Accelerated simulation of power grid reliability indices

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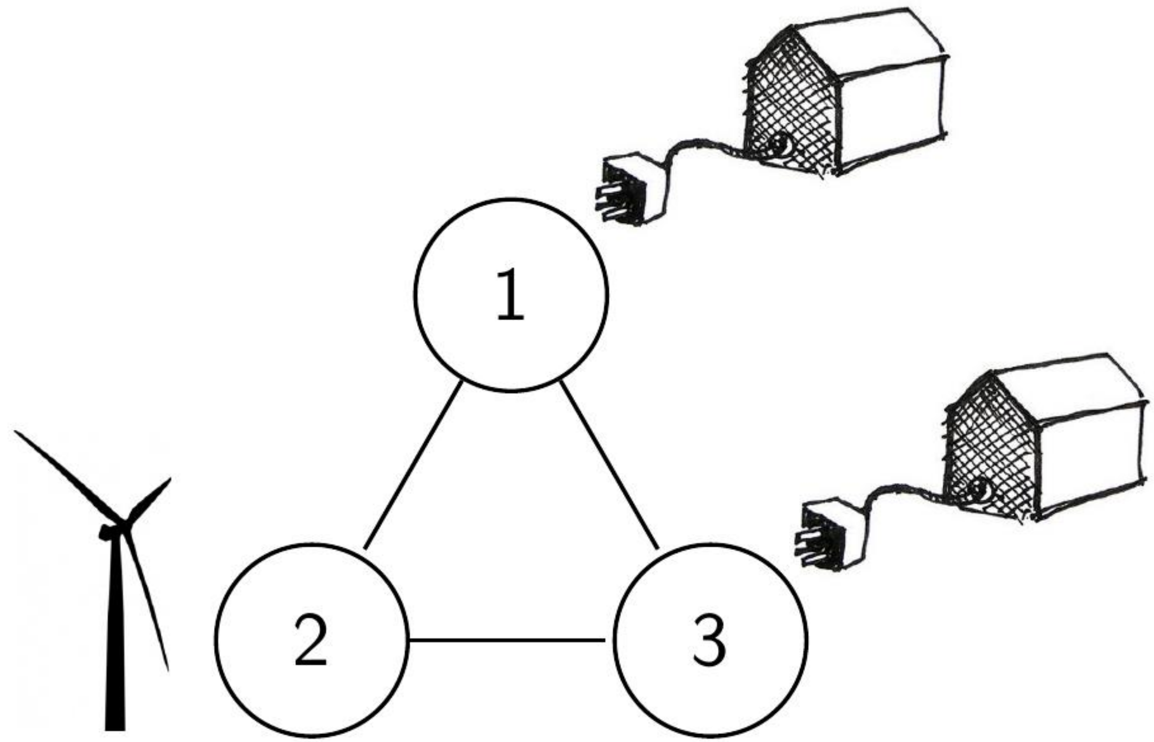
How reliable are modern power grids?



$$V_{min} \leq |V(t)| \leq V_{max}, \qquad \text{at all nodes}$$

$$|I(t)| \leq I_{max}, \qquad \text{at all connections}$$

Research aim

[Li and Billinton 1994]

Probability
Expected duration
Expected number
Expected size (kWh)

of power curtailments during a time interval $\mathcal{T}$

Research aim

[Li and Billinton 1994]

Compute reliability indices

$$\mathbb{E}[J] = \begin{cases} \text{Probability} \\ \text{Expected duration} \\ \text{Expected number} \\ \text{Expected size (kWh)} \end{cases}$$

