Journées atelier FIME

Energy savings and demand response: a mean-field game approach

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14 September 2023

Introduction

Context

- Energy efficiency is called the first fuel in clean energy transitions, as it provides some of the quickest and most cost-effective CO2 mitigation options. In response to the energy crisis countries are prioritising energy efficiency action due to its ability to simultaneously meet affordability, supply security and climate goals.
- To integrate variable renewable energy sources, a flexible consumption (i.e. demand following supply) could - next to energy savings - play an important role. The Energy Union refers to this as the “Energy Efficiency First Paradigm”, explicitly including both energy savings and demand response (European Climate Foundation, 2016).

Introduction

Energy efficiency

- The *Energy Savings Certificate mechanism* was created in 2005 as one of the key tool of the French energy demand side management policy within the context of European objectives. It enables the promotion and stimulation of investments in terms of energy efficiency through a market mechanism.
- In Italy and other European countries (UK, Denmark...), similar types of incentives: *white certificates*.
- The government determines a pluri-annual global energy savings goal (usually of $3, 4$ years).
- This goal is then beared by all energy suppliers, also called ”Obligés”, according to their share of the total supply. To fulfil their obligation, they have to promot energy saving projects to the consumers or face financial penalties.

Introduction

Energy efficiency

Evidence from behavioural economy:

- Financial reward and/or information on social norms or comparisons to other customers motivates energy savings.
- For example, in Dolan and Metcalfe, 2015
  - social norms can reduce consumption by around 6%
  - large financial rewards can reduced consumption by 8%

Existing incentives "Provider → customers":

- Comparison to similar customers (comparable households in the same area)
  - EDF, TotalEnergy, Engie, . . .
- Reward/Bonus when reduction compared to past consumption
  - "SimplyEnergy"[1], "Plüm énergie"[2], "OhmConnect"[3]

[1] www.simplyenergy.com.au/residential/energy-efficiency/reduce-and-reward
[2] www.plum.fr/cagnotte/
[3] www.ohmconnect.com/

Introduction

Energy efficiency

- Our model: A principal (the retailer) aims at designing a rank-based reward function for a population of heterogeneous agents (consumers) which maximizes its profit, taking into account the consumption distribution at the equilibrium.
- Mathematical tool: Stackelberg mean-field games.

Reference:

”A Rank-Based Reward between a Principal and a Field of Agents: Application to Energy Savings” (joint with C. Alasseur, E. Bayraktar, Q. Jacquet), https://arxiv.org/abs/2209.03588

Introduction

Demand response

- Demand response refers to balancing the demand on power grids by encouraging customers to shift electricity demand to times when electricity is more plentiful or other demand is lower, typically through prices or monetary incentives.

Introduction

Demand response

- Our model: We consider an energy system with a large number of consumers who are linked by a Demand side management contract - dynamic pricing and interruptible load feature. The consumers interact through the electricity price and a penalty cost.
- Mathematical tool: mean-field games.

References:

- C. Alasseur, L. Campi, R.D, J. Zeng, ”MFG model with long-lived penalty at random jump times: application to demand side management for electricity contracts”, Annals of Operations Research (2023)
- C. Alasseur, Z. Bensaid, R.D., X. Warin, ”Deep-learning algorithms for coupled FBSDEs with jumps: application to option pricing and a MFG model for smart grids”, work in progress

Part I

MFG model with a long-lived penalty at random jump times: application to demand side management for electricity contracts

Section 1

Model

Model

N-player game formulation

- Step 1. Each consumer $i \in \{1, \ldots, n\}$ wants to minimise its total expected costs:

$$\inf_{\alpha^i \in \mathcal{A}} J_n^i(\alpha) = \inf_{\alpha^i \in \mathcal{A}} \mathbb{E}\left[\int_0^T \left(\underbrace{g(\alpha_t^i, S_t^i, Q_t^i)}_{\text{inconvenience cost}} + \underbrace{l(Q_t^i + \alpha_t^i)}_{\text{demand charge}} \right.\right.$$
$$\left.\left. + \underbrace{c_t^i}_{\text{real time tariff}} + \underbrace{d_t^i}_{\text{divergence cost}} \right) dt + \underbrace{h(S_T^i)}_{\text{terminal cost}} \right],$$

  with $\alpha = (\alpha^1, \ldots, \alpha^n)$.

