
Exact solutions to binary equilibrium problems with compensation and the power market uplift problem

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Huppmann, D., & Siddiqui, S. (2018). An exact solution method for binary equilibrium problems with compensation and the power market uplift problem. *European Journal of Operational Research*, 266(2), 622-638.

Joint work with Daniel Huppmann

June 21, 2019

Even small improvements in power market operation can have huge societal benefits due to increased efficiency

Over the past decade, power market operation was greatly improved by using Mixed-Integer Programs (MIP) instead of Linear Programs (LP)...

- *In 2004, PJM implemented MIP in its day-ahead market [...], with savings estimated at \$100 million/year.*
- *On April 1, 2009, the California ISO (CAISO) implemented its Market Redesign and Technology Update (MRTU), [...] achieving an estimated \$52 million in annual estimated savings by using MIP.*

Quoted from: “Recent ISO Software Enhancements and Future Software and Modelling Plans”, FERC Staff Report, 2011, pages 3-4

In electricity markets, marginal-cost pricing may yield **incentive-incompatible dispatch and losses for generators**

Generators may earn negative profits based on market-clearing prices derived from least-cost unit commitment & dispatch

⇒ Generators leave the market or do not follow the optimal dispatch prescribed by the ISO (“self-scheduling”)

More generally, in markets with non-convexities & indivisibilities, using duals as market-clearing (Walrasian) prices doesn’t quite work

⇒ There may not even exist any Nash equilibrium in many cases

In practice, the ISO pays out compensation to “make whole” individual generators after market clearing (“no-loss rule”)

⇒ Potential for gaming and issues of acceptance by consumers

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A Very Quick Introduction to Continuous Equilibrium Problems

Optimization Problem

$$\begin{array}{ll} \min & f(x) \\ \text{s.t.} & \\ & h(x) \leq 0 \end{array}$$

Karush-Kuhn-Tucker
Conditions

Complementarity Problem

$$\begin{array}{ll} z \geq 0 \\ g(z) \geq 0 \\ z^T g(z) = 0 \end{array}$$

Equilibrium (Optimization Problems)

$$\begin{array}{ll} \min f_1(x_1, x_{-1}) & \min f_2(x_2, x_{-2}) \\ \text{s.t.} & \text{s.t.} \\ h(x_1) \leq 0 & h(x_2) \leq 0 \end{array}$$

$$\begin{array}{ll} \min f_3(x_3, x_{-3}) & \min f_4(x_4, x_{-4}) \\ \text{s.t.} & \text{s.t.} \\ h(x_3) \leq 0 & h(x_4) \leq 0 \end{array}$$

.....

$$\begin{array}{ll} \min f_n(x_n, x_{-n}) \\ \text{s.t.} \\ h(x_n) \leq 0 \end{array}$$

There exists no scalable approach to find Nash equilibria in non-cooperative games with binary decision variables

Current approaches for equilibria in binary games:

- neglect the gaming aspects altogether
- compute all permutations of the binary variables
 - ⇒ quickly grows beyond the bounds of computational tractability
- use a two-stage approach such as “make-whole” payments or convex-hull pricing and the minimum uplift problem
- relax/linearize the binary variables, apply equilibrium methods see Gabriel, Conejo, Ruiz, and Siddiqui (2013)
 - ⇒ doesn't yield exact solution, not clear whether it's truly a NE

We propose an exact solution approach to find equilibria in games with binary variables

Outline of the proposed solution methodology:

- derive KKT conditions for *both states* of each binary variable
- add an objective function as *equilibrium selection* mechanism
- include compensation payments to ensure incentive compatibility as decision variables of the equilibrium selection problem

The reformulated problem...

- ⇒ is a *multi-objective bi-level* optimization program
- ⇒ allows to incorporate the trade-off between market efficiency and compensation payment budget
- ⇒ can be solved as a mixed-binary convex program

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The math – Obtaining “duals” in integer programming

What are duals in integer or binary programming, and why can we use them as market-clearing prices?

