Auctions in the Energy Sector
An Introduction and Survey

David Salant

Toulouse School of Economics
and FTI Consulting

April 2019

Introduction

Introduction

These lectures will introduce some basic results of auction theory, extend some other results and apply them to energy markets.

Auction Theory

1. Auction formats
2. Revenue Equivalence Theorem
3. Revelation principle
4. Martingale Theorem
5. Vickrey Clarke Groves Mechanisms

Energy Markets

1. Energy entitlements and PPAs
2. Transmission rights
3. Renewable procurement
4. Capacity markets
5. Default service

Agenda

1^{st} Lecture:

- Overview of energy auctions
- Examples of how auction design and strategy can matter
- Introduction of the application of game theory and auction theory in energy
- Revenue equivalence theorem
- Pay-as-bid vs. uniform-price auctions

2^{nd} Lecture:

- VCG auctions
- Existence theorems
- Supply function games
- Bidder coordination

Agenda

3^{rd} Lecture:

- Declining price anomaly
- Martingale theorem
- Variants of martingale theorem
- Winner's curse

4^{th} Lecture:

- Multi-product auctions
- The Simultaneous Multiple Round Auction ("SMR" auction or "SMRA") and convergence to optimal allocations
- Combinatorial Clock Auction ("CCA")
- Core selecting auctions

Introduction

Auctions in the Energy Sector

Auctions play a large role in the energy sector:

- Wind farms and other renewable procurement
- Transmission rights
- Generation capacity to ensure "resource adequacy," i.e., adequate capacity to ensure reliable energy supplies
- Default service procurement
- Entitlements and PPAs for generation capacity

Auctions in the Energy Sector - Renewables

- Renewable energy - includes wind farm, photovoltaic, hydro, bio mass, each with different properties.
- Wind-farms - not controllable. Requires sites and usually new transmission facilities.
- Solar - also not controllable and can be disperse.
- Hydro - Limited. Seasonal and weather dependent. Good complement to wind and solar.
- Biomass - a substitute for fossil fuels.

Auctions in the Energy Sector - Renewable Designs

- Generally, auctions are used for selecting subsidies - lowest subsidies win. Bidders sell energy into market. Their expenses are recovered through subsidies.
- Wind farms - most commonly one-shot sealed-bid
  - Auctions can be multi-attribute, e.g., location, availability, etc.
  - How to compare offers?
- Renewable Portfolio Standards - for a variety of renewables
  - Auctions must compare different types of resources
  - How to compare offers? Can use fixed weights or divide budget.

Introduction

Types of Auctions

- Single product auctions
  - Sealed-bid vs. open, ascending price
  - Forward and reverse
  - Private values and common values
- Forward and reverse
- Price and/or quantity and/or other attributes
- Multi-object
  - Combinatorial package
  - Sequential
  - Simultaneous multiple round

Auction "Games"

- Auctions are typically modeled as non-cooperative games in normal, or strategic, forms.
- A normal form game is comprised of 3 components
  1. A set of players $\mathcal{N} = \{1, 2, \ldots N\}$.
  2. A strategy set $\mathcal{S}_j$ for each $j \in \mathcal{N}$.
  3. A payoff function $\pi_j(s_1, \ldots, s_j, \ldots, s_N) : S_1 \times S_2 \times \ldots \times S_N \rightarrow \mathbb{R}$ for each $j \in \mathcal{N}$.
- A Nash (non-cooperative) equilibrium is a n-tuple strategy $\mathbf{s}^* \in S_1 \times S_2 \times \ldots \times S_N$ such that for each $j \in \mathcal{N}, s_j^* \in argmax_{s_j \in S_j} \pi_j(s_j, \mathbf{s}^*_{-j})$.
- In words, at a Nash equilibrium, each player's strategy is a best reply to the strategies of its rivals.

"Games" and the Prisoner's Dilemma

The Prisoner's dilemma is a classic example of a non-cooperative game. In this case, each player's best reply is independent of its rival's decision, i.e., this is a *dominant strategy* equilibrium.

Row & Column Player	Left	Right
Top	$(-5, -5)$	$(-10, -1)$
Bottom	$(-1, -10)$	$(-8, -8)$

Auctions as Games

- In an auction "game":
  - i. The bidders are the set of players.
  - ii. The allowed bids are the strategy sets.
  - iii. Payoff functions are profits, monetary benefits or utilities of the bidders.
- Nash, using the Kakutani fixed point theorem, proved existence of equilibrium in pure or mixed strategies for any non-cooperative games with finite sets of pure strategies. This proof relies on
  1. Strategy sets being compact and convex - unit simplex in Nash
  2. Payoffs are continuous
- Nash's approach can be used to prove existence of equilibrium for some types of auctions - more later. However, in many auctions, the payoff functions are discontinuous.

Revenue Equivalence Theorem ("RET") - Vickrey, Riley and Samuelson, Myerson

Four main types of single object auctions:

- 1^{st} price sealed-bid - highest bidder wins and pays its own bid amount.
- 2^{nd} price sealed-bid - highest bidder wins and pays the 2^{nd} highest bid amount.
- Ascending price (English) - the price starts low and gradually increases until no bidder is willing to raise price further.
- Descending price (Dutch) - the price starts high and gradually decreases until a bidder indicates willingness to accept the price.

Bid Strategy

- English and 2^{nd} price auctions:
  - In an English auction, a bidder will only stop bidding when price reaches the bidder's value. Thus, the optimal stopping rule, or strategy, is to bid up to value.
  - In a 2^{nd} price auction, it is a *dominant* strategy for each bidder to bid its value. Bidding less means a bidder may lose when it could have won at a price below its value. Bidding more means a bidder can risk overpaying when the bidder should not win.
  - Thus, if rivals' bids convey no information, English and 2^{nd} price auctions are strategically equivalent.
- Dutch and 1^{st} price auctions:
  - In both 1^{st} price auctions and Dutch auctions, a bidder wants to bid below its value - weighing the benefits of saving money by bidding less against the reduced chance of winning.
  - Thus a bidder's optimal strategy is the same in Dutch auctions and 1^{st} price auctions.
- In private value case, revenue (and outcomes) are the same in all four auctions

Revenue Equivalence Theorem (RET) - sketch of proof

There are several key steps in proving RET:

1. The first step is to apply *Revelation Principle* to restrict analysis to *direct revelation mechanisms (DRMs)*, i.e., "mechanisms" in which bidders are only asked to report values. For any feasible auction mechanism there is an equivalent DRM. The idea is that incentive compatibility allows backing out values from bids, so might as well start with asking bidders to report values.
2. The second step is to characterize optimal bid strategy. In case of DRM, this means that there is an "incentive compatibility" requirement under which bidders won't want to misreport values.
3. The third step is to note that for any DRM, the slope of each bidder's expected payoff with respect to its underlying value is the same across auctions.

RET

Theorem 1

(Klemperer, p. 43). Assume each of N risk-neutral potential buyers has a privately known value independently drawn from a common distribution, $F(v)$, which is strictly increasing and atomistic on $[\underline{v}, \bar{v}]$. Also, suppose no buyer wants more than one of the k available and indivisible objects. Then any auction mechanisms in which (i) the objects always go to the k buyers with the highest values and (ii) any bidder with value at $\underline{v}$ expects zero surplus must yield the same expected auction revenue and result in each buyer with value v making the same expected payment.

RET - sketch of proof

- Restricting attention to DRM. For each bidder, let $P(v)$ denote its probability of winning by reporting v, let $x(v)$ denote a its payment if it wins by reporting v.
- Incentive compatibility requires that if a bidder has true value v, then it won't want to report $v + \Delta$, i.e.,

$$\Big[v - x(v)\Big] P(v) \geq \Big[v - x(v + \Delta)\Big] P(v + \Delta), \forall \Delta$$

or

$$\Big[v - x(v)\Big] P(v) \geq \Big[v + \Delta - x(v + \Delta)\Big] P(v + \Delta) - P(v + \Delta)\Delta,$$

or

$$\Big[v - x(v)\Big] P(v) - \Big[v + \Delta - x(v + \Delta)\Big] P(v + \Delta) \geq -\Delta P(v + \Delta),$$

or

- $S(v) - S(v + \Delta) \geq -\Delta P(v + \Delta)$, where $S(v) = [v - x(v)]P(v)$.

RET - sketch of proof, continued

- Similarly,

$$\Big[v + \Delta - x(v + \Delta)\Big] P(v + \Delta) \geq \Big[v + \Delta - x(v)\Big] P(v),$$

or

$$\Big[v + \Delta - x(v + \Delta)\Big] P(v + \Delta) \geq \Big[v - x(v)\Big] P(v) + \Delta P(v),$$

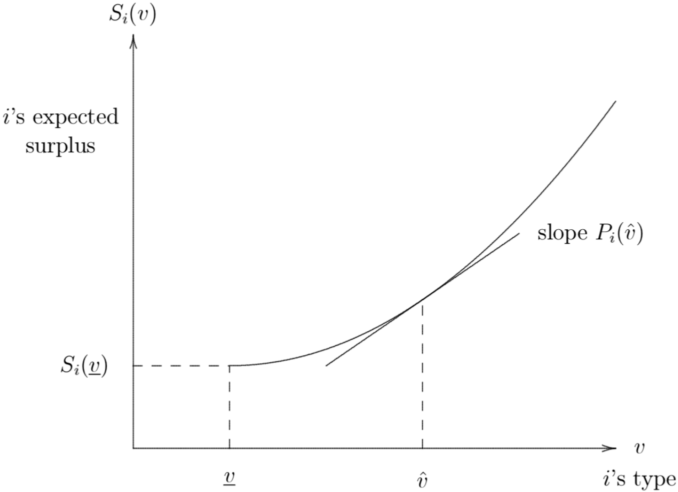
or

$$\Big[v + \Delta - x(v + \Delta)\Big] P(v + \Delta) - \Big[v - x(v)\Big] P(v) \geq \Delta P(v),$$

so

$$-\Delta P(v) \geq S(v) - S(v + \Delta) \geq -\Delta P(v + \Delta)$$

- Taking limits as $\Delta \to 0$ implies $S'(v) = P(v)$.


$S_i(v)$
i's expected surplus
$S_i(\underline{v})$
slope $P_i(\hat{v})$
v
$\underline{v}$
$\hat{v}$
i's type

Risk Averse Bidders (Maskin and Riley 1984)

- Bidders are not risk-neutral when their utilities are not quasilinear in payment.
- Suppose a bidder with type θ has payoff $u(-t, \theta)$ if it wins and pays price t and has payoff $w(-t)$ if it loses and pays t.
- If $u_1 > 0$, $w_1 > 0$, $u_{11} < 0$, $w_{11} < 0$, then both u and w are concave functions of income, which implies risk aversion.
- When bidders are risk averse, the pay-as-bid auction generates greater expected revenue than the uniform-price auction.
- The revenue dominance of pay-as-bid auction is intensified if the seller is also risk-averse.
- This is because a risk-averse bidder has an incentive to smooth its net surplus between winning and losing by bidding more aggressively in the pay-as-bid auction: bidding more aggressively reduces the risk of losing at the cost of reducing the payoff upon winning.

Pay-as-Bid vs. Uniform-Price Auctions in Electricity Market

- The RET suggests that pay-as-bid and uniform-price auctions should be the same.
- In 2000, the California Power Exchange appointed a panel to investigate whether shifting from uniform to pay-as-bid pricing in electricity market would provide power purchasers substantial relief from soaring prices.
- Kahn et al. (2001) advised against the proposal and pointed out that the critical assumption behind this proposal is that generators will bid just as they had before after the shift in pricing rule. However, one absolute certainty is that they will not.
- "The immediate consequence of this shift would be a radical change in bidding behavior that would introduce new inefficiencies, weaken competition in new generation, and impede expansion in capacity. "

Pay-as-bid v.s. Uniform-price Auctions in Electricity Market

- Under uniform-pricing, suppliers bid their marginal costs for every block of power that they offer, earning the difference between their marginal costs and market-clearing price to recover their fixed costs.
- As a result, power will be dispatched in merit order of generators from the lowest to the highest marginal cost. Power is supplied at the minimum cost at each point in time.
- Under pay-as-bid pricing, suppliers will not bid their marginal costs as only receiving marginal costs on successful bids will not cover their fixed costs. They bid instead at what they *expect* to be the market-clearing price.
- The *only* difference between the average prices actually realized under the two auctions would be the extent to which the suppliers' predictions are correct under pay-as-bid pricing.