  We represent the DSM contract with two parts:

  (i) RTP: real time pricing $\mapsto$ *interaction in the control*
  (ii) interruptible load = divergence cost $\mapsto$ *interaction in the control*

- Step 2. Find a *consensus* $\mapsto$ Nash equilibria

Model

N-player game formulation

Real time tariff:

$$c_t^i = (Q_t^i + \alpha_t^i)p\left(\underbrace{\frac{1}{n+n'}\sum_{j=n+1}^{n'} Q^j}_{\text{standard consumers}} + \underbrace{\frac{1}{n+n'}\sum_{j=1}^{n}(Q_t^j + \alpha_t^j)}_{\text{consumers with DSM contract}}\right).$$

Model

N-player game formulation

Interruptible load contract. When activated, the aim of the interruptible load contract is that the global consumption of the active consumers during θ. The divergence cost has the form:

$$d_t^i = J_t^\theta(\tilde{Q}_t^i + \alpha_t^i - \bar{\alpha})f\left(\frac{1}{n}\sum_{j=1}^{n}(Q_t^j + \alpha_t^j) - \bar{\alpha}_t\right)$$

- with f a convex growing function such as $f(0) = 0$
- J_t^θ equal to one during interruptible load contract activation and 0 otherwise.
- $dR_t = dt - R_{t-}\, dN_t^0,\ R_0 = 2\theta,$
- $J_t^\theta = \mathbf{1}_{R_t \leq \theta}$

Section 2

Mean-field game

Mean-field game

Formulation.

- W^0 and W two independent Brownian motions
- N^0 is a doubly stochastic Poisson processes with intensity process (λ_t) which is $\mathbb{F}^{W_0}$-adapted.
- $\tilde{N}_t := N_t - \int_0^t \lambda_s ds$ the compensated Poisson processes
- $\mathbb{F} = (\mathcal{F}_t)_{t \in [0,T]}$ be the (complete) natural filtration generated by (W, W^0, N^0, s_0, q_0, q_0^{st}).
- $\mathbb{F}^0 = (\mathcal{F}_t^0)_{t \in [0,T]}$ be the (complete) natural filtration generated by (W^0, N^0).

Mean-field game

Formulation

$$\begin{cases} dQ_t &= \mu(Q_t, t)\,dt + \sigma(Q_t, t)\,dW_t + \sigma^0(Q_t, t)\,dW_t^0, \quad Q_0 = q_0, \\ dQ_t^{st} &= \mu^{st}(Q_t^{st}, t)\,dt + \sigma^{st}(Q_t^{st,j}, t)\,dW_t^0, \quad Q_0^{st} = q_0^{st}, \\ dS_t &= \alpha_t dt, \quad S_0 = s_0. \end{cases}$$

with $\alpha \in \mathcal{A}$, $\mathcal{A}$ the set of $\mathbb{F}$-adapted real-valued processes $a = \{a_t\}$ such that $\mathbb{E}\left[\int_0^T |a_u|^2 du\right] < \infty$.

For a $\mathbb{F}$-adapted process $\xi = (\xi_t)$, denote $\widehat{\xi}_t := \mathbb{E}[\xi_t | \mathcal{F}_t^0]$.

Mean-field game

MFG problem. Let $\xi = (\xi_t)_{t\in[0,T]}$ be a given $\mathbb{F}^0$-adapted process.

$$J^{MFG}(\alpha;\xi) = \mathbb{E}\left[\int_0^T \left(g(\alpha_t, S_t, Q_t) + l(Q_t+\alpha_t) + (Q_t+\alpha_t)p\left(\pi\widehat{Q}_t^{st} + (1-\pi)(\widehat{Q}_t+\xi_t)\right)\right.\right.$$
$$\left.\left. + J_t^\theta(Q_t+\alpha_t-\bar{\alpha})f\left(\widehat{Q_t}+\xi_t-\bar{\alpha}_t\right)\right) dt + h(S_T)\right],$$

where $\alpha = (\alpha_t)_{t\in[0,T]}$ is an *admissible* control process which belongs to $\mathcal{A}$, the set of all real-valued $\mathbb{F}$-adapted processes such that $\mathbb{E}[\int_0^T \alpha_t^2\,dt] < \infty$.

$$V^{MFG}(\xi) = \inf_{\alpha\in\mathcal{A}} J^{MFG}(\alpha;\xi).$$

The goal is to find a process $\alpha^\star = (\alpha_t^\star)_{t\in[0,T]}$ such that

$$J^{MFG}(\alpha^\star;\xi) = V^{MFG}(\xi)$$

and

$$\widehat{\alpha}_t^\star = \xi_t, \text{ a.s. for all } t\in[0,T].$$

Such a process $\alpha^\star$ is called a *mean-field Nash equilibrium*.