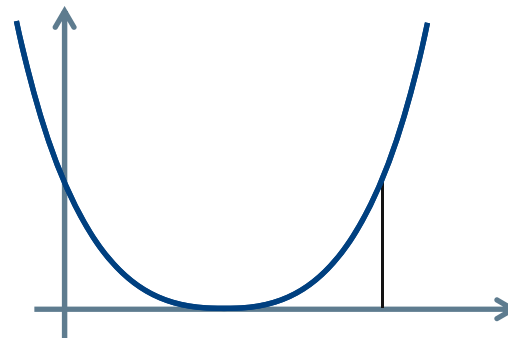
Using optimization for determining prices relies on duality

⇒ But duality in integer programs is difficult to establish

⇒ The notion of a “marginal relaxation” doesn’t quite make sense

Look at a stylized example:

$$\begin{array}{ll} \min_x & (x - 0.5)^2 \\ \text{s.t.} & x \in \{0, 1\} \end{array}$$



If you solve this program in GAMS (or most numerical solver packages), you will get some “duals” reported

⇒ Where do these values come from?

Duals in integer programs using a two-step procedure

O'Neill et al. (2005) proposed a two-step approach:

1. Solve the MIP using standard methods (Problem 1)
2. Solve the linearized LP model (Problem 2), fixing discrete/binary variables at optimal level x^* as determined by Problem (1)

$$\begin{aligned} \min_{x,y} f(x,y) \quad (1) \\ \text{s.t.} \\ g(x,y) \leq 0 \\ x \in \{0,1\}^n \\ y \in \mathbb{R}^m \end{aligned}$$

$$\begin{aligned} \min_{x,y} f(x,y) \quad (2) \\ \text{s.t.} \\ g(x,y) \leq 0(\lambda) \\ x = x^* \quad (\mu) \\ (x,y) \in \mathbb{R}^{n+m} \end{aligned}$$

- \Rightarrow The dual variables (λ^*, μ^*) to Problem (2) can be interpreted as market-clearing, Walrasian prices!
- \Rightarrow But these are not actually the correct prices for Problem (1)!

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An exact solution method – Introducing the “switch value”

Rather than focusing on relaxations of binary variables, let's look at the loss from deviation (“switch value”)

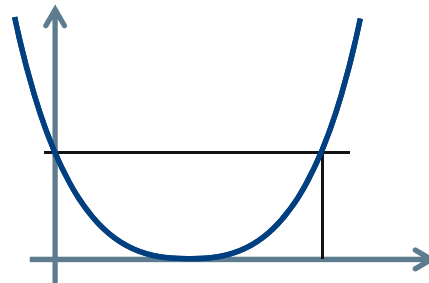
We introduce the term “switch value κ ” to describe the absolute (not marginal) loss when deviating from the optimal value of x^* :

$$f(x^*, y^*) = f(x^\times, y^\times) - \kappa$$

where $x^\times = 1 - x^*$ and $y^\times = \arg \min_y f(x^\times, y)$

Applying this idea to the simple example:

$$\begin{array}{ll} \min_x & (x - 0.5)^2 \\ \text{s.t.} & x \in \{0, 1\} \end{array}$$



$$\mu(x^* = 0) = -1$$

$$\mu(x^* = 1) = 1$$

$$\kappa = 0$$

⇒ The switch variable can be interpreted as a shadow price!

⇒ It can be used as a measure of “disequilibrium” (Çelebi & Fuller)

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Collecting definitions for binary games and notions of equilibrium

We introduce the notion of a binary quasi-equilibrium to describe incentive-compatible outcomes with compensation

Definition: *Binary game*

We have a set of players $i \in I$,
each seeking to solve a binary problem:

$$\begin{cases} \min_{\substack{x_i \in \{0,1\} \\ y_i \in \square^m}} f_i(x_i, y_i, y_{-i}(x_{-i})) \\ \text{s.t.} \quad g_i(x_i, y_i) \leq 0 \end{cases} \quad (\lambda_i)$$

Definition: *Equilibrium in a binary game*

A (Nash) equilibrium in binary variables is a feasible vector $(x_i^*, y_i^*)_{i \in I}$
such that: $f_i(x_i^*, y_i^*, y_{-i}^*(x_{-i}^*)) \leq f_i(x_i^\times, y_i^\times, y_{-i}^*(x_{-i}^*)) \quad \forall i \in I$

