Optimal make-take fees for market making regulation

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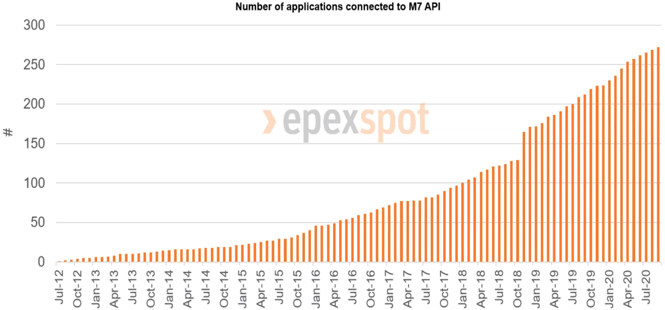
Robot trading on electricity market

The **number of applications** connected to the EPEX SPOT M7 API is **steadily growing**



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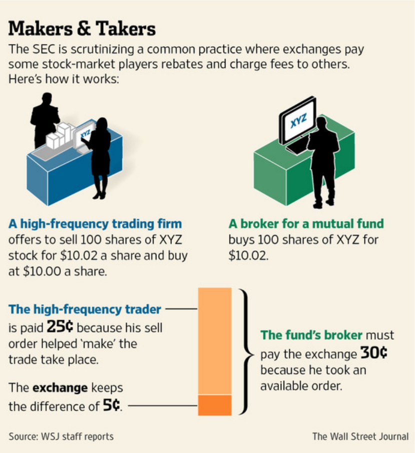
Trading Makers-Takers fees... towards Fintech



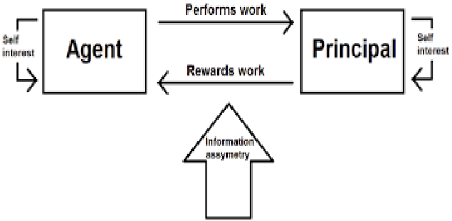
Delegation problem: accounting for moral hazard

X: value of an output process owned by Principal

Agent devotes effort a, thus impacting distribution of $X \Longrightarrow X^a$

- cost of effort $c(a)$
- compensation ξ : contract

Choose ξ so that Agent devotes effort in the interest of Principal



Market makers, and brokers trading

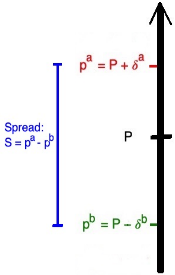
- Fundamental price $\{P_t\}_{t\geq 0}$: $dP_t = \sigma dW_t$

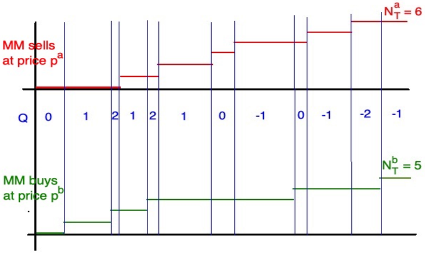
- **Market Maker** sets bid-ask prices $p_t^b = P_t - \delta_t^b$ and $p_t^a = P_t + \delta_t^a$

- N_t^b, N_t^a: # trades, unit jump point process with intensities

$$\lambda_t^b = \lambda(\delta_t^b) \quad \text{and} \quad \lambda_t^a = \lambda(\delta_t^a), \quad \text{with} \quad \lambda(x) = Ae^{-\frac{k}{\sigma}(x+c)}$$

$\Longrightarrow$ MM inventory $Q_t = N_t^b - N_t^a$, where





Market makers, and brokers trading

MM and Platform have constant absolute risk aversion

$$U_A(x) = -e^{-\gamma x}, \quad U_P(x) = -e^{-\eta x}$$

• MM chooses bid and ask prices:

$$V_A(\xi) \quad := \quad \sup_{\delta=(\delta^b,\delta^a)} \mathbb{E}^{\delta} U_A\Big(\xi + \int_0^T p_t^a dN_t^a - p_t^b dN_t^b + Q_T P_T\Big)$$

• Given optimal response $\delta^*(\xi)$, **Platform** chooses optimal contract

$$V_P \quad = \quad \sup_{\xi \in \Xi_R} \mathbb{E}^{\delta^*(\xi)} U_P\big(-\xi + c(N_T^a + N_T^b)\big)$$

c: fee paid by broker $\Longrightarrow$ c affects the arrival process...