Mean-field game

Characterization of mean field Nash equilibria

- Let $\hat{\xi}$ be a given $\mathbb{F}^0$-adapted $\mathbb{R}$-valued process and $x_0 = (s_0, q_0)$ be a a vector of RV independent of $\mathbb{F}$. Assume that $\alpha \mapsto J(\alpha, \hat{\xi})$ is strictly convex. If there exists a control $\alpha^\star \in \mathcal{A}$ which minimizes the map $\alpha \mapsto J^{MFG}(\alpha, \hat{\xi})$ and if $(S^{\alpha^\star}, Q)$ is the state process associated to the initial condition x_0 and control $\alpha^\star$, then there exists a unique solution $(Y^\star, q^{0,\star}, q^\star, \nu^{0,\star})$ of the following BSDE with jumps:

$$-dY_t^\star = \partial_s g(\alpha, S_t^{\alpha^\star}, Q_t)\,dt - q_t^{0,\star}\,dW_t^0 - q_t^\star\,dW_t - \nu_t^\star\,d\widetilde{N}_t - \nu_t^{0,\star}\,d\widetilde{N}_t^0,$$
$$Y_T^\star = \partial_s h(S_T^{\alpha^\star}), \quad (1)$$

satisfying the coupling condition

$$\partial_\alpha g(\alpha_t^\star, S_t^{\alpha^\star}, Q_t) + \partial_\alpha l(Q_t + \alpha_t^\star) + p_t\left(\widehat{Q}_t + \hat{\xi}_t\right) + Y_t^\star + J_t^\theta f\left(\widehat{Q}_t + \hat{\xi}_t - \bar{\alpha}\right) = 0. \quad (2)$$

Mean-field game

Characterization of mean field Nash equilibria

- Conversely, assume that there exists $\left(\alpha^\star, S^{\alpha^\star}, Y^\star, q^{0,\star}, q^\star, \nu^{0,\star}\right)$ satisfying the above coupling condition, as well as the FBSDE for (S, Y), then $\alpha^\star$ is the optimal control minimizing the map $\alpha \mapsto J^{MFG}(\alpha, \hat{\xi})$ and $S^{\alpha^\star}$ is the optimal trajectory.

  If additionally $\widehat{\alpha}_t^\star = \hat{\xi}_t$ a.s. for all $t \in [0, T]$, then $\alpha^\star$ is a Mean-field Nash equilibrium.

Mean-field game

Semi explicit characterisation of the MFG Nash equilibrium in the linear-quadratic case

- The *MFG equilibrium* has the representation:

$$\alpha_t^\star = \frac{1}{A+K}\Big(-KQ_t - p_0 - \pi p_1 \widehat{Q}_t^{st} - p_1(1-\pi)(\widehat{Q}_t + \widehat{\alpha}_t)$$
$$-\phi_t S_t^{\alpha^\star} - \psi_t - \left(f_0 + f_1\left(\widehat{Q}_t + \widehat{\alpha}_t - \alpha^{tg}\right)\right) J_t^\theta\Big),$$

  where ϕ solves a Ricatti BSDE with jumps and ψ a linear BSDE with jumps.

- In the linear-quadratic setting, we get the existence of an ε_n-Nash equilibrium $(\alpha^{\varepsilon,1}, \ldots, \alpha^{\varepsilon,n})$, where

$$\alpha_t^{\varepsilon,i} = \frac{1}{A+K}\Big(-KQ_t^i - p_0 - \pi p_1 \widehat{Q}_t^{st} - p_1(1-\pi)(\widehat{Q}_t + \widehat{\alpha}_t)$$
$$-\phi_t^i S_t^{\alpha^\star} - \psi_t^i - \left(f_0 + f_1\left(\widehat{Q}_t + \widehat{\alpha}_t - \alpha^{tg}\right)\right) J_t^\theta\Big).$$

Section 3

Mean-field control problem

Mean-field control problem

The central planner problem: MFC problem. The mean-field type control problem corresponds to the problem of a *central planner* who wants to optimise the objective of the global population (standard and DSM consumers). The objective functional takes the following form

$$
\begin{aligned}
J^{MFC}(\alpha) =& \mathbb{E}\Bigg[(1-\pi)\int_0^T \Big(g(\alpha_t, S_t^\alpha, Q_t) + (Q_t+\alpha_t)p\Big(\pi \widehat{Q}_t^{st} + (1-\pi)(\widehat{Q}_t + \widehat{\alpha}_t)\Big) \\
& + l(Q_t+\alpha_t) + J_t^\theta(\tilde{Q}_t + \alpha_t - \alpha^{tg}) f\Big(\widehat{\tilde{Q}_t} + \widehat{\alpha}_t - \alpha^{tg}\Big)\Big)\, dt \\
& + (1-\pi)h(S_T^\alpha) + \pi\int_0^T \Big(Q_t^{st} p\Big(\pi \widehat{Q}_t^{st} + (1-\pi)(\widehat{Q}_t + \widehat{\alpha}_t)\Big) + l(Q_t^{st})
\end{aligned}
\tag{3}
$$

The optimization problem of the central planner writes as follows:

$$
V^{MFC} = \inf_{\alpha \in \mathcal{A}} J^{MFC}(\alpha). \tag{4}
$$

Similarly to the MFG setting, the equilibria can be characterized in terms of the solution of a coupled FBSDE system.