Pay-as-bid v.s. Uniform-price Auctions in Electricity Market

Kahn et al. (2001) pointed out that inefficiency is likely to arise under pay-as-bid pricing because of forecasting errors:

- Pay-as-bid introduces some inevitable reduction in efficiency, as all bids will exceed the marginal costs of all blocks of power by amounts that depend upon the *varying estimates* of suppliers on the final market clearing price.
- A lower marginal cost bidder who overestimates the market clearing price will lose to a higher marginal cost bidder who estimates the market clearing price more conservatively, resulting in departure from merit order dispatch of their plants.

Pay-as-bid v.s. Uniform-price Auctions in Electricity Market

In addition to forecasting errors, inefficiency can also arise from rational strategic bidding behavior under pay-as-bid pricing:

- When a bidder is likely to have lower cost than the other on average, the bidder who is likely to have higher cost will rationally bid with a smaller markup while the bidder who is likely to have lower cost will rationally incorporate a larger markup, which also leads to inefficiency.
- The greater the number of separate generating companies, the greater will be the number of instances in which a higher-marginal-cost generator be selected over a lower-cost one.
- Shifting to pay-as-bid pricing will also impose an unnecessary cost of forecasting market prices on all bidders.

Position Auctions (Varian 2007)

- Position auctions are used by search engines such as Google and Yahoo for selling sponsored advertising slots.
- The problem is to assign agents $a = 1, \cdots, A$ to slots $s = 1, \cdots, S$. Agent a's value for slot s is $u_{as} = v_a x_s$, in which v_a is agent a's value per click. x_s is the click-through rate (CTR) of slot s.
- Assume CTR is decreasing in ranking: $x_1 > x_2 > \cdots > x_S$.
- Each agent bids b_a. The highest slot is assigned to the agent with the highest bid. The second highest slot is assigned to the agent with the second highest bid, etc.
- Each bidder who receives slot s pays the bid of the agent immediately below him for each click, $p_s = b_{s+1}$.
- The net profit of agent a who receives slot s is $(v_a - p_s)x_s = (v_a - b_{s+1})x_s$.
- Further, with marker power, the pay-as-bid and uniform price auctions are no longer equivalent

Equilibrium of Position Auction

- Consider the position auction as a simultaneous move game with complete information.
- In equilibrium, each agent prefers his current slot to any other slot.
- To move up to a higher slot, an agent needs to beat the bid that the agent show currently occupies the slot is making.
- To move to a lower slot, an agent needs to beat the price that the agent who currently occupies the slot is paying.
- A Nash equilibrium (NE) set of prices satisfies

$$\begin{aligned}(v_s - p_s)x_s &\geq (v_s - p_t)x_t \quad \text{for} \quad t > s \\ (v_s - p_s)x_s &\geq (v_s - p_{t-1})x_t \quad \text{for} \quad t < s\end{aligned}$$

  where $p_t = b_{t+1}$.
- There are a range of bids that satisfy these inequalities.

Symmetric Nash Equilibrium

A symmetric Nash equilibrium (SNE) set of prices satisfies

$$(v_s - p_s)x_s \geq (v_s - p_t)x_t, \quad \text{for all } t \text{ and } s,$$

Any SNE satisfies the following properties:

- Non-negative surplus: $v_s \geq p_s$ for all s
- Monotone values: $v_{s-1} \geq v_s$ for all s
- Monotone prices: $p_{s-1}x_{s-1} > p_s x_s$ for all s
- SNE is a subset of NE
- One step solution: If a set of bids satisfies the SNE inequalities for $s+1$ and $s-1$, then it satisfies the inequalities for all s.

Therefore, if an agent in slot s does not want to move up to $s-1$ or move down to $s+1$, it does not want to move to any other slot $s^{'} \neq s$.

Characterization of SNE

- Since the agent in position s does not want to move down one slot:

$$(v_s - p_s)x_s \geq (v_s - p_{s+1})x_{s+1}$$

- Since the agent in position $s+1$ does not want to move up one slot:

$$(v_{s+1} - p_{s+1})x_{s+1} \geq (v_{s+1} - p_s)x_s$$

- Put these two inequalities together gives

$$v_s(x_s - x_{s+1}) + p_{s+1}x_{s+1} \geq p_s x_s \geq v_{s+1}(x_s - x_{s+1}) + p_{s+1}x_{s+1}$$

$$v_{s-1}(x_{s-1} - x_s) + b_{s+1}x_s \geq b_s x_{s-1} \geq v_s(x_{s-1} - x_s) + b_{s+1}x_s$$

- This gives the upper and lower bounds of SNE bids:

$$b_s^U x_{s-1} = v_{s-1}(x_{s-1} - x_s) + b_{s+1}x_s$$
$$b_s^L x_{s-1} = v_s(x_{s-1} - x_s) + b_{s+1}x_s$$

Logic of SNE Bounds

- Suppose the agent in slot s is considering bidding b_s to beat the agent in slot $s-1$. At equilibrium b_s^*, the agent in slot s should be indifferent between winning $s-1$ and s in the worse case scenario when he just beats the agent in slot $s-1$ by a tiny amount and pays close to b_s:

$$(v_s - b_s^*)x_{s-1} = (v_s - b_{s+1})x_s$$

- Solving for b_s^* gives the lower bound of SNE.
- Suppose the agent in slot s is considering bidding b_s to squeeze the profit of the agent in slot $s-1$ so that he might prefer to move down to s. At equilibrium b_s^*, the agent in slot $s-1$ should be indifferent between staying in $s-1$ and moving down to s:

$$(v_{s-1} - b_s^*)x_{s-1} = (v_{s-1} - b_{s+1})x_s$$

- Solving for b_s^* gives the upper bound of SNE.

Bounds on Values

- Can we derive bounds on unobserved values from the observed equilibrium prices?
- The characterization of SNE implies

$$
\begin{aligned}
v_1 &\geq \frac{p_1 x_1 - p_2 x_2}{x_1 - x_2} \geq \\
v_2 &\geq \frac{p_2 x_2 - p_3 x_3}{x_2 - x_3} \geq \\
\ldots \\
v_S &\geq p_S
\end{aligned}
$$

- There exists a pure strategy NE if and only if each interval $\left[\frac{p_{s-1}x_{s-1} - p_s x_s}{x_{s-1} - x_s}, \frac{p_s x_s - p_{s+1}x_{s+1}}{x_s - x_{s+1}}\right]$is non-empty.
- The inequalities imply that the marginal cost of a click must increase as you move to higher positions.

Graphic Interpretation

- An expenditure profile graphs $p_s x_s$ on the vertical axes and graphs x_s on the horizontal axes .
- The slope of the line segments connecting each vertex are the marginal costs of moving up one position, which is increasing as you move to higher positions.
- An isoprofit line connects all CTR-expenditure bundles that gives the same profit to agent s:

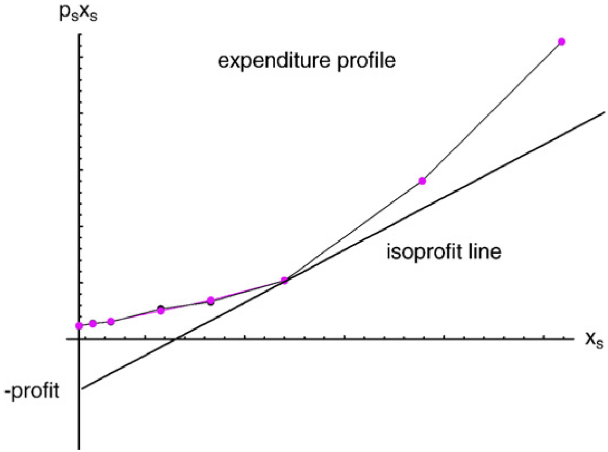
$$\bar{\pi}_s = v_s x_s - p_s x_s$$

- Taking total differentiation gives the slope of each isoprofit line

$$0 = v_s dx_s - d(p_s x_s)$$
$$\frac{d(p_s x_s)}{dx_s} = v_s$$

- Therefore, the isoprofit line is a straight line with slope v_s.

Graphic Interpretation



- Each bidder wants to choose the position with the lowest expenditure on the isoprofit line.

Introduction

Renewable Auctions - What Can Go Wrong -1

- In 1993, the California Public Utility Commission ("CPUC") implemented a requirement for the three main public utilities to purchase long term supplies from "Qualifying Facilities" (QF) through auctions.
- The QF facilities were "renewables" intended to substitute for fossil fuels.
- Each facility offer included "peak period capacity cost", "energy related capacity cost", and "energy cost", as well as capacity and availability.
- The score for each facility was a weighted average of these costs, where the weights were based on availability.
- The lowest (best) offers won, and the prices paid was based on the lowest losing score, i.e., a second price rule.
- The winning bidders almost all had **negative** energy prices. The second price logic was misapplied.

Entitlement Auctions - What Can Go Wrong -2

In 2001, the Texas utilities auctioned entitlements (like VPPs auctioned by EDF), in a single simultaneous auction. The Reliant and TXU contracts were identical for shares of a nuclear plant.

Table 9.6
Texas capacity auctions: fall 2001

	Reliant South	TXU South	Δ	%Δ
1 year base load	$7.32	$10.59	$3.27	45%
2 year base load	$6.59	$10.33	$3.74	57%
1 year base cyclic	$0.75	$1.79	$0.95	127%

Introduction

Vickrey-Clarke-Groves ("VCG") Auctions

- Basic question is wether there are types of auctions in which bidders will report true values.
- Vickrey was first to study this problem, showing that bidding true values is a dominant strategy for single object auctions.
- Riley - Samuelson and Clarke-Groves generalized the analysis.
- Clarke-Groves and Green and Laffont provided "mechanism design" approach.
- Green and Laffont show that under certain conditions, the VCG mechanism is the only mechanism in which "players" reporting true values is a dominant strategy.

Auctions and Mechanism Design

- Solving for an optimal auction is a form of a *mechanism design* problem.
- A mechanism design problem is one in which a "planner" or auctioneer sets the rules of a non-cooperative game to optimize the outcome. The decision problem can be how to allocate a resource, such as generation or transmission capacity.
- The mechanism is characterized by
  1. Set of players (or bidders)
  2. Set of possible decisions (allocations in an auction)
  3. Strategy sets, or set of what players can report or bid.
  4. Decision rule and payments based on reports
- An auction mechanism is one in which the strategy sets are bids, and the outcome is an allocation and payments.
- In a *Direct Revelation Mechanism* agents, or bidders, are asked to report values.

The Revelation Principle

- One of the key insights in the analysis of auction design is that one can restrict attention to Direct Revelation Mechanisms.
- While bidders will not always report true values, one can still back-out values from bids, assuming bidders are maximizing their own payoffs. Hence,

Lemma 2

The Revelation Principle *(Myerson) Given any feasible auction mechanism, there exists an equivalent feasible direct revelation mechanism which gives to the seller and all bidders the same expected utilities as in the given mechanism.*

VCG Mechanisms (Green and Laffont (1977)

- Considers Direct Revelation Mechanisms,("DRM") where players have utility of the form $U_i(K, t_i) = v_i(K) + t_i$, , where K is the decision and t_i the transfer to player i.
- This means that utility is additively separable in money.
- Players (bidders) report valuations. The valuations can be for a single item, if, for example, the mechanism is a single object auction. Or it can be different combinations of multiple items.
- The outcome is then the allocation which maximizes the sum of reported values among feasible combinations.
- Let $w_i(K)$ denote $i's$ reported valuation as a function of the decision and $t_i[K^*[(w)]$ be the associated transfer to player i, where $K^*(w)$ is optimal decision with a vector of reported values being $w(K)$.

