MFG model with a long-lived penalty at random jump times: application to demand side management for electricity contracts

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- Addition of 260 GW of renewables in 2020 (which represents 80% of all added capacities) [1].
- Almost 2800 GW of renewables worldwide (36% of total capacities), 730 GW is wind, 714 GW is solar [1].

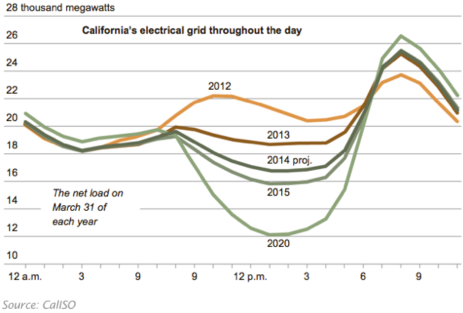


Figure: Source: CAISO

[1] IRENA, RENEWABLE CAPACITY STATISTICS 2021

Ducks do happen in reality!

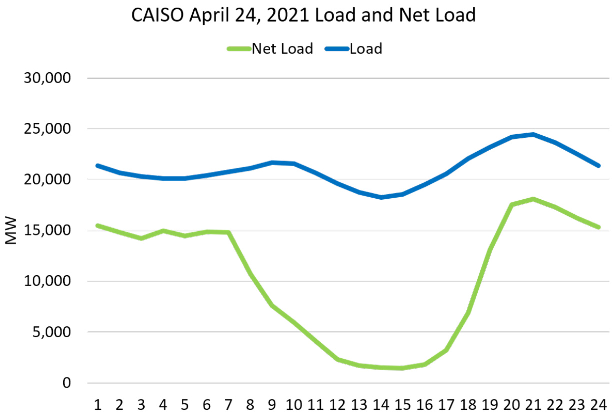


Figure: Source: CAISO

The power system requires more flexiblities.

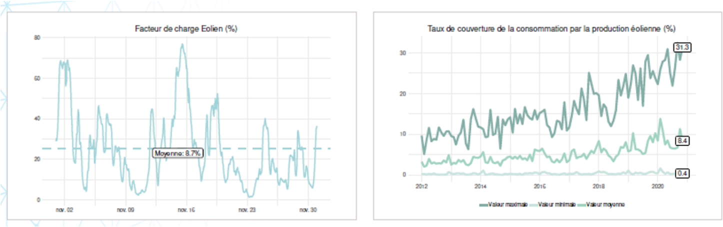


Figure: source: RTE, bilan mensuel novembre 2020

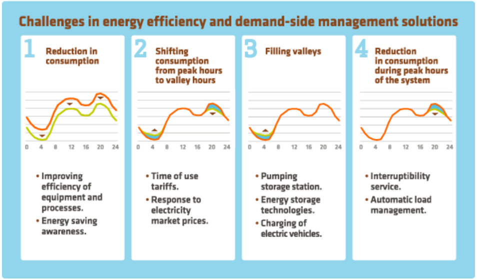
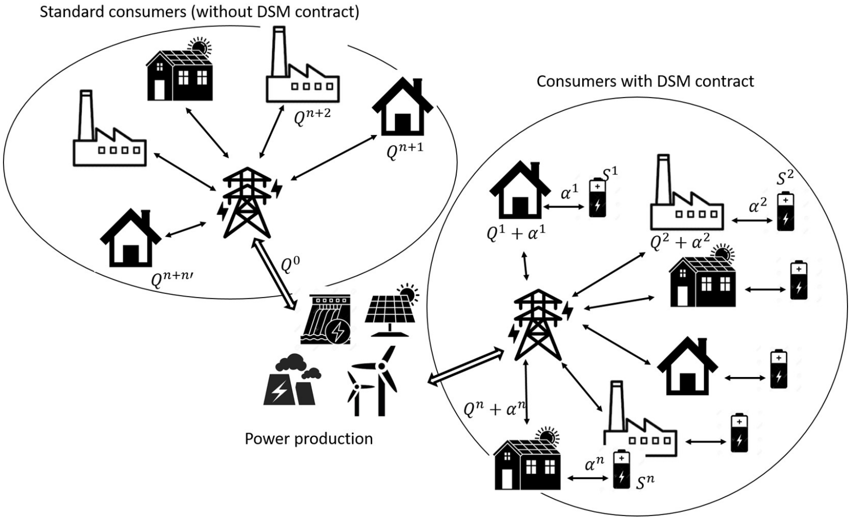


Figure: Source: Red Electrica

Clean Energy Package: each final customer should be entitled to choose a dynamic electricity price contract


