Efficient swing and gas storage hedging

Fime Seminar

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Schedule

Motivation

Risk management aims at evaluating risk for complicated portfolios.
To integrate dynamic hedging in the Monte Carlo assets simulations, it is necessary to be able to evaluate hedging strategies along price trajectories.

- Classical approach based on finite difference used each day on each simulation for the asset :
  1. very costly
  2. number of calculations : at least number of hedging products,
  3. some extra parameters to fit.
- More advance method : Tangent process:
  1. more efficient,
  2. still costly : calculation needed every day for each simulation

Need for a method not involving daily calculation along trajectories.

Tangent process for european options

Classical Black Scholes asset under neutral risk measure (zero risk free rate):

$$dS_t = \sigma S_t dW_t \quad t \leq T,$$

European option pay off g, maturity T :

$$P_t = \mathbb{E}\left[g(S_T) \mid \mathcal{F}_t\right] \quad \text{for} \quad t \leq T \quad \mathbb{P}-\text{a.s.} ,$$

$$P_t = \mathbb{E}\left[g(S_t \exp\left(-\frac{1}{2}\sigma^2(T-t) + \sigma(W_T - W_t)\right)) \mid \mathcal{F}_t\right] \quad \text{for} \quad t \leq T \quad \mathbb{P}-\text{a.s.} ,$$

Delta

$$\begin{aligned} \Delta_t &= \frac{\partial P_t}{\partial S_t} \\ &= \mathbb{E}\left[g'(S_T) \exp\left(-\frac{1}{2}\sigma^2(T-t) + \sigma(W_T - W_t)\right) \mid \mathcal{F}_t\right] \end{aligned}$$

Tangent process for european options

$$\Delta_t = \mathbb{E}\left[g'(S_T)Y_T \mid \mathcal{F}_t\right]/Y_t$$

where Y_t **tangent process**

$$\begin{cases} dY_t &= \sigma Y_t dW_t, \\ Y_0 &= 1, \end{cases}$$

Example

For an european put option with strike k :$g'(x) = -1_{x<K}$.

$$\Delta_t = \mathbb{E}\left[-1_{S_T<K} Y_T \mid \mathcal{F}_t\right]/Y_t$$

Conditonal tangent process for european options

- $\hat{\mathbb{E}}[\cdot \mid \mathcal{F}_{t_i}]$ conditional expectation operator
- M the number of Monte Carlo simulations, κ number of time steps of size π,
- $S_i^j = S_{t_i}^j, i = 0..\kappa, j = 1$ to M : j th asset simulation at time step i.
- Y_i^j tangent process for simulation j at time step i.

Hedge from $t = 0$ till maturity european put option with Monte Carlo along trajectories.

Conditional tangent process calculation for european options by backward resolution

Algorithm 1 Algorithm to calculate value and conditional deltas for european put option

$CF^j = (K - S^j_\kappa)1_{S^j_\kappa < K}$ for $j = 1$ to M // final cash flow
$\Delta^j = -Y^j_\kappa 1_{S^j_\kappa < K}$ for $j = 1$ to M // delta
for $i = \kappa - 1, 0$ **do**
  Calculate and store conditional deltas $\Delta_i = \hat{\mathbb{E}}[\Delta | \mathcal{F}_{i\pi}]/Y_i$
end for
Final value $\frac{\sum_j^M CF^j}{M}$, delta $\frac{\sum_j^M \Delta^j}{M}$ at $t = 0$.

Hedge european put options in simulation

Algorithm 2 Algorithm to simulate and hedge an european put option

Require: Option and asset parameters, conditional delta calculated in optimization part
Ensure: Simulate the option exercise, portfolio with hedging

$PFH^j = 0., \forall j \in [1, M]$ // Portfolio with hedge initialisation
for $i = 0$ to $\kappa - 1$ **do**
  Get back $\Delta_i(.)$
  for $j = 1 \dots M$ **do**
    $PFH^j - = \Delta_i(S_i^j)(S_{i+1}^j - S_i^j)$
  end for
end for
for $j = 1 \dots M$ **do**
  $PFH^j + = (K - S_\kappa^j) 1_{S_\kappa^j < K}$
end for

Tangent process for american options

$$P_t \quad = \quad \sup_{\tau \in \mathcal{T}_{[t,T]}} \mathbb{E}\left[g(\tau, S_\tau) \mid \mathcal{F}_t\right] \quad \text{for} \quad t \leq T \quad \mathbb{P}-\text{a.s.}\ , \qquad (1)$$

Conditional delta :

$$\Delta_t = \mathbb{E}\left[g'(S_{\tau_t}) Y_{\tau_t} \mid \mathcal{F}_t\right] (Y_t)^{-1} \ ,\ t \leq T\ . \qquad (2)$$

Proof.