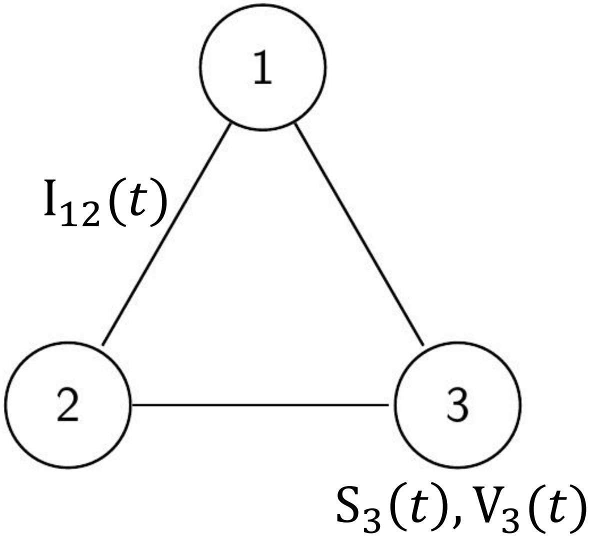
of power curtailments during a time interval $\mathcal{T}$,

given the distribution of $S(t)$ of nodal power injections.

Model

A grid with
- N nodes
- $M \leq N(N-1)/2$ connections

Discrete time domain $\mathcal{T} = \{0, t_1, t_2, \ldots, T\}$



Vectors of stochastic processes:
- nodal power injections $\{S(t), t \in \mathcal{T}\}$ with values $S(t) \in \mathbb{C}^N$
- nodal voltages $\{V(t), t \in \mathcal{T}\}$ with values $V(t) \in \mathbb{C}^N$
- connections currents $\{I(t), t \in \mathcal{T}\}$ with values $I(t) \in \mathbb{C}^M$

Model

[Grainger and Stevenson 1994]

Power flow equations

For every $t \in \mathcal{T}$, solve the nonlinear algebraic system

$$\begin{aligned} P_j &= \sum_k |V_j||Y_{jk}||V_k|\cos(\theta_{jk} + \delta_k - \delta_j), \\ Q_j &= -\sum_k |V_j||Y_{jk}||V_k|\sin(\theta_{jk} + \delta_k - \delta_j), \end{aligned}$$

for all $V_j = |V_j|e^{i\delta_j}$, given admittances $Y_{jk} = |Y_{jk}|e^{i\theta_{jk}}$, $P_j = \mathrm{Re}\{S_j\}$, $Q_j = \mathrm{Im}\{S_j\}$.

Ohm's law $\Rightarrow$ connection currents I_{jk}

Crude Monte Carlo is inefficient

Event $C = \{\text{a curtailment during } \mathcal{T}\}$ is rare!

$$\hat{P}_n := \frac{1}{n}\sum_{i=1}^{1} \mathbf{1}_{\{C \text{ in sample i}\}} \text{ for } \mathbb{P}(C).$$

Relative variance

$$\frac{\text{Var } \hat{P}_n}{\mathbb{P}(C)^2} = \frac{1-\mathbb{P}(C)}{n\mathbb{P}(C)} \to \infty \quad \text{as } \mathbb{P}(C) \to 0.$$

[#CMC samples] x [# time steps] x [duration power flow solver] = LONG!

Splitting technique to estimate $\mathbb{P}(C)$

[L'Ecuyer et al. 2006], [Garvels 2000]

Decompose $\mathbb{P}(C) = \prod \mathbb{P}(T_k < T | T_{k-1} < T),$

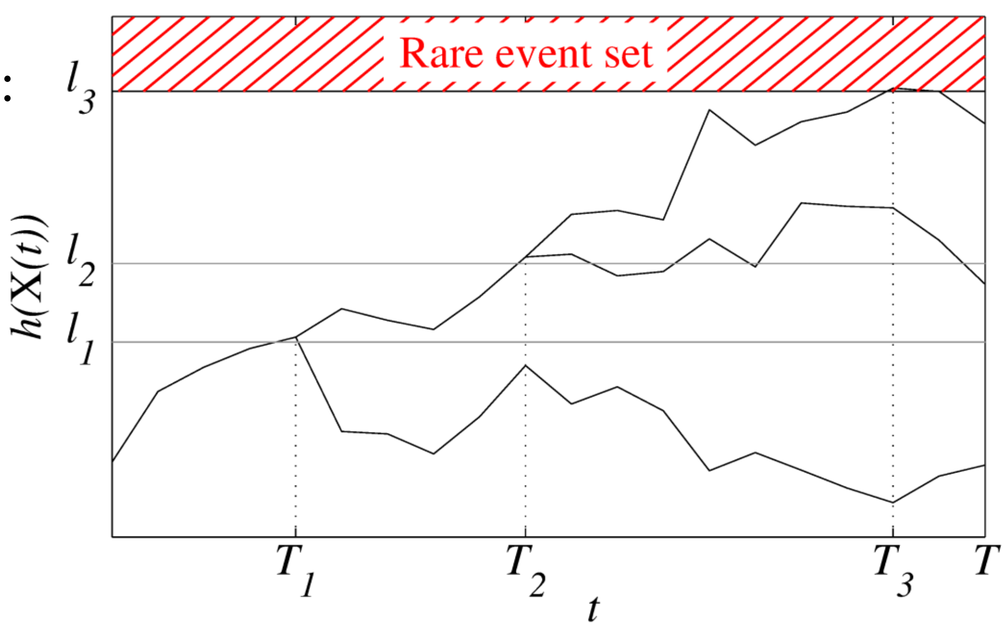
with $T_k := \min\{t \in \mathcal{T} : h(\mathrm{X}(t)) \geq l_k\},$

