Computation of Convex Hull Prices
in Electricity Markets with Non-Convexities
Using Dantzig-Wolfe Decomposition

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Contents

Electricity Markets

- Pricing in Electricity Markets with Non-Convexities
  - Day-Ahead Market
    - Unit Commitment problem
    - Commercial state-of-the-art: Mixed Integer Linear Programming
  - Non-convexities: due to commitment costs and technical constraints, indivisibilities.
  - There may be no **market-clearing** prices!
  - Standard marginal cost pricing may result in losses even for truthful bidders
    - Prices may not be adequate to cover for start-up/minimum-load costs
- **Several Approaches** proposed to define prices in this context (keeping marginal costs as prices, and/or providing side-payments to market participants, and/or "inflating marginal costs" to obtain revenue adequate prices).

- G. Liberopoulos and P. Andrianesis, "Critical review of pricing schemes in markets with non-convex costs," Oper. Res., vol. 64, no. 1, pp. 17-31, 2016.

Convex Hull Pricing [Preliminaries]

- **Unit Commitment problem**

$$\min_{\mathbf{x},\mathbf{y}} f(\mathbf{x},\mathbf{y}) = \sum_i f_i(\mathbf{x}_i,\mathbf{y}_i),$$

subject to:

System constraints,
e.g., power balance:

$$\sum_i x_{i,t} = D_t, \ \forall t,$$

Generation unit constraints,
e.g., min/max limits,
ramp rates,
min up/down times, etc.:

$$(\mathbf{x}_i,\mathbf{y}_i) \in \mathbf{Z}_i, \ \forall i.$$

$f_i(\cdot)$: Cost function of unit i

$x_{i,t}$: **Continuous** variables, e.g., power output of unit i, at time period t

$y_{i,t}$: **Discrete** variables, e.g., status (on/off) of unit i, at time period t

D_t : Demand at time period t

$\mathbf{Z}_i$: Set of constraints of unit i

Convex Hull Pricing [Preliminaries]

- **Lagrangian Dual** of the Unit Commitment Problem

$$\max_{\boldsymbol{\lambda}} q(\boldsymbol{\lambda}),$$

where: $$q(\boldsymbol{\lambda}) = \inf_{(\mathbf{x}_i, \mathbf{y}_i) \in \mathbf{Z}_i, \forall i} L(\mathbf{x}, \mathbf{y}, \boldsymbol{\lambda}),$$

$$L(\mathbf{x}, \mathbf{y}, \boldsymbol{\lambda}) = \sum_i f_i(\mathbf{x}_i, \mathbf{y}_i) - \sum_t \lambda_t \left(\sum_i x_{i,t} - D_t \right).$$

Unit Commitment:

$$\min_{\mathbf{x}, \mathbf{y}} f(\mathbf{x}, \mathbf{y}) = \sum_i f_i(\mathbf{x}_i, \mathbf{y}_i),$$

subject to:

$$\sum_i x_{i,t} = D_t, \ \forall t,$$

$$(\mathbf{x}_i, \mathbf{y}_i) \in \mathbf{Z}_i, \ \forall i.$$

- **Convex Hull prices** are obtained by the solution of the Lagrangian Dual of the Unit Commitment problem.

- P. R. Gribik, W. W. Hogan, and S. L. Pope, "Market-clearing electricity prices and energy uplift," Working Paper, John F. Kennedy School of Government, Harvard University, 2007.

Convex Hull Pricing [Preliminaries]

- **Equivalent convexified primal formulation**

$$\min_{\mathbf{x},\mathbf{y}} \sum_i f_i^{**}(\mathbf{x}_i, \mathbf{y}_i),$$

subject to:

$$\sum_i x_{i,t} = D_t, \ \forall t, \longrightarrow \lambda_t,$$

$$(\mathbf{x}_i, \mathbf{y}_i) \in conv(\mathbf{Z}_i), \ \forall i.$$

Unit Commitment:

$$\min_{\mathbf{x},\mathbf{y}} f(\mathbf{x},\mathbf{y}) = \sum_i f_i(\mathbf{x}_i, \mathbf{y}_i),$$

subject to:

$$\sum_i x_{i,t} = D_t, \ \forall t,$$

$$(\mathbf{x}_i, \mathbf{y}_i) \in \mathbf{Z}_i, \ \forall i.$$

- **Convex Hull prices** are obtained by the solution of the UC problem, replacing the objective function by its convex envelope, and the feasible set of each unit by its convex hull.

- P. R. Gribik, W. W. Hogan, and S. L. Pope, "Market-clearing electricity prices and energy uplift," Working Paper, John F. Kennedy School of Government, Harvard University, 2007.

Convex Hull Pricing [Preliminaries] (Parenthesis)

- **Current Marginal Cost Pricing?**

$\mathbf{y}^*$: Optimal values of discrete variables

$$\min_{\mathbf{x}} f(\mathbf{x},\mathbf{y}^*) = \sum_i f_i(\mathbf{x}_i,\mathbf{y}_i^*),$$

subject to:

$$\sum_i x_{i,t} = D_t, \ \forall t, \longrightarrow \lambda_t,$$

$$(\mathbf{x}_i,\mathbf{y}_i^*) \in \mathbf{Z}_i, \ \forall i.$$

Unit Commitment:

$$\min_{\mathbf{x},\mathbf{y}} f(\mathbf{x},\mathbf{y}) = \sum_i f_i(\mathbf{x}_i,\mathbf{y}_i),$$

subject to:

$$\sum_i x_{i,t} = D_t, \ \forall t,$$

$$(\mathbf{x}_i,\mathbf{y}_i) \in \mathbf{Z}_i, \ \forall i.$$

- **Marginal Costs (Locational Marginal Prices)** are obtained by the solution of the Linear Programming problem that results after fixing the discrete variables to their optimal values.
- If generation units incur losses under these prices, they are compensated with make-whole payments.

Convex Hull Pricing [Preliminaries] (Parenthesis)

- **How about Integer Relaxation?**

$\mathbf{y}$: Continuous variable (relaxed)

$$\min_{\mathbf{x},\mathbf{y}} f(\mathbf{x},\mathbf{y}) = \sum_i f_i(\mathbf{x}_i,\mathbf{y}_i),$$

subject to:

$$\sum_i x_{i,t} = D_t, \ \forall t, \longrightarrow \lambda_t,$$

$$(\mathbf{x}_i,\mathbf{y}_i) \in \mathbf{Z}_i, \ \forall i,$$

$$0 \le y_{i,t} \le 1, \ \forall i,t.$$ (assume relaxed binary)

Unit Commitment:

$$\min_{\mathbf{x},\mathbf{y}} f(\mathbf{x},\mathbf{y}) = \sum_i f_i(\mathbf{x}_i,\mathbf{y}_i),$$

subject to:

$$\sum_i x_{i,t} = D_t, \ \forall t,$$

$$(\mathbf{x}_i,\mathbf{y}_i) \in \mathbf{Z}_i, \ \forall i.$$

- **Integer Relaxation prices** are obtained by the solution of the Linear Programming problem that results after relaxing the discrete variables.
- Integer Relaxation is at most as tight as the Lagrangian Dual (usually less tight).
- **Extended Locational Marginal Prices** currently relax fast-start units (limited set).

- H. Chao, “Incentive for efficient pricing mechanism in markets with non-convexities,” J. Reg. Econ., vol 56, pp. 33–58, 2019.

Convex Hull Pricing [Preliminaries]

- **Key Property**: Convex Hull prices **support** an arbitrary **market solution**, with **minimum uplift**. This uplift equals the **duality gap** between the market (primal) solution and the optimal solution of the Lagrangian Dual.

Support the market solution?

Make participant (generation unit) indifferent between:
(i) following the market schedule, and
(ii) self-scheduling.

- P. R. Gribik, W. W. Hogan, and S. L. Pope, "Market-clearing electricity prices and energy uplift," Working Paper, John F. Kennedy School of Government, Harvard University, 2007.
- W. W. Hogan and B. J. Ring, "On minimum-uplift pricing for electricity markets," Working Paper, John F. Kennedy School of Government, Harvard University, 2003.

Convex Hull Pricing [Preliminaries]

- **Key Property**: Convex Hull prices **support** an arbitrary **market solution**, with **minimum uplift**. This uplift equals the **duality gap** between the market (primal) solution and the optimal solution of the Lagrangian Dual.