Standard consumers (without DSM contract)
Q^{n+2}
Q^{n+1}
$Q^{n+n\prime}$
Q^0
Consumers with DSM contract
S^1
α^1
$Q^1+\alpha^1$
S^2
α^2
$Q^2+\alpha^2$
$Q^n+\alpha^n$
α^n
S^n
Power production

Objective function

Each consumer $i \in \{1, \ldots, n\}$ wants to minimise its total expected costs:

- payment of the variable part of his energy contract indexed on its energy consumption
- payment of the fixed part of his energy contract indexed on its subscribed power
- DSM contract satisfaction
- inconvenience due to consumption modification

We chose to represent a DSM contract with two parts:

- RTP: real time pricing
- interruptible load = divergence cost

Real time tariff

Spot price is sensitive to the global power demand.

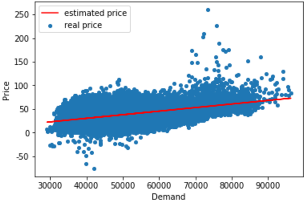


Figure: source: ENTSOE and Epexspot

The real time tariff:

$$c_t^i = (Q_t^i + \alpha_t^i)p\left(\underbrace{\frac{1}{n+n'}\sum_{j=n+1}^{n'} Q^j}_{\text{standard consumers}} + \underbrace{\frac{1}{n+n'}\sum_{j=1}^{n}(Q_t^j + \alpha_t^j)}_{\text{consumers with DSM contract}}\right).$$

or more simply $c_t^i = (Q_t^i + \alpha_t^i)p_t\left(\frac{1}{n}\sum_{j=1}^{n}(Q_t^j + \alpha_t^j)\right)$

When activated, the aim of the interruptible load contract is that the global divergence $\sum_i \alpha_t^i$ equals $\bar{\alpha}$ during θ. The divergence cost has the form:

$$d_t^i = J_t^\theta(\tilde{Q}_t^i + \alpha_t^i - \bar{\alpha}) f\left(\frac{1}{n}\sum_{j=1}^{n}(\tilde{Q}_t^j + \alpha_t^j) - \bar{\alpha}\right)$$

- with f a convex growing function such as $f(0) = 0$
- J_t^θ equal to one during interruptible load contract activation and 0 otherwise.
- $dR_t = dt - R_{t^-} dN_t^0,\ R_0 = 2\theta,$
- $J_t^\theta = \mathbf{1}_{R_t \leq \theta}$
- $\tilde{Q}_t^i = Q_t^i - \mathbb{E}\left[Q_t^i\right]$

Each consumer $i \in \{1, \dots, n\}$ wants to minimise its total expected costs:

$$\inf_{\alpha^i \in \mathcal{A}} J_n^i(\alpha) = \inf_{\alpha^i \in \mathcal{A}} \mathbb{E}\left[\int_0^T \left(\underbrace{g(\alpha_t^i, S_t^i, Q_t^i)}_{\text{inconvenience cost}} + \underbrace{l(Q_t^i + \alpha_t^i)}_{\text{demand charge}} + \underbrace{c_t^i}_{\text{real time tariff}} + \underbrace{d_t^i}_{\text{divergence cost}} \right) dt + \underbrace{h(S_T^i)}_{\text{terminal cost}} \right],$$

with $\alpha = (\alpha^1, \dots, \alpha^n)$.

$\Longrightarrow$

- interaction of controls in real time tariff and divergence cost
- jump and delay in the divergence cost

- W^0 and W two independent Brownian motions
- N^0 and N two independent Poisson processes with intensities λ^0 and λ.
- $\tilde{N}$ the compensated Poisson processes
- $\mathbb{F} = (\mathcal{F}_t)_{t\in[0,T]}$ be the (complete) natural filtration generated by (W, W^0, N, N^0, s_0, q_0).
- $\mathbb{F}^0 = (\mathcal{F}_t^0)_{t\in[0,T]}$ be the (complete) natural filtration generated by (W^0, N^0).

$$
\begin{aligned}
dQ_t = &\quad \mu(Q_t, t)dt + \sigma(Q_t, t)dW_t + \beta(Q_{t-}, t)d\tilde{N}_t + \sigma^0(Q_t, t)dW_t^0, \quad Q_0 = q_0,\\
dQ_t^{st} = &\quad \mu^{st}(Q_t^{st}, t)dt + \beta(Q_{t-}^{st}, t)d\tilde{N}_t + \sigma^{st}(Q_t^{st}, t)dW_t^0, \quad Q_0^{st} = q_0^{st},\\
dS_t = &\quad \alpha_t dt, \quad S_0 = s_0.
\end{aligned}
$$