Avellaneda & Stoikov '08 corresponds to $\xi = 0$

Optimal MM compensation

Let

$$u(t,q) = \sum_{p\geq 0} \frac{[C_1'(T-t)]^p}{p!} \sum_{j\geq 0} \frac{[C_1'(T-t)]^j}{j!} e^{-C_1(T-t)(q+j-p)^2} \mathbf{1}_{\{|q+j-p|\leq \bar{q}\}},$$

with constants C_1, C_1'. Then,

Optimal contract is

$$\widehat{\xi} \;=\; U_A^{-1}(R) + \int_0^T \widehat{Z}_t^a dN_t^a + \widehat{Z}_t^b dN_t^b + \widehat{Z}_t^P dP_t + \Big(\frac{1}{2}\gamma\sigma^2\big(\hat{Z}_t^P + Q_t\big)^2 - H(\widehat{Z}_t, Q_t)\Big) dt$$

where $\widehat{Z}_t^P = \frac{-\gamma}{\eta+\gamma} Q_t$: inventory risk sharing

$$\text{and } \widehat{Z}_t^i = c + \frac{1}{\eta}\Big[\ln\Big(\frac{u(t,Q_t)}{u(t,Q_t+\varepsilon_i)}\Big) - \zeta_0\Big], \quad i = b, a, \quad \varepsilon_b = 1, \ \varepsilon_a = -1,$$

$$\zeta_0 := -\log\Big(1 - \frac{1}{(1+\frac{k}{\sigma\gamma})(1+\frac{k}{\sigma\eta})}\Big)$$

Effect of the exchange optimal incentive policy

Parameters values from Guéant, Lehalle and Fernandez-Tapia:

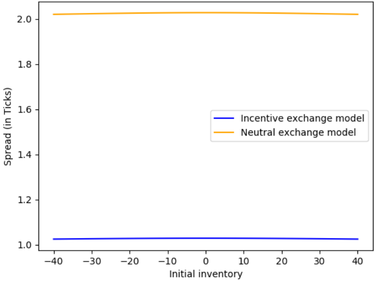
$$T = 600s, \quad \sigma = 0.3\text{Tick}.s^{-1/2}, \quad A = 0.9s^{-1}, \quad k = 0.3s^{-1/2},$$
$$\bar{q} = 50 \text{ unities}, \quad \gamma = 0.01\text{Tick}^{-1}, \quad \eta = 1\text{Tick}^{-1}, \quad c = 0.5\text{Tick}.$$

Impact of the incentive policy on the spread

The optimal spread is given by $\widehat{S}_t = \widehat{\delta}_t^a + \widehat{\delta}_t^b$ with

$$\widehat{\delta}_t^i = \delta_t^i(\widehat{\xi}) = -\widehat{Z}_t^i + \frac{1}{\gamma}\log\left(1+\frac{\sigma\gamma}{k}\right), \quad i=a,b$$

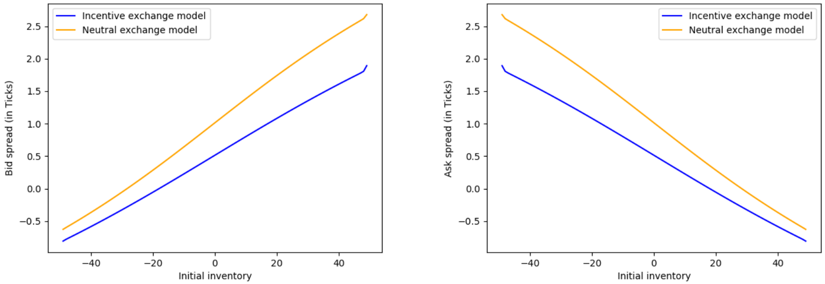
Incentive contract induces spread to be cut by half



Optimal initial spread with/without the exchange incentive policy in terms of initial inventory Q_0.

Impact of the incentive policy on the spread

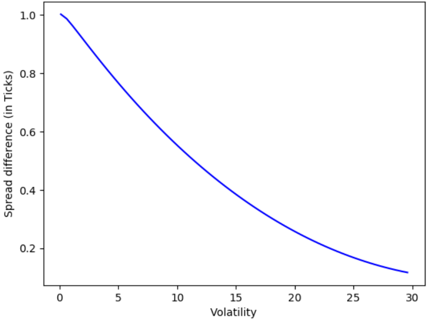
Incentive contract induces bid and ask spreads to be cut by half



Optimal initial bid (left) and ask (right) spread component
with/without the exchange incentive policy
in terms of initial inventory Q_0.

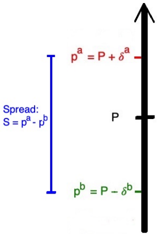
Impact of the volatility on the incentive policy

Incentive contract effect decreases with volatility...



Initial optimal spread difference (with/without incentive) in terms of the volatility σ.

Regulation implication: how to choose the constant fee c



Bid-ask spread $\widehat{S}_t$ is explicit...