Support the market solution?

Make participant (generation unit) indifferent between:
(i) following the market schedule, and
(ii) self-scheduling.

Define Profit:

$$\varphi_i(\mathbf{x}_i, \mathbf{y}_i, \boldsymbol{\lambda}) = \sum_t \lambda_t x_{i,t} - f_i(\mathbf{x}_i, \mathbf{y}_i)$$

Market Schedule $\longrightarrow$ $\varphi_i(\mathbf{x}_i^{Market}, \mathbf{y}_i^{Market}; \boldsymbol{\lambda})$ (i)

Self-schedule for given $\boldsymbol{\lambda}$ $\longrightarrow$ $(\mathbf{x}_i^{Self}, \mathbf{y}_i^{Self}) = \underset{(\mathbf{x}_i, \mathbf{y}_i) \in \mathbf{Z}_i}{\arg\max} \left[\varphi_i(\mathbf{x}_i, \mathbf{y}_i; \boldsymbol{\lambda}) \right]$ $\longrightarrow$ $\varphi_i(\mathbf{x}_i^{Self}, \mathbf{y}_i^{Self}; \boldsymbol{\lambda})$ (ii)

Uplift? Additional payments required to compensate for Lost Opportunity Costs (*LOC*)

$$LOC_i = \underbrace{\varphi_i(\mathbf{x}_i^{Self}, \mathbf{y}_i^{Self}; \boldsymbol{\lambda})}_{\text{(ii)}} - \underbrace{\varphi_i(\mathbf{x}_i^{Market}, \mathbf{y}_i^{Market}; \boldsymbol{\lambda})}_{\text{(i)}}$$

- P. R. Gribik, W. W. Hogan, and S. L. Pope, "Market-clearing electricity prices and energy uplift," Working Paper, John F. Kennedy School of Government, Harvard University, 2007.
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Convex Hull Pricing [Preliminaries]

- **Key Property**: Convex Hull prices **support** an arbitrary **market solution**, with **minimum uplift**. This uplift equals the **duality gap** between the market (primal) solution and the optimal solution of the Lagrangian Dual.

Support the market solution?

Make participant (generation unit) indifferent between:
(i) following the market schedule, and
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Define Profit:

$$\varphi_i(\mathbf{x}_i,\mathbf{y}_i,\boldsymbol{\lambda}) = \sum_t \lambda_t x_{i,t} - f_i(\mathbf{x}_i,\mathbf{y}_i)$$

Market Schedule $\longrightarrow$ $\varphi_i(\mathbf{x}_i^{Market},\mathbf{y}_i^{Market};\boldsymbol{\lambda})$ (i)

Self-schedule for given $\boldsymbol{\lambda}$ $\longrightarrow$ $(\mathbf{x}_i^{Self},\mathbf{y}_i^{Self}) = \arg\max_{(\mathbf{x}_i,\mathbf{y}_i)\in\mathbf{Z}_i}\left[\varphi_i(\mathbf{x}_i,\mathbf{y}_i;\boldsymbol{\lambda})\right]$ $\longrightarrow$ $\varphi_i(\mathbf{x}_i^{Self},\mathbf{y}_i^{Self};\boldsymbol{\lambda})$ (ii)

Uplift? Additional payments required to compensate for Lost Opportunity Costs (*LOC*)

$$LOC_i = \underbrace{\varphi_i(\mathbf{x}_i^{Self},\mathbf{y}_i^{Self};\boldsymbol{\lambda})}_{\text{(ii)}} - \underbrace{\varphi_i(\mathbf{x}_i^{Market},\mathbf{y}_i^{Market};\boldsymbol{\lambda})}_{\text{(i)}}$$

Duality gap = minimum uplift?

$$f - q^* = \inf_{\boldsymbol{\lambda}}\left(\sum_i LOC_i\right)$$

- P. R. Gribik, W. W. Hogan, and S. L. Pope, "Market-clearing electricity prices and energy uplift," Working Paper, John F. Kennedy School of Government, Harvard University, 2007.
- W. W. Hogan and B. J. Ring, "On minimum-uplift pricing for electricity markets," Working Paper, John F. Kennedy School of Government, Harvard University, 2003.

Convex Hull Pricing [Computational Approaches]

- Main Computational Approaches employed so far:

  - Subgradient methods

(Lagrangian Dual) $$\max_{\boldsymbol{\lambda}} q(\boldsymbol{\lambda}),$$

where: $$q(\boldsymbol{\lambda}) = \inf_{(\mathbf{x}_i, \mathbf{y}_i) \in \mathbf{Z}_i, \forall i} L(\mathbf{x}, \mathbf{y}, \boldsymbol{\lambda}),$$

$$L(\mathbf{x}, \mathbf{y}, \boldsymbol{\lambda}) = \sum_i f_i(\mathbf{x}_i, \mathbf{y}_i) - \sum_t \lambda_t \left(\sum_i x_{i,t} - D_t \right),$$

  - Extended formulations (convex hull)

(Convexified Primal) $$\min_{\mathbf{x}, \mathbf{y}} \sum_i f_i^{**}(\mathbf{x}_i, \mathbf{y}_i),$$

subject to: $$\sum_i x_{i,t} = D_t, \ \forall t,$$

$$(\mathbf{x}_i, \mathbf{y}_i) \in conv(\mathbf{Z}_i), \ \forall i.$$

Convex Hull Pricing [Computational Approaches]

- **Subgradient methods** (first "early" approaches)
  - Solve the Lagrangian Dual
  - MISO initially tried this approach, but…
    - Convergence difficulties.
    - Required customized algorithms.
    - Would it always work?
    - Effort abandoned.

$$\max_{\boldsymbol{\lambda}} q(\boldsymbol{\lambda}),$$

where: $$q(\boldsymbol{\lambda}) = \inf_{(\mathbf{x}_i, \mathbf{y}_i) \in \mathbf{Z}_i, \forall i} L(\mathbf{x}, \mathbf{y}, \boldsymbol{\lambda}),$$

$$L(\mathbf{x}, \mathbf{y}, \boldsymbol{\lambda}) = \sum_i f_i(\mathbf{x}_i, \mathbf{y}_i) - \sum_t \lambda_t \left(\sum_i x_{i,t} - D_t \right).$$

- C. Wang, P. B. Luh, P. Gribik, L. Zhang, and T. Peng, "A subgradient based cutting plane method to calculate convex hull market prices," in Proc. 2009 IEEE PES GM,, Calgary, AB, Canada, 26–30 July 2009.
- C. Wang, T. Peng, P. B. Luh, P. Gribik, and L. Zhang, "The subgradient Simplex cutting plane method for extended locational marginal prices," in IEEE Trans. Power Syst., vol. 28, no. 3, pp. 2758–2767, 2013.
- G. Wang, U. V. Shanbhag, T. Zheng, E. Litvinov, and S. Meyn, "An extreme-point subdifferential method for convex hull pricing in energy and reserve markets — Part I: Algorithm structure," IEEE Trans. Power Syst., vol 28, no. 3, pp. 2111–2120, 2013. ——, Part II: Convergence analysis and numerical performance," IEEE Trans. Power Syst., vol 28, no. 3, pp. 2121–2127, 2013.

Convex Hull Pricing [Computational Approaches]

- **Extended Formulations** (latest stream of works)
  - Characterize the convex envelope of the cost functions, and the convex hull of the constraints sets.
    - Usually yield approximate, not exact, convex hull prices.
    - Problematic constraints (e.g., ramps)
    - Result in Linear Programs at least impractical to solve.
    - Depend on specific formulations of constraints, on a case-by-case basis.
    - Difficult to implement, complicate modifications (e.g., additions of new units).
    - Lack intuition of the price formation.

$$\min_{\mathbf{x},\mathbf{y}} \sum_i f_i^{**}(\mathbf{x}_i, \mathbf{y}_i),$$

subject to:

$$\sum_i x_{i,t} = D_t, \ \forall t,$$

$$(\mathbf{x}_i, \mathbf{y}_i) \in conv(\mathbf{Z}_i), \ \forall i.$$

- B. Hua and R. Baldick, "A convex primal formulation for convex hull pricing," IEEE Trans. Power Syst., vol 32, no. 5, pp. 3814–3823, 2017.
- Y. Yu, Y. Guan, and Y. Chen, "An extended integral unit commitment formulation and an iterative algorithm for convex hull pricing," IEEE Trans. Power Syst., vol 35, no. 6, pp. 4335–4346, 2020.
- Y. Chen, R. O'Neill, and P. Whitman, "A Unified approach to solve convex hull pricing and average incremental cost pricing with large system study," Working Paper, 2020.
- D. A. Schiro, T. Zheng, F. Zhao, and E. Litvinov, "Convex hull pricing in electricity markets: Formulation, analysis, and implementation challenges," IEEE Trans. Power Syst., vol. 31, no. 5, pp. 4068–4075, 2016.
- B. Knueven, J. Ostrowski, A. Castillo, and J.-P. Watson, "A computationally efficient algorithm for computing convex hull prices," SAND2019-10896 J, Sandia National Labs, Albuquerque, NM, Sep. 2019.

