Electricity intraday price modeling with marked Hawkes processes

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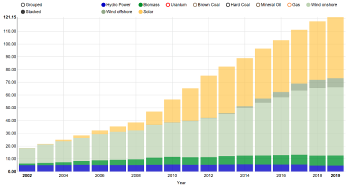
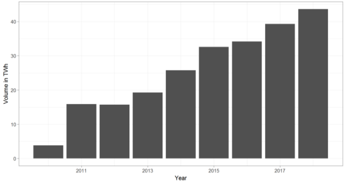
Motivation

- Renewable production increases in Europe.
- This production is difficult to forecast when the spot price is settled.
- Producers need to buy or sell electricity on the intraday market.
- Intraday markets also allow to increase the value of storage assets.



Renewable capacity evolution in Germany [1]



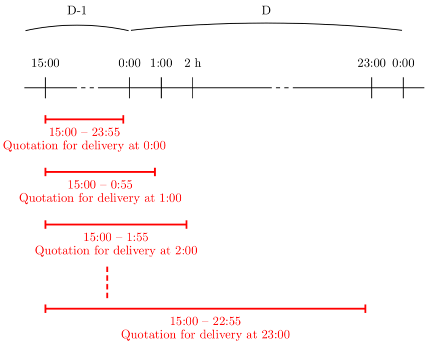
Yearly transaction volumes on the German intraday market[1]

[1] Source: Auction and continuous market for power: organization and microstructure, Clara Balardy.

Motivation – what are intraday markets?

EPEX Spot German intraday market, organized in continuous trading:

- Opens at 15:00 the day before;
- Possibility to buy/sell physical delivery contracts for the 24 periods 0:00–1:00, ..., 23:00–24:00;
- Closes 5 minutes before beginning of delivery.



Objectives

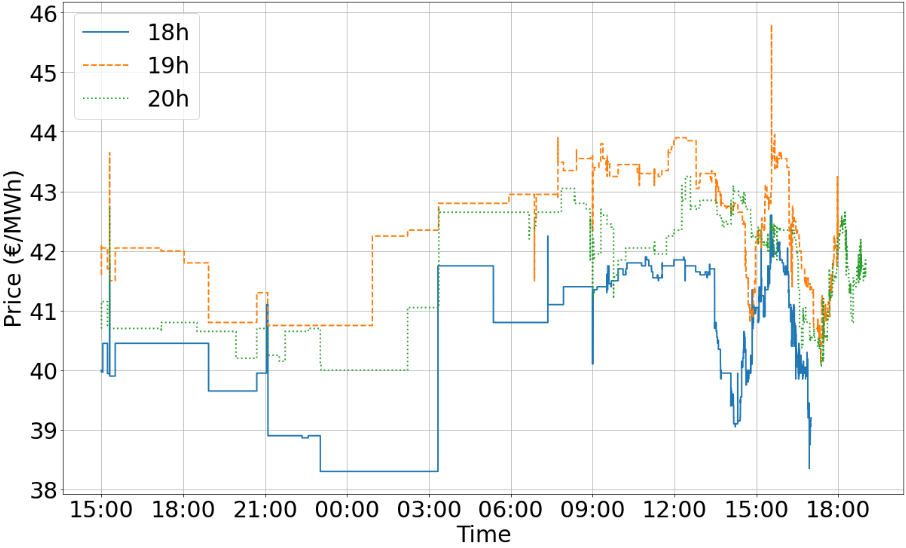
- To assess the quality of trading strategies or to value assets, need for a price model that captures risks on the market.
- Needs to represent different stylized facts that we identify.
- Few literature on intraday markets modeling:
  - Favetto (2019); Graf von Luckner and Kiesel (2020) : order arrivals modeling
  - Kiesel and Paraschiv (2017) : econometric analysis
- We propose a price model with a focus on the representation of the volatility.

