Optimal control of storage and short-term price formation in electricity market

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Journées ateliers FIME

Motivation

Context

- Necessity to **massively replace** fossil-fired power plants by renewable technologies $\longrightarrow$ intermittent nature of the latter: this also requires a large-scale use of electricity storage
- Pumped Hydroelectric Energy Storage (PHES) = 96% of total electricity storage in the world

Aim of this work

- Define a realistic and tractable model to study the problem of the optimal strategy for a price taker PHES
- Study its impact on the short-term equilibrium in the electricity market in different frameworks

Literature

Small literature

- Literature on optimal control of storage (Carmona and Ludkovski, Cruise and Zachary, etc.)
- a review on the development of PHES by Barbour et al
- Price formation on electricity market, work inspired by a paper by Aïd et al

Overview

Plan

Storage

Consider a PHES whose state process, noted Q, represents the **tank level** and follows the SDE:

$$dQ_t = -q_t dt + \rho dW_t^1. \tag{1}$$

q_t, the control process, is the **injection rate** (positive or negative). Here, ρ describes the random amount of energy lost/gained because of external factors (example: drought).
The price P is considered stochastic and exogenous. The stochastic optimal control problem for a single agent reads:

$$V(0, Q_0) = \inf_q \mathbb{E}\left[\int_0^T -(P_s q_s - \alpha q_s^2) + \frac{\beta}{2}(Q_s - Q_0)^2 ds + \frac{\gamma}{2}(Q_T - Q_0)^2\right].$$

with γ,β,α strictly positive numbers.

Storage

Proposition

The **closed loop** optimal control process can be expressed as follows :

$$\boxed{q_t = \sqrt{\frac{\beta}{\alpha}} f(t,T)(Q_t - Q_0) - \mathbb{E}\left[\int_t^T f_1(t,T,s)\frac{P_s}{2\alpha}ds|\mathcal{F}_t\right] + \frac{P_t}{2\alpha}}$$

with $u = \frac{\sqrt{\alpha\beta}-\gamma}{\sqrt{\alpha\beta}+\gamma}$ and f, f_1 auxiliary functions depending on $\alpha, \beta, \gamma and T$.

Overview

Global framework

With this knowledge, the focus shifts to viewing the electricity market as a platform for interaction between the energy demand D and three types of players:

Players

- **Renewable producers**: they always bid their full, stochastic, capacity R_t
- **Conventional producers**: they use a supply function $C(P_t)$, supposedly known and depending only on the electricity price
- **Storage facilities**: they have their optimal strategy found earlier q_t. Here there is only one big storage representing the aggregation of every small player.

The price process is defined in this way :

$$P_t = \inf \{P : D_t \leq R_t + C(P) + q_t\} \wedge \bar{P}$$

with $\bar{P}$ an upper bound for the electricity price.

Overview

Overview

Assumptions

- We ignore the random events that could affect the tank level of the PHES.
- There is no renewable production
- The energy demand is deterministic
- C is a linear function, $C(P) = C_0 + CP$

$\longrightarrow$ q is deterministic
$\longrightarrow$ P is deterministic

Toy model

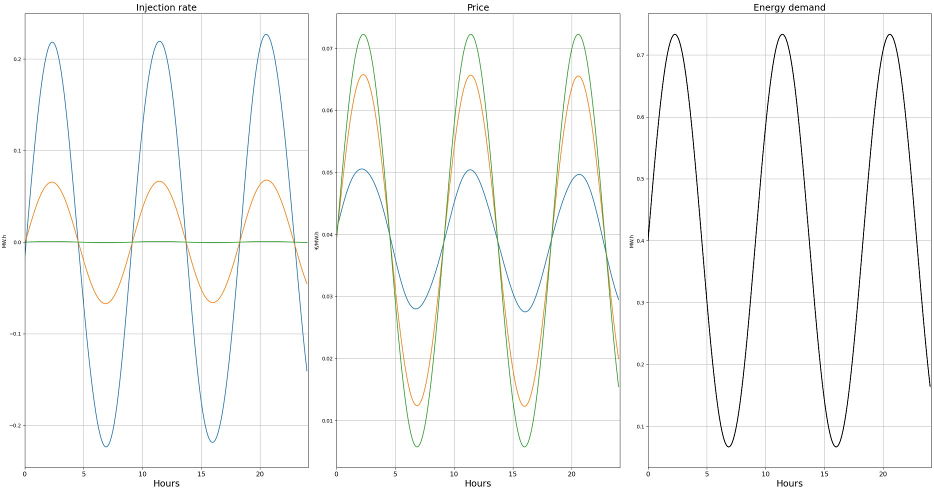
The main positive aspect of this toy model is that we can derive an explicit expression for the injection rate and the electricity price :

