Optimal Dynamic Regulation of Carbon Emissions Market

Journées Ateliers du Laboratoire FiME
Septembre, 2021

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Question

- What are the optimal dynamic allocation schemes to reach a given expected emissions reduction over a finite time horizon?

Model

- A regulator wishes to reduce the emissions of a set of N firms over a period of time $(0, T)$.
- Each firm i emissions E_t^i increase at a constant rate μ_i and are prone to random shocks $\sigma_i W^i$ potentially correlated to a common economic shock W^0.
- In the Business As Usual scenario (BAU), total expected emissions at time T are

$$\mathbb{E}\Big[\sum_{i=1}^{N} E_T^i\Big] =: \mathbb{E}\big[E_T\big] = N\bar{\mu}T, \quad \bar{\mu} := \frac{1}{N}\sum_{i=1}^{N}\mu_i.$$

- The regulator wishes to reduce the emissions to

$$L := \rho N\bar{\mu}T, \quad \rho \in (0,1).$$

Model (contd)

- At time $t = 0$, the regulator opens a bank account for each i and credit (or debit) the account by the value A_0^i of allowances.
- A_t^i is the cumulative allocation given to firm i at time t.
- Dynamics of firm's i bank account X_t^i

$$dX_t^i = -dE_t^{i,\alpha^i} + \beta_t^i dt + dA_t^i, \quad X_0^i = A_0^i,$$
$$dE_t^{i,\alpha^i} = (\mu_i - \alpha_t^i)dt + \sigma_i dW_t^i.$$

with α_t^i the abatement effort rate, and β_t^i the trading rate.

- For given vector of allocation schemes (A^i) and emission allowance price P_t, the objective of the firm is to minimize the total expected cost of
    - abatement cost $c_i(\alpha) := h_i\alpha + \frac{1}{2}\frac{\alpha^2}{\eta_i}$
    - trading cost $\beta_t^i P_t$
    - penalty cost of non compliance of zero position of the bank account at terminal time $\lambda(X_T^i)^2$,

  which resumes in

$$\inf_{\alpha^i,\beta^i} J^i(\alpha^i,\beta^i) := \mathbb{E}\Big[\int_0^T \Big(c_i(\alpha_t^i) + \beta_t^i P_t\Big)dt + \lambda\big(X_T^i\big)^2\Big],$$

- Market price P_t is determined by usual market clearing condition :

$$\sum_{i=1}^N \beta_t^i = 0.$$

Regulator's optimisation problem

Find dynamic allocation schemes (A^i) that minimise total abatement costs and terminal penalty costs while ensuring that emissions are reduced to the level $L = \rho N \bar{\mu} T$.

$$\inf_A R(A) := \mathbb{E}\Big[\sum_{i=1}^N \int_0^T c_i(\hat{\alpha}_t^i)dt + \lambda(\hat{X}_T^i)^2\Big],$$

$$\mathbb{E}\Big[\sum_{i=1}^N E_T^{i,\hat{\alpha}^i}\Big] = L = \rho T N \bar{\mu},$$

where $\hat{\alpha}^i$ and $\hat{X}^i$ are the optimal abatement rates and bank account dynamics at market equilibrium.

Optimal regulations

(i) The solutions to the regulator optimisation problem are non-unique and characterised by the minimisation of the price volatility and the condition that $\mathbb{E}[\bar{A}_T] = N\bar{\mu}T + \ell(\rho)$ where $\ell(\rho) < 0$ is a constant that depends on the parameter of the model.

(ii) The cumulative allocations given by

$$A_t^i = \underbrace{\ell(\rho)}_{\text{init. debt}} + \underbrace{\mu_i t}_{\text{BAU trend}} + \underbrace{\sigma_i W_t^i}_{\text{eco. shock}}, \quad i = 1, \dots, N,$$

form a solution.

(iii) The equilibrium price and abatement efforts are constant given by

$$\hat{P}_0 = \frac{\bar{H}}{\bar{\eta}} + (1-\rho)\frac{\bar{\mu}}{\bar{\eta}}, \quad \hat{\alpha}_0^i = \eta_i(\hat{P}_0 - h_i),$$

where $\bar{H} := \frac{1}{N}\sum_{i=1}^N \eta_i h_i$, $N\bar{h} := \sum_i h_i$, and $N\bar{\eta} := \sum_i \eta_i$.

Optimal regulations in words

The regulator insures the firms
from economic business cycles
to make them exert constant abatement efforts
and thus, avoid costly adjustment costs.