Convex Hull Pricing [Proposal]

- **Key Idea**:
  - ***Generalized Linear Programming***, *a.k.a.* ***Dantzig-Wolfe decomposition***, *a.k.a.* ***Column Generation*** solves the Lagrangian Dual, equivalently the convexified primal!
    - T. L. Magnanti, J. F. Shapiro, and M. H. Wagner, "Generalized linear programming solves the dual," Manag. Sci., vol. 22, no. 11, pp. 1195–1203, 1976.
    - A. M. Geoffrion, "Lagrangian relaxation for integer programming," Mathem. Program. Study, pp. 82–114, 1974.
    - G. B. Dantzig and P. Wolfe, "Decomposition Principle for Linear Programs," Oper. Res., vol. 8, no. 1, pp. 101-111, 1960.
  - Main motivation: Crew-scheduling problems!
    - C. Barnhart, E. L. Johnson, G. L. Nemhauser, M. W. P. Savelsebergh, and P. H. Vance, "Branch-and-Price: Column generation for solving huge integer programs," Oper. Res., vol. 46, no. 3, pp. 316–329, 1998.
    - F. Vanderbeck, "On Dantzig-Wolfe decomposition in integer programming and ways to perform branching in a branch-and-price algorithm," Oper. Res., vol. 48, no. 1, pp. 111–128, 2000.
    - M. E. Lubbecke and J. Desrosiers, "Selected Topics in Column Generation," Oper. Res., vol. 53, no. 6, pp. 1007–1023, 2005.

Convex Hull Pricing [Proposal]

- **Method**

Define **feasible schedule** n of unit i : $z_i^n \rightarrow (\hat{\mathbf{x}}_i^n, \hat{\mathbf{y}}_i^n) \in \mathbf{Z}_i,$

with cost: $\hat{c}_i^n = f_i(\hat{\mathbf{x}}_i^n, \hat{\mathbf{y}}_i^n).$

Unit Commitment problem:

$$\min_{\mathbf{z}} g(\mathbf{z}) = \sum_{i,n} \hat{c}_i^n z_i^n,$$

subject to:

$$\sum_{i,n} \hat{x}_{i,t}^n \, z_i^n = D_t, \ \forall t,$$

$$\sum_n z_i^n = 1, \ \forall i,$$

$$z_i^n \in \{0,1\}, \ \forall i, n.$$

Convex Hull Pricing [Proposal]

- **Method**

Define **feasible schedule** n of unit i : $z_i^n \rightarrow (\hat{\mathbf{x}}_i^n, \hat{\mathbf{y}}_i^n) \in \mathbf{Z}_i$,

with cost: $\hat{c}_i^n = f_i(\hat{\mathbf{x}}_i^n, \hat{\mathbf{y}}_i^n)$.

Unit Commitment problem:

$$\min_{\mathbf{z}} g(\mathbf{z}) = \sum_{i,n} \hat{c}_i^n z_i^n,$$

subject to:

$$\sum_{i,n} \hat{x}_{i,t}^n \; z_i^n = D_t, \ \forall t,$$

$$\sum_{n} z_i^n = 1, \ \forall i,$$

LP relaxation

$$\cancel{z_i^n \in \{0,1\}, \ \forall i, n.} \quad \longrightarrow \quad z_i^n \geq 0, \ \forall i, n.$$

Convex Hull Pricing [Proposal]

- **Method**

Define **feasible schedule** n of unit i : $z_i^n \rightarrow (\hat{\mathbf{x}}_i^n, \hat{\mathbf{y}}_i^n) \in \mathbf{Z}_i$,

with cost: $\hat{c}_i^n = f_i(\hat{\mathbf{x}}_i^n, \hat{\mathbf{y}}_i^n)$.

Unit Commitment problem: **(LP relaxation)**

$$\min_{\mathbf{z}} g(\mathbf{z}) = \sum_{i,n} \hat{c}_i^n z_i^n,$$

subject to:

$$\sum_{i,n} \hat{x}_{i,t}^n \, z_i^n = D_t, \ \forall t, \longrightarrow \lambda_t$$

$$\boxed{\sum_n z_i^n = 1, \ \forall i, \quad \textbf{Convexity constraint}}$$

$$z_i^n \geq 0, \ \forall i, n.$$

Convex Hull Pricing [Proposal]

- **Method** (@ iteration k)

Restricted Master Problem

$$\min_{\mathbf{z}} g^k(\mathbf{z}) = \sum_{i,n\in N_i^k} \hat{c}_i^n z_i^n,$$

subject to:

$$\sum_{i,n\in N_i^k} \hat{x}_{i,t}^n \, z_i^n = D_t, \ \forall t, \longrightarrow \lambda_t^k$$

$$\sum_{n\in N_i^k} z_i^n = 1, \ \forall i, \longrightarrow \pi_i^k$$

$$z_i^n \geq 0, \ \forall i, n \in N_i^k.$$

If negative reduced cost (rc), then, add schedule to RMP.

Sub-problem of unit i

$$\min_{\mathbf{x}_i,\mathbf{y}_i} rc_i(\mathbf{x}_i,\mathbf{y}_i) = f_i(\mathbf{x}_i,\mathbf{y}_i) - \sum_t \lambda_t^k x_{i,t} - \pi_i^k$$

subject to: $(\mathbf{x}_i,\mathbf{y}_i) \in \mathbf{Z}_i$

Convex Hull Pricing [Proposal]

- **Method** (@ iteration k)

Restricted Master Problem

$$\min_{\mathbf{z}} g^k(\mathbf{z}) = \sum_{i,n \in N_i^k} \hat{c}_i^n z_i^n,$$

subject to:

$$\sum_{i,n \in N_i^k} \hat{x}_{i,t}^n \, z_i^n = D_t, \ \forall t, \longrightarrow \lambda_t^k$$

$$\sum_{n \in N_i^k} z_i^n = 1, \ \forall i, \longrightarrow \pi_i^k$$

$$z_i^n \geq 0, \ \forall i, n \in N_i^k.$$

- Constraints of Units: $(\mathbf{x}_i, \mathbf{y}_i) \in \mathbf{Z}_i$
  - Bounded feasible sets (always true).
  - MILP representation yields finite convergence.
- Exact convex hull prices.
- Valid Lagrangian Dual bounds.
- Highly parallelizable.
- Highly generalizable!

If negative reduced cost (rc), then, add schedule to RMP.

Sub-problem of unit i

$$\min_{\mathbf{x}_i, \mathbf{y}_i} rc_i(\mathbf{x}_i, \mathbf{y}_i) = f_i(\mathbf{x}_i, \mathbf{y}_i) - \sum_t \lambda_t^k x_{i,t} - \pi_i^k$$

subject to: $(\mathbf{x}_i, \mathbf{y}_i) \in \mathbf{Z}_i$

Convex Hull Pricing [Proposal]

- **Economic Interpretation**

Restricted Master Problem

$$\min_{\mathbf{z}} g^k(\mathbf{z}) = \sum_{i,n \in N_i^k} \hat{c}_i^n z_i^n,$$

subject to:

$$\sum_{i,n \in N_i^k} \hat{x}_{i,t}^n \, z_i^n = D_t, \ \forall t, \longrightarrow \lambda_t^k$$

$$\sum_{n \in N_i^k} z_i^n = 1, \ \forall i, \longrightarrow \pi_i^k$$

$$z_i^n \geq 0, \ \forall i, n \in N_i^k.$$

If negative reduced cost (rc), then, add schedule to RMP.