Data

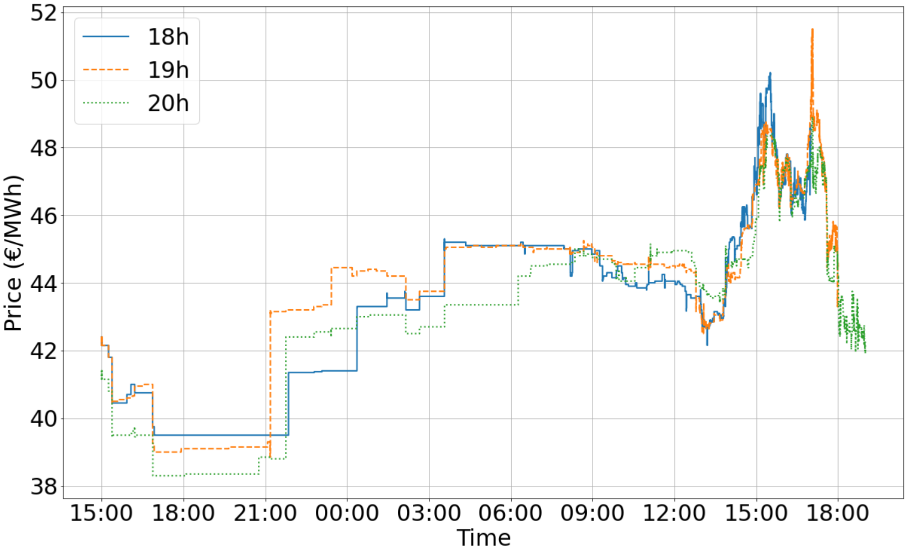
- German electricity intraday mid-prices between July and September 2017 for products with a delivery period of one hour.
- Mid-prices built using order book data from EPEX Spot.
- Mid-prices sampled at the second frequency for simplicity (available at milliseconds frequency).
- Market opens at 3 p.m. the day before delivery and closes 5 minutes before delivery...
- Yet, one hour before delivery, cross-border trading is not possible anymore.
- Also, thirty minutes before delivery, transactions are only possible into each of the four control areas in Germany and not across them.

$\Longrightarrow$ We only consider prices until one hour before delivery.

Data: 2017-07-11



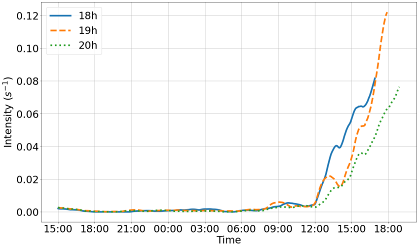
Data: 2017-08-30



Outline

Outline

Increasing intensity of arrival price changing times



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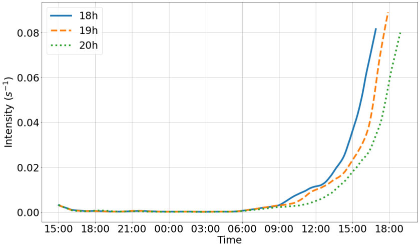
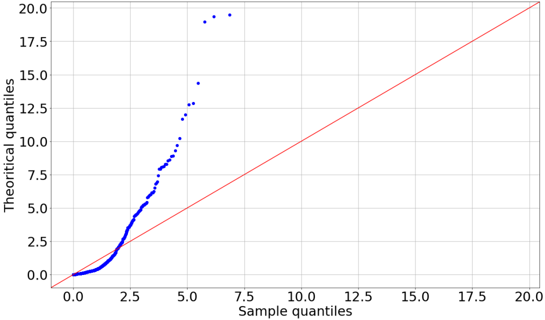


Average

Estimated intensity of price changing times with an Epanechnikov kernel and a window of 300 seconds

- Quasi null activity at the beginning of the trading session...
- then an exponential increase near the end of the trading period.