Link between MFC and MFG

Lemma

Let $\alpha^\star$ be a mean-field optimal control for the problem with pricing rules p_{MFC} and f_{MFC}. Then it is a mean-field Nash equilibrium for the MFG problem with pricing rules p_{MFG} and f_{MFG}:

$$p_{MFG}(x) = p_{MFC}(x) + xp'_{MFC}(x),$$
$$f_{MFG}(x) = f_{MFC}(x) + xf'_{MFC}(x).$$

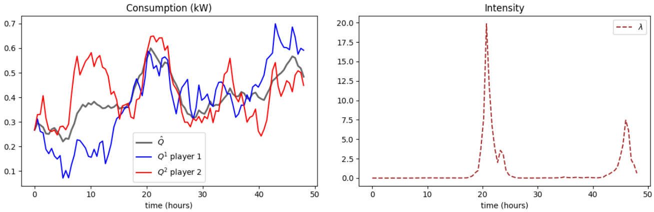
The reverse also holds true.

Section 4

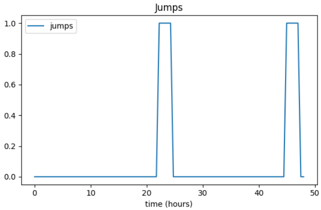
Numerical results

Numerical results

Scenario considered

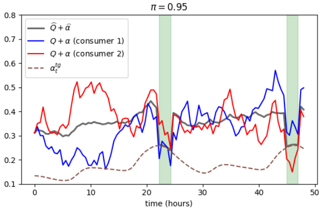


One trajectory of $\widehat{Q}$ and Q (in kW) for two different consumers (left) and one trajectory of λ_t^0 (right) along time (in half-hours).

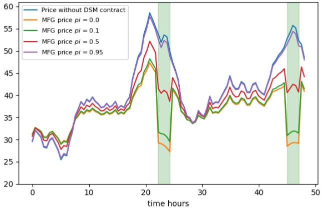


One trajectory of the jump process (J_t).

Numerical results

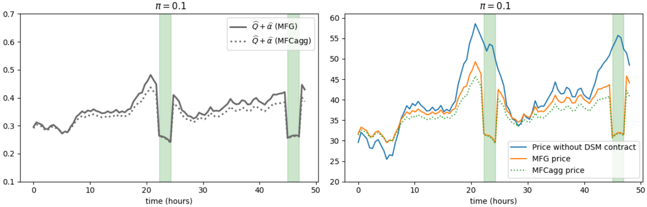


One trajectory of $\widehat{Q} + \widehat{\alpha}$ and $Q + \alpha$ (in kW) for two different consumers (left) in the MFG setting.



Trajectories of the price p for four different proportions of active consumers in the MFG setting.

Numerical results



Trajectories of price p (right) and $\widehat{Q} + \widehat{\alpha}$ in kW (left) for MFG setting (plain lines) compared to MFC setting (dotting lines)

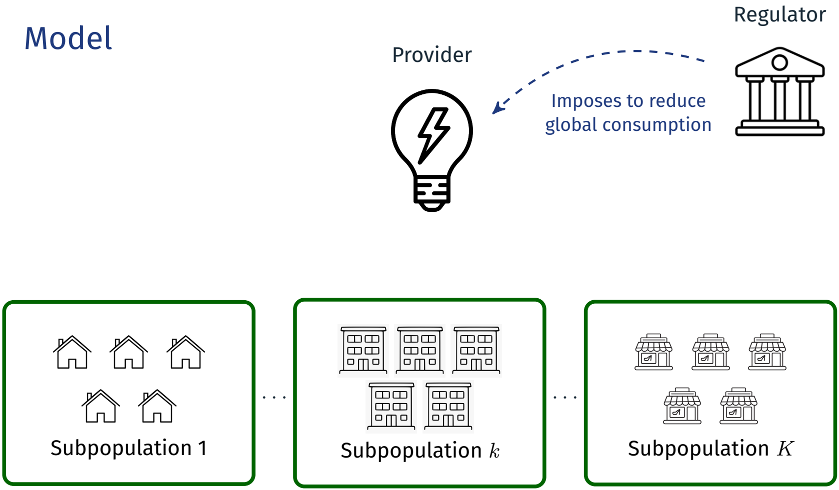
Part II

A Rank-Based Reward between a Principal and a Field of Agents: Application to Energy Savings

