Une méthode de contrôle champ moyen pour le chargement de grandes flottes de véhicules électriques

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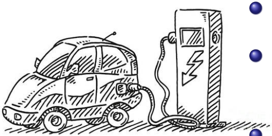
Outline

1. Mean Field model
2. The optimization problem
3. Results

Motivations and context

A central planner wants to optimally charge a huge fleet of EVs over a finite time horizon $[0, T]$. Different constraints must be taken into account:

- Satisfy EV owner requirements and avoid battery aging.
- Exploit EVs flexibility, in particular Vehicle-to-Grid (V2G).



- State variable : $X_t = (I_t, S_t)$
- I_t mode of charging (fast charging, idle, V2G...)
- S_t level of battery

The central planner optimizes the charging of the fleet by controlling the mode of charging of each EV (the discrete variable).

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⚠ Curse of dimensionality !

The mean field model

Description of the process

- I_t is a jump process with values in $I = \{\text{charging}, \text{idle}, \text{V2G} \ldots\}$, switching spontaneously, with intensity α:

$$\mathbb{P}\left[I_{t+\delta t} = j | I_t = i, S_t = s\right] \approx \alpha_{ij}(t,s)\delta t.$$

- S_t follows a deterministic dynamics between two consecutive jumps:

$$\frac{d}{dt} S_t = b(I_t, S_t) \quad \forall t \in [T_k, T_{k+1})$$

- $X_t = (I_t, S_t) \in I \times [0,1]$ is a PDMP(b, α)

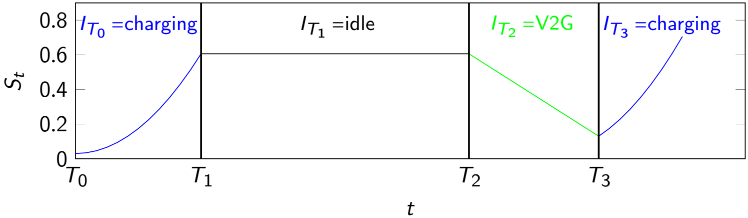

Figure 1: Evolution of the hybrid state variable $X_t = (I_t, S_t)$ over the time

On the PDMP literature:

- These processes are introduced rigorously in [Davis, 1984].
- Multiple applications in system **reliability and maintenance** [De Saporta and Zhang, 2013], **oil production** [Zhang et al., 2014], **biology** [Lin and Buchler, 2018], **insurance** [Marciniak and Palmowski, 2016], **communication networks**[Hespanha, 2005] etc...
- Existence of a large literature on the optimal control of PDMP using **dynamic programming** [Costa et al., 2016, De Saporta et al., 2017, Huang and Guo, 2019, Verms, 1985] or **BSDE representation** [Bandini, 2018].
- Existence of a growing literature on the analysis of the **mean field limit** of population of PDMPs [Diez, 2020, Monmarché, 2018].

Macroscopic description of the fleet

Let $m(t) \in \mathscr{P}(I \times [0,1])$: distribution of EVs at time $t \in [0,T]$ and $m^0 \in \mathscr{P}(I \times [0,1])$ the initial distribution.

By the mean field assumption (m, α) is a weak solution on $[0,T] \times I \times [0,1]$ of the continuity equation:

$$\begin{aligned} &\partial_t m_i + \partial_s (m_i b_i) = -\sum_{j \in I, j \neq i} (\alpha_{i,j} m_i - \alpha_{j,i} m_j), \\ &m_i(0) = m_i^0, \end{aligned} \tag{CE}$$

where the charging rate $b \in C^1(I \times [0,1])$ is given and $\alpha_{i,j}(t,s)$ is the jump intensity of EVs from state (i,s) to state (j,s) at time t.

The optimization problem

Control problem settings

- Congestion constraints: let $D \in C^0(I \times [0,T], \mathbb{R}_+^*)$:

$$\underbrace{\int_0^1 m_i(t,ds)}_{\text{mass of EVs in mode } i} \leq D_i(t) \quad \forall (i,t) \in I \times [0,T] \tag{1}$$

- Objective function J:

$$\begin{aligned} J(m,\alpha) := & \int_0^T \int_0^1 \sum_{i \in I} \underbrace{c_i(t,s) m_i(t,ds)}_{\text{electricity cost/reward}} + \sum_{j \in I} \underbrace{L(\alpha_{i,j}(t,s)) m_i(t,ds)}_{\text{transfers cost}} dt \\ & + \sum_{i \in I} \int_0^1 \underbrace{g_i(s) m_i(T,ds)}_{\text{final cost}} \end{aligned} \tag{2}$$

with $c \in C^1([0,T] \times I \times [0,1]), g \in C^1(I \times [0,1])$, L being strongly convex and continuously differentiable on its domain $\mathbb{R}^+$.

We study

$$\boxed{\begin{array}{l}\inf\limits_{(m,\alpha)} J(m,\alpha) \\ \text{s.t. } (m,\alpha) \text{ is a weak sol. (CE) and satisfies (1)}\end{array}} \qquad (P)$$

Lemma

Problem (P) admits a solution.