Definition: *Quasi-equilibrium in a binary game with compensation*

A (Nash) quasi-equilibrium in binary variables is a feasible vector $(x_i^*, y_i^*)_{i \in I}$
and a vector of compensation payments $(\zeta_i)_{i \in I}$
such that: $f_i(x_i^*, y_i^*, y_{-i}^*(x_{-i}^*)) - \zeta_i \leq f_i(x_i^\times, y_i^\times, y_{-i}^*(x_{-i}^*)) \quad \forall i \in I$

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The core idea of our solution approach

We compute the optimal value w.r.t. the continuous variables for both states of the binary variable simultaneously

Assume that KKT conditions are necessary and sufficient w.r.t. continuous variables y_i for fixed binary x_i and given rivals actions y_{-i} , then...

...we compute the optimal response for *both states* of variable x_i

assuming $x_i=1$:

$$0 = \nabla_{y_i} f_i(1, \tilde{y}_i^{(1)}, y_{-i}(x_{-i})) + \tilde{\lambda}_i^{(1)} \nabla_{y_i} g_i(1, \tilde{y}_i^{(1)}) \quad , \quad \tilde{y}_i^{(1)} \text{ (free)}$$

$$0 \geq g_i(1, \tilde{y}_i^{(1)}) \quad \perp \quad \tilde{\lambda}_i^{(1)} \geq 0$$

and assuming $x_i=0$:

$$0 = \nabla_{y_i} f_i(0, \tilde{y}_i^{(0)}, y_{-i}(x_{-i})) + \tilde{\lambda}_i^{(0)} \nabla_{y_i} g_i(0, \tilde{y}_i^{(0)}) \quad , \quad \tilde{y}_i^{(0)} \text{ (free)}$$

$$0 \geq g_i(0, \tilde{y}_i^{(0)}) \quad \perp \quad \tilde{\lambda}_i^{(0)} 1 \geq 0$$

And then, we check which strategy is optimal by comparing pay-offs:

$$f_i(1, \tilde{y}_i^{(1)}, y_{-i}(x_{-i})) \leq f_i(0, \tilde{y}_i^{(0)}, y_{-i}(x_{-i}))$$

We use the switch value to the incentive-compatibility check to replace the cumbersome “if-then” conditions

The “if-then” conditions to determine the individually optimally binary decision are very painful to compute in large-scale problems:

$$f_i(\mathbf{1}, \tilde{y}_i^{(1)}, y_{-i}(x_{-i})) < f_i(\mathbf{0}, \tilde{y}_i^{(0)}, y_{-i}(x_{-i})) \Rightarrow x_i^* = 1$$

$$f_i(\mathbf{1}, \tilde{y}_i^{(1)}, y_{-i}(x_{-i})) > f_i(\mathbf{0}, \tilde{y}_i^{(0)}, y_{-i}(x_{-i})) \Rightarrow x_i^* = 0$$

$$f_i(\mathbf{1}, \tilde{y}_i^{(1)}, y_{-i}(x_{-i})) = f_i(\mathbf{0}, \tilde{y}_i^{(0)}, y_{-i}(x_{-i})) \Rightarrow x_i^* = \{0, 1\}$$

We use the switch value κ and introduce a compensation payment ζ :

$$f_i(\mathbf{1}, \tilde{y}_i^{(1)}, y_{-i}) + \kappa_i^{(1)} - \zeta_i^{(1)} - \kappa_i^{(0)} + \zeta_i^{(0)} = f_i(\mathbf{0}, \tilde{y}_i^{(0)}, y_{-i})$$

$$\kappa_i^{(1)} + \zeta_i^{(1)} \leq x_i \tilde{K}$$

$$\kappa_i^{(0)} + \zeta_i^{(0)} \leq (1 - x_i) \tilde{K}$$

$$\kappa_i^{(1)}, \kappa_i^{(0)}, \zeta_i^{(1)}, \zeta_i^{(0)} \in \mathbb{R}_+$$

We solve for a binary equilibrium using a two-stage problem by introducing an upper-level “market operator” player

We introduce an additional player called “market operator”

⇒ not exclusively related to electricity, but rather a “coordinator”

⇒ can also be interpreted as an equilibrium selection mechanism

The market operator acts as an upper-level player, optimizing:

$$\min F\left((x_i, y_i)_{i \in I}\right) + G\left((\zeta_i)_{i \in I}\right)$$

while guaranteeing feasibility, optimality & incentive compatibility for each player.