We denote by $\tilde{Q}_t = Q_t - \mathbb{E}[Q_t]$, $t \in [0, T]$ and for a $\mathbb{F}$-adapted process $\xi = \{\xi_t\}$, denote $\hat{\xi}_t := \mathbb{E}[\xi_t|\mathcal{F}_t^0]$

MFG problem: Let $\xi = (\xi_t)_{t\in[0,T]}$ be a given $\mathbb{F}^0$-adapted process.

$$J^{MFG}(\alpha;\xi) = \mathbb{E}\left[\int_0^T \left(g(\alpha_t, S_t, Q_t) + l(Q_t+\alpha_t) + (Q_t+\alpha_t)p_t\left(\widehat{Q}_t + \xi_t\right)\right.\right.$$
$$\left.\left. + J_t^\theta(\tilde{Q}_t + \alpha_t - \bar{\alpha}) f\left(\widehat{\tilde{Q}_t} + \xi_t - \bar{\alpha}\right)\right) dt + h(S_T)\right],$$

where $\alpha = (\alpha_t)_{t\in[0,T]}$ is an *admissible* control process which belongs to $\mathcal{A}$, the set of all real-valued $\mathbb{F}$-adapted processes such that $\mathbb{E}[\int_0^T \alpha_t^2 dt] < \infty$ and $\mathbb{E}[|\alpha_\tau|\mathbf{1}_{\tau<\infty}] < \infty$ for all $\mathbb{F}^0$-stopping times τ with values in $[0,T]\cup\{+\infty\}$.

$$V^{MFG}(\xi) = \inf_{\alpha\in\mathcal{A}} J^{MFG}(\alpha;\xi).$$

The goal is to find a process $\alpha^\star = (\alpha_t^\star)_{t\in[0,T]}$ such that

$$J^{MFG}(\alpha^\star;\xi) = V^{MFG}(\xi)$$

and

$$\widehat{\alpha}_t^\star = \xi_t, \text{ a.s. for all } t\in[0,T].$$

Such a process $\alpha^\star$ is called a *mean-field Nash equilibrium*.

MFC problem: Let $\xi = (\xi_t)_{t\in[0,T]}$ be a given $\mathbb{F}^0$-adapted process.

$$
\begin{aligned}
J^C(\alpha) =& \mathbb{E}\Bigg[(1-\pi)\int_0^T \Big(g(\alpha_t, S_t, Q_t) + (Q_t+\alpha_t)p_t\left(\widehat{Q}_t + \widehat{\alpha}_t\right) \\
&+ l(Q_t+\alpha_t) + J_t^\theta(\tilde{Q}_t + \alpha_t - \bar{\alpha})f\left(\widehat{\tilde{Q}_t} + \widehat{\alpha}_t - \bar{\alpha}\right)\Big)\,dt + (1-\pi)h(S_T) \\
&\pi\int_0^T \left(Q_t^{st}p_t\left(\widehat{Q}_t + \widehat{\alpha}_t\right) + l(Q_t^{st})\right)dt\Bigg].
\end{aligned}
$$

$$
V^C = \inf_{\alpha\in\mathcal{A}} J^C(\alpha). \tag{1}
$$

MFG and MFC are characterised by FBSDE systems (stochastic maximum principle) and MFC equilibrium is unique by strict convexity of the criterion.

Proposition

Consider the solution $\alpha^\star_{MFC}$ of MFC problem with a pricing rule p_{MFC} and f_{MFC}. Then $\alpha^\star_{MFC}$ is a mean field nash equilibrium for the MFG problem with pricing rule

$$p_{MFG}(x) = p_{MFC}(x) + x p'_{MFC}(x) \; ,$$
$$f_{MFG}(x) = f_{MFC}(x) + x f'_{MFC}(x) \; .$$

Remark 1: Uniqueness of MFC implies the uniqueness of the MFG equilibrium.

Remark 2 : For the numerics, we use those relationships to compute the solution of the MFC by using the same code for computing both equilibria.

Numerical examples - State variables based on historical Australian data

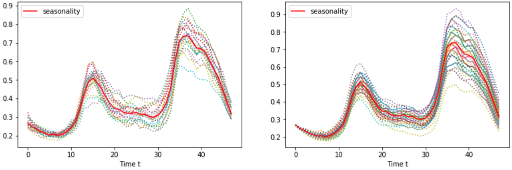


Figure: Trajectories of $\widehat{Q}$ (in kW) with estimated seasonality over 48 half-hours in a weekday in July.