Use the fact that $\frac{\partial P_t}{\partial S_t} Y_t$ is a martingale + smooth paste condition

$\frac{\partial P_t}{\partial S_t} Y_t = \mathbb{E}\left[\frac{\partial P_\tau}{\partial S_\tau} Y_\tau \mid \mathcal{F}_t\right]$ □

Price model

$$\frac{dF(t,T)}{F(t,T)} = \sigma(t)e^{-a(T-t)}dz_t,$$

Uncertainty and variance :

$$\begin{aligned} V(t1,t2) &= \int_0^{t_1} \sigma_(u)^2 e^{-2a(t_2-u)}, \\ W_t &= \int_0^{t} \sigma(u) e^{-a(t-u)} dz_u, \end{aligned} \tag{3}$$

$$F(t,T) = F(t_0,T)e^{-\frac{1}{2}V(t,T) + e^{-a(T-t)}W_t}.$$

$$S_t = \lim_{T\downarrow t} F(t,T)$$

Suppose here daily hedging possible

Sensibility of European option to future product $F(t, T)$

European option with pay off g maturity date T

$$\begin{aligned} \Delta_t^T &= \frac{\partial P_t}{\partial F(t,T)} \\ &= \mathbb{E}\left[g'(S_T) Y_T^T \mid \mathcal{F}_t\right] / Y_t^T \end{aligned}$$

with future tangent process

$$Y_t^T = e^{-\frac{1}{2}V(t,T) + e^{-a(T-t)} W_t}$$

Remark

Conditional Delta for european/american call style option is the expectation of the volume exerciced multiplied by future tangent process.

Swing caracterization

- Call swing with strike K, daily constraints q_{min},q_{max} and global constraints Q_{min}, Q_{max}, κ dates to exercice
- $V^*(t, S, c)$ the optimal volume exercised at date t, with $S_t = S$ and quantity already exercized c (calculated by Longstaff Schwartz),
- Optimal swing consumption trajectory starting with consumption c for $S_{t_i} = S$ at date t_i^-

$$\left\{ \begin{array}{lcl} C_i^{*,i}(S,c) & = & c, \\ C_m^{*,i}(S,c) & = & c + \sum_{k=i}^{m-1} V^*(t_k, S_{t_k}, C_k^{*,i}(x,S)) \text{ for } 0 \leq i \leq m \leq \kappa, \end{array} \right.$$

Conditional Delta for swing

- Introduce the D function where $D(t_i, t_m, x, c)$ is the optimal volume seen at date t_i for price x and consumption c to exercice at date t_m multiplied by futur tangent process

$$D(t_i, t_m, x, c) \quad = \quad V^*(t_m, X_{t_m}, C_m^{*,i}(x, c)) Y_{t_m}^{t_m}. \qquad (4)$$

Remark

$D(t_i, t_m, x, c)$ is depending on the trajectory of S_t between t_i and t_m.

- Conditional delta at date t_i for delivery at date t_m , $m = i + 1, \ldots \kappa - 1$ is given by

$$\Delta(t_i, t_m, x, c) = \mathbb{E}[D(t_i, t_m, X_{t_i}, c) \mid X_{t_i} = x] / Y_{t_i}^{t_m}. \qquad (5)$$

Discretized version

For $m > i$ (all delivery dates superior to current date)

$$\hat{\Delta}_i^m(c, S_i^j) \quad = \quad \hat{\mathbb{E}}[Y_{t_m}^{t_m} \hat{V}^*(t_m, S_{t_m}, \hat{C}_m^{*,i}(S_i^j, c)) \mid S_t = S_i^j] / Y_{t_i}^{t_m}$$

where

- $\hat{V}^*$ is the volume estimated by dynamic programming,
- $\hat{\mathbb{E}}$ the approximated conditional expectation operator,

Remark

$\hat{D}(t_i, t_m, S_{t_i}^j, c_l) = \hat{V}^*(t_m, S_{t_m}^j, \hat{C}_m^{*,i}(S_{t_i}^j, c_l)) Y_{t_m}^{t_m,j}$ *can be calculated during valorization by dynamic programming on each trajectory.*