Non Poissonian arrival price changing times



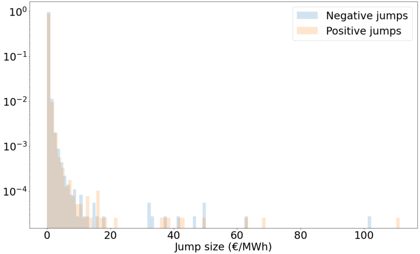
QQ-plot between the time-changed jump time intervals and an exponential distribution for the trading session of August, 30th, 2017 and for maturity 18h

For an inhomogeneous Poisson process with cumulated intensity $\Lambda(t) = \int_0^t \lambda(s)ds$ jumping at times $(\tau_i)_i$,

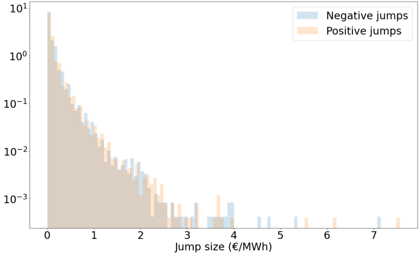
$$\Lambda(\tau_i) - \Lambda(\tau_{i-1}) \overset{iid}{\sim} \mathcal{E}(1).$$

- Inhomogeneous Poisson process modeling not suitable.

Jump sizes distribution (1/2)



All trading session

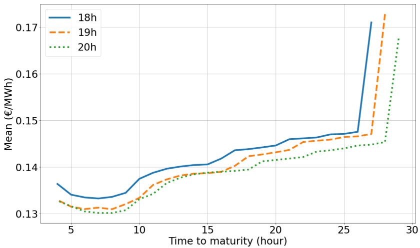


From 9 hours before maturity

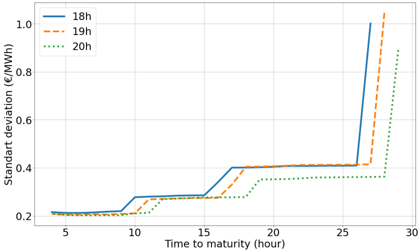
Positive and negative jump size distributions with a log scale on the y-axis, for maturity 18h

- Positive and negative jumps seem to have the same law (confirmed if we consider only the first two moments).
- Time dependency in the distribution of jumps with big jumps at the beginning, featuring a lack of liquidity.

Jump sizes distribution (2/2)



Mean



Standard deviation

Mean and standard deviation of jump sizes (positive and negative considered indifferently) against time to maturity: x-axis corresponds to the number of hours before maturity at which the estimation starts

From now on, one considers only data from 9 hours before maturity.

Volatility estimation

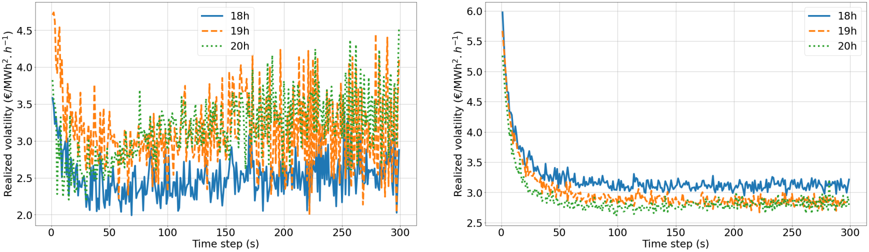
- Classical estimator of volatility of $f_t = \int_0^t \mu_s ds + \int_0^t \sigma_s dW_s$:

$$C(T, \Delta_n) = \frac{1}{T} \sum_{i=1}^{\lfloor \frac{T}{\Delta_n} \rfloor} \left(f_{i\Delta_n} - f_{(i-1)\Delta_n} \right)^2 \xrightarrow[\Delta_n \to 0]{} \frac{1}{T} \int_0^T \sigma_s^2 ds.$$

- One then wants to consider the highest frequency Δ_n^{-1}.
- Presence of microstructure noise in high-frequency financial data:
  - volatility estimator unstable when frequency is very high ;
  - mean reverting behavior of price.