⇒ This player can effectively consider the trade-off between market efficiency and compensation payments!

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The two-stage program to obtain binary (quasi) equilibria

The market operator incorporates the trade-off of efficiency vs. compensation, subject to a binary (quasi-)equilibrium

$$\min_{\substack{x_i, y_i, \tilde{y}_i^{(\bar{x}_i)}, \tilde{\lambda}_i^{(\bar{x}_i)} \\ \kappa_i^{(\bar{x}_i)}, \zeta_i^{(\bar{x}_i)}}} F\left((x_i, y_i)_{i \in I}\right) + G\left((\zeta_i^{(\bar{x}_i)})_{i \in I}\right)$$

$$\text{s.t. } \nabla_{y_i} f_i\left(\mathbf{1}, \tilde{y}_i^{(1)}, y_{-i}\right) + (\tilde{\lambda}_i^{(1)})^T \nabla_{y_i} g_i\left(\mathbf{1}, \tilde{y}_i^{(1)}\right) = 0$$

$$0 \leq -g_i\left(\mathbf{1}, \tilde{y}_i^{(1)}\right) \perp \tilde{\lambda}_i^{(1)} \geq 0$$

$$\nabla_{y_i} f_i\left(\mathbf{0}, \tilde{y}_i^{(0)}, y_{-i}\right) + (\tilde{\lambda}_i^{(0)})^T \nabla_{y_i} g_i\left(\mathbf{0}, \tilde{y}_i^{(0)}\right) = 0$$

$$0 \leq -g_i\left(\mathbf{0}, \tilde{y}_i^{(0)}\right) \perp \tilde{\lambda}_i^{(0)} \geq 0$$

$$f_i\left(\mathbf{1}, y_i^{(1)}, y_{-i}\right) + \kappa_i^{(1)} - \zeta_i^{(1)} - \kappa_i^{(0)} + \zeta_i^{(0)} = f_i\left(\mathbf{0}, y_i^{(0)}, y_{-i}\right)$$

$$\kappa_i^{(1)} + \zeta_i^{(1)} \leq x_i \tilde{K}$$

$$\kappa_i^{(0)} + \zeta_i^{(0)} \leq (1 - x_i) \tilde{K}$$

$$\tilde{y}_i^{(0)} - x_i \tilde{K} \leq y_i \leq \tilde{y}_i^{(0)} + x_i \tilde{K}$$

$$\tilde{y}_i^{(1)} - (1 - x_i) \tilde{K} \leq y_i \leq \tilde{y}_i^{(1)} + (1 - x_i) \tilde{K}$$

$$x_i \in \{0, 1\}, (y_i, \tilde{y}_i^{(\bar{x}_i)}) \in \mathbb{R}^{3m}, (\lambda_i^{(\bar{x}_i)}, \kappa_i^{(\bar{x}_i)}, \zeta_i^{(\bar{x}_i)}) \in \mathbb{R}_+^{2k+4}$$

Multi-objective function of equilibrium selection mechanism

Optimal decision y_i for each player i given $x_i = 1$

Optimal decision y_i for each player i given $x_i = 0$

Comparison of pay-off for each player

Translation mechanism of individually optimal strategy to equilibrium outcome

3

Properties of the market operator's problem

The market operator's problem is an exact solution method for binary equilibria – and in many cases, it's a linear program!

Theorem: *Exact reformulation of a binary equilibrium program*

Any feasible solution to the market operator's program
is a (quasi-) equilibrium in a binary equilibrium problem

⇒ Using the market operator's objective function towards “optimality”

Theorem: *Quadratic mixed-binary program with linear constraints*

Under certain conditions, the market operator's program can be
reformulated as a linear or quadratic mixed-binary program
with linear constraints

⇒ These conditions hold for the power market uplift problem,
fossil fuel & resource markets, agriculture, investment games, etc.