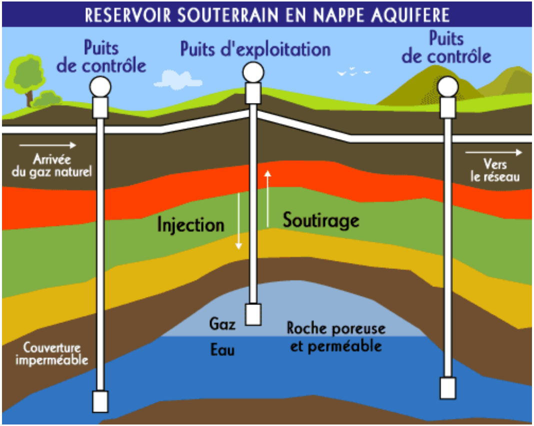
Gas storage valuation formulation

Financial valuation facing market price :

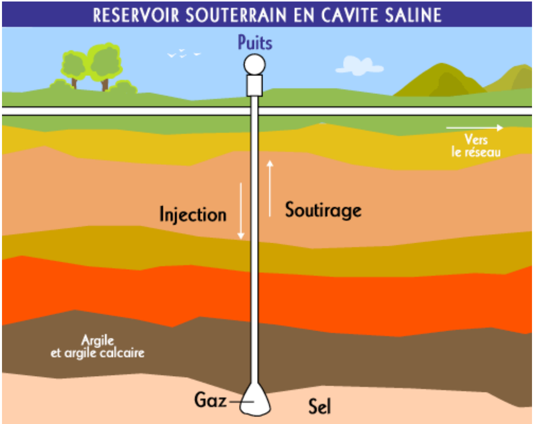
- Optimize profit from gas storage management
- Arbitrage on market :
  - Buy when prices are high (summer, weeke end)
  - Withdraw when prices are high (winter, Monday-friday)
- Find an optimal asset management under constraints.

$\Longrightarrow$ Optimal switching problem under constraint

Aquifer reservoir



Salt cavity


RESERVOIR SOUTERRAIN EN CAVITE SALINE
Puits
Vers
le réseau
Injection
Soutirage
Argile
et argile calcaire
Gaz
Sel

State variable

- S_t, Spot market prices for all dates $t \in [0, T]$ (if one factor model)
- I_t, Stock level in $t \in [0, T]$, so that $dI_t = e_t q_t dt$

Control on the system : $\Leftrightarrow$ asset management strategy

- q_t, Injected or withdrawed gas in $t \in [0, T]$, so that $q_t \geq 0$
- e_t, Working regime indicator, such that:

$$e_t = \begin{cases} +1 & \text{if in } t\text{, injection regime} \\ 0 & \text{if in } t\text{, idle regime} \\ -1 & \text{if in } t\text{, withdrawal regime} \end{cases}$$

$(\tau_n)_{n \geq 0}$, regime changing dates :

- τ_{2k}, injection dates :$e_{\tau_{2k}^-} = -1$
- τ_{2k+1}, withdrawal dates : $e_{\tau_{2k+1}^-} = +1$

Profit spawn by gas storage management

- Profit $t \in [0, T]$: $f_t(e_t, I_t, S_t, q_t) = -e_t q_t S_t - q_t k(e_t, I_t)$,

$$f_t(e_t, I_t, S_t, q_t) = \begin{cases} -q_t\,(S_t + k(+1, I_t)) & \text{if injection regime in } t \\ 0 & \text{if idle in } t \\ +q_t\,(S_t - k(-1, I_t)) & \text{if withdrawal regime in } t \end{cases}$$

  o $k(+1, I_t) > 0$ et $k(-1, I_t) > 0$, withdrawal and injection cost dealing with :
    – compressor cost
    – gas treatment (drying..)

- Penalisation at maturity T:
$$g_T(I_T, S_T) = -CS_T\,|I_T - I_f|\,,\ \text{o } C > 0$$

Not dealing here with switching cost.

Gas storage caracteristics

- $Q_{\max} > 0$, cavity volume
- $I_{\min}(t)$ et $I_{\max}(t)$, minimal and maximal tunnel to respect,

$$0 \leq I_{\min}(t) \leq I_t \leq I_{\max}(t) \leq Q_{\max}, \forall t \in [0, T]$$

- Flow constraints

  $q_{\min} > 0$ et $q_{\max} > 0$, maximal withdrawal and injection capacity, depending on:

  - stock level (cavity pressure)
  - time (compressor maintenance)

$$-q_{\min}(t, I_t) \leq e_t q_t \leq q_{\max}(t, I_t), \forall t \in [0, T]$$

Not dealing with other thermodynamical constraints :

- accounting for cavity creep ,
- avoid hydrate formation (integral constraint on flow)

Gas storage valuation :

- Markovian state for one factor model : $(I_t, S_t)_{0 \leq t \leq T}$
- Management strategy: $(e_t, q_t)_{0 \leq t \leq T}$
- Expected profit:

$$J(e,q) = \mathbb{E}\left[\int_0^T f_t(e_t, I_t, S_t, q_t)dt + g_T(I_T, S_T)\right]$$

Optimization problem :

$$\sup_{(e_t,q_t)_{0 \leq t \leq T}} J(e,q)$$

under constraints

Market representation

Two factor model

long term volatility	29 % / $\sqrt{year}$
long term mean reverting	0 / $year$
short term volatility	94 % / $\sqrt{year}$
short term mean reverting	7.4 / $year$
correlation	-0.13

Figure: Gas model parameters

Product dynamic

We assumed daily hedging.