Profit maximization if self-scheduling under λ_t^k

$$\max_{\mathbf{x}_i, \mathbf{y}_i} \left[\sum_t \lambda_t^k x_{i,t} - f_i(\mathbf{x}_i, \mathbf{y}_i) \right]$$

subject to: $(\mathbf{x}_i, \mathbf{y}_i) \in \mathbf{Z}_i$

Equivalent sub-problem

Sub-problem of unit i

$$\min_{\mathbf{x}_i, \mathbf{y}_i} rc_i(\mathbf{x}_i, \mathbf{y}_i) = f_i(\mathbf{x}_i, \mathbf{y}_i) - \sum_t \lambda_t^k x_{i,t} - \pi_i^k$$

subject to: $(\mathbf{x}_i, \mathbf{y}_i) \in \mathbf{Z}_i$

Convex Hull Pricing [Proposal]

- **Economic Interpretation**

Restricted Master Problem

$$\min_{\mathbf{z}} g^k(\mathbf{z}) = \sum_{i,n\in N_i^k} \hat{c}_i^n z_i^n,$$

subject to:

$$\sum_{i,n\in N_i^k} \hat{x}_{i,t}^n z_i^n = D_t,\ \forall t, \longrightarrow \lambda_t^k \longrightarrow$$ Tentative price at time *t*, iteration *k*

$$\sum_{n\in N_i^k} z_i^n = 1,\ \forall i, \longrightarrow \pi_i^k \longrightarrow [-\pi_i^k]$$ Tentative profit of unit *i*, at iteration *k*

$$z_i^n \geq 0,\ \forall i, n \in N_i^k.$$

Profit maximization if self-scheduling under λ_t^k

$$\max_{\mathbf{x}_i,\mathbf{y}_i} \left[\sum_t \lambda_t^k x_{i,t} - f_i(\mathbf{x}_i,\mathbf{y}_i) \right]$$

subject to: $(\mathbf{x}_i,\mathbf{y}_i) \in \mathbf{Z}_i$

Equivalent sub-problem

Sub-problem of unit *i*

$$\min_{\mathbf{x}_i,\mathbf{y}_i} rc_i(\mathbf{x}_i,\mathbf{y}_i) = f_i(\mathbf{x}_i,\mathbf{y}_i) - \sum_t \lambda_t^k x_{i,t} - \pi_i^k$$

subject to: $(\mathbf{x}_i,\mathbf{y}_i) \in \mathbf{Z}_i$

If negative reduced cost (*rc*), then, add schedule to RMP.

- W. J. Baumol and T. Fabian, "Decomposition, pricing for decentralization and external economies," Manag. Sci., vol. 11, no. 1, pp. 1–32, 1964

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Convex Hull Pricing [Proposal]

- **Economic Interpretation**

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$$\min_{\mathbf{z}} g^k(\mathbf{z}) = \sum_{i,n\in N_i^k} \hat{c}_i^n z_i^n,$$

subject to:

$$\sum_{i,n\in N_i^k} \hat{x}_{i,t}^n \; z_i^n = D_t, \; \forall t, \longrightarrow \lambda_t^k \longrightarrow$$ Tentative price at time *t*, iteration *k*

$$\sum_{n\in N_i^k} z_i^n = 1, \; \forall i, \longrightarrow \pi_i^k \longrightarrow [-\pi_i^k]$$ **Tentative profit of unit *i*, at iteration *k***

$$z_i^n \geq 0, \; \forall i, n \in N_i^k.$$

Profit maximization if self-scheduling under λ_t^k

$$\max_{\mathbf{x}_i,\mathbf{y}_i} \left[\sum_t \lambda_t^k x_{i,t} - f_i(\mathbf{x}_i,\mathbf{y}_i) \right]$$

subject to: $(\mathbf{x}_i,\mathbf{y}_i) \in \mathbf{Z}_i$

Equivalent sub-problem

$$rc_i(\mathbf{x}_i,\mathbf{y}_i) = \left[-\pi_i^k\right] - \left[\sum_t \lambda_t^k x_{i,t} - f_i(\mathbf{x}_i,\mathbf{y}_i) \right]$$

Self-scheduling profit > Tentative profit

If **negative reduced cost (*rc*),** then, add schedule to RMP.

Sub-problem of unit *i*

$$\min_{\mathbf{x}_i,\mathbf{y}_i} rc_i(\mathbf{x}_i,\mathbf{y}_i) = f_i(\mathbf{x}_i,\mathbf{y}_i) - \sum_t \lambda_t^k x_{i,t} - \pi_i^k$$

subject to: $(\mathbf{x}_i,\mathbf{y}_i) \in \mathbf{Z}_i$

- W. J. Baumol and T. Fabian, "Decomposition, pricing for decentralization and external economies," Manag. Sci., vol. 11, no. 1, pp. 1–32, 1964

Numerical Illustrations [Stylized Examples]

- A simple example [Schiro et al. 2016, Ex. 1]
  - Two Generators: (A) and (B) serve 35 MW load, single period.

Consider trivial schedules: $z_A^1 \rightarrow \hat{x}_A^1 = 10; \hat{c}_A^1 = 500;$

$$z_B^1 \rightarrow \hat{x}_B^1 = 0; \hat{y}_B^1 = 0.$$

Unit Commitment Problem (MILP):

$$\min_{x_A, x_B, y_B} \; f = 50x_A + 10x_B,$$

subject to:

$$x_A + x_B = 35,$$

$$10 \le x_A \le 50,$$

$$x_B = 50y_B, \quad y_B \in \{0,1\}.$$

RMP(1): $\min_{z_A^1, z_B^1, s} \; g^1 = 500z_A^1 + 0z_B^1 + 1000s$

subject to: $10z_A^1 + 0z_B^1 + s = 35, \rightarrow \lambda^1 (= 1000)$

$$z_A^1 = 1, \rightarrow \pi_A^1 (= -9500)$$

$$z_B^1 = 1, \rightarrow \pi_B^1 (= 0)$$

- D. A. Schiro, T. Zheng, F. Zhao, and E. Litvinov, “Convex hull pricing in electricity markets: Formulation, analysis, and implementation challenges,” IEEE Trans. Power Syst., vol. 31, no. 5, pp. 4068–4075, 2016.

Numerical Illustrations [Stylized Examples]

- A simple example [Schiro et al. 2016, Ex. 1]
  - Two Generators: (A) and (B) serve 35 MW load, single period.

Sub-problems:

[tentative] –[self]

$$\max_{10\le x_A\le 50}\left(1000x_A - 50x_A\right) \longrightarrow x_A = 50 \longrightarrow rc_A = 9500 - 47500 = -38000 < 0$$

$$\longrightarrow z_A^2 \rightarrow \hat{x}_A^2 = 50; \hat{c}_A^2 = 2500.$$

$$\max_{\substack{x_B = 50 y_B,\\ y_B\in\{0,1\}.}}\left(1000x_B - 10x_B\right) \longrightarrow x_B = 50 \longrightarrow rc_B = 0 - 49500 = -49500 < 0$$

$$\longrightarrow z_B^2 \rightarrow \hat{x}_B^2 = 50; \hat{c}_B^2 = 500.$$

RMP(1): $\min_{z_A^1, z_B^1, s} g^1 = 500z_A^1 + 0z_B^1 + 1000s$

subject to: $10z_A^1 + 0z_B^1 + s = 35, \rightarrow \lambda^1(=1000)$

$$z_A^1 = 1, \rightarrow \pi_A^1(=-9500)$$

$$z_B^1 = 1, \rightarrow \pi_B^1(=0)$$

- D. A. Schiro, T. Zheng, F. Zhao, and E. Litvinov, “Convex hull pricing in electricity markets: Formulation, analysis, and implementation challenges,” IEEE Trans. Power Syst., vol. 31, no. 5, pp. 4068–4075, 2016.

Numerical Illustrations [Stylized Examples]

- A simple example [Schiro et al. 2016, Ex. 1]
  - Two Generators: (A) and (B) serve 35 MW load, single period.