Signature plot

$$\delta \mapsto C(T,\delta) = \frac{1}{T}\sum_{i=1}^{\lfloor \frac{T}{\delta} \rfloor} \left(f_{i\delta} - f_{(i-1)\delta}\right)^2$$



2017-08-30 Average

- Same behavior than financial data, see Bacry et al. (2013a).
- Instability at high-frequencies, fast decreasing then stabilization.

Outline

Point process modeling

- Consider a sequence of arrival times $0 < \tau_1 < \tau_2 < \ldots$ defined on $(\Omega, \mathcal{F}, \mathbb{P})$ endowed with a filtration $(\mathcal{F}_t)_{t \geq 0}$ (complete and right continuous).
- Let $(J_i)_{i \geq 1}$ be a sequence of positive i.i.d. r.v. defined on $(\Omega, \mathcal{F}, \mathbb{P})$, with $J_i \sim J$ and $\mathbb{E}(J^2) < \infty$.
- Mark the arrival times $(\tau_i)_i$: $(\tau_i^+)_i$ (price increase) and $(\tau_i^-)_i$ (price decrease), associated with $(J_i^+)_i$ and $(J_i^-)_i$.

Hawkes modeling

Hawkes modeling on $[0, T]$:

$$\begin{pmatrix} \lambda_t^+ \\ \lambda_t^- \end{pmatrix} = \mu\left(\frac{t}{T}\right)\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \int_0^t \varphi(t-s)\begin{pmatrix} J_s dN_s^+ \\ J_s dN_s^- \end{pmatrix}$$

with

- $\mu : [0,1] \to \mathbb{R}_+$ a non decreasing bounded function,
- $\varphi : \mathbb{R}_+ \to \mathbb{R}^{2,2}$ a locally bounded function with positive components such that spectral radius of $\mathbb{E}(J)\int_0^\infty |\varphi(u)|du$ is less than 1.

The intraday price is given by

$$f_t = f_0 + f_t^+ - f_t^-$$

with

$$\begin{pmatrix} f_t^+ \\ f_t^- \end{pmatrix} = \int_0^t \begin{pmatrix} J_s dN_s^+ \\ J_s dN_s^- \end{pmatrix}.$$

Parametrization

- $\mu = t \mapsto \mu_0 e^{\kappa t}$: models the increasing intensity
- $\varphi = \begin{pmatrix} 0 & \varphi_{\text{exp}} \\ \varphi_{\text{exp}} & 0 \end{pmatrix}$ with $\varphi_{\text{exp}} : t \mapsto \alpha e^{-\beta t}$, α, $\beta > 0$ and $\alpha \mathbb{E}(J) < \beta$: good candidate to represent the signature plot (see Bacry et al. (2013a))

- Simple parameterisation with only four parameters.
- Tractable model with nice theoretical properties.
- A priori, allows one to represent the different characteristics of the prices.

Estimation

Maximisation of llh, equal for one trading session to $\mathcal{L}^- + \mathcal{L}^+$ to

$$\mathcal{L}^{\mp} = \int_0^T \log(\lambda_t^{\mp}) dN_t^{\mp} + \int_0^T (1 - \lambda_t^{\mp}) dt,$$

see (Daley and Vere-Jones, 2003, Proposition 7.2III), that is

$$\mathcal{L}^{\mp} = \sum_{i=1}^{N_T^{\mp}} \log\left(\mu_0 e^{\kappa \frac{\tau_i^{\mp}}{T}} + \sum_{j=1}^{N^{\pm}_{\tau_i^{\mp}}} \alpha J_j^{\pm} e^{-\beta\left(\tau_i^{mp} - \tau_j^{pm}\right)}\right) +$$

$$T - \frac{\mu_0 T}{\kappa}\left(e^{\kappa} - 1\right) - \sum_{i=1}^{N_T^{\pm}} \frac{\alpha}{\beta} J_i^{\pm}\left(1 - e^{-\beta\left(T - \tau_i^{\pm}\right)}\right).$$

Initialization:

- for μ_0, α, β, minimization of the L^2 distance between the theoretical signature plot in the case $\kappa = 0$ and the empirical one;
- $\kappa = 0.1$.