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Comparing our method to the standard approach

We believe that our method is at least as good in terms of social welfare as the currently applied two-stage approach

Proposition:

Our approach is always at least as good than the two-stage approach following O'Neill et al. (2005), irrespective of which market rules apply

Idea for the proof:

Our method:

$$\begin{aligned} \min \quad & F\left((x_i, y_i)_{i \in I}\right) + G\left((\zeta_i)_{i \in I}\right) \\ \text{s.t.} \quad & \text{feasibility, optimality,} \\ & \text{incentive compatibility} \end{aligned}$$

Two-stage approach:

$$\begin{aligned} \min \quad & G\left((\zeta_i)_{i \in I}\right) \\ \text{s.t.} \quad & \min F\left((x_i, y_i)_{i \in I}\right) \text{ s.t. feasibility} \\ & \text{incentive compatibility} \end{aligned}$$

Using an integrated model is, in principle, at least as good as solving a two-stage model

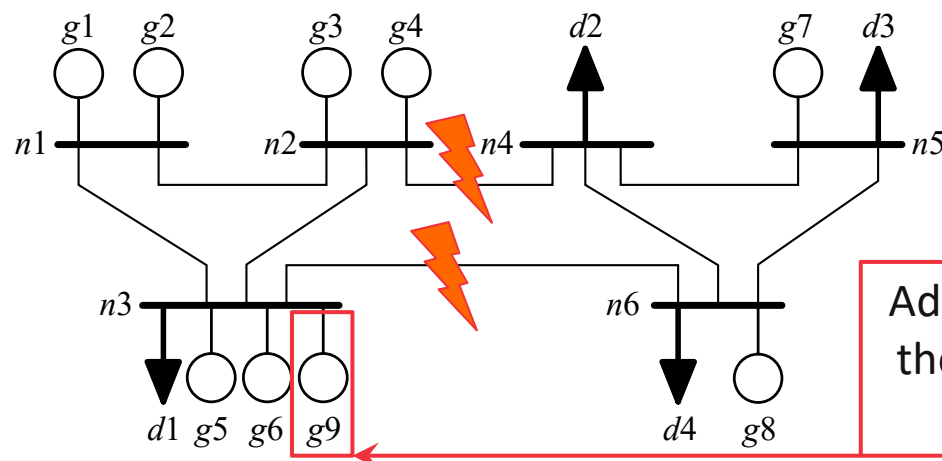
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An application: the power market uplift problem (I)

Power markets are a great field of application for our method

We use the nodal-pricing power market uplift problem example from Gabriel, Conejo, Ruiz, and Siddiqui (2013):

- 6 nodes, 9 generators, 4 load units, 2 periods (high/low demand)
- Each generator has on-off decisions, start-up/shut-down costs, and minimum generation constraints (if active)
- Two zones, with transmission bottlenecks on lines N2-N4 and N3-N6



Additional generator compared to the model by Gabriel et al. (2013) to have a more salient example

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An application: the power market uplift problem (II)

There are different market rules in real-world power markets and our approach is flexible to implement a variety of those

The power market uplift problem:

- ⇒ can be reformulated and solved as a mixed-binary linear program
- ⇒ a variety of market rules can be implemented as linear constraints

We compare three different market rule implementations:

- Two-step method following O'Neill et al. (2005), **standard approach**; taking into account a no-loss rule for every generator
- A **game-theoretic** solution (binary equilibrium with compensation)
- A regulatory framework that no generator should lose money, but only active generators receive compensation (**no-loss & active**)

Standard

New
approach

4

Illustrative results for the power market uplift problem

Compensation isn't necessary for incentive-compatibility
and overly restrictive market rules reduce overall efficiency