Sub-problems:

[tentative] –[self]

$$\max_{10 \le x_A \le 50} \left(1000x_A - 50x_A\right) \longrightarrow x_A = 50 \longrightarrow rc_A = 9500 - 47500 = -38000 < 0$$

$$\longrightarrow \boxed{z_A^2 \rightarrow \hat{x}_A^2 = 50; \hat{c}_A^2 = 2500.}$$

$$\max_{\substack{x_B = 50 y_B, \\ y_B \in \{0,1\}.}} \left(1000x_B - 10x_B\right) \longrightarrow x_B = 50 \longrightarrow rc_B = 0 - 49500 = -49500 < 0$$

$$\longrightarrow \boxed{z_B^2 \rightarrow \hat{x}_B^2 = 50; \hat{c}_B^2 = 500.}$$

RMP(2): $$\min_{z_A^1, z_B^1, z_A^2, z_B^2, s} g^2 = 500z_A^1 + 0z_B^1 + 2500z_A^2 + 500z_B^2 + 1000s$$

subject to: $$10z_A^1 + 0z_B^1 + 50z_A^2 + 50z_B^2 + s = 35, \rightarrow \lambda^2 (=10)$$

$$z_A^1 + z_A^2 = 1, \rightarrow \pi_A^2 (= 400)$$

$$z_B^1 + z_B^2 = 1, \rightarrow \pi_B^2 (= 0)$$

- D. A. Schiro, T. Zheng, F. Zhao, and E. Litvinov, "Convex hull pricing in electricity markets: Formulation, analysis, and implementation challenges," IEEE Trans. Power Syst., vol. 31, no. 5, pp. 4068–4075, 2016.

Numerical Illustrations [Stylized Examples]

- A simple example [Schiro et al. 2016, Ex. 1]
  - Two Generators: (A) and (B) serve 35 MW load, single period.

Sub-problems:

[tentative] –[self]

$$\max_{10 \le x_A \le 50} \left(10x_A - 50x_A\right) \longrightarrow x_A = 10 \longrightarrow rc_A = -400 - (-400) = 0$$

$$\max_{\substack{x_B = 50 y_B, \\ y_B \in \{0,1\}.}} \left(10x_B - 10x_B\right) \longrightarrow rc_B = 0 - 0 = 0$$

RMP(2): $$\min_{z_A^1, z_B^1, z_A^2, z_B^2, s} g^2 = 500z_A^1 + 0z_B^1 + 2500z_A^2 + 500z_B^2 + 1000s$$

subject to: $$10z_A^1 + 0z_B^1 + 50z_A^2 + 50z_B^2 + s = 35, \rightarrow \lambda^2 (=10)$$

$$z_A^1 + z_A^2 = 1, \rightarrow \pi_A^2 (= 400)$$

$$z_B^1 + z_B^2 = 1, \rightarrow \pi_B^2 (= 0)$$

- D. A. Schiro, T. Zheng, F. Zhao, and E. Litvinov, "Convex hull pricing in electricity markets: Formulation, analysis, and implementation challenges," IEEE Trans. Power Syst., vol. 31, no. 5, pp. 4068–4075, 2016.

Numerical Illustrations [Stylized Examples]

- A simple example [Schiro et al. 2016, Ex. 1]
  - Two Generators: (A) and (B) serve 35 MW load, single period.

$$\lambda^2 = \lambda^{CH} = 10$$

$$g^2 = g^* = 750 \quad \left(z_A^1 = 1,\ z_B^1 = 0.5,\ z_B^2 = 0.5.\right)$$

$$\text{Uplift} = f^{MILP} - g^* = 1750 - 750 = 1000$$

RMP(2): $$\min_{z_A^1, z_B^1, z_A^2, z_B^2, s} g^2 = 500 z_A^1 + 0 z_B^1 + 2500 z_A^2 + 500 z_B^2 + 1000 s$$

subject to: $$10 z_A^1 + 0 z_B^1 + 50 z_A^2 + 50 z_B^2 + s = 35, \to \lambda^2 (=10)$$

$$z_A^1 + z_A^2 = 1, \to \pi_A^2 (=400)$$

$$z_B^1 + z_B^2 = 1, \to \pi_B^2 (=0)$$

- D. A. Schiro, T. Zheng, F. Zhao, and E. Litvinov, “Convex hull pricing in electricity markets: Formulation, analysis, and implementation challenges,” IEEE Trans. Power Syst., vol. 31, no. 5, pp. 4068–4075, 2016.

Numerical Illustrations [Stylized Examples]

- Another "simple" example [Chen et al. 2020, Ex. 2]
  - Two Generators, 3-hours, ramp constraints.

Unit Commitment

$$\min \sum_{t=1}^{3} 10 \cdot p_{1,t} + \sum_{t=1}^{3} (30 \cdot u_{2,t} + 50 \cdot p_{2,t} + 1000 \cdot v_{2,t})$$

Limit constraints:

$0 \le p_{1,t} \le 100 \quad for\ 1 \le t \le 3$ (a1)

$20u_{2,t} \le p_{2,t} \le 35\, u_{2,t} \quad for\ 1 \le t \le 3$ (a2)

Ramping constraints:

$p_{2,t} - p_{2,t-1} \le 5u_{2,t-1} + 22.5v_{2,t} \quad for\ 1 \le t \le 3$ (a3)

$p_{2,t-1} - p_{2,t} \le 5u_{2,t} + 35e_{2,t} \quad for\ 2 \le t \le 3$ (a4)

Binary constraints:

$u_{2,t} - u_{2,t-1} = v_{2,t} - e_{2,t} \quad for\ 1 \le t \le 3$

$with\ u_{2,0} = 0\ for\ initially\ off$ (a5)

$v_{2,t} \le u_{2,t} \quad for\ 1 \le t \le 3$ (a6)

$v_{2,t} \le 1 - u_{2,t-1} \quad for\ 1 \le t \le 3$ (a7)

Power balance constraint:

$p_{1,t} + p_{2,t} = LD_t \quad for\ 1 \le t \le 3$ (a8)

$v_{2,t}, u_{2,t}, e_{2,t}\ are\ binary \quad for\ 1 \le t \le 3$ (a9)

Extended Formulation

The extended formulation is:

$$\min \textstyle\sum_{t=1}^{3} 10 \cdot p_{1,t} + 1000 \cdot \left(\sum_{tk\in\{02,03,13\}} y_{2,tk} + \sum_{t\in\{1,2,3\}} w_{2,t}\right) + 30 \cdot \left(\sum_{t\in\{0,1,2\}} \sum_{s\in[t+1,3]} w_{2,t} + \sum_{tk\in\{02,03,13\}} \sum_{s\in[t+1,k-1]} y_{2,tk}\right) + 50 \cdot \left(\sum_{t\in\{0,1,2\}} \sum_{s\in[t+1,3]} qw_{2,t}^{s} + \sum_{tk\in\{02,03,13\}} \sum_{s\in[t+1,k-1]} qy_{2,tk}^{s}\right)$$

Limit constraints

$0 \le p_{1,t} \le 100 \quad for\ 1 \le t \le 3$ (b1)

$20w_{2,t} \le qw_{2,t}^{s} \le 35\, w_{2,t} \quad t \in [0,2], s \in [t+1,3]$ (b2)

$20y_{2,tk} \le qy_{2,tk}^{s} \le 35\, y_{2,tk} \quad tk \in \{02,03,13\}, s \in [t+1,k-1]$ (b3)

Ramping constraints

$qy_{2,tk}^{t+1} \le 22.5\, y_{2,tk},\ qw_{2,t}^{t+1} \le 22.5\, w_{2,t}$ (b4)

$qy_{2,03}^{2} - qy_{2,03}^{1} \le 5\, y_{2,03},\ qy_{2,03}^{1} - qy_{2,03}^{2} \le 5\, y_{2,03},$

$qw_{2,t}^{s+1} - qw_{2,t}^{s} \le 5\, w_{2,t},\ t \in [0,2], s \in [t+1,3]$

$qw_{2,t}^{s} - qw_{2,t}^{s+1} \le 5\, w_{2,t},\ t \in [0,2], s \in [t+1,3]$

Binary constraints

$-o_{2,0} + y_{2,02} + y_{2,03} + w_{2,0} = 0, \quad -o_{2,1} + y_{2,13} + w_{2,1} = 0,$

$-o_{2,2} + w_{2,2} = 0, \quad y_{2,02} - z_{2,22} - z_{2,23} = 0,$

$y_{2,03} + y_{2,13} - z_{2,33} = 0, \quad o_{2,0} + o_{2,1} + o_{2,2} \le 1$

The final dispatch MW of Gen2:

$p_{2,1} = qy_{2,02}^{1} + qy_{2,03}^{1} + qw_{2,0}^{1}$

$p_{2,2} = qy_{2,03}^{2} + qy_{2,13}^{2} + qw_{2,0}^{2} + qw_{2,1}^{2}$

$p_{2,3} = qw_{2,0}^{3} + qw_{2,1}^{3} + qw_{2,2}^{3}$

Power balance constraint:

$p_{1,t} + p_{2,t} = LD_t \quad for\ 1 \le t \le 3$ (a8)

$o_{2,t}$: representing Gen2 staying off through t and starting up at the beginning of t+1, for t=0,1,2

$w_{2,t}$: representing Gen2 starting at the beginning of t+1 and staying on until the end, for t=0,1,2.