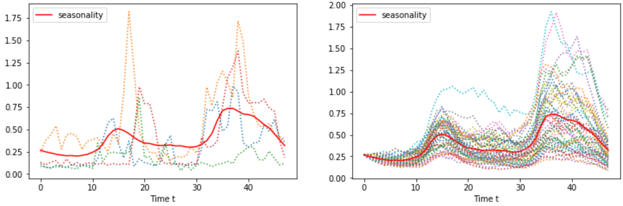


Figure: Trajectories of Q (in kW) with estimated seasonality over 48 half-hours in a weekday in July.

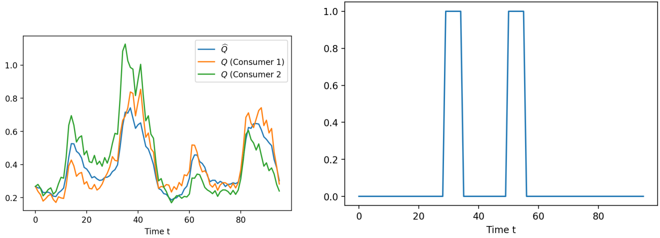


Figure: One trajectory of $\widehat{Q}$ and Q (in kW) for two different consumers (left) and one trajectory of J (right) along time (in half-hours).

Numerical results for Real Time Tariff and no DSM

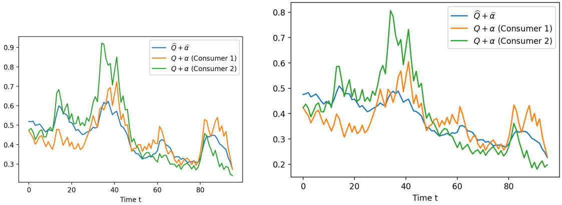


Figure: Trajectories of $\widehat{Q}+\widehat{\alpha}$ and $Q+\alpha$ (in kW) for two different consumers for MFG (left) and corresponding trajectories for MFC (right) along time (in half-hours).

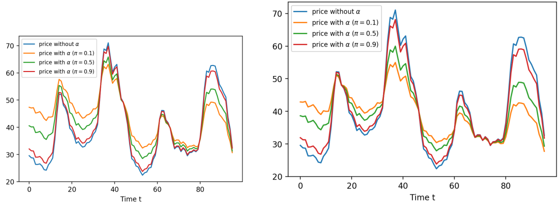


Figure: Trajectories of price p for three different proportions of active consumers for MFG (left) and corresponding trajectories for MFC (right) along time (in half-hours).

Numerical results for DSM and Real Time Tariff.

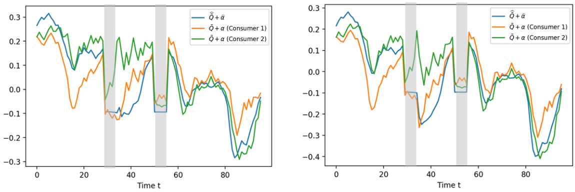


Figure: Trajectories of $\widehat{\tilde{Q}} + \widehat{\alpha}$ (in kW) and $\tilde{Q} + \alpha$ for two different consumers for MFG (left) and for MFC (right) along time (in half-hours).

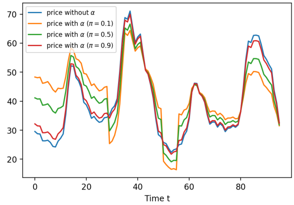


Figure: Trajectories of price p for different proportion π of standard consumers in the system in the MFG setting (jumps episodes are highlighted in grey) along time (in half-hours).

Main results:

- MFG of controls is interesting for several applications for power system with distributed local energy generation and flexibilities.
- MFG of controls with jumps and delay approach provides an analytically and numerically tractable setting to analyze the model of DSM contract.
- With quadratic cost structure and linear pricing rule, we provide quasi-explicit solutions and existence + unicity results for the equilibrium.
- A numerical implementation is proposed and provides interesting results.
- Centralised optimization can be decentralized: extended MFG can linked to suitable Mean Field Type Control (MFC) problem (central planner point of view)

- Electrical system and MFG:
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- Extended MFG and MFG with common noise : R. Carmona and F. Delarue (2013), (2015), (2017); R. Carmona and F. Delarue and D. Lacker (2016), D. Gomes & al. (2013), (2016), P. Cardaliaguet and C.A. Lehalle (2017)
- Games with delay: R. Carmona, JP. Fouque, SM. Mousavi, LH. Sun(2018), Bensoussan & al. (2016)
- MFG with jumps: C. Benazzoli, L. Campi, and L. Di Persio (2019 and 2020), Z. Li, A. M. Reppen, and R. Sircar (2019)
- Linear Quadratic case: A. Bensoussan, Yong (2013), Pham (2016), Graber (2016), Sun(2015)
- Numerics: . Lejay, E. Mordecki, and S. Torres (2014), R. Dumitrescu and C. Labart (2016)