VCG Mechanisms - continued

Definition 1

A **Groves Mechanism** *is a DRM with a transfer rule of the form:*

$$t_i[w(\cdot)] = \sum_{j \neq i} w_j[K^*(w(\cdot))] + h_i[w_{-i}(\cdot)] \tag{1}$$

- Each function $h_i[w_{-i}(\cdot)]$ is an additional payment for i that only depends on the reported values of its rivals, $w_{-i}(\cdot)$
- A special case of the Groves Mechanism is called the *P*ivot Mechanism where all the $h_i[w_{-i}(\cdot)]$'s are zero.
- In general, a bidder's payment only depends on the outcome, K^* and rivals bids.
- Thus, the only way a bidder's bid affects its payment is in the impact on the outcome.

Incentive Compatibility and the Revelation Principle

Definition 2

A revelation mechanism is strongly individually incentive compatible *(s.i.i.c) if the truth is a dominant strategy for each player, i.e., i.e., for all $i, w_i(\cdot), w_{-i}(\cdot)$, and $v_i(\cdot)$*

$$U_i(v_i, w_{-i}) \geq U_i(w_i, w_{-i}) \tag{2}$$

Theorem 3

A Groves Mechanism is s.i.i.c.

Theorem 4

A s.i.i.c. Direct Revelation Mechanism is a Groves Mechanism.

So, the only mechanisms which induce bidders true values are Groves mechanisms, a.k.a. Vickrey- Clarke-Groves (“VCG”) auction mechanisms.

A VCG Example

Bidder/ no. of blocks	1 block	2 blocks	3 blocks
Bidder 1	10	20	22
Bidder 2	10	16	19
Bidder 3	8	13	15

- The above table provides bidder values in a VCG auction for 3 blocks with 3 bidders in which the h_i's are all zero. In other words, this is a pivot mechanism.
- Assuming bidder offer values, Bidder 1 will win two blocks and Bidder 2 will win 1 block.
- Vickrey prices are determined by looking at how each bidder affects the values the other bidders derive (Bidder 1 gets 20, and Bidder 2, 10) in this example.
- Bidder 1's VCG price $= 16 + 8 - 10 = 14$.
- Similarly, Bidder 2's VCG price $= 20 + 8 - 20 = 8$.

Another VCG Example

Bidder/ no. of blocks	1 block	2 blocks	3 blocks
Bidder 1	10	20	29
Bidder 2	10	13	15
Bidder 3	4	6	7

Table: VCG outcome not in core

- The above table provides a different set of bidder values in a VCG auction for 3 blocks with 3 bidders.
- Again Bidder 1 wins two blocks Bidder 2 wins one block.
- Absent Bidder 1, Bidder 2 wins 2 blocks and Bidder 3 one block.
- So, Bidder 1's VCG price $= 13 + 4 - 10 = 7$.
- Similarly, Without Bidder 2, Bidder 1 wins three blocks. Thus Bidder 2's VCG price $= 29 - 20 = 9$.
- Thus, Bidder 1 pays 7 for two blocks and Bidder 2 pays 9 for one block!

Some Other not VCG, 2^{nd} Price Auctions

New Zealand 8 MHz UHF TV license auction

Lot	Winner	High Bid	2nd Bid
1	Sky Network TV	$2,371,000	$401,000
2	Sky Network TV	$2,273,000	$401,000
3	Sky Network TV	$2,273,000	$401,000
4	BCL	$255,124	$200,000
5	Sky Network TV	$1,121,000	$401,000
6	Totalisator A.B.	$401,000	$100,000
7	United Christian	$685,200	$401,000

More 2^{nd} Price Auction Examples

AMPS-A Auction

Bidder	Bid amount
Telecom New Zealand	NZ$101,200,000
First City Capital	NZ$11,158,800
Imagineering Telecommunications	NZ$1,388,000

TACS-A Auction

Bidder	Bid amount
Bell South	NZ$85,552,101
Telecom New Zealand	NZ$25,200,000
Racal-Vodafone Ltd	NZ$1,000,000
Broadcast Communications Ltd.	NZ $2,000

More NZ Examples

TACS-B Auction

Bidder	Bid amount
Bell South*	NZ$85,552,101
OTC International*	NZ$13,250,000
Telecom New Zealand*	NZ$7,000,000
Broadcast Communications Ltd.	NZ$5,000
Michael Oliver Thaisen	NZ$300

Introduction

Equilibrium Existence

.

- An auction is a non-cooperative game.
- An equilibrium solution is such that each firm's strategy is a best reply to those of its rivals.
- Not all auctions have equilibrium in *pure* or even *mixed* strategies.
- Many standard auctions have no pure strategy equilibrium.
- Interpretation of mixed strategy equilibrium is a challenge.

Equilibrium Existence Examples

1. A 2^{nd} price auction for a single object - dominant strategies equilibrium exists - each bidder bids value.
2. In 1^{st} price auctions with iid values $\sim U[0,1]$. In this case, to find a symmetric equilibrium, need to find bid functions, $\beta(v)$, where v is a bidder's value such that for each bidder $\beta(v)$ maximizes expected profits when all rivals use the same bid function.
3. Can use RET to solve for equilibrium bid functions in a 1^{st} price auction. Just the order statistics.
4. Consider a pay as bid auction in which (i) there is asymmetric information, (ii) one bidder has a known value of 100, and (iii) the only other bidder has a random value $\sim U[0,80]$ which is only known to that bidder. This auction has no pure strategy equilibrium.

Pure and Mixed Strategies

- In the pay-as-bid auction with asymmetric information, there is no one price for the stronger, S, firm to set that is optimal.
- If S chooses price p_S, with certainty, then if the weaker firm's (W) value is above p_S it will choose $p_W = p_S + \epsilon$,. And it will price equal to value of less otherwise.
- S should instead randomize. This is a "mixed strategy".

Table: Matching "pennies" game only has equilibrium in mixed strategies

Row & Column Player	Left	Right
Top	$(+1, -1)$	$(-1, +1)$
Bottom	$(-1, +1)$	$(+1, -1)$

Pure and Mixed Strategy Equilibrium Examples: (1)

- Consider a pay-as-bid, FPSB auction with 2 bidders, and 1 lot
- Bidder B1 values the lot at 100 Bidder B2 has a value of 50, which is known (approximately) to B1
- Equilibrium (Optimal strategies)
    1. B2 bids 50
    2. B1 bids just above 50 and wins
- Why is this an equilibrium?
    - B1 can't bid lower without risking losing the auction. Also B1 can't bid higher without paying more for the bid unnecessarily
    - B2 doesn't gain anything by bidding lower. And B2 can't bid higher without risking winning the lot for more than it is worth
    - Therefore, nobody gains anything by changing their bid. This is an equilibrium.[1]

[1] With 3 bidders (2 strong and 1 weak) and 3 lots, the analysis is the same.

Pure and Mixed Strategy Equilibrium Examples: (2)

- Now we suppose the above example is modified: B1's value is as before, but now, B2 has value of either 80 or 0.
- Bidder 2 knows its own value ex ante, but B1 only knows that there is a 50% chance of each value.
- If B2's value is 0, it submits no, or a null, bid.
- Here the equilibrium is not deterministic, i.e., mixed rather than. pure strategies.
  - To see why, suppose B1 always enters a bid of $b \in (0, 80)$.
  - Then B2 will bid $b + \epsilon$ whenever $B > v$, and 0 otherwise.
  - But, then bidding b is not optimal for B1. It should bid zero if $b > 50$ or $b + 2\epsilon$ otherwise.
- There is no deterministic equilibrium.

Pure and Mixed Strategy Equilibrium Examples: (3)

- Equilibrium strategies:
  1. B1 bids 0 or ϵ with probability $\frac{3}{8}$.
  2. Otherwise, bidder B1 randomizes tis offer price over $[0, 50]$ using the probability distribution function $F(b) = \frac{30}{80-b}$.
  3. B2 will bid 0 when it has a low value.
  4. When B2 has a high value, it will randomize over $[0, 50]$ using the distribution function $G(b) = \frac{50}{100-b}$.
- Why is this an equilibrium? This works as an equilibrium because expected return is constant:
  - For B1, ALL bids x between 0 and 50 result in a return of $\frac{(100-b)\times(1+G(b))}{2} = 50$.
  - For B2, ALL bids x between 0 and 50 result in a return of $(80 - b) \times F(b) = 30$.
  - Since all bids are between 0 and 50, B1's expected return of 50 and B2's expected return (when they participate) of 30 can't be improved on.

Equilibrium Existence

- Many real auctions have the feature of the above example that there is no equilibrium in pure strategies.
- Supply function games are common in the energy sector, and may have no pure strategy equilibria.
- The above examples raise the question as to whether an auction game has an equilibrium, and if so, what type.
- In other cases, mostly in which bidders can coordinate, there will be multiple equilibria.

Existence Theorems (1)

- John Nash proved one of the first existence theorems for matrix games.
- The proof established that the vector of best reply functions were continuous functions from compact and a convex set into itself.
- The Brouwer fixed point theorem can be applied to show that the best reply function has a fixed point.
- A best reply for one bidder is the strategy it would choose in response to those chosen by rivals.
- Thus, if each bidder's strategy is a best reply to its rivals, the vector of strategies is then a "Nash," non-cooperative equilibrium.
- In many auctions, the payoff functions, are non-concave and the best reply functions are not continuous.

Existence of Equilibrium in Auction Games

- The Nash results were for equilibrium in pure or mixed strategies, with a finite set of pure strategies.
- Application of the Brouwer or Kakutani fixed point theorem to auctions in which bids are continuous variables require concave payoff functions.
- In the asymmetric information auction example, the payoff functions are discontinuous around the pairs of bids where the two players offer the same amounts.
- In the California day-ahead market, bidders were required to submit step function bids, and which have a similar form of jump discontinuity. This is also used in European day-ahead markets.
- Other auction formats include sealed-bids and clock auctions.

Pure and Mixed Strategy Equilibrium Examples: (4)

- The California Independent System Operator (CAISO) required supplies to be step functions.
- The bids could be characterized as then pairs $(p_f, q_f) \in \Re^{20}$, where $p_f \leq p_{f+1}$ and $q_f \leq q_{f+1}$ for all $f = 1, 2, \ldots, 10$.
- The price p_1 is the price offered for the first q_1 MWs, p_2 is the price for second q_2 MWs etc.
- Payoffs are revenues less costs.
- As there are jump discontinuities where two firms offer the same prices, there is no equilibrium in pure strategies.

Main Existence Theorems (Dasgupta and Maskin (1986)

Theorem 5

Let $Ai \subset \Re^m(i = 1, ..., N)$, *be non-empty, convex and compact. If* $\forall i, U_i : A_i \to \Re^1$ *is quasi-concave in* a_i, *upper semi-continuous in* **a** *and graph-continuous, then the game* $[(A_i, U_i); i = 1, \ldots, N]$. *possesses a pure-strategy Nash equilibrium.*

Theorem 6

Let $A \subset \Re^m, (i = 1, \ldots, N)$ *be non-empty and compact. Let* $U, : A \to \Re^1$ *be continuous. Then there exists a mixed-strategy equilibrium for the game* $[(A_i, U_i); i = 1, \ldots, N]$.

Main Existence Theorems (Dasgupta and Maskin (1986)

Theorem 7

Let $\bar{A} \subset \Re^m$. be non-empty and compact, and let $[(A_i, U_i); i = 1, ..., N]$ be a symmetric game, where $\forall i,$, $A_i = \bar{A}$ and $U_i : \bar{A} \times \bar{A} \times \ldots \times \bar{A} \rightarrow \Re^1$ is continuous, except on a subset $A^{**}(i)$ of $\bar{A} \times \bar{A} \times \ldots \times \bar{A}$.. Suppose $\sum_{i=1}^{N} U_i(a)$ is upper semi-continuous, and $\forall i$ each U_i is bounded and satisfies Property (α^*). Then there exists a symmetric mixed-strategy equilibrium $(\mu^*, \ldots, \mu^*)$ with the property that $\forall i$ and $\forall \bar{a}_i \in A_i^{**}(i), \mu(\{\bar{a}_i\}) = 0$.

