A Common Shock Model for multidimensional electricity intraday price modelling with application to battery valuation

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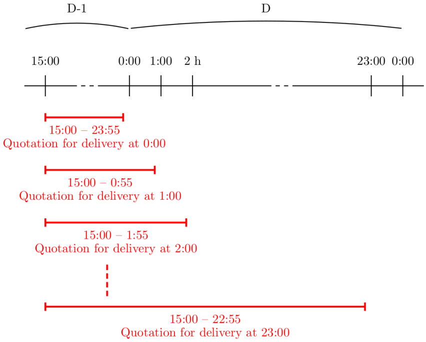
September 2023

What are intraday markets ?

EPEX Spot German intraday market, organized in continuous trading:

- Opens at 15:00 the day before (spot price settled at noon);
- Possibility to buy/sell physical delivery contracts for the 24 periods;
- Closes 5 minutes before beginning of delivery.

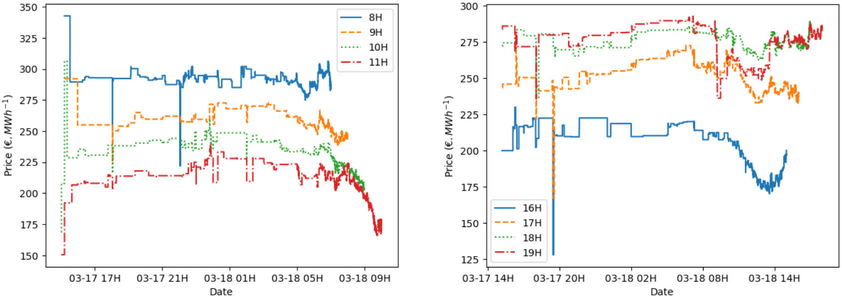


Motivation

- At least 24 products simultaneously traded on the market.
- Need a **multivariate** price model at the trading session level for
  - Storage assets valuation,
  - Optimal strategies computation.
- Very little literature on the subject:
  - Order book arrivals modelling in an univariate setting:
    - Favetto (2019): Hawkes processes
    - Graf von Luckner and Kiesel (2021): Hawkes processes
  - Price modelling in an univariate setting:
    - Deschatre and Gruet (2023): marked Hawkes processes
    - Hirsch and Ziel (2022): forecast model
  - Price modelling in an multivariate setting:
    - Hirsch and Ziel (2023): forecast model
    - **No simulation model**.
- In this work, we propose a multivariate statistical model
  - with a focus on the volatility and on the correlation modelling,
- and use it to value a storage asset.

Data

- German and French electricity intraday transaction prices
- between 2019 and 2022 provided by EPEX.
- Timestamps accurate to 1min (2019), 1s (2020), or 1ms ($\geq$2021).
- Tick size = 0.01 €/MWh.
- Prices until 1 hour before the delivery (change in market rules).

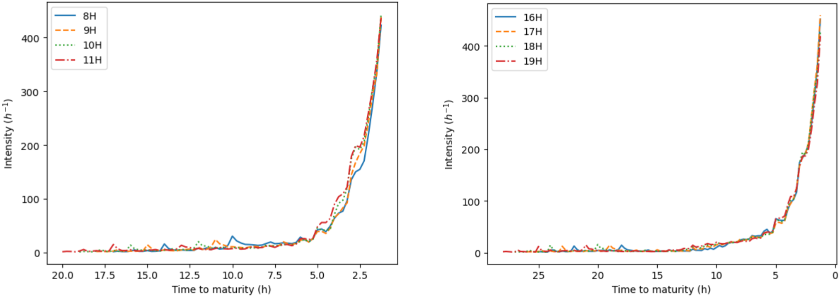


German transaction prices for the trading session on March 18, 2022.

Outline

1. Empirical analysis
2. Model
3. Battery valuation

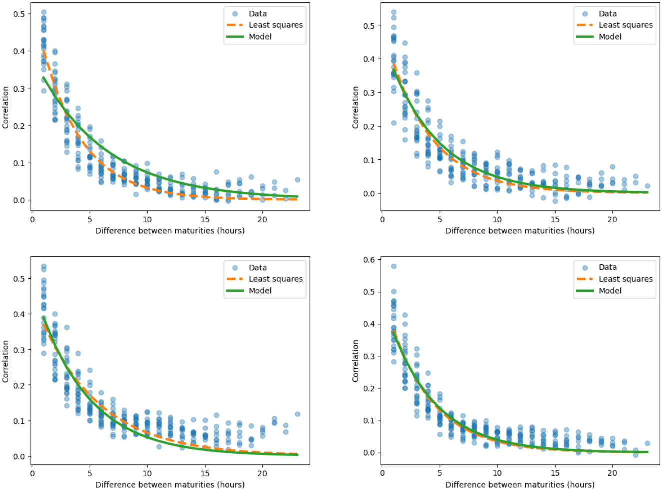
Intensity and volatility: Samuelson effect



Intensity of German transaction price changes in 2022 for some delivery periods (average number of price changes over the different trading sessions of the year in 15-minute windows).