- Day products available from tomorrow to the end of the running week.
- Week products available from next week to the end of the month
- Month products available from next month and up to the next quarter.
- Quarter products are available starting next quarter and until next year.
- no overlapping
- Finally, next year, Y+2, and Y+3 are available.

Gas storage representation (EDFE)

	Fast storage	Seasonal Storage
Working gas capacity (th)	36,600,000 th	32,637,363 th
withdrawal rate (th/day)	4,500,000 th/day	400,000 th/day
injection rate (th/day)	6,000,000 th/day	140,659 th/day
withdrawal cost (p/th)	0.35 p/day	0.31 p/day
injection cost (p/th)	0.35 p/day	0.72 p/day

Figure: Parameters for the fast and seasonal gas storage asset.

Calculation parameters

- number of trajectories in optimization : 70000
- number of meshes 10×7 for regression
- Time simulation : one year for fast storage, three years for seasonal storage,
- storage discretization :
  - 24 meshes for fast storage,
  - 80 meshes for seasonal storage
- Finite difference for a quoted product p

$$
\begin{aligned}
F_\epsilon(t, T) &= F(t, T)(1+\epsilon) \text{ for all } T \in p, \\
&= F(t, T) \text{ if } T \text{ not in } p,
\end{aligned}
$$

Sensibility $\frac{V_\epsilon - V}{\epsilon}$ with $\epsilon = 0.005$

Week product for fast storage

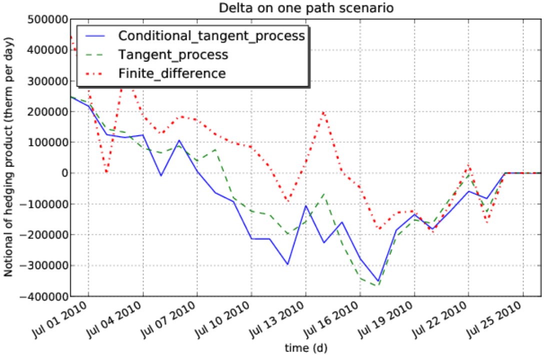
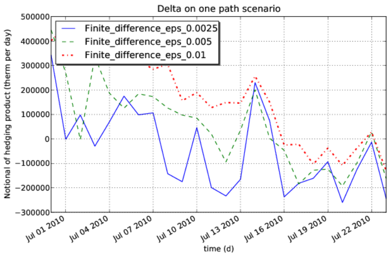
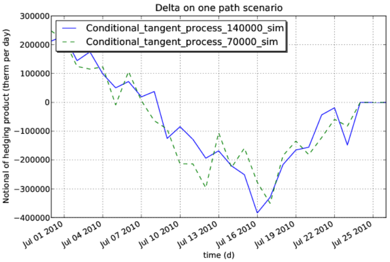


Figure: Fast storage: Hedging strategy for week product with delivery the last week of july 2010

Week product for fast storage (further calculation)



(a) Influence of ϵ parameter in finite difference



(b) Influence of the number of simulations for the conditional tangent process

Figure: Fast storage: Example of delta evolution for the weekly product (july 26th 2010- july 31th 2010) on one scenario.

December product for fast storage

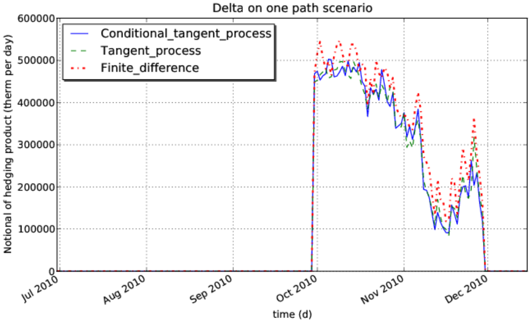


Figure: Fast storage: Example of delta evolution for the monthly product december 2010 on one scenario