- Characterization of the solutions of Problem (P)?
- Regularity of the Lagrange multiplier?
- Numerical approximation?

Constrained Mean field control literature

- Stochastic target problems [Soner and Touzi, 2002]
- Stochastic control problems with expectation constraints [Pfeiffer et al., 2021]
- Constraints of the type : $\Psi(\mathscr{L}(X_t)) \leq 0$ [Daudin, 2021, Germain et al., 2021]
- Local constraints [Cardaliaguet et al., 2016]
- Constraints in Wasserstein spaces [Bonnet, 2019]

Results

Optimality conditions

Theorem (S. 2023)

Assume D is piecewise linear and there exists $\varepsilon^0 > 0$ such that:

$$\varepsilon^0 < \inf_{t\in[0,T]} D_i(t) - m_i^0([0,1]) \quad \forall (t,i) \in [0,T]\times I,$$

then (m,α) is a solution to (P), if and only if there exists (φ,λ,β) in $\mathrm{Lip}([0,T]\times I\times[0,1]) \times L^\infty([0,T]\times I,\mathbb{R}_+) \times (\mathbb{R}_+)^{|I|}$ *such that $\alpha_{i,j} = H'(\varphi_i - \varphi_j)$ and $(\varphi,\lambda,\beta,m)$ is a weak solution of the following system on* $[0,T]\times I\times[0,1]$:

$$\begin{cases} -\partial_t \varphi_i - b_i\partial_s\varphi_i - c_i - \lambda_i + \sum_{j\in I, j\neq i} H(\varphi_j - \varphi_i) = 0 \\ \partial_t m_i + \partial_s(m_i b_i) + \sum_{j\neq i}(H'(\varphi_i-\varphi_j)m_i - H'(\varphi_j-\varphi_i)m_j) = 0 \\ m_i(0,s) = m_i^0(s), \varphi_i(T,s) = g_i(s) + \beta_i \\ \int_0^1 m_i(t,ds) - D_i(t) \leq 0, \lambda_i \geq 0, \beta_i \geq 0 \\ \sum_{i\in I}\int_0^T \left(\int_0^1 m_i(t,ds) - D_i(t)\right)\lambda_i(t)dt + \left(\int_0^1 m_i(T,ds) - D_i(T)\right)\beta_i = 0 \end{cases} \tag{S}$$

where H is the Fenchel conjugate of L and H' its derivative.

Numerical approximation

Find $(\varphi, \lambda, \beta)$

$$\begin{aligned} &\partial_t \varphi_i + b_i \partial_s \varphi_i + c_i + \lambda_i - \textstyle\sum_{j\in I, j\neq i} H(\varphi_j - \varphi_i) \leq 0, \\ &\varphi_i(T) \leq g_i + \beta_i. \end{aligned} \tag{HJ}$$

$$A(\varphi,\lambda,\beta) := \sum_{i\in I} \int_0^1 -\varphi_i(0,s) m_i^0(ds) + \int_0^T D_i(t)\lambda_i(t)dt + D_i(T)\beta_i.$$

$$\boxed{\begin{array}{l} \inf_{(\varphi,\lambda,\beta)} A(\varphi,\lambda,\beta) \\ (\varphi,\lambda,\beta) \text{ weak sol (HJ)} \end{array}} \tag{D}$$

- Time and space discretization of Problem (D).
- Explicit finite difference scheme for the discretization of (HJ).

Use case

- 4h period, $I = \{0,1,2\}$, where 0: idle; and 1: charging, 2: V2G, $g(s) := Ce^{c((0.75-s)^+)^2}$, continuous flow of arriving EVs, possibility to leave the parking at $t = 2h$, congestion constraint :

$$\underbrace{\sum_{i \in I} \int_0^1 b_i(s) m_i(t, ds)}_{\text{power consumption}} \leq D(t)$$

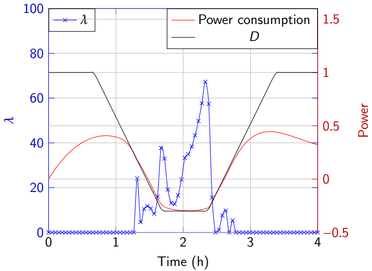

Figure 2: Optimal Lagrangian multiplier λ and consumption of EVs over the time

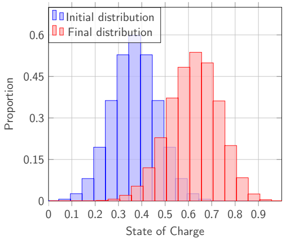

Figure 3: Marginal distribution of the State of Charge (s) at initial and final time

Thank you for your attention!