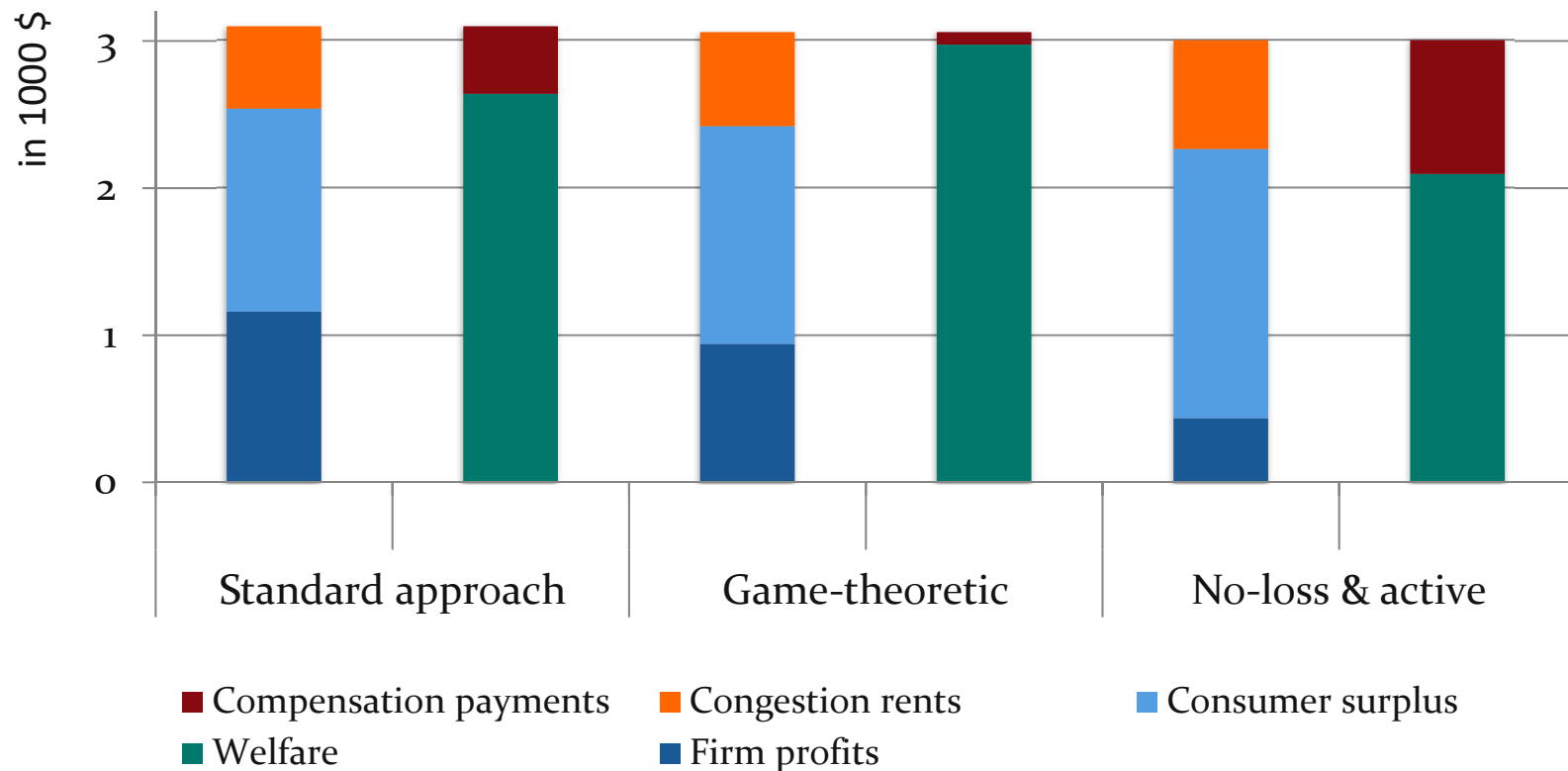
Player		Standard approach			Game-theoretic			No-loss & Active		
	x^{init}	(x_{1i}, x_{2i})	π_i	ξ_i	(x_{1i}, x_{2i})	π_i	ξ_i	(x_{1i}, x_{2i})	π_i	ξ_i
g3	1	(0,0)	-300	300	(0,0)	-300	15	(1,1)	-525	525
g4	1	(1,1)	-160	160	(1,1)	-250	65	(1,0)	-335	335
g5	1	(1,1)	50		(1,1)	-50		(1,1)	-50	50
g6	1	(1,1)	200		(1,1)	100		(1,1)	100	
g7	0	(1,1)	690		(1,1)	715		(1,1)	690	
g8	0	(1,1)	680		(1,1)	730		(1,1)	550	
g9	0	(0,0)	0		(1,1)	-5	5	(0,0)	0	
All			1160	460		940	85		435	910
Other			1940			2120			2570	
Total			3100	460		3060	85		3005	910