Quarter product for fast storage

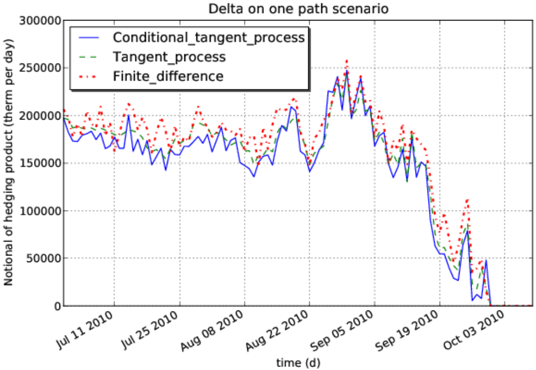


Figure: Fast storage: Example of delta evolution for the quarter product Q4 2010 on one scenario

Distribution results for fast storage

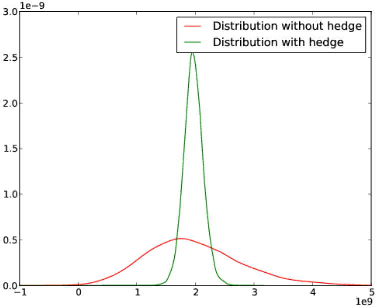


Figure: Fast storage: cash flow distribution with and without hedge computed with the conditional tangent method

Week product for seasonal storage

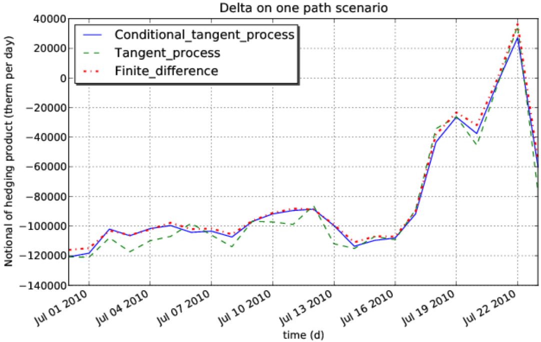


Figure: Seasonal storage: Hedging strategy for week product with delivery the last week of july 2010

December product for seasonal storage

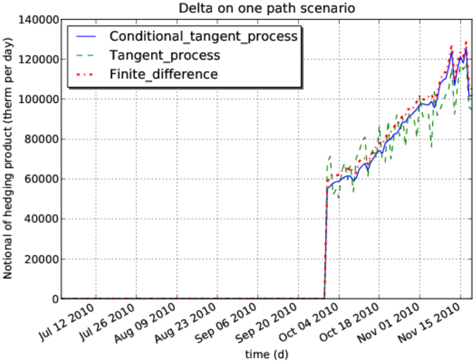


Figure: Seasonal storage: Example of delta evolution for the monthly product december 2010 on one scenario

Quarter product for seasonal storage

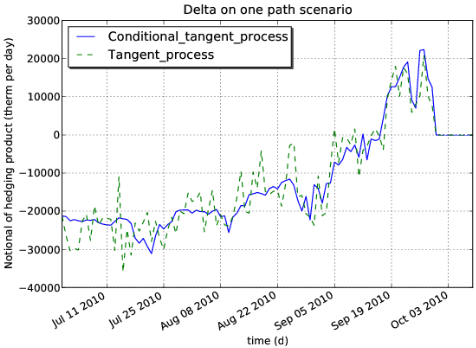


Figure: Seasonal storage: Example of delta evolution for the quarter product Q4 2010 on one scenario

Year product for seasonal storage

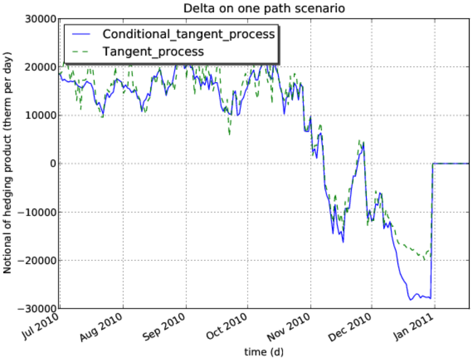


Figure: Seasonal storage: Example of delta evolution for the year 2011 on one scenario

Distribution results for seasonal storage

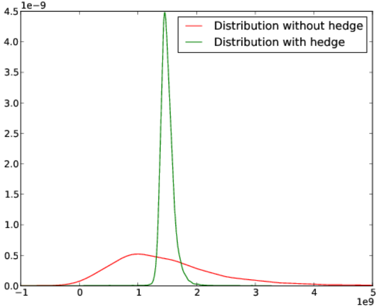


Figure: Seasonal storage: cash flow distribution with and without hedge computed with the conditional tangent method

Conclusion

- More easily fit than Finite Difference,
- Results as good as finite difference,
- Far more efficient in term of calculation cost,
- Can accelerate all swing sensibility.