When $w_{2,t} = 1$, Gen 2 is on for s=t+1, …, 3. Define the dispatch variable as $qw_{2,t}^{s}, s \in [t+1,3]$

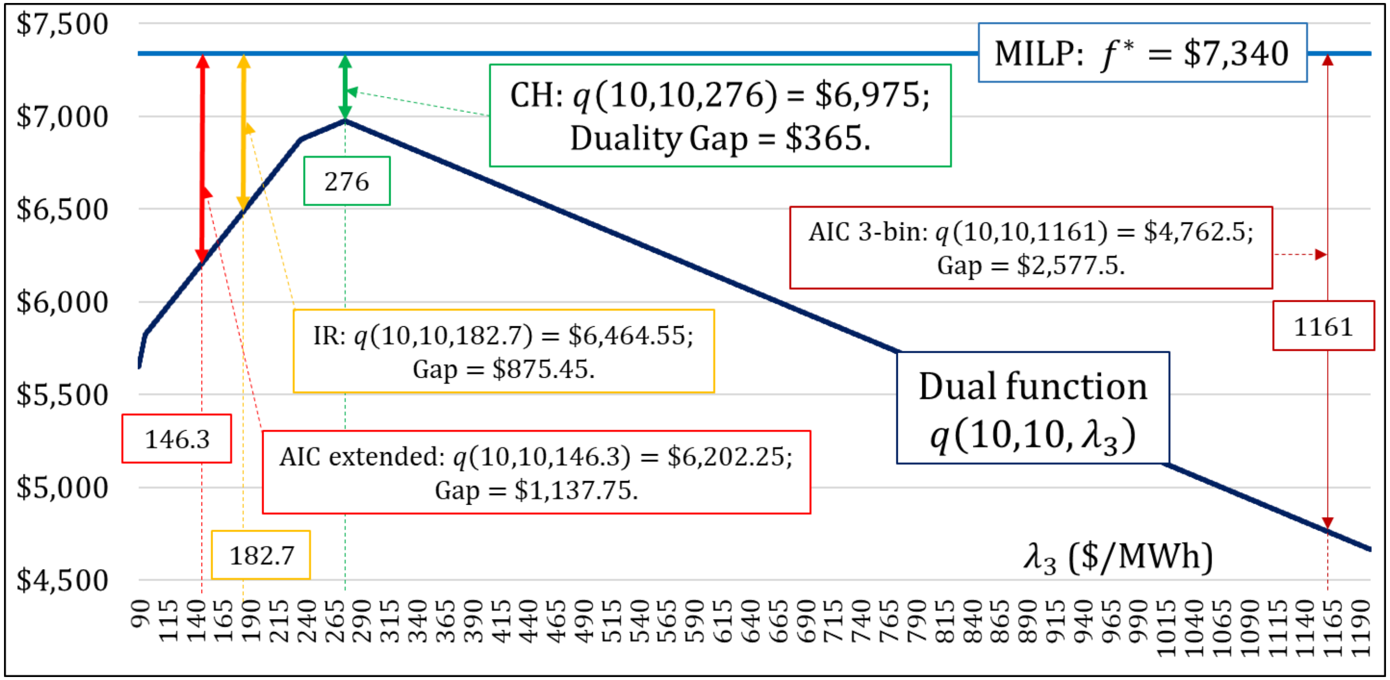
$y_{2,tk}$: representing Gen2 starting at the beginning of t+1 and shutting down at the beginning of k, for $tk \in \{02,03,13\}$. Define the dispatch variable $qy_{2,tk}^{s}, s \in [t+1,k-1]$

$z_{2,tk}$: representing Gen2 shut down at the beginning of t and staying off until the beginning of k+1, for $tk \in \{22,23,33\}$.

- Y. Chen, R. O'Neill, and P. Whitman, "A Unified approach to solve convex hull pricing and average incremental cost pricing with large system study," Working Paper, 2020.

Numerical Illustrations [Stylized Examples]

- Another "simple" example [Chen et al. 2020, Ex. 2]
  - Two Generators, 3-hours, ramp constraints.
  - Column Generation terminates in 4 iterations.

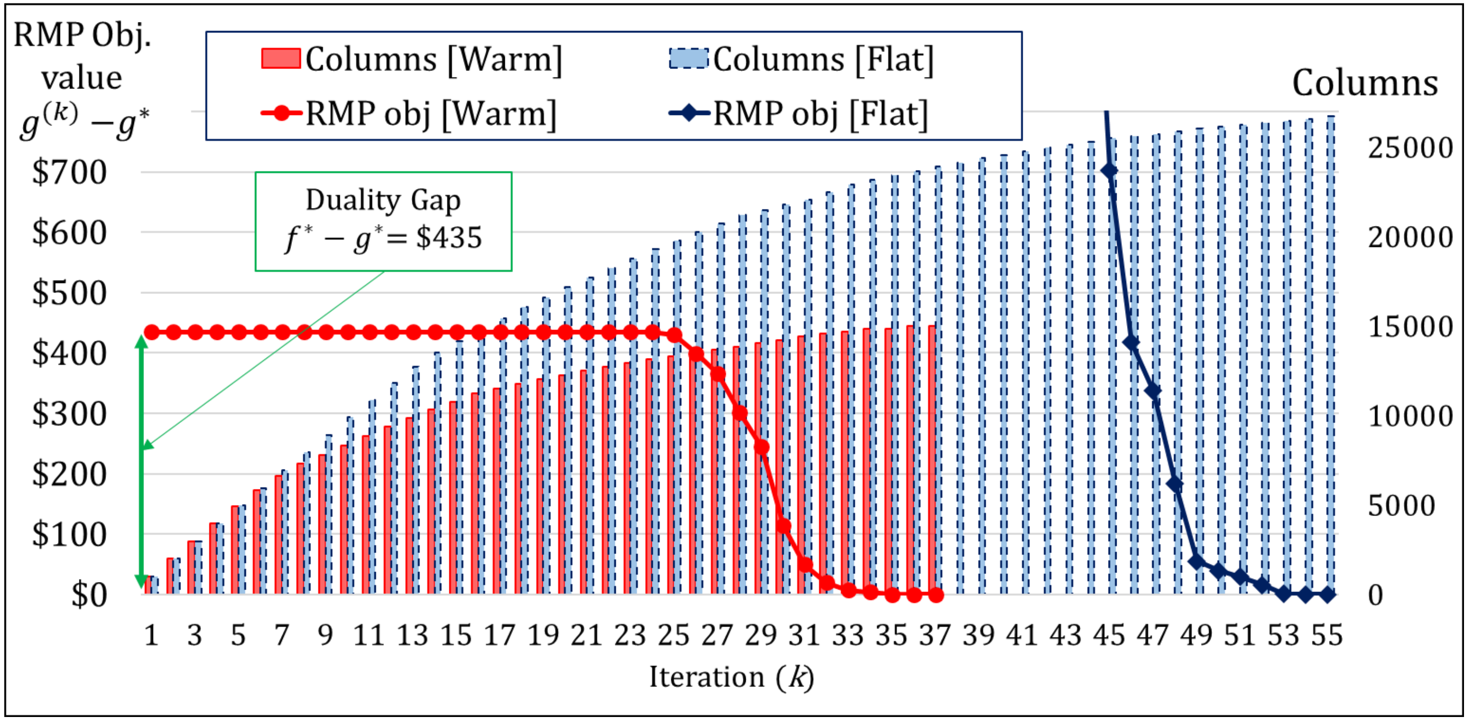


Evaluation of q(λ) for λ = (10,10,λ3), 90 <= λ3 <= 1200.