- Equilibrium price model in Aïd et al. (2022) explains this effect.

Correlation structure: Correlation Samuelson effect



Correlations versus distance between maturities with a sampling time step of 30 minutes for German transaction prices in 2019, 2020, 2021 and 2022. In orange, least-squares estimator with the function $x \mapsto \alpha e^{-\beta x}$.

Outline

How to correlate two Poisson processes ?

- Common Shock Poisson Model Lindskog and McNeil (2003)
- Consider 3 Poisson processes $\tilde{N}_1$, $\tilde{N}_2$, and N_c
- with intensities μ_1, μ_2 and μ_c.

$$\text{Let } N_1 = \tilde{N}_1 + N_c \text{ and } N_2 = \tilde{N}_2 + N_c,$$

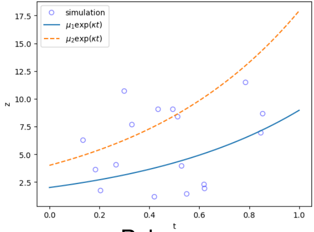
$$\text{Then } \rho(N_1, N_2) = \frac{\mu_c}{\sqrt{(\mu_1 + \mu_c)(\mu_2 + \mu_c)}}.$$

- N_c: **exogenous** shocks creating events for both 1 and 2.
- Different from the Hawkes approach:
  - correlation is **endogenous**: events for 1 can lead to events for 2.

Issue: the common shock has the same intensity in N_1 and N_2 !

Poisson measures

- Let $\pi(ds, dx)$ a Poisson measure with intensity $ds \otimes dx$ on $\mathbb{R}_+^2$
  - see Definition 2.18 in Cont and Tankov (2003).
- Consider $\int_0^t \int_{\mathbb{R}} \mathbf{1}_{x \leq f(s)} \pi(ds, dx)$
  - inhomogeneous Poisson process
  - with integrated intensity $\int_0^t \int_{\mathbb{R}} \mathbf{1}_{x \leq f(s)} dsdx = \int_0^t f(s)ds$.



A simulation of a homogeneous Poisson measure with intensity $ds \otimes dx$.

- Possible to add marks with law $\nu(dy)$ considering
  - a Poisson measure $\pi(ds, dx, dy)$ on $\mathbb{R}_+^2 \times K$
  - with intensity $ds \otimes dx \otimes \nu(dy)$
  - and $\int_0^t \int_{\mathbb{R}} \int_K y \mathbf{1}_{x \leq f(s)} \pi(ds, dx, dy)$.

Model: Common shock approach

$$\underbrace{f_{m,t}}_{\text{price for maturity } m} = \underbrace{f_{m,0}}_{\text{initial price}} + \underbrace{f^{+}_{m,t}}_{\text{sum of price increases}} - \underbrace{f^{-}_{m,t}}_{\text{sum of price decreases}} .$$

π^h_m, π^h Poisson measures on $\mathbb{R}^2_+ \times K$ with intensities $ds \otimes dx \otimes \nu(dy)$.

$$f^h_m = \int_0^t \int_{\mathbb{R}_+} \int_K y \mathbf{1}_{x \leq \mu e^{-\kappa(T_m - t)} \mathbf{1}_{t \leq T_m}} \pi^h_m(ds, dx, dy) + \int_0^t \int_{\mathbb{R}_+} \int_K y \mathbf{1}_{x \leq \mu_c e^{-\kappa(T_m - t)} \mathbf{1}_{t \leq T_m}} \pi^h(ds, dx, dy), \; h = +, -$$

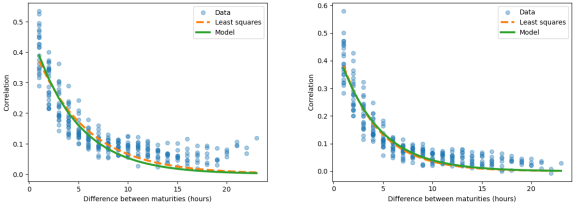
- Specific price movements for a given maturity:
  - trading activity increases with time to maturity.
- Common shock:
  - price movements affecting several maturities,
  - created by a same shock (i.e. failure of an asset affecting several maturities).