importance function $h: \mathbb{R}^{N+M} \mapsto \mathbb{R},$

and $\mathrm{X}(t) := [|\mathrm{V}(t)|, |\mathrm{I}(t)|] \in \mathbb{R}^{N+M}.$

Unbiased splitting estimator:

$$\hat{P} := \prod_k \hat{p}_k$$



Controlling the estimator accuracy

[Amrein and Künsch 2011]

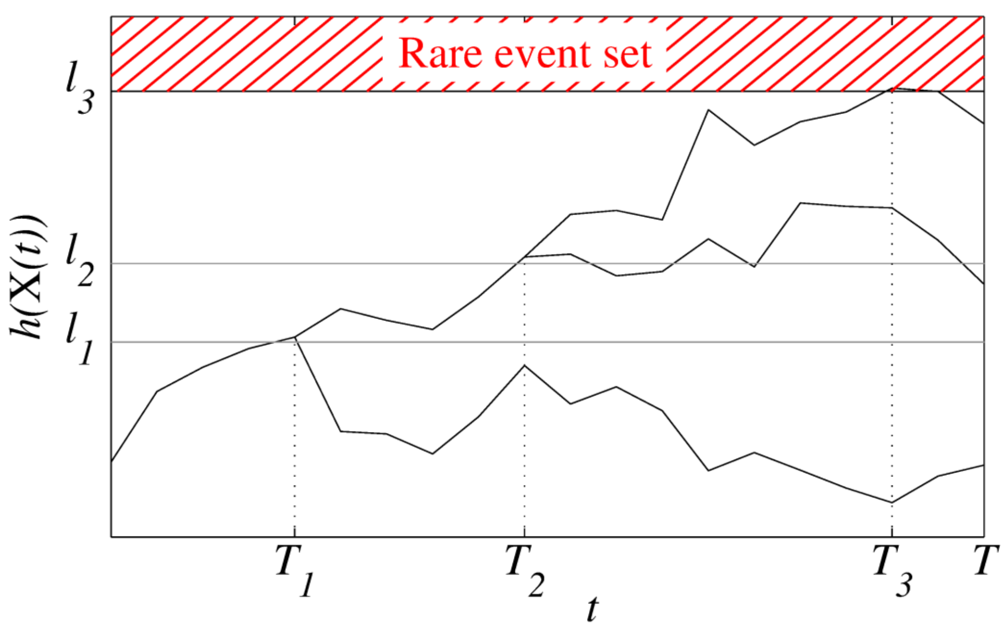
Ideal setting: $\hat{P}$ is work-normalized asymptotically efficient:

$$\frac{\operatorname{Var}\hat{P}}{\mathbb{P}(C)^2} \sim \frac{(\log \mathbb{P}(C))^2}{n}$$

If conditional probabilities do not depend on the entrance states

$$\frac{\operatorname{Var}\hat{P}}{\mathbb{P}(C)^2} \le -1 + \prod_k \frac{r_k - 1}{r_k - 2},$$