$$q_t = -c_1 \frac{\beta}{\alpha} \cosh\left(\sqrt{\frac{\beta}{\alpha}} t\right) + \frac{P_t}{\alpha} + \int_0^t \sqrt{\frac{\beta}{\alpha}} \sinh\left(\sqrt{\frac{\beta}{\alpha}}(t-s)\right) \frac{P_s}{2\alpha} ds$$

$$P(t) = \frac{D_t - C_0 - \frac{\sqrt{\frac{\beta}{\alpha}}}{2\alpha} G(t) + c_1 \frac{\beta}{\alpha} \cosh\left(\sqrt{\frac{\beta}{\alpha}} t\right)}{F + \frac{1}{2\alpha}} \wedge \bar{P}$$

with c_1 a constant depending on α, β and γ, G a given function.

Numerical illustration



Overview

As before, we look at the electricity market as a platform for interactions between three types of players.

Players

- **Renewable producers**: they always bid their full, stochastic, capacity R_t
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In this part, there is only one big storage representing the aggregation of every small player.

Price formation

Stochastic case with one storage agent

The electricity price process is still defined as the solution of

$$P_t = \inf\{P : D_t \leq R_t + C(P) + q_t\} \wedge \bar{P}$$

Theorem

Suppose that the residual demand $\tilde{D}_t := D_t - R_t$ can be written in this way :

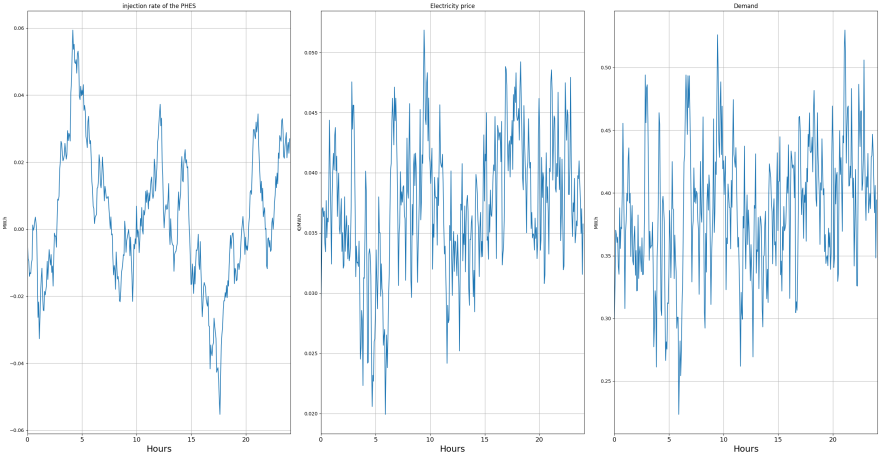
$$d\tilde{D}_t = \mu(t, \tilde{D}_t)dt + \sigma(t, \tilde{D}_t)dW_t^2$$

where μ and σ are uniformly bounded.
Then, there exists a **unique** price process satisfying the previous equation on $[0, T]$.