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Construction

Knowing T_k and $X_{T_k} = (I_{T_k}, S_{T_k})$, one obtains $(T_{k+1}, X_{T_{k+1}})$ as follows:

$$\begin{cases} \text{For any } j \in I \\ T_{k+1,j} := \inf\left\{t \geq T_k : E_{k+1,j} < \int_{T_k}^{t} \alpha_j(r, X_r)dr\right\} \quad \text{where } E_{k+1,j} \sim \text{Exp}(1) \\ T_{k+1} := \min_{j\in I} T_{k+1,j} \\ I_{T_{k+1}} = \min\{j \in I : T_{k+1,j} = T_{k+1}\} \\ S_{T_{k+1}} := \int_{T_k}^{T_{k+1}} b(I_t, S_t)dt \\ X_{T_{k+1}} = (I_{T_{k+1}}, S_{T_{k+1}}) \end{cases}$$

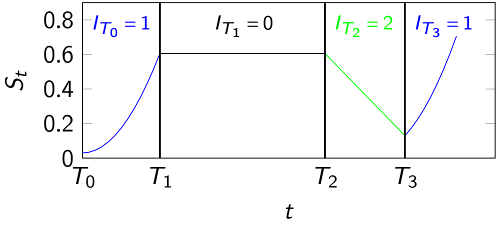

Figure 4: Evolution of the hybrid state variable $X_t = (I_t, S_t)$ over the time

Sketch of the proof

For $\delta, \varepsilon > 0$, we define the penalized problem

$$\begin{array}{|l|}\hline \inf\limits_{(m,\alpha)} J(m,\alpha) + \sum_{i\in I} \int_0^T \frac{1}{\varepsilon}\Psi_i^+(m_i(t))dt + \sum_{i\in I} \frac{1}{\delta}\Psi_i^+(m_i(T)), \\ (m,\alpha) \text{ weak sol. (CE)} \\ \hline \end{array} \qquad (D^{\varepsilon,\delta})$$

where $\Psi_i(\mu) := \mu_i([0,1]) - D_i$

- Optimality conditions of Problem $(D^{\varepsilon,\delta})$?
- Link between the solutions of Problem (P) and Problem $(D^{\varepsilon,\delta})$?

Proposition

Problem $(D^{\varepsilon,\delta})$ *has at least a solution and for any solution* (m,α) *there exists* $(\varphi,\lambda,\beta) \in \mathrm{Lip}([0,T]\times I\times[0,1]) \times L^\infty([0,T]\times I,\mathbb{R}_+) \times (\mathbb{R}_+)^{|I|}$ *such that* $\alpha_{i,j} = H'(\varphi_i - \varphi_j)$ *on* $\{m_i > 0\}$ *and* $(\varphi,\lambda,\beta,m)$ *is a weak solution of the following system on* $[0,T]\times[0,1]\times I$*:*

$$\begin{cases} -\partial_t \varphi_i - b_i \partial_s \varphi_i - c_i - \dfrac{\lambda_i}{\varepsilon} + \sum_{j\in I, j\neq i} H(\varphi_i - \varphi_j) = 0, \\ \partial_t m_i + \partial_s(m_i b_i) + \sum_{j\in I} H'(\varphi_i - \varphi_j) m_i - H'(\varphi_j - \varphi_i) m_j = 0, \\ m_i(0) = m_i^0, \varphi_i(T) = g_i + \dfrac{\beta_i}{\delta}, \end{cases} \qquad (S^{\varepsilon,\delta})$$

and (λ,β) *satisfies*

$$\lambda_i(t) = \begin{cases} 0 & \text{if } \Psi_i(m(t)) < 0, \\ \in [0,1] & \text{if } \Psi_i(m(t)) = 0, \\ 1 & \text{if } \Psi_i(m(t)) > 0, \end{cases} \qquad \beta_i := \begin{cases} 0 & \text{if } \Psi_i(m(T)) < 0, \\ \in [0,1] & \text{if } \Psi_i(m(T)) = 0, \\ 1 & \text{if } \Psi_i(m(T)) > 0. \end{cases}$$

Proposition

There exists $\varepsilon^*,\delta^* > 0$, such that for any $(\varepsilon,\delta) \in (0,\varepsilon^*) \times (0,\delta^*)$, Problems (P) and $(D^{\varepsilon,\delta})$ have the same solutions.

Proof by contradiction:

- Uniform bound on $\|\alpha\|_\infty + \|\partial_s \alpha\|_\infty$, independently of ε and δ.
- For any $\delta < \delta^*$, $\Psi_i(m(T)) \leq 0$.
- Assume for any $\varepsilon > 0$, there exists $t^\varepsilon > 0$ such that $\Psi_i(m(t^\varepsilon)) > 0$
- For any $\varepsilon < \varepsilon^*$ and a.e. $t \in [0,T]$ satisfying $\Psi_i(m(t)) > 0$:

$$\frac{d^2}{dt^2}\Psi_i(m(t)) \geq C \sum_{j \in I} \int_0^1 \left(\frac{1}{C\sqrt{\varepsilon}} - C\right)(\alpha_{i,j} m_i(t) + \alpha_{j,i} m_j(t)) \geq 0$$

- Since $\Psi_i(m^0) < 0$, there exists $\tau \in (0, t^\varepsilon)$ such that $\Psi_i(m(\tau)) > 0$ and $\frac{d}{dt}\Psi_i(m(\tau)) > 0$.
- Then the map $t \mapsto \Psi_i(m(t))$ is strictly increasing on $[\tau, T]$. Then, $\Psi_i(m(T)) > 0$ (contradiction)