Section 5

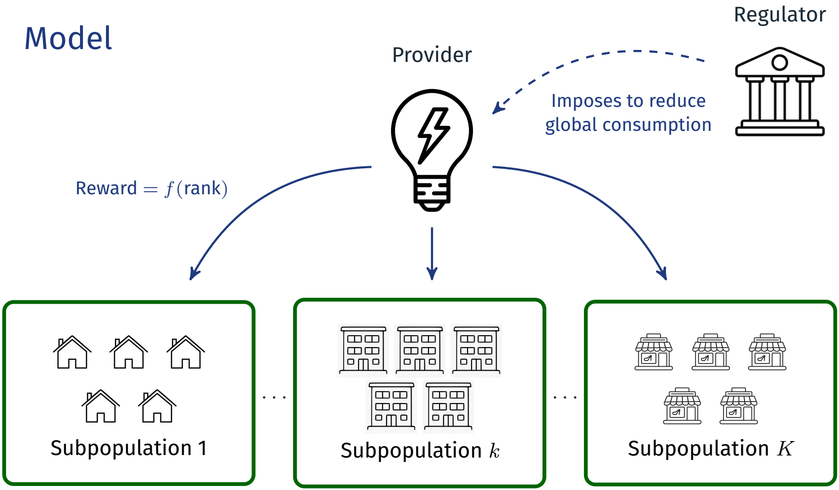
Ranking games model

Model

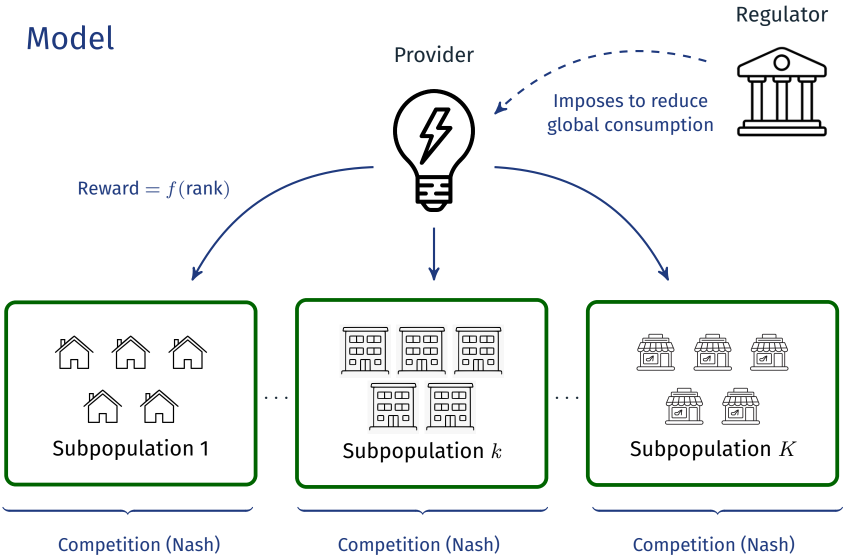


Regulator
Provider
Imposes to reduce global consumption
Subpopulation 1
Subpopulation k
Subpopulation K

Model


Regulator
Provider
Imposes to reduce global consumption
Reward $= f(\text{rank})$
Subpopulation 1
. . .
Subpopulation k
. . .
Subpopulation K

Model



Regulator
Provider
Imposes to reduce
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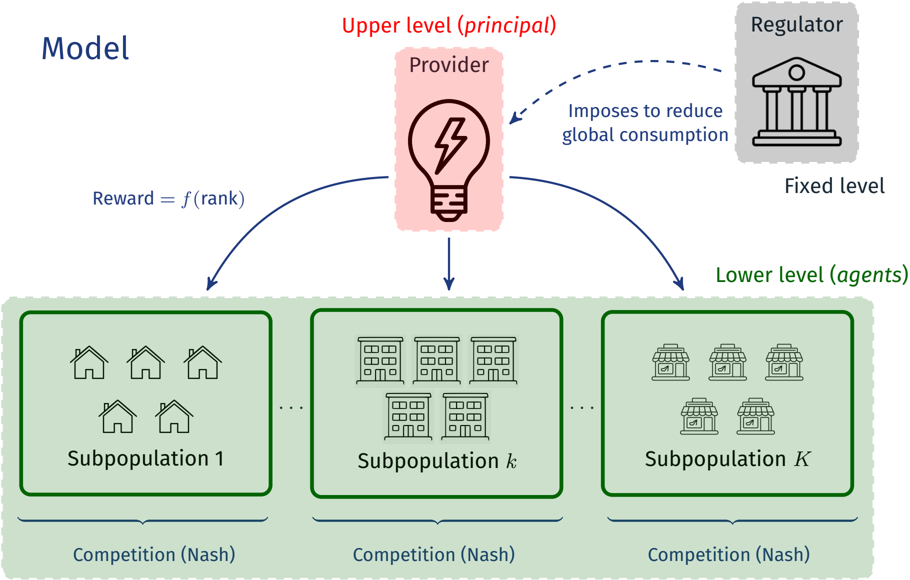
Reward $= f(\text{rank})$
Subpopulation 1
. . .
Subpopulation k
. . .
Subpopulation K
Competition (Nash)
Competition (Nash)
Competition (Nash)