Assume Exchange fixes the transaction cost c so that $\widehat{S}_t = 1$

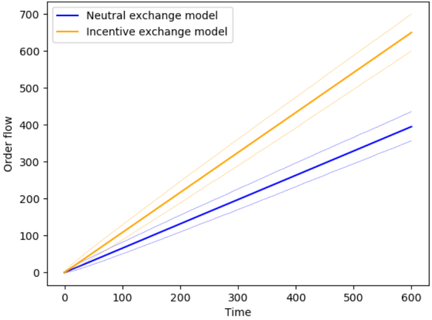
Then, we compute that

$$c \approx \frac{\sigma}{k} - \frac{1}{2}\text{Tick}$$

Impact of the incentive policy on the market liquidity

transactions $= N_T^a + N_T^b$

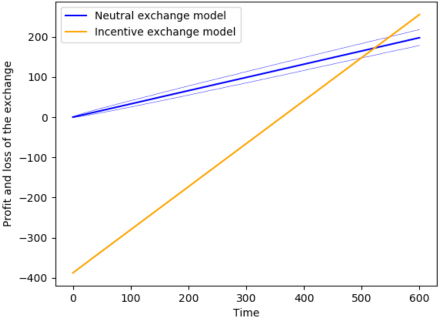
Incentive contract induces more transactions...



Average order flow with 95% confidence interval with/without incentive policy (5000 scenarios).

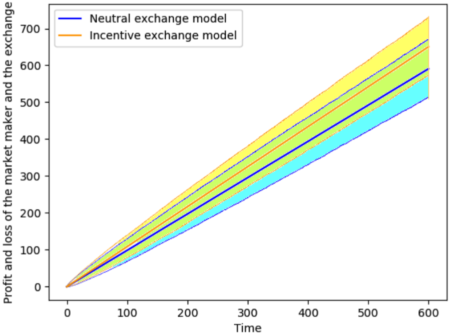
Impact of the incentive policy on the platform P&L

$-\hat{\xi} + c(N_T^a + N_T^b)$



platform P&L with 95% confidence interval
with/without incentive policy (5000 scenarios).

Impact of the incentive policy on the market maker and exchange profit and loss



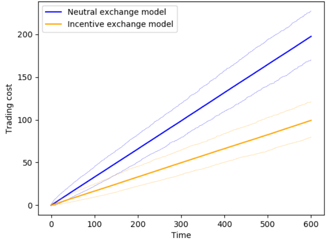
Aggregate P&L of MM and exchange with 95% confidence interval with/without incentive policy (5000 scenarios).

Impact of the incentive policy on trading costs

- One market taker buying a fixed quantity $Q_{final} = 200$ shares

trading cost $\int_0^T \delta_s^a dN_s^a$. with or without incentive

Incentive contract decreases significantly the average trading cost



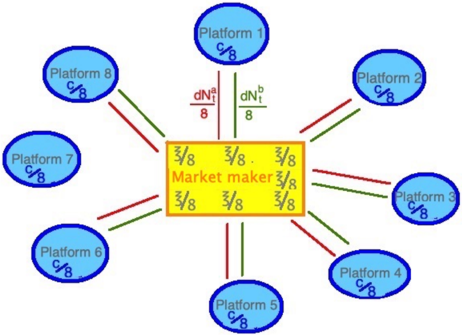
Average trading cost with 95% confidence interval
with/without incentive policy (5000 scenarios).

Summarizing the benefits from optimal contracting

Benefits of the exchange incentive policy

- Smaller spread.
- Increase of the market liquidity.
- Increase of the profit and loss of the MM and the exchange.
- Less transaction costs.

Symmetric platforms in Nash equilibrium



1 Market maker
facing n symmetric platforms

Optimal MM compensation: main result

Theorem

There exists a unique symmetric Nash equilibrium with optimal contract

$$\widehat{\xi}^{(n)} = U_A^{-1}(R) + \int_0^T \widehat{Z}_t^{n,a} dN_t^a + \widehat{Z}_t^{n,b} dN_t^b + \widehat{Z}_t^{n,P} dP_t + \Big(\frac{1}{2}\gamma\sigma^2\big(\hat{Z}_t^{n,P} + Q_t\big)^2 - H\big(\widehat{Z}_t^n, Q_t\big)\Big) dt$$

$\widehat{Z}_t^{n,P} = \frac{-n\gamma}{\eta+n\gamma} Q_t$: inventory risk sharing $\underset{n\to\infty}{\longrightarrow} -Q_t$ (Selling firm effect)

and $\widehat{Z}_t^{n,i} = c + \frac{n}{\eta}\Big[\ln\Big(\frac{u_n(t,Q_t)}{u_n(t,Q_t+\varepsilon_i)}\Big) - \zeta_0\Big]$, $i = b, a$ $\varepsilon_b = 1$, $\varepsilon_a = -1$,