- Y. Chen, R. O'Neill, and P. Whitman, "A Unified approach to solve convex hull pricing and average incremental cost pricing with large system study," Working Paper, 2020.

Numerical Illustrations [Larger Datasets]

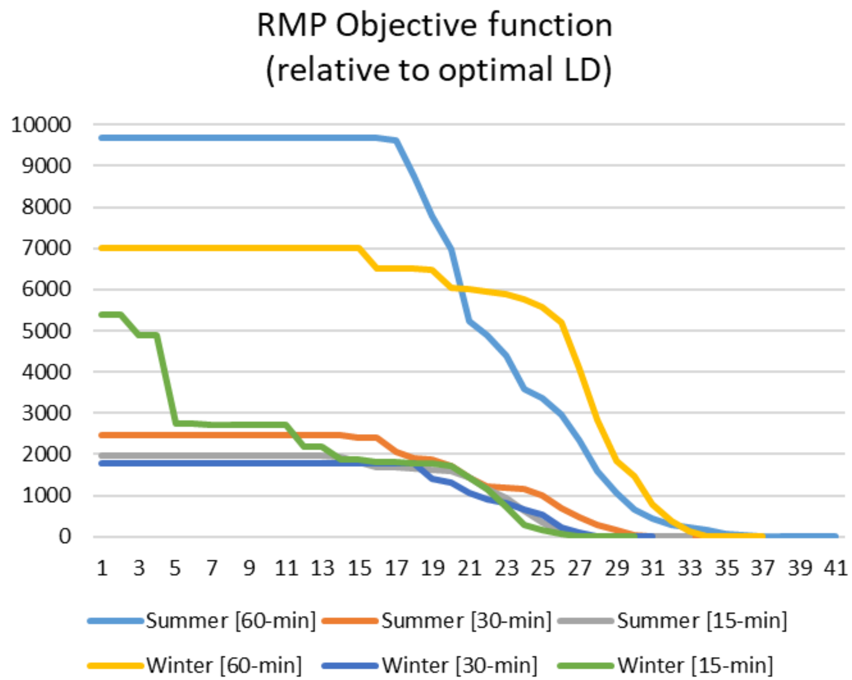
- FERC PJM-like dataset [Krall et al., 2012]
  - ~ 1000 Generators, 24-hours, up to 10 block offers for energy, reserve offers
  - Unit constraints (min/max, min up/down times, ramp up/down, start-up/shut down ramps)

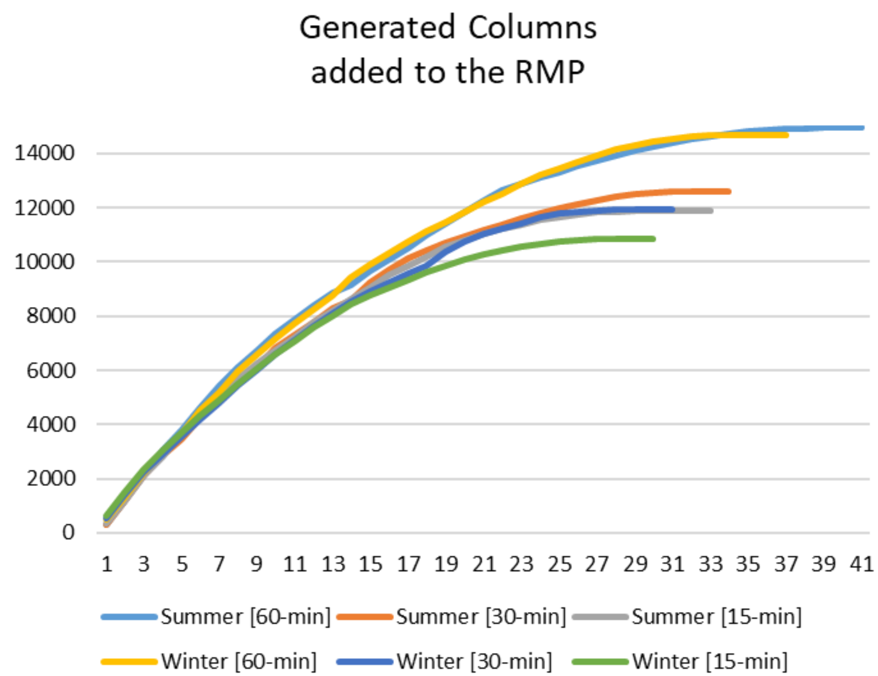


- E. Krall, M. Higgins, and R. P. O'Neill, "RTO unit commitment test system," FERC Staff Report, 2012.

Numerical Illustrations [Larger Datasets]

- FERC PJM-like dataset [Krall et al., 2012]
  - Additional instances (reduced ramp limits)





- E. Krall, M. Higgins, and R. P. O'Neill, "RTO unit commitment test system," FERC Staff Report, 2012.

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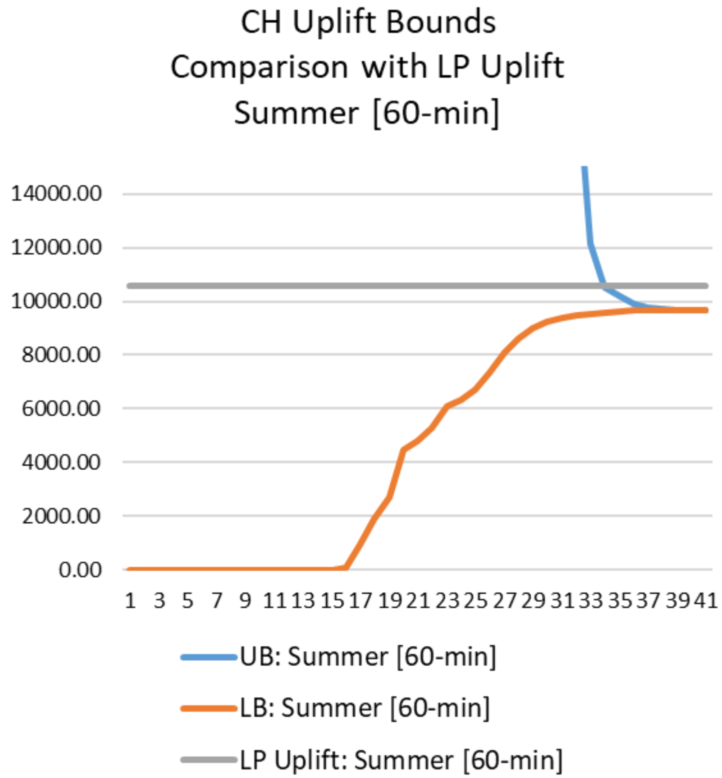

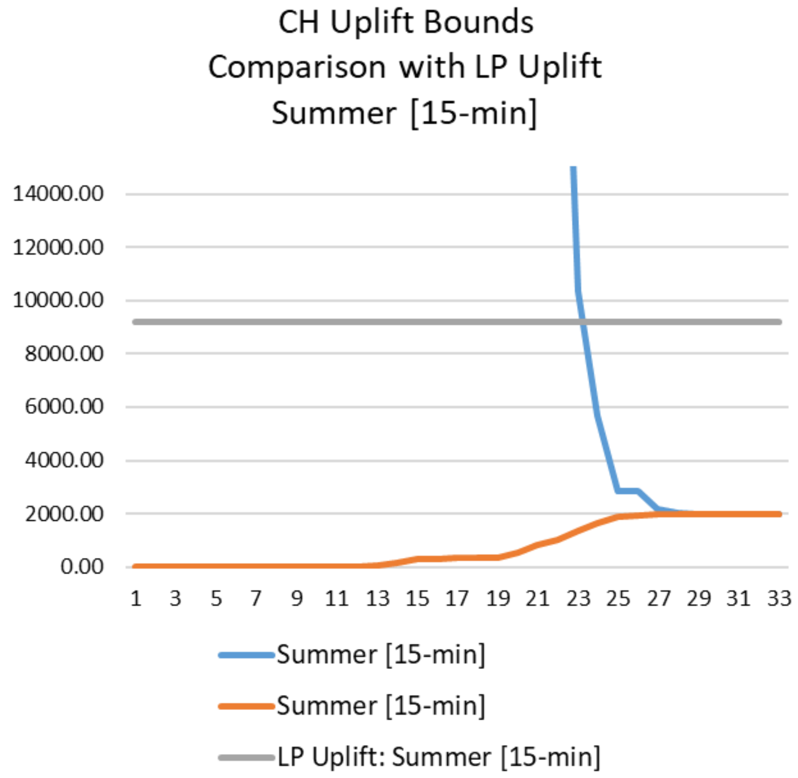

- E. Krall, M. Higgins, and R. P. O'Neill, "RTO unit commitment test system," FERC Staff Report, 2012.