Model



Upper level (*principal*)
Regulator
Provider
Imposes to reduce global consumption
Fixed level
Reward = f(rank)
Lower level (*agents*)
Subpopulation 1
. . .
Subpopulation k
. . .
Subpopulation K
Competition (Nash)
Competition (Nash)
Competition (Nash)

Section 6

Agents' problem

The field of agents at the *lower level*

- The population is divided into K clusters of *indistinguishable* consumers. Each cluster $k \in [K]$ represents a proportion ρ_k.
- $X_k^a(t)$ the *energy consumption* of a representative customer of cluster k, forecasted at time t for consumption at $t < T$:

$$X_k^a(t) = X_k(0) + \int_0^t a_k(s)\,ds + \sigma_k \int_0^t dW^k(s), \quad X_k(0) = x_k^{\text{nom}} \quad , \tag{5}$$

  with
  - $(W_t^k)_{1 \le k \le K}$ a family of K independent Brownian motions
  - a_k is a $\mathbb{F}$- progressively measurable process satisfying $\mathbb{E} \int_0^T |a(s)|^2\,ds < \infty$.

Interpretation:

- ◇ a_k is the consumer's *effort* to reduce his electricity consumption.
- ◇ Without effort ($a \equiv 0$), customers have a mean *nominal* consumption x_k^{nom}, and the terminal p.d.f. of $X_k^a(T)$ is:

$$f_k^{\text{nom}}(x) := \varphi\left(x;\, x_k^{\text{nom}}, \sigma_k\sqrt{T}\right) \quad ,$$

  where $\varphi(\,\cdot\,; \mu, \sigma)$ is the pdf for $\mathcal{N}(\mu, \sigma)$.

Rank-based reward

In the N-players game setting:

- Each subpopulation k contains N_k players
- The terminal ranking of a player i, consuming $X_k^i(T)$, is measured by

$$\frac{1}{N_k}\sum_{j=1}^{N_k} \mathbb{1}_{X_k^j(T) \leq X_k^i(T)} \qquad \left(\begin{array}{c}\textit{empirical cumulative}\\ \textit{distribution}\end{array}\right)$$

$\Rightarrow$ The reward function should be decreasing (Low rank = good energy saver)

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In the mean-field setting:

- If $X_k(T) \sim \mu_k$, the *terminal ranking* of a player consuming x is $r = F_{\mu_k}(x)$

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Assumption: Each sub-population k receives a reward R_k of the form

$$\mathbb{R} \times [0,1] \ni (x,r) \mapsto R_k(x,r) = B_k(r) - px \ , \tag{6}$$

- ◇ We call R the *total reward* and B_k the *additional reward*.
- ◇ $-px$ represents the *natural incentive* to reduce the consumption, coming from the price p to consume one unit of energy
- ◇ When $R_k(x,r)$ is independent of x, the reward is *purely ranked-based*

Mean-field game between consumers

Representative agent's problem (cluster k):

- Given the reward R_k and the terminal consumption distribution $\tilde{\mu}_k$,

$$V_k(R_k, \tilde{\mu}_k) := \sup_a \mathbb{E}\left[R_{k,\tilde{\mu}_k}(X_k^a(T)) - \underbrace{\int_0^T c_k a_k^2(t)\, dt}_{\text{cost of effort}} \right], \qquad (P^{\text{cons}})$$

  where $R_{k,\mu}(x) = R_k(x, F_\mu(x))$.

Interpretation:

- The cost corresponds to the purchase of new equipment (new heating installation, isolation, ...).
- In exchange, the consumer receives $B(r)$, depending on his rank $r = F_{\tilde{\mu}_k}(x)$, where $\tilde{\mu}_k$ is the k-subpopulation's distribution.
- The quantity $V_k(R_k, \tilde{\mu}_k)$ is called the *optimal utility* of an agent of k.

Agents' best response

Characterization of the best response (Bayraktar and Zhang, 2021,Proposition 2.1)

Given $R \in \mathcal{R}_k$ and $\tilde{\mu}_k \in \mathcal{P}(\mathbb{R})$, let

$$\gamma_k(\tilde{\mu}) = \int_{\mathbb{R}} f_k^{\text{nom}}(x) \exp\left(\frac{R_{k,\tilde{\mu}}(x)}{2c_k\sigma_k^2}\right) dx \quad (< \infty) \ . \tag{7}$$

Then, the *optimal terminal distribution* μ_k^* of cluster k has p.d.f.

$$f_{\mu_k^*}(x) = \frac{1}{\beta(\tilde{\mu}_k)} f_k^{\text{nom}}(x) \exp\left(\frac{R_{k,\tilde{\mu}_k}(x)}{2c_k\sigma_k^2}\right) \ , \tag{8}$$

and the optimal value is then $V_k(R_k, \tilde{\mu}_k) = 2c_k\sigma_k^2 \ln \gamma_k(\tilde{\mu}_k)$.

