demand (GW)	-0.4	0.3	0.63	0.46	0.79

Is there a way to make a distinction between risk premium and error model?

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Conclusion

Conclusions

- SRN electricity spot price model with a scarcity function
- allows futures and derivatives pricing and hedging
- nevertheless, only fuels dependancies can he hedged...
- ... and present work only treated hourly futures

Perspectives

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Perspectives

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- comparison with 'real' quoted futures dynamics
- comparison with calibration procedure
- American options for investment problem

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Perspectives comparison with "real" quoted futures dynamics comparison with calibration procedure Amorican options, for investment problem

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- comparison with "real" quoted futures dynamics
- comparison with calibration procedure
- American options for investment problem

Conclusion

Conclusions

- SRN electricity spot price model with a scarcity function
- allows futures and derivatives pricing and hedging
- nevertheless, only fuels dependancies can he hedged...
- ... and present work only treated hourly futures

Perspectives

- comparison with "real" quoted futures dynamics
- comparison with calibration procedure
- American options for investment problem

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