Property (α^*). $\forall \bar{a}_i \in A_i^{**}(i)$, $\exists$ a non-atomic measure ν on B^m such that for all $\boldsymbol{a}_{-i} \in A_{-i}^{**}(\bar{a}_i)$

$$\int_{B^m} [\lim_{\theta \to 0} \inf U_i(\bar{a}_i + \theta e, \boldsymbol{a}_{-i}) d\nu(e)] \geqq U_i(\bar{a}_i, \boldsymbol{a}_{-i}),$$

where the inequality is strict if

$$\boldsymbol{a}_{-i} = \underbrace{(\bar{a}_i, \ldots, \bar{a}_i)}_{(N-1)\text{ times}}.$$

Introduction

FTR and Interconnector Auctions

- Auctions are used to allocate interconnector capacity/transmission rights.
- Transmission flows tend to be seasonal, and the flow is from energy rich regions, say N, to expensive energy regions, say S.
- When energy costs differ across the two regions, competition from the lower cost region will affect the energy suppliers in the higher cost one.
- The impact on competition will affect bidding in an auction for Financial Transmission Rights ("FTR"s)
- And often there is concentration of generation assets in the high cost region, which means that the outcome of an open auction without restrictions may be sub-optimal.

FTR Auctions and Market Power

- Consider the case in which there are two regions, N and S
- There is one generator in each region. The generator in S has higher costs than N.
- A limited transmission capacity connects the two regions.
- All consumption is in S.
- Then, an auction for FTRs in which both generators in N and S can bid is unlikely to result in a socially optimal outcome.
- Caps on the allocation of FTRs can be imposed to achieve socially optimal outcomes.

Introduction

Sequential auction overview

Main Issues

I In many markets, such as energy markets, auction managers run auctions in advance of transaction date, and face decisions about how far in advance to conduct auction for all or part of auction volumes.

II Question addressed here is how decision about timing of auction can affect bidder incentives and the outcome. And bidders are assumed *know* that the auction manager *must* purchase a fixed transaction by a definite date.

III Key assumption is that bidder valuations can change over time, but that at any date, the valuations are *stochastically equivalent*

IV If the auction manager must make an ex ante decision about dividing auction volume, (s)he will want to spread it out, leaving a lesser amount for the last auction.

Examples of Sequential Auctions

Ashenfelter (1989) noticed that in sequential auctions of fine wines, prices tended to decline.

Distribution of price patterns for identical wines sold in same auctions (number of auctions)

	Christie's London	Sotheby's London	Christie's Chigaco	Butterfield's San Franc.	Total
Later price higher	271	143	90	20	524 (11%)
Later price lower	628	430	183	41	1282 (28%)
Later price equal	1498	1073	226	39	2836 (61%)

	Chateau Palmer 1961			Croft 1927			Chateau Margaux 1952			Quinta de Noral 1934		
	Lot size	Price	Price/ bottle	Lot size	Price	Price/ bottle	Lot size	Price	Price/ bottle	Lot size	Price	Price/ bottle
Lot 1	12	920	77	12	800	67	12	480	40	10	400	40
Lot 2	12	800	67	12	800	67	12	480	40	12	500	42
Lot 3	12	700	58	12	750	63	12	480	40	12	500	42
Lot 4				12	650	54	24	480	20	12	480	40
Lot 5				12	650	54	24	480	20	12	480	40
Lot 6				12	650	54	20	480	24			
Lot 7				12	650	54						

Source: "Liquid Assets", The International Guidet o Fine Wines,I ssue No. 4, Spring 1988.

Types of Sequential Auctions

- The wine auctions studied by Ashenfelter have been modeled as a sequence of auctions with a fixed set of buyers whose values remain the same across auctions.
- Weber (1983) showed that under some conditions, the auction price in a sequence of auctions is a Martingale. Milgrom and Weber (1999) suggest price should increase and not decrease.
- Sequential auctions with information revelation (Wolfstetter)
- Sequential auctions for complements (Levin, Krishna and Rosenthal, spectrum auctions). Also, complements over time, and sunk investments.
- Bidders with risk aversion (McAfee and Vincent)

Sequential Auctions with Stochastically Equivalent Values

- Assume a fixed set of bidders, who can each win at most one lot.
- In each auction, values are selected from an identical distribution, but a bidder's value in one auction is independent from its value in the previous auction, that is, the distribution of individual bidder valuations in each auction is *stochastically equivalent*. Bernhardt and Scoones, and Engelbrecht-Wiggans have previously studied this case.
- In many auctions, bidders valuations do change, even between auctions held close together.
- The *key assumption* is that the best losing bidder in one auction, cannot expect to win with probability one in the next auction. Stochastic equivalence allows straightforward calculations.

Why Stochastic Equivalence?

- Model is one of auctions for future delivery or performance. This type of model has been studied in other settings, such as Lang and Rosenthal's (1991) paper on contractor bidding, and empirical on procurement (Hong and Shum (2002)).
- In energy sector, "full-requirements" energy procurement auctions are usually for forward delivery, and at times, the auction manager has the flexibility to defer a purchase decision (Loxley and Salant).
- Capacity auctions in energy sector are also conducted in advance of needed construction.
- Many markets have both forward and spot markets. Buyers or sellers often have the ability to defer decision to spot market. Here it is assumed that both the buyer (auctioneer) and the sellers (the bidders) are strategic players who can influence prices.
- Most of the results are for two fixed auction dates, but this lecture also considers a sequence of auctions.

Sequential Auctions and Information

- Often bidders obtain information over time.
- The information obtained can affect distribution of values, or can cause values or costs to change.
- Resolution of uncertainty can increase or decrease variance of distribution of values. This can lead to declining prices as noticed by Ashenfelter.
- This is not what is assumed here - the main assumption is that costs or values can shift.
- Shifts can occur due to informational accumulation across auctions, or other exogenous changes.

Sequential Auctions and Weber's Martingale Theorem

- Consider an example where the auctioneer wants to purchase (or sell) two identical lots or blocks. An example is of a Regional Transmission Organization who might need to purchase $1,000$ MW of new capacity and will need to purchase two new plants.

Sequential Auctions and Weber's Martingale Theorem

- Consider an example where the auctioneer wants to purchase (or sell) two identical lots or blocks. An example is of a Regional Transmission Organization who might need to purchase $1,000$ MW of new capacity and will need to purchase two new plants.
- Option 1 is to conduct a single auction for both lots.
- Option 2 is to conduct two auctions, one lot in each auction.
- Option 3, which might not always be feasible, is to run two auctions, and purchase both lots in the first auction, if bids are very competitive, one lot in each if the bids in the first auction are moderately competitive, and otherwise wait until the last auction.

Sequential Auctions and Weber's Martingale Theorem

- Consider an example where the auctioneer wants to purchase (or sell) two identical lots or blocks. An example is of a Regional Transmission Organization who might need to purchase $1,000$ MW of new capacity and will need to purchase two new plants.
- Option 1 is to conduct a single auction for both lots.
- Option 2 is to conduct two auctions, one lot in each auction.
- Option 3, which might not always be feasible, is to run two auctions, and purchase both lots in the first auction, if bids are very competitive, one lot in each if the bids in the first auction are moderately competitive, and otherwise wait until the last auction.
- Question: Which option minimizes costs? And, if Option 3 is feasible, what rule should be used to minimize expected costs?

The Martingale Theorem

- Suppose there are B bidders and k identical items for sale. Let $V(t^b)$ the b^{th} highest value. Bidders can win only one item each.

The Martingale Theorem

- Suppose there are B bidders and k identical items for sale. Let $V(t^b)$ the b^{th} highest value. Bidders can win only one item each.
- Let I_n denote the information available after the sale of the n^{th} item.

The Martingale Theorem

Theorem 8

Suppose k identical items are sold one-at-a-time in sequence of auctions with the same B bidders participating in all auctions. Suppose, too that each bidder can only win one item, and that in each round, a bidder's bid is an increasing function of the bidder's type or value. Then

$$E[p_n|I_{n-1}] = E[v^{k+1}|I_{n-1}]$$

If the auctions are all first or second price auctions, and prices are publicly announced, then the sequence of prices forms a Martingale, i.e., $E[p_n|p_{n-1}, p_{n-2}, \ldots, p_1] = p_{n-1}$, where p_n is the price in the n^{th} auction.

Sequential Auctions with Varying Costs

- Most often, bidders make decisions after one auction that affects their costs, or values, in subsequent auctions.
  1. In energy procurement auctions, bidders execute contracts, either to hedge positions if they win, sell off energy rights or unwind positions if they lose.
  2. Contractors who lose one project can redeploy resources elsewhere.
  3. In forward auctions, for oil leases, spectrum licenses, and other large, durable investments, firms with capital constraints will adjust portfolios win or lose.
- In addition, firms acquire information that affect costs or values.
- This means that firms j an k with costs $c_j < c_k$ in one auction, may find that $c_j > c_k$ in the second auction.

Sequential Auctions and Stochastically Equivalent Costs

- In practice, over time, and as the delivery period approaches, costs will shift.
- Even when average costs don't change, rank of individual firm costs can shift.
- Moreover, the distribution of costs or values can get wider or narrower over time.
- In the energy sector, portfolios of contract are constantly changing with trades, and load shifts.
- One model of this is to assume that a bidder has two independent draws from the same distribution in a sequence of two auctions. This is called stochastically equivalent auctions.

Equilibrium with Stochastic Equivalence

- Suppose each bidder can win at most one of the two plants.

Equilibrium with Stochastic Equivalence

- Suppose each bidder can win at most one of the two plants.
- If there is one auction, and N bidders to start, then as above, the expected price is $\frac{3}{N+1}$.

Equilibrium with Stochastic Equivalence

- Suppose each bidder can win at most one of the two plants.
- If there is one auction, and N bidders to start, then as above, the expected price is $\frac{3}{N+1}$.
- Now suppose there are two auctions, and that the first auction winner cannot participate in the second auction.

Equilibrium with Stochastic Equivalence

- Suppose each bidder can win at most one of the two plants.
- If there is one auction, and N bidders to start, then as above, the expected price is $\frac{3}{N+1}$.
- Now suppose there are two auctions, and that the first auction winner cannot participate in the second auction.
- Then, the expected value of the second auction price will be $\frac{2}{N}$. This provides the winner an expected profit of $\frac{1}{N}$.

Equilibrium with Stochastic Equivalence

- Suppose each bidder can win at most one of the two plants.
- If there is one auction, and N bidders to start, then as above, the expected price is $\frac{3}{N+1}$.
- Now suppose there are two auctions, and that the first auction winner cannot participate in the second auction.
- Then, the expected value of the second auction price will be $\frac{2}{N}$. This provides the winner an expected profit of $\frac{1}{N}$.
- Therefore, in the first auction, the low cost winner should win, but at a price of $\frac{2}{N+1} + \frac{1}{N(N-1)}$. This last term is the expected profit this bidder would earn from waiting for the second auction.

Equilibrium with Stochastic Equivalence

- Suppose each bidder can win at most one of the two plants.
- If there is one auction, and N bidders to start, then as above, the expected price is $\frac{3}{N+1}$.
- Now suppose there are two auctions, and that the first auction winner cannot participate in the second auction.
- Then, the expected value of the second auction price will be $\frac{2}{N}$. This provides the winner an expected profit of $\frac{1}{N}$.
- Therefore, in the first auction, the low cost winner should win, but at a price of $\frac{2}{N+1} + \frac{1}{N(N-1)}$. This last term is the expected profit this bidder would earn from waiting for the second auction.
- The expected costs from one auction are $\frac{6}{N+1}$, whereas the expected costs from holding two auctions are $\frac{2}{N} + \frac{2}{N+1} + \frac{1}{N(N-1)}$.