Definition (mean-field equilibrium): $\mu_k \in \mathcal{P}(\mathbb{R})$ is an *equilibrium* if it is a fixed-point of the *best response* map

$$\Phi_k \ : \ \tilde{\mu}_k \mapsto \mu_k^* \ ,$$

with μ_k^* given by (8).

Nash Equilibrium

For purely ranked-based reward (Bayraktar and Zhang, 2021, Theorem 3.2)

An equilibrium ν_k exists and is *unique* and the quantile is given by

$$q_{\nu_k}(r) = x_k^{\text{nom}} + \sigma_k \sqrt{T} N^{-1} \left(\frac{\int_0^r \exp\left(-\frac{B_k(z)}{2c_k\sigma_k^2}\right) dz}{\int_0^1 \exp\left(-\frac{B_k(z)}{2c_k\sigma_k^2}\right) dz} \right) . \tag{9}$$

Theorem

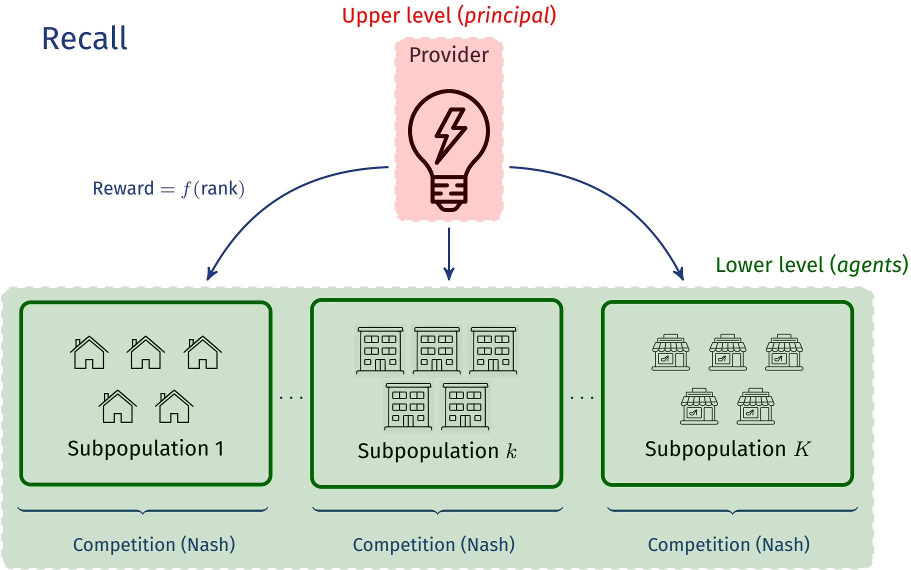
Let $R_k(x,r) = B_k(r) - px$. Then, the equilibrium μ_k is *unique*, and satisfies

$$q_{\mu_k}(r) = q_{\nu_k}(r) - \frac{pT}{2c_k} , \tag{10}$$

where ν_k is the (unique) equilibrium distribution for $p = 0$ (purely ranked-based reward), defined in (9).

$\Rightarrow$ add of a linear part in "x" acts as a shift on the probability density function.

Recall



Upper level (*principal*)
Provider
Reward $= f(\text{rank})$
Lower level (*agents*)
Subpopulation 1
Subpopulation k
Subpopulation K
Competition (Nash)
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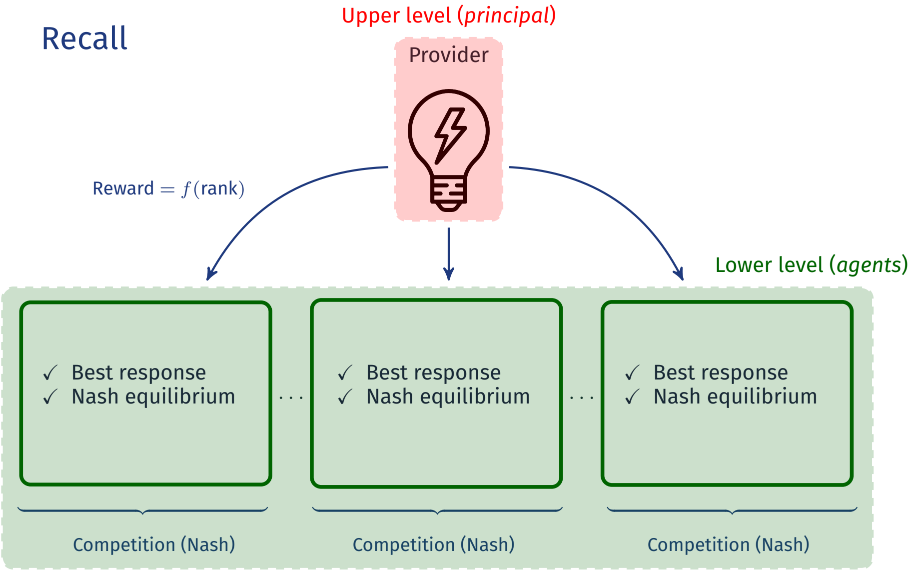
Recall