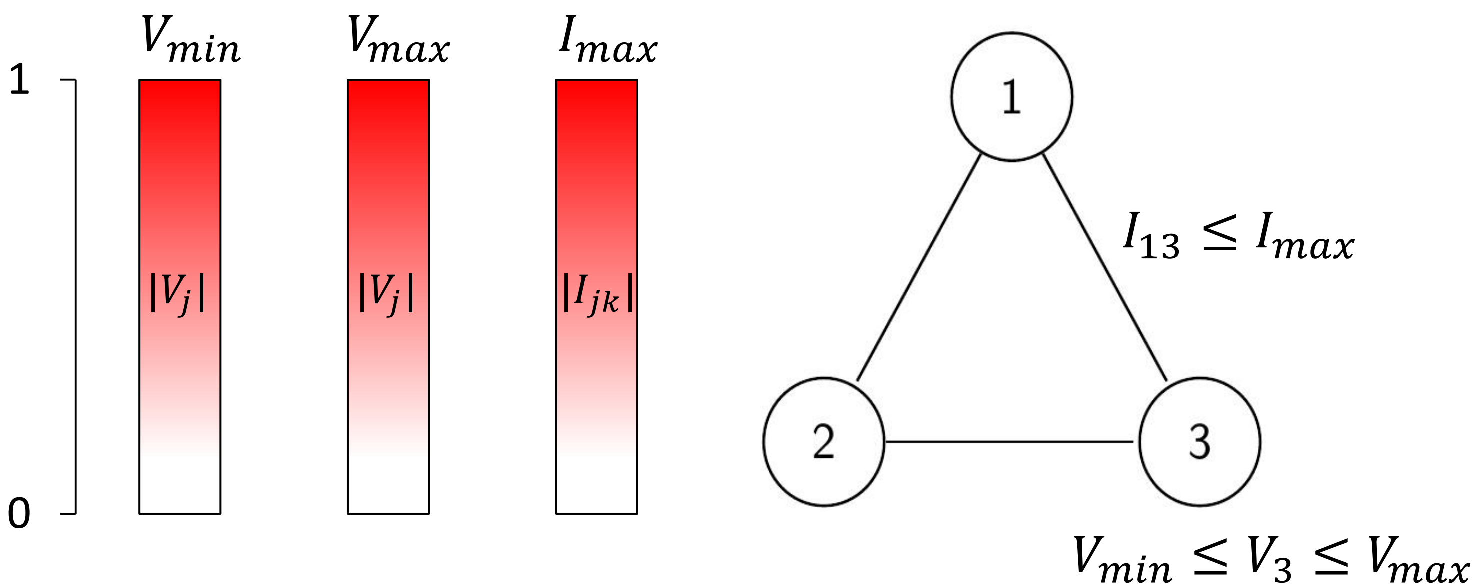
with r_k the (fixed) number of successes at level k.



CWI

Importance function

$$h(\mathrm{X}(t)) \coloneqq \max_j \frac{\mathrm{X}_j(t) - \mathrm{L}_j}{\mathrm{U}_j - \mathrm{L}_j}$$



Other grid reliability indices

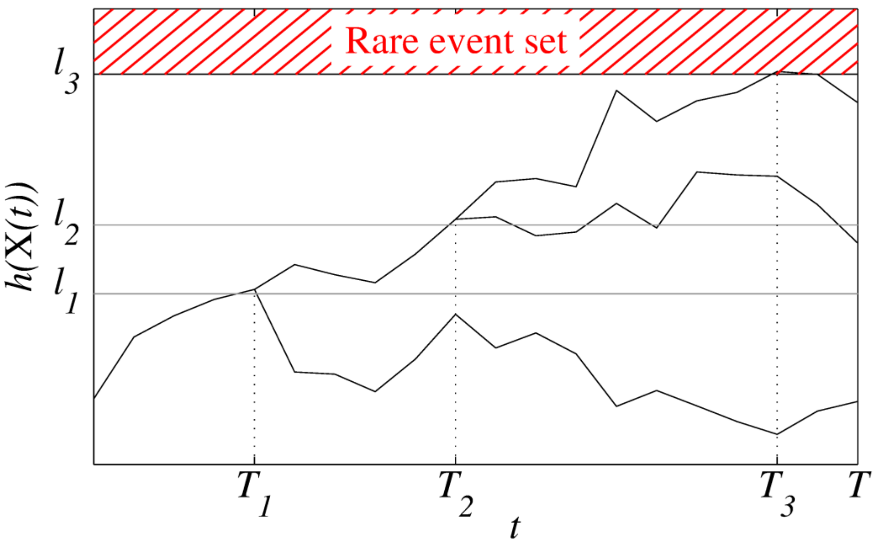
[Wadman, Crommelin and Frank 2013]