Estimation: Results

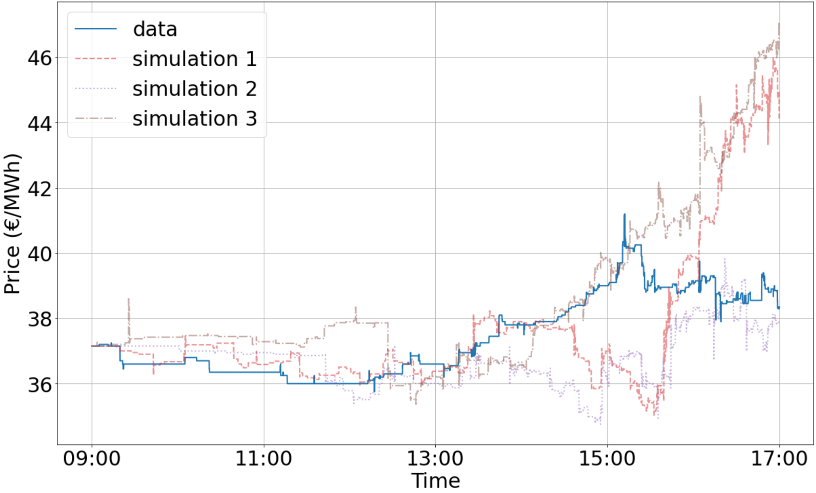
Estimation on the whole dataset by taking the sum of the individual llh.

Maturity	μ_0 (h^{-1})	κ	α (h^{-1})	β (h^{-1})	$\mathbb{E}(J)$	$\mathbb{E}(J^2)$
18h	2.49	3.51	864.39	237.30	0.13	0.066
19h	3.01	3.50	2344.97	639.64	0.13	0.061
20h	3.06	3.51	3100.46	859.11	0.13	0.058

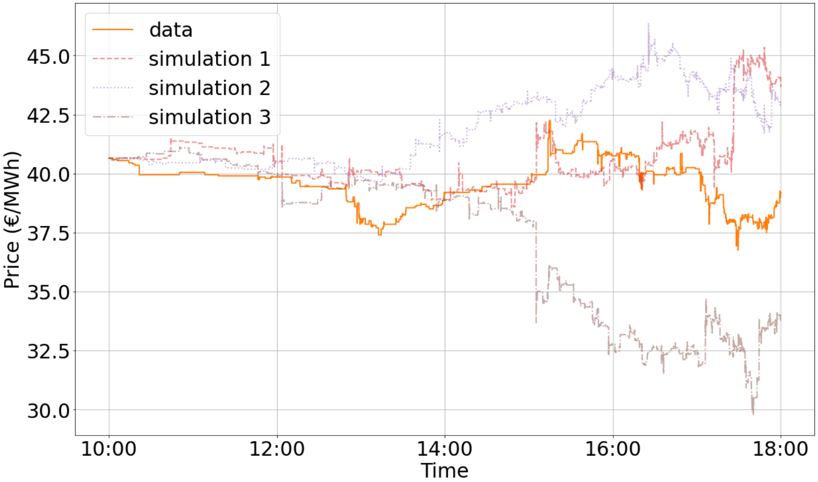
We do not estimate the law of J.