One Auction or Two?

- When values are stochastically equivalent, two auctions result in lower costs whenever $N \geq 3$.
- There are two effects

One Auction or Two?

- When values are stochastically equivalent, two auctions result in lower costs whenever $N \geq 3$.
- There are two effects
  1. An efficiency effect - there are cost savings from distributing the purchases over time.
  2. A strategic effect - competition in the first auction will be reduced if bidders can possibly win more in the second auction.

One Auction or Two?

- When values are stochastically equivalent, two auctions result in lower costs whenever $N \geq 3$.
- There are two effects
  1. An efficiency effect - there are cost savings from distributing the purchases over time.
  2. A strategic effect - competition in the first auction will be reduced if bidders can possibly win more in the second auction.
- The strategic effect implies that the best option is to spread the auction volume across auctions.

Dividing $K > 2$ Objects Across Two Auctions

- More generally, with K objects, X purchased in the first auction and $Y = K - X$ in the second, and N bidders, the price in the second of two auctions will be

$$p^2 = \frac{Y+1}{N-X+1} = \frac{K-X+1}{N-X+1} \tag{3}$$

- This will result in expected surplus for each participant in the second auction of $\frac{Y}{2(N-X+1)}$
- The above implies that the price in the first auction will be

$$p^1 = \frac{X+1}{N+1} + \frac{Y}{2(N-X)(N-X+1)} \tag{4}$$

- The ex ante optimal K will minimize expected costs of $C(X, K) =$ $X\left(\frac{X+1}{N+1} + \frac{K-X+1}{2(N-X)(N-X+1)}\right) + (K-X)\left(\frac{K-X+1}{N-X+1}\right)$

How Price Varies Across Auctions Depends on Auction Split

- The expressions (3) and (4) determine expected prices in each auction.
- A larger first auction procurement, X, and therefore a smaller second auction procurement, Y, tend to result in decreasing prices and vice versa.
- Other factors can affect whether prices increase or decrease.
  - When bidders are risk averse and uncertainty is reduced between auction dates, then bidders will tend to bid less (more aggressively) in the later auction.
  - Costs can shift - but if the shocks affect all bidder costs in the same way (e.g., fuel prices), then the expected prices will be the same across auctions.
- As prices vary based on the split of the K needed units into $X + Y$, the auction manager can select X and Y to minimize total expected costs.

The Optimal Split in the Two Auction Case

Proposition 9

In an auction of K objects to N bidders, there exists $N^{'} > 0$ such that, if $N > N^{'}$, then $0 < X^* < K$ where X^* is the optimal amount sold in the first auction.

Proposition 10

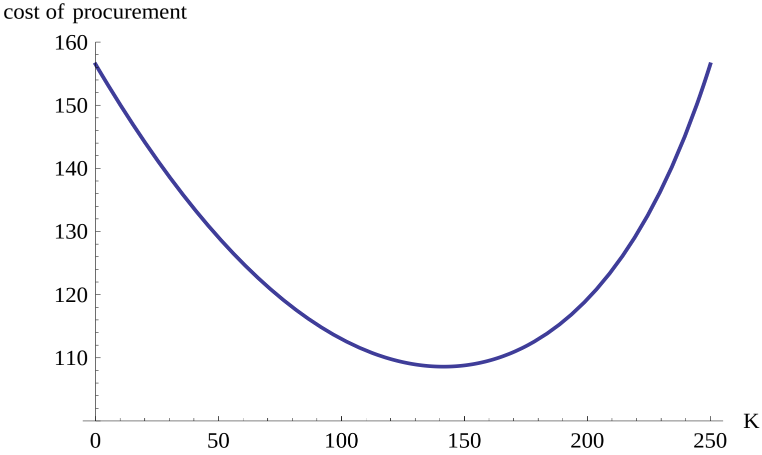
There exist $N^{'} > 0$ and $0 < \alpha^{'} < \alpha^{''} < 1$ such that, if $N > N^{'}$ and $\frac{K}{N} < \alpha^{'}$ (respectively $\frac{K}{N} > \alpha^{'}$), then $X^* > \frac{K}{2}$, (respectively $X^* < \frac{K}{2}$.)

Ex Ante Optimal Division: Two Auctions, 400 Bidders, 250 Lots

Generally, somewhat more than half the volume should be purchased in the first auction. This second example considers the case in which N = 400, K = 250

x	y	p_1	p_2	Total Costs
0	250	0.198	0.63	156.48
100	150	0.377	0.501	112.98
140	110	0.442	0.425	108.60
141	109	0.443	0.423	108.598202
142	108	0.445	0.421	108.598178
143	107	0.446	0.419	108.60
145	105	0.449	0.414	108.63
150	100	0.457	0.402	109.79

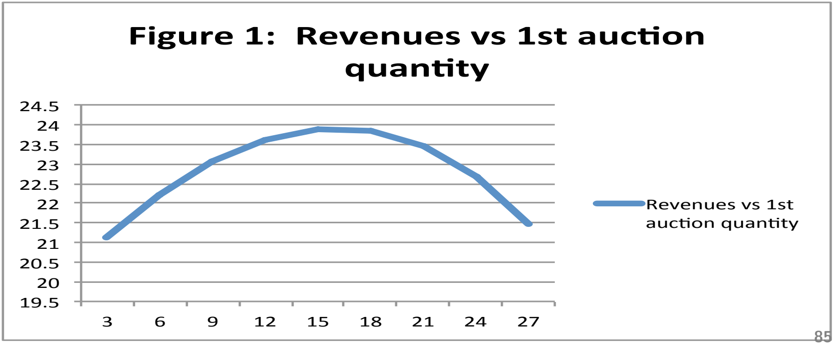
Optimal Split - Reverse Auction: 400 Bidders and 250 Lots



Optimal Split in Two Forward Auctions

Similar results apply in forward auctions. Consider case with 90 bidders and 30 lots. In this case revenues are

$$\begin{aligned} R(X,K) = \quad & \max\{0, X\left(\tfrac{N-X}{N+1} - \tfrac{(K-X)(K-X+1)}{2(N-X)(N-X+1)}\right). \\ + \quad & (K-X)\left(\tfrac{N-K}{N-X+1}\right)\}. \end{aligned}$$



When Division of Auction Volume Does Not Matter

The Martingale theorem still applies as long as the best losing bidder in the first auction will win the second with probability one.

Proposition 11

Suppose first auction values, v_{j1}, are stochastically and independently distributed and the second auction values, $v_{j2} = v_{j1} + z$ for each j, where z is some random shock, and that this is all known to all bidders. Suppose that the price in each auction is the highest losing offer. Then the second period price is $v_2^{K+1}(N - X) = v_1^{K+1}(N) + z$ and the first period price is $E[v_2^{K+1}]$, where $E[v_2^{K+1}]$ is the ex ante expected value v_2^{K+1} conditional on losing auction 1 and assuming the conditional expected value, $E[v_2^{K+1}|v]$ of v_2^{K+1} is independent of v.

Proposition 11 states that the expected price in auction 2 given auction 1 price is $E[v_1^{K+1} + z|p_1] = E[z] + E[v_2^{K+1}] = p_1 + E[z]$. When $E[z] = 0$, then the expected price in auction 2 is the realized value of the auction 1 price.

Equilibrium Prices in the Two Auction Case

Lemma 12

Suppose bidder valuations are stochastically independent draws from the same distribution in each auction, and that each bidder can win at most one unit. As above, let X denote the number of units the auction manager sells in the first auction, and $Y = K - X$ the number of units sold in the second auction. Also, suppose that the price in each auction is the highest losing offer. Then, the expected value of the price in the second auction is

$$E(p_2) = E[v_2^{Y+1}], \tag{5}$$

and the expected value of the price in the first auction is

$$E(p_1) = E\left[v_1^{X+1}\right] - E\left[\max\{0, v_{j2} - v_2^{Y+1}\}\right] \tag{6}$$

.

Prices and Split of Auction Volume

Equilibrium prices in the each auction depends on the split of the auction volume.

Proposition 13

Suppose, as above, N bidders are competing for X units in auction 1 and $Y = K - X$ units and auction 2. Also, suppose, as above, that bidders can each purchase at most one unit, and bidder valuations in each auction are stochastically independent. Then, the auction 1 price will be higher (respectively lower) than the auction 2 price whenever

$$E\left[v_1^{X+1}\right] - E\left[\max\{0, v_{j2} - v_2^{Y+1}\}\right] > E\left[v_2^{Y+1}\right] \tag{7}$$

A higher X, and therefore a lower Y, will increase p_2. The increase in X will also decrease p_1 when valuations are drawn from a uniform distribution on $[0, 1]$.

Closed Loop Equilibrium

When the auction manager can decide to adjust amount purchased then the amount it will want to pay for each unit will not exceed the expected cost of deferring that purchase to the second auction. and even with strategic adjustments of bids, the auction manager can still improve on the "open loop" case.

Proposition 14

Suppose the assumptions of Lemma 1 are satisfied. Suppose that the auction manager can set unit specific reserve prices in the first auction. Then, the expected revenues will be maximized when the unit specific reserve prices p^r_X for unit X in the first auction is set as follows:

$$\begin{aligned} p^r_X = & \frac{1}{X}\Big\{(K-X+1)v_2^{K-X+2}(N-X+1) \\ - & (K-X)v_2^{K-X+1}(N-X) \\ + & (X-1)\big[v_1^X(N) - E\Big[\max\{0, v_{j2} - v_2^{K-X+2}(N-X+1)\}\Big]\big]\Big\} \end{aligned} \quad (8)$$

Closed vs. Open Loop Comparison

- The following is an example of how per unit reserve prices can affect expected procurement costs in a reverse auction.
- Suppose, bidder costs are uniform on $[0, 1]$ in each auction.
- When there are two lots, $K = 2$, the auction manager would never want to purchase more than $\frac{2}{N}$ or the first unit for more than $\frac{3}{N+1}$. And more generally, the auction manager should, if possible,
  1. Never purchase at all when there are no bids below the expected per unit cost of deferring the purchase of both units to a second auction.
  2. Purchase only one lot in the first auction, if the purchase price of the first lot is below the expected cost of a second lot in the second auction, and the price of the second lot in the first auction is above its expected cost in the second auction.
  3. Purchase *both* lots in the first auction when the price of the second lot in the first auction is below the expected price of a first lot in the second auction.
- Such rules *have* been used in prior procurement auctions.

Declining Prices?

- The auction manager may have the ability to space its sales over several dates. In this section, it is assumed that there are K auctions with $N > K$ bidders, and each auction is for exactly one unit.
- The following example illustrates equilibrium prices with 3 lots and $N > 3$ bidders.

Table: Price changes across 3 auctions

Number of bidders	5	20	40	80
1^{st}auction price	.5125	.8991	.9499	.9750
2^{nd} auction price	.5167	.8971	.9493	.9748
3^{rd} auction price	.5	.8947	.9487	.9747
Total revenues	1.53	2.69	2.85	2.92

Declining Prices and Large Numbers Bidders

Proposition 15

Suppose, there is a sequence of K auctions $N > K$ bidders, and that the auction manager sells one unit in each auction. Also suppose that no bidder can win more than one auction. And also, as above, suppose that each bidder's valuation in each auction is an independent draw from the uniform distribution on $[0, 1]$. Then the last auction price will necessarily be lower than the next to last auction price.

Declining Prices - Sketch of Proof

Idea of Proof

- Consider auction $K-1$. A bidder in that auction has a probability $\frac{1}{N-K+1}$ of winning auction K. The expected surplus in auction K is $\frac{1}{N-K+2}$.
- The highest value losing bidder will have expected value of $\frac{N-K-1}{N-K+1}$ in auction K, and $\frac{N-K}{N-K+2} > \frac{N-K-1}{N-K+1}$ in auction $K-1$.
- Only where there valuation of the highest losing bid can increase, which may be unlikely, can price increase at the end.