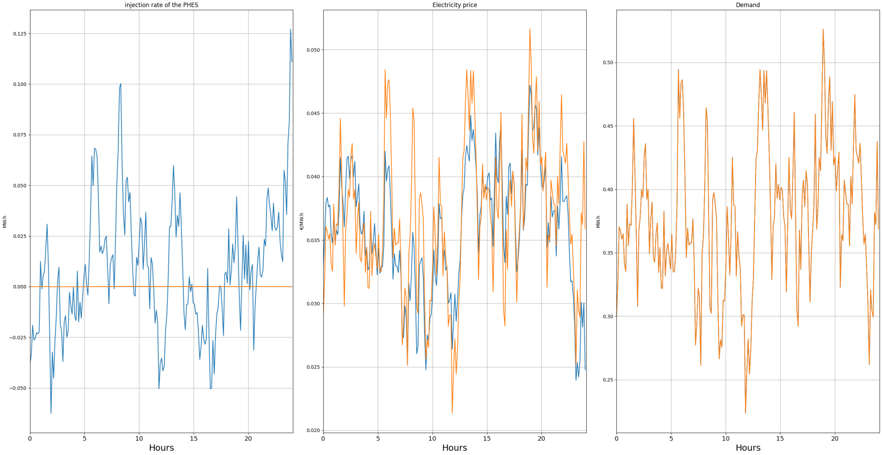
Numerical illustration

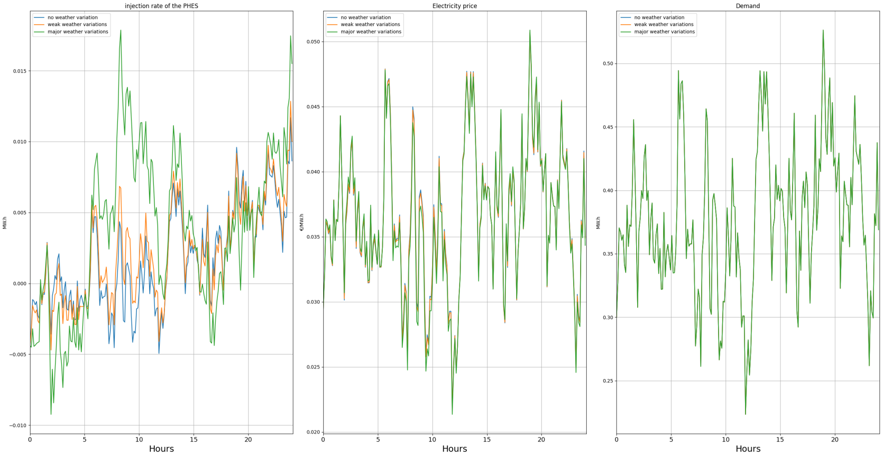
In this numerical illustration, the residual demand follows a Jacobi process:

$$d\tilde{D}_t = \theta(\tilde{D}_t - \mu)dt + \sqrt{\tilde{D}_t(1-\tilde{D}_t)}dW_t^2$$

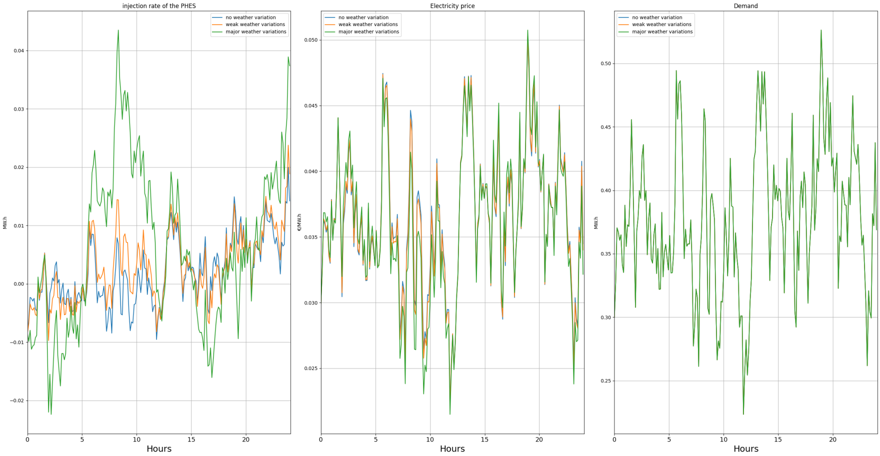


Numerical illustration : impact of storage compared to no storage




injection rate of the PHES
no weather variation
weak weather variations
major weather variations
MW.h
Hours
Electricity price
no weather variation
weak weather variations
major weather variations
€/MW.h
Hours
Demand
no weather variation
weak weather variations
major weather variations
MW.h
Hours

Numerical illustration : impact of weather intensity



Overview

Price formation

Stochastic case with N storage agents

Let us assume the presence of N different categories of storage, each with unique parameters and characteristics.

Agent j

$$dQ_t^j = -q_t^j dt + \rho^j dW_t^{1,j}$$

He solves the following problem:

$$V(0, Q_0^j) = \inf_{q_t^j} \mathbb{E} \int_t^T -(P_t q_t^j - \alpha^j q_t^{j\,2}) + \frac{\beta^j}{2}(Q_t - Q_0)^2 dt + \frac{\gamma^j}{2}(Q_T^j - Q_0^j)^2$$

and his optimal strategy is

$$q_t^j = \sqrt{\frac{\beta^j}{\alpha^j}} f^j(t, T, t)(Q_t^j - Q_0^j) - E\left[\int_t^T f^j(t, T; r)\frac{P_s}{2\alpha^j} ds | \mathcal{F}_t^j\right] + \frac{P_t}{2\alpha^j}$$

In this case, the price process is defined in this way:

$$P_t = \inf \left\{ P : D_t \leq R_t + C(P) + \sum_j q_t^j \right\} \wedge \bar{P}$$

Theorem

Under some conditions on the $(\alpha^j, \beta^j)_{0<j\leq N}$ there exists a unique price process resulting from the above definition in $[0, T]$.

More restrictive assumptions than the one-agent case.

Conclusion

Conclusion

- We introduced a tractable model to derive with classic methods an explicit expression for the optimal strategy of a storage system, considering both deterministic and stochastic exogenous price processes.
- We proved the existence and uniqueness of the price process resulting from short-term equilibrium between the energy demand, renewable production, conventional producers, and storage players.
- We observed in the deterministic case that increasing storage capacity led to a compression of electricity prices
- The more storage there is on the electricity market, the less volatility there is from renewable energy producers.

Ongoing work

Introducing a mean-field framework to better describe the variety of storage agents.