Simulation: Illustration for maturity 18h

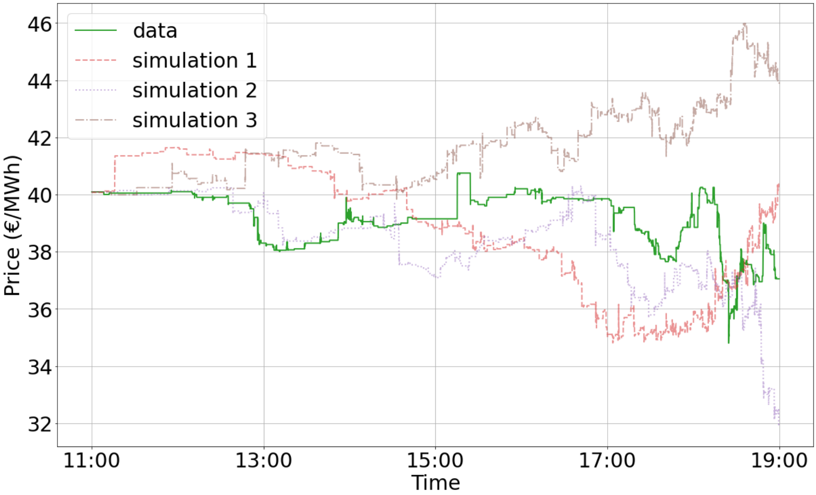
Simulation with thinning algorithm Ogata (1981) bootstrapping jump sizes.



Simulation: Illustration for maturity 19h

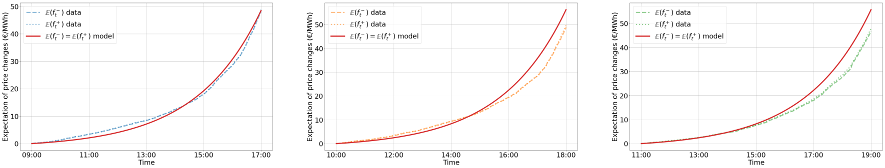


Simulation: Illustration for maturity 20h



Expectation

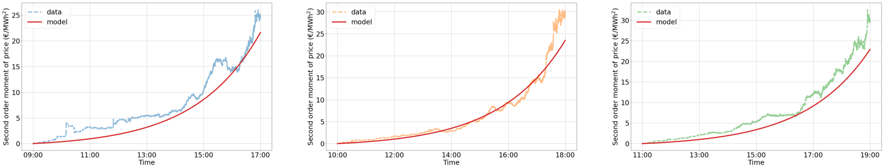
$$\mathbb{E}(f_t^+) = \mathbb{E}(f_t^-) = \mu_0 \mathbb{E}(J) \left(\frac{\beta + \frac{\kappa}{T}}{\frac{\kappa}{T}\left(\beta - \alpha\mathbb{E}(J) + \frac{\kappa}{T}\right)} e^{\kappa \frac{t}{T}} \right.$$
$$\left. + \frac{\alpha\mathbb{E}(J)}{(\beta - \alpha\mathbb{E}(J))\left(\beta - \alpha\mathbb{E}(J) + \frac{\kappa}{T}\right)} e^{-(\beta - \alpha\mathbb{E}(J))t} - \frac{\beta}{\frac{\kappa}{T}(\beta - \alpha\mathbb{E}(J))} \right)$$



18h 19h 20h

Second order moment

Closed formula for the second order moment.



18h 19h 20h

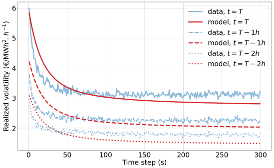
Sketch of the proof:

- Find integro-differential equation for characteristic function of (f_t^+, f_t^-) using cluster representation of Hawkes processes as in El Euch and Rosenbaum (2019).
- Then derive integro-differential equations for moments that can be solved.

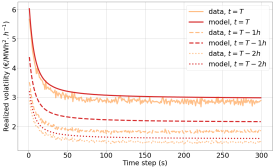
Signature plot

$$C(t,\delta) = \frac{1}{t}\left(\mathbb{E}(f^2_{\lfloor \frac{t}{\delta} \rfloor \delta}) - f_0^2\right) -$$

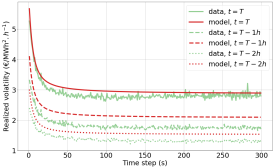
$$\frac{1}{t}\frac{\left(1 - e^{-(\beta+\alpha\mathbb{E}(J))\delta}\right)}{(\beta+\alpha\mathbb{E}(J))}\sum_{i=0}^{\lfloor \frac{t}{\delta} \rfloor - 1}\left(\frac{d\mathbb{E}(f_s^2)}{ds}(i\delta) - 2\mathbb{E}(J^2)\mathbb{E}(\lambda^+_{i\delta})\right).$$



18h



19h



20h

- Generalization of the results of Bacry et al. (2013a): we include random jumps and time-dependent intensity baseline.
- Increasing of the signature plot when time approaches to delivery: Samuelson effect for each frequency.