Forward Sales

- In many cases firms must line up resources long before delivery.
- Fonterra, a New Zealand dairy cooperative produces a variety of milk products - whole milk powder (WMP),skim powder (SMP), butter fats (BF), and sells these products in auctions generally 3 - 5 months in advance of shipment.
- Fonterra has processing facilities has limitations in what combinations of WMP, SMP and BF are feasible.
- Fonterra will sell auctions for forward delivery, and generally each delivery date contract may be available at three different auction dates before it is shipped.
- The Fonterra auctions allows shifting of volumes from longer term auctions to nearer term ones.

Energy Procurement - Capacity Markets

- Reliability of the energy grid is a public good.
- However, private, bilateral transactions will not necessarily ensure adequate reserves for the entire grid. There is a classic free-rider problem.
- Individual buyers on the grid cannot possibly ensure reliability by any financial means other than by ensuring total system reliability
- Therefore, grid operators and distribution companies have been directed to offer subsidies or other inducements, for new capacity.
- And price spikes, that may reward investors are not generally allowed even to provide inducements for ensuring reliability

Capacity Markets

- New capacity, for fossil fuels, or renewables, require significant lead time.
- Auctions for new capacity are often conducted up to 5 years in advance of needed plants.
- An Auction Manager or grid operator can have a chance to defer purchases in the earlier auctions.
- Developer cost is tied to financing and other market conditions, and these can vary quite quickly.

Energy Procurement

- Energy utilities have to purchase electricity from independent generators in many jurisdictions.
- Except in California during the CALPX meltdown and energy crisis, utilities normally try to limit how much is purchased in "day-ahead" and "real-time" or "hour-ahead" markets. Some spot purchases are always required for short term load fluctuations.
- Utilities in some jurisdictions will seek contracts for full requirements service up to 3, or even 5, years in advance of service.
- Current auctions in a number of jurisdictions allow firms to defer purchases.

Strategic Withholding

- If one bidder can supply both units, it may have an incentive to withhold supply, or offer higher prices than two separate bidders would offer.
- Suppose, more specifically, that there is one bidder who can supply both lots.
- Let $c_1 \leq \tilde{c}_1$ denote the costs of the two lots that the large bidder can provide, and $c_f \leq \tilde{c}_f$ denote the costs of the two small bidders (the subscript f denotes the "field").

Strategic Withholding Incentives

There are six possible cases, and a large bidder will have incentives to withhold in cases 1, 2 and 4.

1. The large bidder has the two lowest cost units. $[(c_1, \tilde{c}_1, c_f, \tilde{c}_f)]$
2. The large bidder has the first and third lowest cost units.$[(c_1, c_f, \tilde{c}_1, \tilde{c}_f)]$
3. The large bidder has the first and fourth lowest cost units.$[(c_1, c_f, \tilde{c}_f, \tilde{c}_1)]$
4. The large bidder has the second and third lowest cost units. $[(c_f, c_1, \tilde{c}_1, \tilde{c}_f)]$
5. The large bidder has the second and fourth lowest cost units.$[(c_f, c_1, \tilde{c}_f, \tilde{c}_1)]$, and
6. The large bidder has the two highest cost units.$[(c_f, \tilde{c}_f, c_1, \tilde{c}_1)]$

Introduction

Supply and Demand Function Electricity Market Auctions

- Many electricity markets require bidders to submit demand functions in forward auctions, or supply functions, in reverse auctions.
- Many US RTOs uses supply/demand function auction designs. Examples include auction for financial transmission rights and capacity.. Also used in much of Europe.
- Supply function bids can allow bidders to match their offers with characteristics of facilities used to serve supply.
- Pricing rule can be uniform or discriminatory, (pay-as-bid).
- Clock auctions are an alternative to supply function bids. But, they only differ when bidders learn from rivals' bids during bidding process.

Supply Function Equilibria without Uncertainty

First, suppose there is no uncertainty in demand.

- Let $Q = D(p)$ be the industry demand curve. Let $\hat{p}$ be the price at which $D(\hat{p}) = 0$. Assume $D(p)$ is continuous and concave.
- There are two firms $k \in \{i, j\}$ in the industry with identical cost function $C(q)$. Assume $C^{'}(q) \geq 0$ and $C^{''}(q) \geq 0$ for all $q \geq 0$.
- A strategy for firm k is a twice continuously differentiable function mapping price to output, $S^k : [0, \hat{p}] \rightarrow (-\infty, \infty)$.
- Firms i and j choose supply functions simultaneously. The market clears at p^* s.t. $D(p^*) = S^i(p^*) + S^j(p^*)$.

Supply Function Equilibria without Uncertainty

- A pure strategy Nash equilibrium consists of a pair of functions $S^i(p)$ and $S^j(p)$ such that $S^k(.)$ maximizes k's profits given the rival $m \neq k$ chooses $S^m(.)$, for both $k = i, j$.
- Consider an output pair $(\bar{q}_i, \bar{q}_j)$ at which the market price $\bar{p} = D^{-1}(\bar{q}_i + \bar{q}_j)$ exceeds both $C^{'}(\bar{q}_k)$, $k = i, j$. How to support this point as an equilibrium?
- We need to find $S^i(p)$ and $S^j(p)$ that pass through $(\bar{p}, \bar{q}_i)$ and $(\bar{p}, \bar{q}_j)$, respectively.
- Moreover, given $S^j(p)$, $(\bar{p}, \bar{q}_i)$ should maximize i's profit along its residual demand curve, $D(p) - S^j(p)$. Similar for j.

Supply Function Equilibria without Uncertainty

Given $S^j(p)$, i's profit-maximizing price solves

$$\max_p p[D(p) - S^j(p)] - C(D(p) - S^j(p))$$

first-order condition:

$$D(p) - S^j(p) + [p - C^{'}(D(p) - S^j(p))][D^{'}(p) - S^{j'}(p)] = 0$$

In order to support $(\bar{q}_i, \bar{q}_j, \bar{p})$ as an equilibrium, we must have $\bar{q}_j = S^j(\bar{p})$ and $\bar{q}_i = D(\bar{p}) - S^j(\bar{p})$, which yields

$$\bar{q}_i + [\bar{p} - C^{'}(\bar{q}_i)][D^{'}(\bar{p}) - S^{j'}(\bar{p})] = 0$$

$$S^{j'}(\bar{p}) = \frac{\bar{q}_i}{\bar{p} - C^{'}(\bar{q}_i)} + D^{'}(\bar{p})$$

The second-order condition is satisfied if $S^{j''}(p) \geq 0$.

Supply Function Equilibria without Uncertainty

Therefore, an equilibrium $S^i(p), S^j(p)$ can be characterized as

$$S^{j'}(\bar{p}) = \frac{\bar{q}_i}{\bar{p} - C'(\bar{q}_i)} + D'(\bar{p}), \quad S^{j''}(\bar{p}) \geq 0$$
$$S^{i'}(\bar{p}) = \frac{\bar{q}_j}{\bar{p} - C'(\bar{q}_j)} + D'(\bar{p}), \quad S^{i''}(\bar{p}) \geq 0$$

Suppose both supply functions have nonnegative slopes at $\bar{p}$. Then

- Extending $S^i(.)$ and $S^j(.)$ linearly over $[0, \hat{p})$ makes $\bar{p}$ a global profit-maximum for each firm.
- Global profit maximization at $\bar{p}$ is also ensured by any increasing, continuously differentiable functions that at $\bar{p}$ are tangent to the linear supply functions and that for all other prices specify outputs that are positive and larger than the linear ones.
- The market-clearing price is unique.

Thus, $(\bar{q}_i, \bar{q}_j)$ can be supported by an infinite number of supply function pairs.

Supply Function Equilibria under Uncertainty

Next, consider a setting with exogenous uncertainty in demand.

- Let industry demand be subject to an exogenous shock ϵ, in which ϵ is a scalar random variable distributed on $[\underline{\epsilon}, \bar{\epsilon}]$: $Q = D(p, \epsilon)$.
- For all (p, ϵ), let $-\infty < D_p < 0$, $D_{pp} \leq 0$, and $D_\epsilon > 0$.
- Let $e(Q, p)$ denote the value of shock ϵ for which industry demand is Q at price p, $Q = D(p, e(Q, p))$.
- There are two firms $k \in \{i, j\}$ with identical cost functions $C(q)$. Assume $C^{'}(q) > 0 \ \forall q > 0$; $C^{''}(q) > 0 \ \forall q \geq 0$.
- A strategy for firm k is a supply function mapping price into output level: $S^k : [0, \infty) \rightarrow (-\infty, \infty)$.
- Firms choose $S^k(p)$ simultaneously, without knowing ϵ. After realization of ϵ, supply function are implemented by each firm producing at a point $(p^*(\epsilon), S^k(p^*(\epsilon)))$ such that $D(p^*(\epsilon)) = S^i(p^*(\epsilon)) + S^j(p^*(\epsilon))$.

Supply Function Equilibria under Uncertainty

- A pure strategy Nash equilibrium is a pair of supply functions $S^i(p)$ and $S^j(p)$ such that $S^k(.)$ maximizes k's expected profit, given the distribution of ϵ, given rival's strategy $S^m(.)$, for $k, m = i, j$, $m \neq k$.
- Consider firm i first. Given rival's strategy $S^j(p)$, i's residual demand at any price is $D(p, \epsilon) - S^j(p)$.
- Assume i's set of profit-maximizing points in price-quantity space along i's residual demand curves as ϵ varies can be described as a supply function $S^i(p)$. Then we can replace maximization of expected profits by maximization of profits for each realization of ϵ.
- Firm i solves

$$\max_p p[D(p, \epsilon) - S^j(p)] - C(D(p, \epsilon) - S^j(p))$$

Supply Function Equilibria under Uncertainty

- First-order condition implies

$$D(p,\epsilon)-S^j(p)+[p-C'(D(p,\epsilon)-S^j(p))][D_p(p,\epsilon)-S^{j'}(p)]=0$$

- If the objective function is strictly concave in p, then FOC implicitly determines i's unique profit maximizing price $p_i^0(\epsilon)$ and corresponding $q_i^0(\epsilon)=D(p_i^0(\epsilon),\epsilon)-S^j(p_i^0(\epsilon))$, for every value of ϵ.
- If $p_i^0(\epsilon)$ is invertible, then this locus can be written as a function from price to quantity: $q_i=S^i(p)=q_i^0((p_i^0)^{-1}(p))$.
- $S^i(p)$ intersects i's residual demand once and only once for each ϵ, at $p_i^0(\epsilon)$.
- Replacing ϵ by $e(S^i(p)+S^j(p),p)$ in the FOC yields

$$S^i(p)+[p-C'(S^i(p))]\,[D_p(p,e(S^i(p)+S^j(p),p))-S^{j'}(p)]=0$$

Supply Function Equilibria under Uncertainty

Restrict attention to symmetric equilibria, then
$S^i(p) = S^j(p) = S(p)$,

$$\begin{aligned} S'(p) &= \frac{S(p)}{p - C'(S(p))} + D_p(p, e(2S(p), p)) \\ &= \frac{S(p)}{p - C'(S(p))} + D_p(p) \end{aligned}$$

This differential equation can be rewritten as a two-dimensional autonomous system

$$\begin{aligned} S'(t) &= S + D_p(p)(p - C'(S)) \\ p'(t) &= p - C'(S) \end{aligned}$$

Any trajectory that solves the differential equation in the region corresponding to possible realizations of the demand curve, and that satisfies second-order conditions as well, is a Nash equilibrium.