We decompose

$$\mathbb{E}[J] = \underbrace{\mathbb{P}(C)}_{\hat{P}}\,\underbrace{\mathbb{E}[J|C]}_{\widehat{J^C}}.$$

Estimate $\mathbb{E}[J]$ by $\hat{P}\hat{J}^C$, with

$$\hat{J}^C = \frac{1}{n}\sum_{i=1}^{n}\hat{J}_i^C.$$

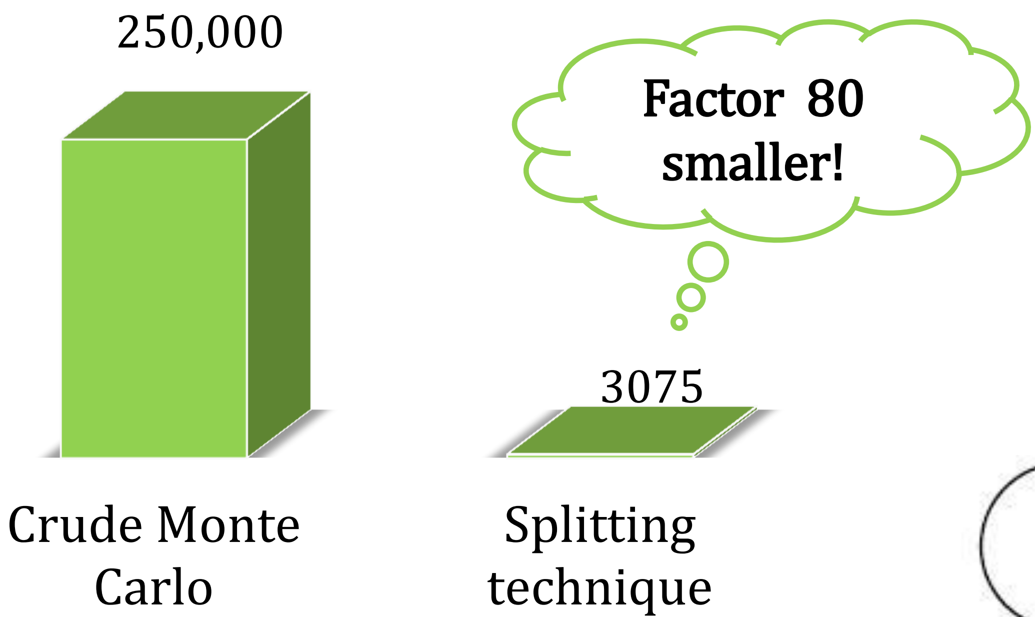


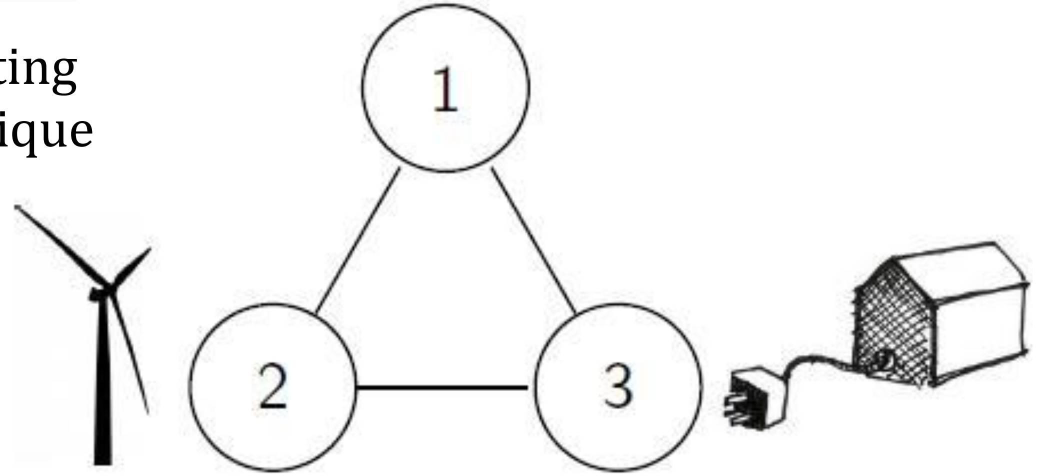
Again, we control the accuracy:

$$\frac{\mathrm{Var}\,\hat{P}\hat{J}^C}{\mathbb{E}[J]^2} = \frac{\mathrm{Var}\,\hat{P}}{\mathbb{P}(C)^2} + \frac{\mathrm{Var}\,\hat{J}^C}{\mathbb{E}[J|C]^2} + \frac{\mathrm{Var}\,\hat{P}}{\mathbb{P}(C)^2}\frac{\mathrm{Var}\,\hat{J}^C}{\mathbb{E}[J|C]^2}.$$

Workload decreases dramatically

Required samples for a 95% CI





LDT: Large Devations Theory

Vector of N SDEs

$$dS^{\varepsilon}(t) = b(S^{\varepsilon}(t))dt + \varepsilon L dW(t), \qquad \varepsilon > 0$$

Good rate function

$$I(f) \coloneqq \frac{1}{2}\int_0^T \|L^{-1}(f' - b(f))\|^2 dt$$

Rare event…

$$\mathbb{P}\{\exists \tau \leq T: p(S^{\varepsilon}(\tau)) = P_{max}\}$$

LDT: Large Devations Theory

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Rare event…

$$\lim_{\varepsilon \downarrow 0} -\varepsilon \log \mathbb{P}\{\exists \tau \leq T: p(S^{\varepsilon}(\tau)) = P_{max}\} = \inf_{f:\{\exists \tau \leq T: p(f(\tau)) = P_{max}\}} I(f) =: I^*$$

… has approximation

$$\mathbb{P}\{\exists \tau \leq T: p(S^{\varepsilon}(\tau)) = P_{max}\} \sim e^{-\varepsilon I^*}$$

LDT: finding critical connections

Compute decay rate I^* for all connections

SDEs Ornstein-Uhlenbeck:

$$dS^\varepsilon(t) = D(\mu - S^\varepsilon(t))dt + \varepsilon L dW(t).$$

Then

$$I^* = \inf_{f:\{\exists \tau \leq T : p(f(\tau)) = P_{max}\}} \frac{1}{2}\int_0^T \|L^{-1}(f' + Df - D\mu)\|^2 dt$$

$$= \inf_{\tau \leq T} g(\tau),$$

with

$$g(\tau) := \inf_{f:\{p(f(\tau)) = P_{max}\}} \frac{1}{2}\int_0^\tau \|L^{-1}(f' + Df - D\mu)\|^2 dt.$$

LDT: the optimization

$$g(\tau) := \inf_{f:\{p(f(\tau))=P_{max}\}} \frac{1}{2}\int_0^\tau \|L^{-1}(f' + Df - D\mu)\|^2 dt. \qquad (1)$$

1. Necessary conditions: Euler–Lagrange
2. OU $\Rightarrow f$ fullfills 2nd order ODEs
3. Assuming DC power flow

$$p(S^\varepsilon(\tau)) := v^\top S^\varepsilon(t) = P_{max}$$

then (1) becomes quadratic programming with 1 equality constraint.

Remains:

$$I^* = \inf_{\tau \leq T} g(\tau).$$

LDT based splitting technique

Importance function

$$h(t,x) = 1 - \frac{I^*(t,x)}{I^*\big(0,S(0)\big)}.$$

Proxies for I^*

- Num. optimizing τ
- Assuming $\tau_{opt} = T$
- Assuming fixed constant end point