Why?

- At equilibrium, the market price for emission is a martingale: on average, it is constant.
- The emission reduction is only determined by the average effort, which is determined only by the average price.
- Once the average effort is determined to reach a given reduction target, the regulator can only act on the volatility of the price to reduce the total social cost.
- Indeed, price volatility creates ups and downs in the abatement efforts of the firms which are costly because of limited flexibility.

Where is the benefit
of a dynamic allocation scheme
compared to a simple static initial allocation?

Static allocation scheme (EU TS Phase I and II)

- Denote Δ^{stat} the difference between the social cost with a static allocation and the social cost under an optimal dynamic allocation.
- We have

$$\Delta^{\text{stat}} := \frac{N\sigma^2}{2\eta} \ln \left[1 + 2\lambda\eta T\right], \qquad N^2\sigma^2 := \sum_{i=1}^{N} \sigma_i^2 + 2\sum_{i<j} \rho_{ij}\sigma_i\sigma_j.$$

- If the world is deteministic ($\sigma = 0$) or perfectly flexible ($\eta \to \infty$), there is no interest in dynamic allocation.
- Suppose there are N firms with identical σ_i and k_i. Denote $\bar{\rho}$ the common correlation. Here, $N^2\sigma^2 = N\bar{\sigma}^2 + \bar{\rho}\bar{\sigma}N(N-1)$.
- Hence, if there are no common shocks, when $N \to \infty$, the per unit difference cost Δ^{stat}/N goes to zero, making also dynamic schemes useless.

- Dynamic allocation provides insurance to firms from common economic uncertainty inducing costly adjustment.

Illustration of the dynamics

- In $T = 10$ years, in a market of $N = 6$ aggregated sectors,
- the regulator wants to reduce the emissions by 20%, i.e. $\rho = 0.8$,
- in a market where the average growth rate of emissions is $N\bar{\mu} = 2$ Gt/year,
- with a volatility of $\sigma_i = 0.2/\sqrt{N}$ Gt/year and per firm,
- and average abatement cost $\bar{h} = 25$ €/t,
- and equal adjustment cost $\eta = 6\,10^8$ t^2/€ (after Gollier (2020)),
- and a equal dependence on the common shocks of $k_i = 0.9$
- and a terminal penalty parameter of $\lambda = 7.5\,10^{-7}$ €/t^2.

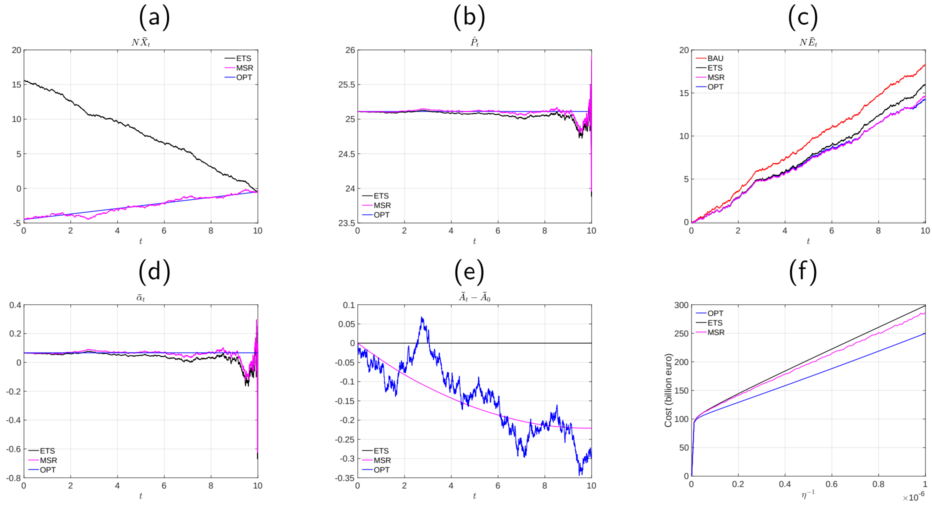

Figure: Simulation of one trajectory under different dynamic policies. (a) the total bank accounts, (b) the equilibrium market prices, (c) total emission, (d) average abatement effort and (e) net allocation minus initial allocation. (f) social cost as a function of η

Perspectives

- Base-line emission reduction (already done with Maria Arduca, post-doc at LUISS)
- Indirect emissions and individual allowances (on going with Maria)
- Non-observability of abatement efforts and moral hazard.