Supply Function Equilibrium in Wholesale Electricity Markets

- Larson and Salant (2003) develop and test a simulation model that closely approximates real electricity markets.
- They examine the extent to which activity in the simulated market can be represented by a supply function equilibrium.
- In their simulation model, there are two generators $i = 1, 2$ competing in supply functions for a single day. Each generator controls a number of discrete units.
- Suppose each generator i's total cost of providing q units of power is $\frac{1}{2} c_i q^2$, in which c_i is a cost parameter for i.
- Assume there is a price grid, so that each generator's supply bid consists of the number of units it is willing to supply at each point on the price grid.

Model

- Supply and demand for each day are cleared in 24 hourly markets. Each generator submits a single supply function binding over all of the hourly markets.
- The demand curve for each hour is linear. Variation in load over the day is captured by different intercepts for each hour. The demand curve and its variation are public knowledge.
- There is a uniform price for all units dispatched in each hour.
- Each unit has a small, independent probability of being out of service for the day. Before market-clearing, each generator's supply bid is amended to account for out of service units.
- Assume each generator can observe the residual demand curve it faced the previous day.

The Learning Dynamics

- A generator calculates the profit-maximizing price and quantity on each hourly residual demand curve. If these price-quantity pairs trace out an upward sloping supply curve, then this is the generator's profit-maximizing response to the previous day's residual demand.
- Let s_i^t denote i's supply function if it could offer fractions of a unit at different prices. Let b_i^t denote i's bid which has a whole number of units at each price.
- A generator's updated strategy is a weighted average of its desired bid previous day and its best response:

$$\begin{aligned} \Delta s_i^t &= s_i^t - s_i^{t-1} \\ &= \beta(BR_i(b_{-i}^{t-1}) - s_i^{t-1}) \\ b_i^t &= round(s_i^t) \end{aligned}$$

where β is the degree to which a generator thinks its rival's most recent behavior indicates how it will bid today.

Simulations

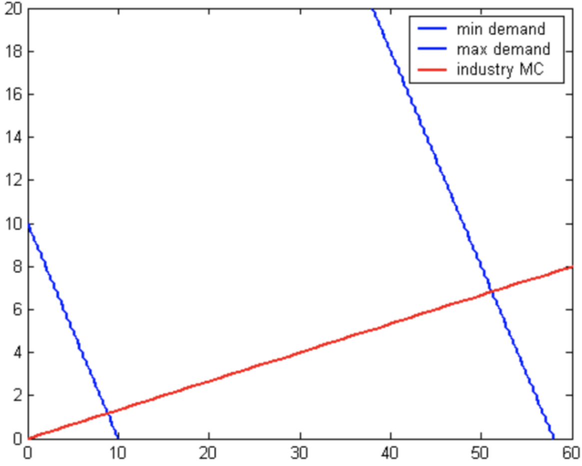
- Let $c_1 = 0.2$ and $c_2 = 0.4$ in the cost functions.
- Assume prices lie on a 100 point grid between 0 and 20, and hourly demand curve $D_h(p)$ is given by

  $$D_h(p) = 10 + z(h) - p$$

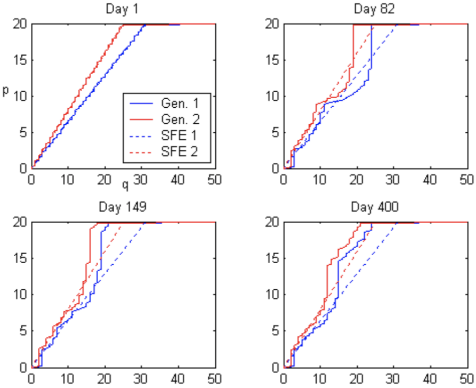
  for $h \in \{1, 2, \cdots, 24\}$.
- Load varies linearly over the day, $z(h) = 2h$.
- The supply function equilibrium (SFE) in this case can be calculated as

  $$q_1 = 1.56p \quad q_2 = 1.26p$$

Maximum and Minimum Hourly Demand

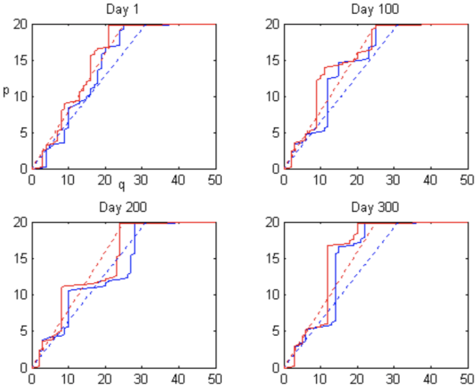


Generators' Bid Functions Over Time



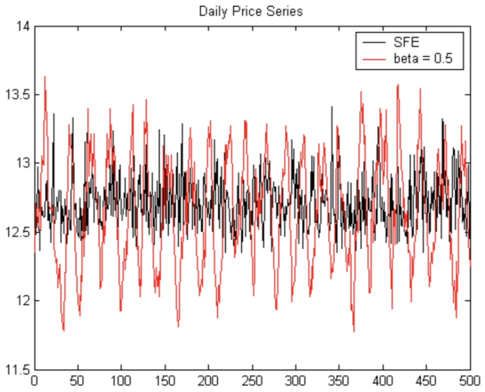
- While the supply schedules remain in the neighborhood of SFE, they do not converge to it.
- Supply schedules cycle between phases with relatively more and less competitive pricing.

Evolution of the Market from Different Initial Conditions



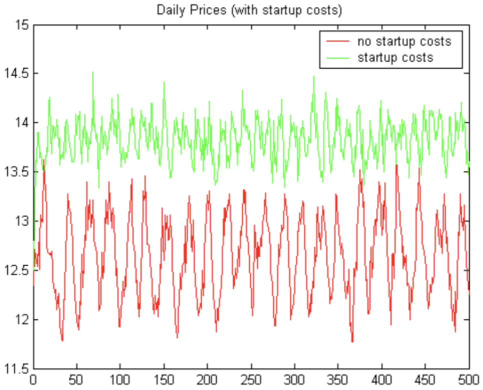
- The variance of average daily price is around 3 times as great as when the market starts at equilibrium.
- Any initial price volatility in the market tends to be preserved by the strategic adjustments of the generators.

Price Series with High Responsiveness to Recent Conditions



- The price series generated by affine SFE bidding is overlaid.
- Both the additional volatility and cyclicality induced by the best response dynamics are pronounced.

Price Series with Start-up Costs



- Suppose the generator has a fixed startup cost, making its cost function non-convex. Then it recovers some, but not all, of these costs through higher prices.
- Price volatility is substantially dampened.

Introduction

Asymmetric Information and the Winner's Curse (Milgrom and Weber (1982)

TABLE 1—BIDS BY SERIOUS COMPETITORS IN RESENT SALES
(All bids in millions of dollars)

Offshore Louisiana, 1967 Tract SS 207	Santa Barbara Channel, 1968 Tract 375	Offshore Texas, 1968 Tract 506	Alaska North Slope, 1969 Tract 059
32.5	43.5	43.5	10.5
17.7	32.1	15.5	5.2
11.1	18.1	11.6	2.1
7.1	10.2	8.5	1.4
5.6	6.3	8.1	0.5
4.1		5.6	0.4
3.3		4.7	
		2.8	
		2.6	
		0.7	
		0.7	
		0.4	
Ratio of Highest to Lowest Bid			
10	7	109	26

- Large bidders had a wide range of values

Mineral Rights Model

- Each of n bidders have two types of information, bidder specific possible signals $X_i, i = 1, \ldots, n$, and other signals, $S_j, j = 1, \ldots, m$.. Let $X = X_1, \ldots, X_n$ and $S = S_1, \ldots, S_m$.
- Each bidder's utility or payoff is $V_i = u(S, X_i, X_{-i})$.
- Assume
  - The function $u(\cdot)$ is continuous and non-decreasing in all components.
  - For each $i, E[V_i] < \infty$.
  - The joint density $f(s, x)$ is symmetric in its last n arguments.
  - The variables $S_1, S_2, \ldots, S_m, X_1, X_2, \ldots, X_n$ are affiliated.
- In what follows let $Y_1, Y_2, \ldots, Y_{n-1}$ denote the largest, second largest, to smallest among $X_2, X_3, \ldots, X_n$.

Affiliated Values

- In many auctions values are uncertain. Classic example is oil leases. But transmission rights, capacity, and spectrum licenses all have that property.
- In many such auctions , bidder values are affiliated, i.e., for $x, y \in \Re^n$, x, y have common density, then $f(x \wedge y) \times f(x \vee y) \geq f(x) \times f(y)$.
- In such auctions, what a bidder learns from rival bids can affect optimal bidding strategies.
- A bidder who fails to account for the fact that winning suggests that the bidder had a relatively high forecast.
- A bidder will thus risk over-paying if if does not discount for possibility of having overly optimistic forecast.
- This is the "Winner's Curse."

Properties of Affiliated RV's

1. If $f(S, X)$ is affiliated and symmetric in $X_2, \ldots, X_n$, then $S_1, \ldots, S_m, X_1, Y_1, \ldots, Y_{n-1}$ are affiliated
2. If $Z_1, \ldots, Z_k$ are affiliated and $g_1, g_2, \ldots, g_k$ are all monotone, then $g_1(Z_1), \ldots, g(Z_k)$
3. If $Z_1, \ldots, Z_k$ are affiliated, then If $Z_1, \ldots, Z_{k-1}$ are affiliated.
4. Suppose $Z_1, \ldots, Z_k$ are affiliated and H is any non-decreasing function. Then the functions t $h(a_1, b_1 : a_2, b_2; \ldots, a_k, b_k)$ be defined by

$$\begin{aligned} h(a_1, b_1 : a_2, b_2; \ldots, a_k, b_k) &= \\ & E[H(Z_1, \ldots, Z_k)| \\ . & a_1 \leq Z_1 \leq b_1, \ldots, a_k \leq Z_k \leq b_k] \end{aligned}$$

is non-decreasing in all of its arguments. Further the functions $h(z_1, \ldots, z_l) = E[H(Z_1, \ldots, Z_k)|z_1, \ldots, z_l]$ are all non-decreasing for all $l = 1, 2, \ldots, k$.

2^{nd} Price Auctions with Affiliated Values

- Let $b_j = b_j(X_j) \geq 0$ denote bidder $j's$ bidding strategy as a function of X_j for $j \neq 1$..
- Let $W = \max_{j \neq 1} b_j$. Then bidder 1 will then want to pick its bid, b so as to solve

$$\max_b \left\{ E\left[(V_1 - W) \mathbb{1}_{W<b} | x_1 \right\} \right.$$

- Let $v(x, y)$ be defined by

$$v(x, y) = E\left[V_1 | X = x, Y_1 = y \right]$$

Theorem 16

The n-tuple of strategies in which each bidder uses $b^*(x) = v(x, x)$ is an equilibrium point of the second price auction.

English Auctions with Affiliated Values

- Let

$$b_0^*(x) = E\Big[V_1 | X_1 = x, Y_1 = x, \ldots, Y_{n-1} = x\Big] \tag{9}$$

- Then iteratively define $b_1^*(x), \ldots, b_{n-2}^*(x)$ by

$$\begin{aligned} b_k^*(x) &= E\Big[V_1 | X_1 = x, Y_1 = x, \ldots, Y_{n-1} = x, \\ &\quad b_{k-1}^*(Y_{n-k} | p_1, \ldots, p_{k-1}) = p_k, \\ &\quad b_0^* = (Y_{n-1}) = p_1\Big] \end{aligned} \tag{10}$$

- The above is an inductive approach to determine optimal bid functions.

1^{st} vs 2^{nd} Price Auctions

Theorem 17

The n-tuple of bid functions defined by (9) *and* (10) *is an equilibrium point of the English auction.*

Theorem 18

The expected price in the English auction is not less than that in the 2^{nd} *price auction.*

First Price Sealed-Bid Auctions

Let

$$b^{*'}(x) = \Big[v(x,x) - b^*(x)\Big]\frac{f_{Y_1}(x|x)}{F_{Y_1}(x|x)}$$

Theorem 19

The n-tuple $(b^*, \ldots, b^*)$ is an equilibrium point of the first auction where

$$b^*(x) = \int_{\underline{x}}^{x} v(\alpha,\alpha)\, dL(\alpha|x),$$

and

$$L(\alpha|x) = exp\Big[-\int_{\alpha}^{x}\frac{f_{Y_1}(s|s)}{F_{Y_1}(s|s)}ds.\Big]$$

Theorem 20

The expected price in the 2^{nd} price auction is as least as large as in the 1^{st} price auction.