Model: properties

- f_m^h is a compound Poisson process
  - with intensity $(\mu + \mu_c)\, e^{-\frac{\kappa}{2}(T_m - t)} \mathbf{1}_{t \leq T_m}$,
  - and jump sizes with law ν.
- $\implies$ price volatility $= \sqrt{2 \frac{\mu + \mu_c}{\kappa} \int_K y^2 \nu(dy)} e^{-\frac{\kappa}{2}(T_m - t)} \mathbf{1}_{t \leq T_m}$.
- Price correlation between maturity T_m and T_l: $\frac{\mu_c}{\mu_c + \mu} e^{-\frac{\kappa}{2}|T_m - T_l|}$.
- The common shocks only affect successive maturities:
  - starting with the nearest,
  - and the probability to affect p maturities decreases with p.
- Only three parameters (plus the jump law ν):
  - $\frac{\kappa}{2}$ is the speed of volatility increase (Samuelson effect)
  - and the speed of correlation decrease (Samuelson correl. effect);
  - μ is the intensity of independent transactions;
  - μ_c is the intensity of events resulting from common shocks.

Estimation: moment-based method

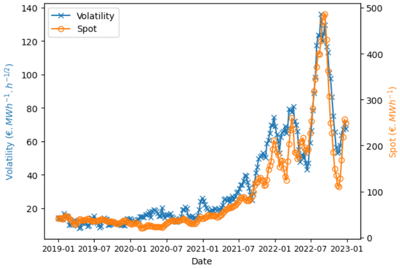
- κ is estimated from expectations for each time t (intensity),
- while $\mu + \mu_c$ is estimated from the integrated variance,
- and $\frac{\mu_c}{\mu+\mu_c}$ from the integrated correlation.

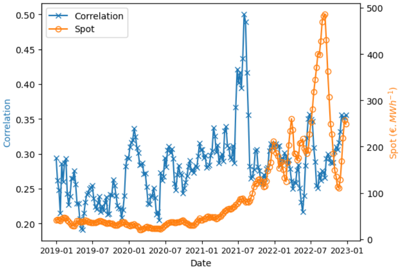


Correlations versus distance between maturities with a sampling time step of 30 minutes for German transaction prices in 2021 and 2022. In orange, least-squares estimator with the function $x \mapsto \alpha e^{-\beta x}$ and in green, correlation curve from the model.

- Green curve and orange curves are very close:
  - justifies the use of a single parameter to model both SE and SCE.

Estimation





Volatility proxy σ and correlation proxy ρ estimated each week from the last 28 trading sessions of the intraday market against the average spot price during the estimation period for Germany.

$$\sigma = \sqrt{\frac{2(\mu + \mu_c)}{\kappa} \int_K y^2 \nu(dy)},$$

$$\rho = \frac{\mu_c}{\mu + \mu_c} e^{-\frac{\kappa}{2}\Delta T}, \ \Delta T = 1h.$$

Outline

Method and results

- One decision per hour at time T_i to buy/sell product $T_i + 1$h.
- Strategy is bang-bang:
  - optimal decision is to inject/withdraw everything or do nothing.
- Use of dynamic programming (StOpt) but dimension is high:
  - regression on the last $p = 5$ prices is a good approximation.
- Model parameters estimated each week from the last 28 days.
- Benchmark: strategy maximising the value using spot price.
- Also consider the model diffusion approximation (intensity $\to \infty$):
  - Gives similar results, consistent with Abeille et al. (2023).

Year	Spot	Poisson	Diffusion
2019	19781	28505	28605
2020	22105	29753	29626
2021	49113	63696	63314
2022	121030	152955	152870

3MWh battery value with efficiency 0.92 in Germany (backtest).

Conclusion, limits and perspectives

- First multidimensional simulation model for intraday prices.
- Only 3 parameters to represent volatility and correlation structure.
- Model easy to estimate and to simulate.

- Do not represent microstructure noise (signature plot, Epps)
  - leads to arbitrary choices in the estimation procedure.
- Same jump sizes for the different maturities when common shock.

- Purely endogenous correlation approach with Hawkes processes
  - PhD thesis of A. Lotz;
- Endo and exo correlation modeling
  - work with E. Bacry, M. Hoffmann, J.F. Muzy, R. Ruan;
- Battery valuation and liquidity costs
  - work with E. Cogneville and X. Warin.

Thank you for your attention.

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