Generators g1 and g2 are not active in any case; "Other": consumer & congestion rent

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Illustrative results for the power market uplift problem (II)

In this example, the market rules to “protect” generators actually induce a substantial welfare shift towards consumers



Rents by stakeholder group in the three settings (method/market setting cases)

4

Computational aspects of the exact solution approach

The number of binary variables increases only linearly in the number of generators and hours

The main challenge in computing equilibria in binary variables is the exponential increase in the number of binary variables

⇒ size of the trial problem: 6 nodes, 9 generators, 4 load units, 2 hours

- Standard approach (welfare-optimal unit-commitment)

⇒ number of binary variables: $|T| \cdot |I| = 18$

- Brute-force enumeration of equilibria

⇒ number of equilibrium problems: $2^{|T||I|} > 262k$

- Exact binary quasi-equilibrium solution method

⇒ number of binary variables: $|T| \left(2(|I| + |J| + |L| + |N| - 1) + |I| \right) = 122$

We propose a novel and very flexible method to find
Nash equilibria in binary games with compensation

We develop a method to find solutions to binary equilibrium problems

We introduce the term “quasi-equilibrium” for market results that are incentive-compatible only if compensation is paid to some players

Under very general conditions, the problem can be solved
as a mixed-binary linear/quadratic program using standard methods

The method allows to include a multitude of market rules and regulations
to replicate real-world settings and considerations

⇒ No-loss rules, compensation only for active generators, etc.

The GAMS code is publicly available under a CC license at
http://danielhuppmann.github.io/binary_equilibrium/



Thank you very much for your attention!



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The method opens up a host of future research opportunities towards real-world applications and new algorithms

Applying the method to real-world size problems

- ⇒ illustrate the trade-off between market efficiency & compensation
- ⇒ can our method realize welfare gains similar to the switch to MIP?

Using the switch value for better numerical algorithms

- ⇒ stopping criteria, guidance in branch-and-bound, etc.

Yield a better understanding of gaming opportunities in power sector

- ⇒ our method can explicitly compare game-theoretic aspects

Extend the method to Generalized-Nash games, integer variables, etc.

We believe that the binary equilibrium approach offers a suitable way to model control power and market behaviour

Three complications for control power and unit commitment:

- There is an interdependence between the spot market and reserved capacity for providing control power (Ortner, 2015)
- Generators must be compensated not only for incurred costs, but also for opportunity costs between offers to the two markets
- Strong duality fails in integer programming, hence dual variables cannot be interpreted as market-clearing (equilibrium) prices

Modelling approach based on binary equilibrium:

- ⇒ Extend current model for control power (capacity reservation)
- ⇒ Re-interpret compensation as control power capacity remuneration to guarantee incentive compatibility for generators

Recent research focuses on additional constraints or relaxations of integrality or complementarity (optimality)

- Björndal and Jornsten (2008)
- Gabriel, Conejo, Ruiz, and Siddiqui (2013)
- Todd (2015)
- And others...

3

The major question mark with regard to previous approaches

The method proposed by Gabriel et al. (2013) relies on a linearization of the binary variables

A player's optimization problem...

$$\begin{aligned} \min_{\substack{x_i \in \{0,1\} \\ y_i \in \square^m}} & f_i(x_i, y_i, y_{-i}(x_{-i})) \\ \text{s.t.} & g_i(x_i, y_i) \leq 0 \quad (\lambda_i) \end{aligned}$$

...and the first order optimality conditions of the linearized problem:

$$\begin{aligned} 0 &= \nabla_{x_i} f_i(x_i, y_i, y_{-i}(x_{-i})) + \lambda_i \nabla_{x_i} g_i(x_i, y_i) \quad , \quad x_i \text{ (free)} \\ 0 &= \nabla_{y_i} f_i(x_i, y_i, y_{-i}(x_{-i})) + \lambda_i \nabla_{y_i} g_i(x_i, y_i) \quad , \quad y_i \text{ (free)} \\ 0 &\geq g_i(x_i, y_i) \quad \perp \quad \lambda_i \geq 0 \end{aligned}$$

⇒ But it is not clear whether stationary points of the linearized problem are indeed Nash equilibria of the original problem!