Summary Winner's Curse

- When there are affiliated values, the English auction results in the highest revenues, followed by the 2^{nd} price auction and then the 1^{st} price auction.
- Note that revenue equivalence fails with affiliated values.
- Further, information provided bidders raises expected revenues in all formats.
- This logic is similar to the ranking result. Bidders have most information in English auction, second most in 2^{nd} price auction (know they won't win unless highest value) and least in the 1^{st} price auction.
- Strategic equivalence of Dutch and 1^{st} price auctions still holds.

Introduction

Multi-unit and multi-object auctions

- Default service procurement and sale of transmission rights or entitlements are often by means of multi unit or multi object auctions.
- Bidders in for one product may wish to switch depending price. In which case, auction should allow bidders opportunity to arbitrage price differentials.
- Products can also be complements.
  - For example, wheeling power from A to C via B might require separately purchasing inter-connector rights from A to B and from B to C.
  - Where there are complements, the amount a bidder might want to pay for one product will depend on what else it wins.
  - This gives rise to an *exposure problem.*

Simultaneous Ascending (or Descending) Price Auctions ("SAA"s)

- Two main variants of Simultaneous Ascending Auctions are the Simultaneous Multiple Round ("SMR") and Simultaneous Ascending Price Clock auctions.
- The Simultaneous Multiple Round ("SMR") auction was designed to allow bidders to manage the exposure problem and arbitrage prices of substitutes.
- In both, multiple units of multiple products are auctioned in a sequence of rounds.
- Prices increase (decrease in reverse auction) from one round to the next for all products in excess demand (excess supply in a reverse auction)
- Bidders can rebid, switch or reduce activity in one round in response to the price adjustment from the previous round.
- Auction ends when there is no longer excess demand (supply in a reverse auction) for any product.

SAA Examples in Electricity

- Several SAA variants are being used in the energy sector.
- First instance was NJ BGS auction, a *Simultaneous Descending Price Clock Auction (SDCA)* for energy procurment.
- SDCA also been used in Ohio, Illinois, Italy.
- Basic format is that there are multiple products, prices start high, and tick down for products for which there is excess supply.
- Activity rules require bidders to maintain or decrease offers as prices fall.
- Auction ends when there is no excess supply for any product.

SAA Variants

- The difference between the SMR and the Clock Auction is that bidders name prices in an SMR auction and the auctioneer names prices, and bidders only indicate quantities in a clock auction.
- Almost all SAAs include activity rules - bidders must be "active" on an increasing fraction of what they hope to win
- SMR auctions determine provisional winning bidders after each round. Most clock auctions do not.
- Almost all SAAs include activity rules - but these can be more or less restrictive.
- Provisions for "over-shooting" and tie-breaking vary, as do rules for price increments.
- Caps and set-asides are sometimes included to promote competition post-auction.
- Qualification and performance guaranties vary a great deal.

Activity Rules in SAAs

- Bidders are typically required to state initial demand at announced starting prices when applying to participate in an auction.
- The lots may be all considered the same value for measuring activity, in which case a bidder's initial demand establishes its initial "eligibility."
- In other cases, lots might differ, e.g. a 25 MW lot might count half of a 50 MW lot. In this case, lots are assignment "bidding units" or "eligibility points." Bidder applications establish maximum points a bidder can win.
- In each round a bidder cannot place bids for a set of lots whose combined value exceeds its eligibility points at the start of the round.
- A bidder's eligibility to start round, r, E_r, is its maximum of activity in round $r-1$, A_{r-1} divided by activity requirement α, and its eligibility in round $r-1$.

Clock Auction vs. Demand Function Auctions

- Clock auction can vary in a number of ways, including price increments, information disclosure and closing rules.
- A clock auction in which bidders are provided no information about rivals between price increments and in which the auction ends when demand is zero is equivalent to a sealed-bid auction in which bidders are asked to report demand curves.
- When bidders see rival bids, or at least know rival demand within a range, then bidders will have additional information not available in deciding how to bid in a sealed-bid auction.
- So, a clock auction should result in higher revenues than a sealed-bid when bidders' values are affiliated.

SAA Auction Dynamics

- As the activity requirement is

$$E_r = \max\{\frac{A_{r-1}}{\alpha}, E_{r-1}\}$$

- Each bidder's eligibility, or potential demand, can either stay the same or decrease, as prices rise across rounds.
- Bidder's can switch products, too.
- An SAA will close when there is no longer excess demand on any product.
- The eligibility ratio, i.e., the ratio of total eligibility to supply, is a measure of how much excess demand is in the auction in any round.

Entitlement Auctions - What Can Go Wrong -2

In 2001, the Texas utilities auctioned entitlements (like VPPs auctioned by EDF), in a single simultaneous auction. The Reliant and TXU contracts were identical for shares of a nuclear plant.

Table 9.6
Texas capacity auctions: fall 2001

	Reliant South	TXU South	Δ	%Δ
1 year base load	\$7.32	\$10.59	\$3.27	45%
2 year base load	\$6.59	\$10.33	\$3.74	57%
1 year base cyclic	\$0.75	\$1.79	\$0.95	127%

Entitlement Auctions - What Can Go Wrong -3

The second Texas entitlements simultaneous ascending auction was no more successful than the first. The Reliant and TXU contracts were identical for shares of a nuclear plant.

Table 9.7
Texas capacity auctions: spring 2002

	Reliant South	CPL South	Δ	%Δ
June base load—capacity only	\$13.13	\$6.85		41.3%
June base load—capacity + fuel	\$22.57	\$18.76	\$3.81	
July base load—capacity only	\$6.76	\$10.60		45.3%
July base load—capacity + fuel	\$22.57	\$18.76	\$3.06	
August base load—capacity only	\$16.76	\$9.85		55.6%
August base load—capacity + fuel	\$26.86	\$22.79	\$4.07	

Activity Rule Issues

- Activity rules can limit arbitrage opportunities.
- In the case of the Texas PGCs bidders were not allowed to switch between TXU and Reliance.
- When there are different sized lots, e.g, 25 MW and 100 MW lots, even with an activity rule that allows arbitrage between the small and large lots, but one that includes provisional winning bids, a bidder may "lose" its bid for 1 – 3 small lots and not be able to switch back to the one large lot. This was the case in a Spanish auction for airwaves in 2011 where the sum of regional license prices was approximately one-third more than the equivalent nationwide license. This was due to limitations of the activity rules.
- Where there are no provisional winning bids, as is the case in some clock auctions, there is a risk of "over-shooting".

Importance of Activity Rules

TOTAL Table 2b 2,6 GHz Blocks				
Block	MHz	Band	Winner	Final Price (M€)
C1_i	2x10MHz	2,6GHz	OSP	22.21
C1_ii	2x10MHz	2,6GHz	OSP	22.88
C1_iii	2x10MHz	2,6GHz	TEF	21.79
C1_iv	2x10MHz	2,6GHz	TEF	22.65
C2_i	2x5MHz	2,6GHz	VOD	9.81
C2_ii	2x5MHz	2,6GHz	VOD	9.79
C2_iii	2x5MHz	2,6GHz	VOD	10.15
D2_R1 a R19	2x10MHz	2,6GHz	Various winners*	24.08
D2_R1 a R19	2x5MHz	2,6GHz	VOD	29.32
*Jazz, Ono, TeleCLM, R, Euskaltel, Telecable				
			TEF	44.44
TOTAL	2x20 MHz		OSP	45.10
€			VOD	59.07

Straightforward Bidding

- *Straightforward bidding* in an SAA in where a bidder always submit bids in each round of an SAA on the set of licenses that maximizes the bidder's expected payoff at the posted prices.
- Bidders can have an incentive to withhold demand. For example, a bidder who drops one block of demand may find that this so reduces price on the other blocks it purchases so as to generates savings on the other units it purchases to make it worthwhile.
- A bidder can have difficulty bidding straightforwardly when it views some of the items in the auction as complements.

SAA Properties

Theorem 21

Straightforward bidding is a feasible strategy for bidder j for all initial prices, all fixed increments and all feasible price paths iff all goods are substitutes for bidder j.

Theorem 22

(Milgrom p.272) Assume all goods are substitutes for all bidders and that all bidders bid straightforwardly. Then, the auction ends within a finite number of rounds. The final provisional winning bids and allocation of goods are a competitive equilibrium for an economy in which bidder valuations are within some small $\epsilon > 0$ of the bidders' true valuations for all bidders. The final assignment maximizes total welfare within a single bid increment.

The Combinatorial Clock Auction ("CCA")

- The CCA is a clock auction, which can include multiple units of multiple productions, and has some additional features.
- A CCA typically has three stages
  1. A primary or clock stage - which is a multi-production clock auction for "generic blocks"
  2. A supplementary round - a sealed-bid for other packages not bid on during the clock stage
  3. An assignment round - for determine which "concrete blocks" bidders will be assigned.
- The clock phase is typically subject to activity rules. The supplementary round bids are typically constrained by revealed preference type activity rules - bids for additional packages have be "consistent" with clock phase bids.
- Prices are core adjusted Vickrey prices.

Vickrey Auction Outcome

Table: Blocks won and prices for main blocks in Swiss 2012 multi-band auction

Band/ bidder	Swisscom	Sunrise	Orange
800 MHz	2	2	29
900 MHz	3	3	1
1800 MHz	6	4	5
2100 MHz	6	2	4
Amount paid (000)	359,846 CHF	481,720 CHF	154,702 CHF

Swisscom won more and paid less.

VCG Auctions and the Core

- The outcome of the above table with a non-core outcome, is such that the auctioneer ran receive more money and bidders 2 and 3 can all be made better off.
- When a subgroup of all bidders + the auctioneer can do better, then the outcome is not in the *core*.
- Let (N, z) is a *cooperative game* representing the auction, where z is the *characteristic function* of the game, representing how much any subset of bidders can get. $N = \{0, 1, ..., n\}$ where player 0 is the auctioneer
- Typically, $z(S) = 0$ if $0 \notin S$.

Core Selecting Auctions

Definition 3

An imputation $\pi = (\pi_{\mathbf{0}}, \pi_{\mathbf{1}}, \ldots, \pi_{\mathbf{n}})$ *is in the* core *if for any* $S \subset N$*, and other feasible* $\tilde{\pi}$*. it must be the case that* $\sum_{j \in S} \pi_j \geq \sum_{j \in S} \tilde{\pi}_j$.

- Notice that a core selecting auction mechanism will only select outcomes that maximize $\sum w_i(K)$, where the w_i's are the reported valuations and K is the decision.
- However, VCG auctions do not always result in core-allocations

Vickrey and the Core

Table: Two blocks - two bidders

Bidder/ no. of blocks	1 block	2 blocks
Bidder 1	0	20
Bidder 2	15	30

Table: Two blocks - two bidders + shill: Bidder 2 does better bidding with a shill.

Bidder/ no. of blocks	1 block	2 blocks
Bidder 1	0	20
Bidder 2a	25	25
Bidder 2b	25	25

Vickrey and the Core (cont'd)

- In the above example, if Bidder 2 were to bid as one entity in a VCG auction, it would outbid Bidder 1, and have to pay 20, the amount Bidder 1 offered for both blocks.
- If Bidder 2 were to split into two bidders (one a shill), and overstate its values, each can bid 25, and the VCG prices would be zero for both Bidder 2's shills.
- This is not a core outcome.

Theorem 23

An efficient direct auction mechanism has the property that no bidder can ever earn more than its Vickrey payoff by disaggregating and bidding with shills if and only if it is a core selecting auction mechanism.