Numerical Illustrations [Larger Datasets]

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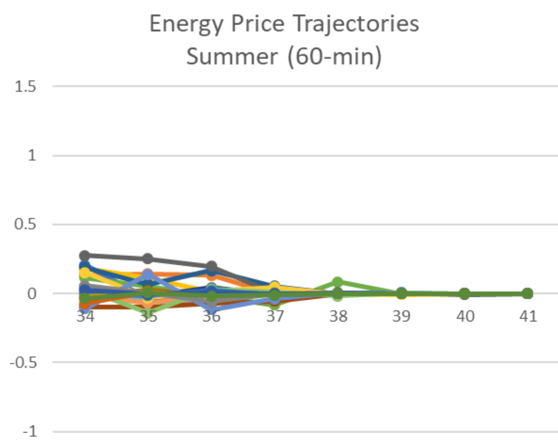
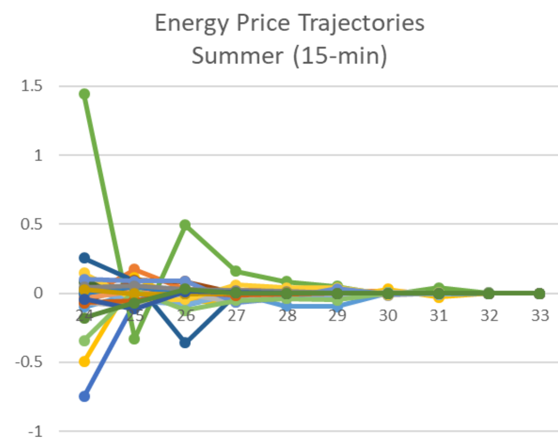
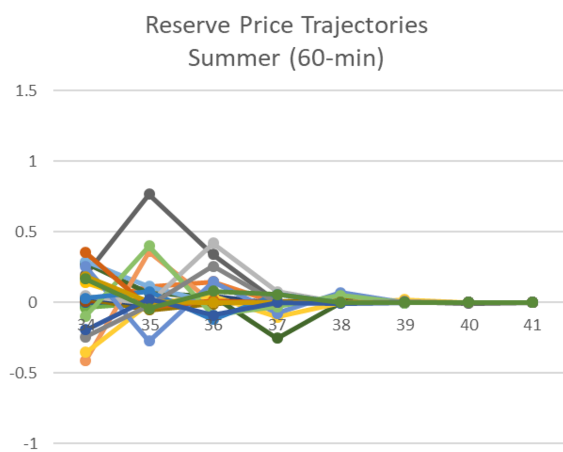
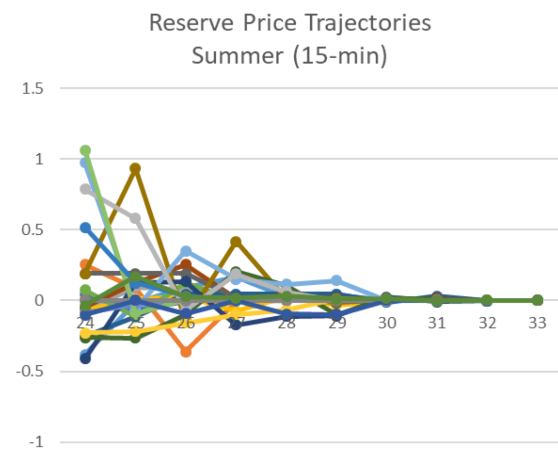
  - Additional instances (reduced ramp limits)



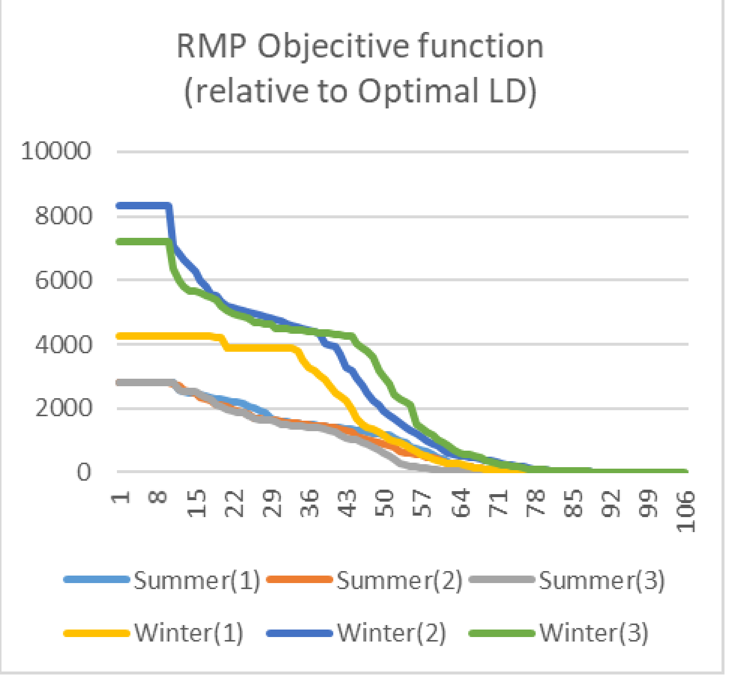


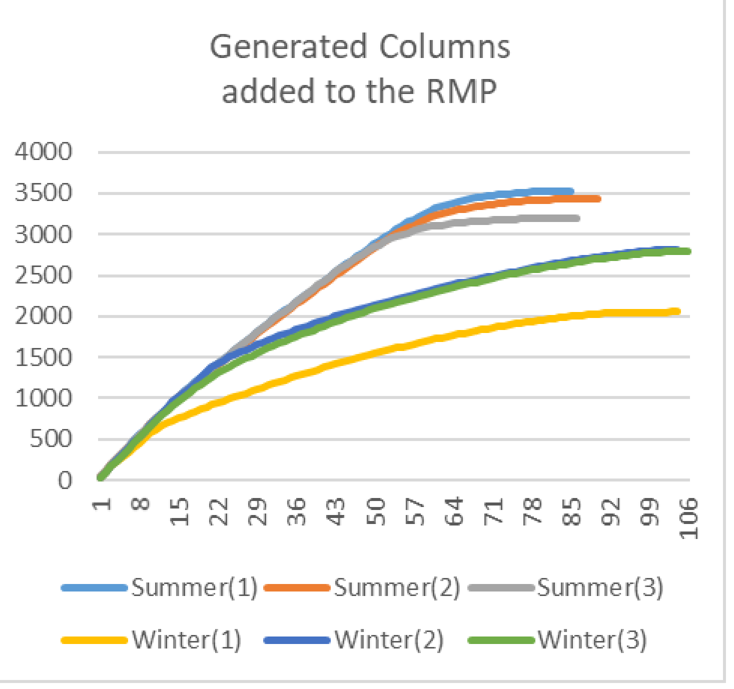




Numerical Illustrations [Larger Datasets]

- IEEE RTS GMLC dataset [Barrows et al., 2020]
  - 72 thermal generators, 73 nodes and 120 lines (includes transmission constraints), 24 hours, 3-block energy offers, reserves, and same unit constraints with FERC dataset.





- Barrows et al., The IEEE Reliability Test System: A Proposed 2019 Update, IEEE Trans. Power Syst. Vol 35, no. 1, 2020, pp. 119-127.

BU

Thank you for your attention!

Questions?

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P. Andrianesis, D. Bertsimas, M.C. Caramanis, W.W. Hogan, "Computation of Convex Hull Prices in Electricity Markets with Non-Convexities using Dantzig-Wolfe Decomposition," IEEE Transactions on Power Systems, 2021, doi: 10.1109/TPWRS.2021.3122000.