Signature plot: asymptotics

Microscopic scale: $\delta \to 0$

$$C^{micro}(t) = 2\mathbb{E}(J^2)\frac{\mathbb{E}\left(\int_0^t \lambda_s^+ ds\right)}{t}.$$

Macroscopic scale: $\delta \to \infty$, $\frac{\delta}{t} \to 0$

$$C^{macro}(t) \sim \frac{2\mathbb{E}(J^2)}{\left(1 + \frac{\alpha\mathbb{E}(J)}{\beta}\right)^2\left(1 - \frac{\alpha\mathbb{E}(J)}{\beta}\right)}\frac{\int_0^t \mu(\frac{s}{T})ds}{t}.$$

When $t \to \infty$,

$$C(t,\delta) \sim \frac{2\mathbb{E}(J^2)\int_0^t \mu(\frac{s}{T})ds}{t\left(1 - \frac{\alpha\mathbb{E}(J)}{\beta}\right)}\left(R^2 + \left(1 - R^2\right)\left(\frac{1 - e^{-(\beta+\alpha\mathbb{E}(J))\delta}}{(\beta + \alpha\mathbb{E}(J))\,\delta}\right)\right)$$

with $R^2 = \frac{1}{\left(1+\frac{\alpha\mathbb{E}(J)}{\beta}\right)^2}$.

Outline

1. Empirical stylized facts
2. Model
3. Price at macroscopic scale

Central limit theorem

Assume $\mathbb{E}(J^4) < \infty$.

$$\left(\frac{1}{\sqrt{T}}\left(f_{vT} - f_0\right)\right)_{v\in[0,1]} \to \left(\sqrt{\frac{2E(J^2)}{\left(1+\frac{\alpha\mathbb{E}(J)}{\beta}\right)^2\left(1-\frac{\alpha\mathbb{E}(J)}{\beta}\right)}}\int_0^v \sqrt{\mu(s)}dW_s\right)_{v\in[0,1]}$$

in law for the Skorokhod topology when $T \to \infty$, where W is a 1-dimensional Brownian motion.

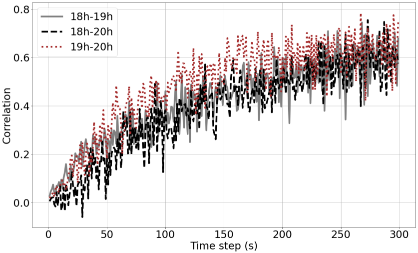
- Diffusive behavior at macroscopic scale.
- Samuelson effect: macroscopic volatility increases when time gets closer to delivery.
- More results are given that generalize limit theorems of Bacry et al. (2013b) considering time dependent baseline intensity and random jumps.

Conclusion

- Highlighting of the presence of microstructure noise in intraday electricity markets;
- Proposition of a price model allowing to represent the different empirical stylized facts, in particular the signature plot;
- Closed formula for moments and signature plot (at different dates);
- Diffusive limit at macroscopic scale;
- Samuelson effect identified for each frequency and in the diffusive limit.

Perspectives

- A more complete analysis and modeling of jumps distribution.
- Multidimensional modeling for the different maturities.



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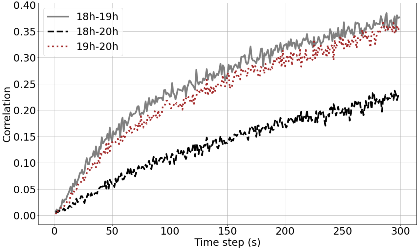

Average

Epps effect

Thank you for your attention.

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