Upper level (*principal*)
Provider
Reward = f(rank)
Lower level (*agents*)
✓ Best response
✓ Nash equilibrium
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✓ Best response
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✓ Best response
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Competition (Nash)
Competition (Nash)
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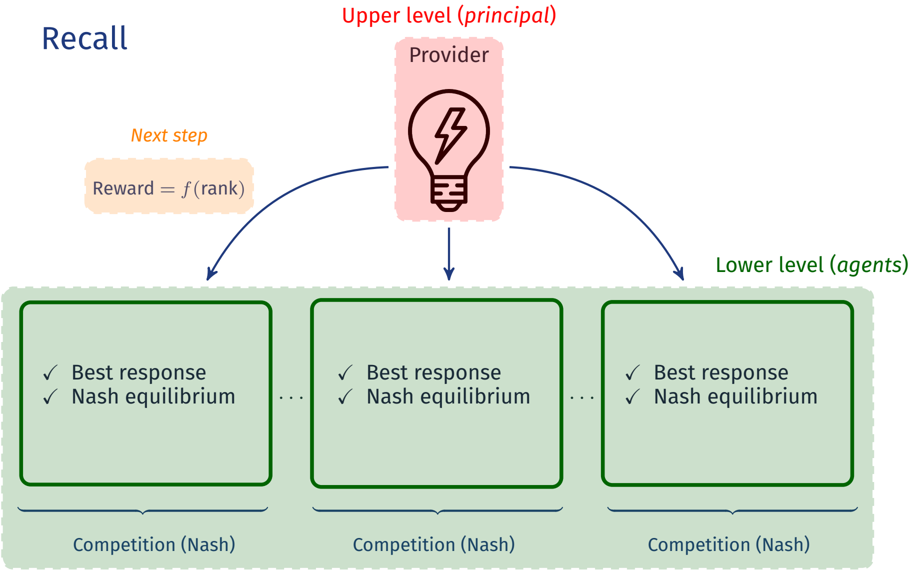
Recall


Upper level (*principal*)
Provider
Next step
Reward = f(rank)
Lower level (*agents*)
✓ Best response
✓ Nash equilibrium
· · ·
✓ Best response
✓ Nash equilibrium
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✓ Best response
✓ Nash equilibrium
Competition (Nash)
Competition (Nash)
Competition (Nash)

Section 7

Retailer's problem

Retailer's problem

- Assumption: $B_k(r) = x_k^{\text{nom}} \beta(r)$ (common reward across sub-populations).
- Notation: For an equilibrium $(\mu_k)_{k \in [K]}$, the mean consumption is $m_{\mu_k} = \int_0^1 q_{\mu_k}(r)\, dr$, and the overall mean consumption is $m_\mu = \sum_{k \in [K]} \rho_k m_{\mu_k}$.
- Principal's problem:

$$\max_{\beta \in \mathcal{B}} \left\{ (p - c_p) m_\mu - s(m_\mu) - \sum_{k \in [K]} \rho_k x_k^{\text{nom}} \int_0^1 \beta(r)\, dr \;\middle|\; \begin{array}{l} R_k(x,r) = x_k^{\text{nom}} \beta(r) - px \\ \mu_k = \epsilon_k(R_k) \\ V_k(R_k, \mu_k) \geq V_k^{\text{pi}} \end{array} \right\} \qquad (P^{\text{ret}})$$

where
  - $\mathcal{B}$ is the set of *bounded* and *decreasing* rewards,
  - $\mu_k = \epsilon_k(B)$ the *agents' equilibrium* given additional reward $B(\cdot)$,
  - $s(\cdot)$ denotes the *penalty* imposed by the regulator (to favor consumption reduction),
  - c_p denotes the *production cost* of energy,
  - V^{pi} is the *reservation utility* (utility when $B \equiv 0$)

In the sequel, we denote by $\kappa(\cdot)$ the function $\kappa : m \mapsto s(m) + c_p m$.

Optimal reward – Homogeneous population $(K = 1)$

Principal's problem:

$$\max_{\beta \in \mathcal{B}} \left\{ (p - c_p) m_\mu - s\,(m_\mu) - x^{\text{nom}} \int_0^1 \beta(r)\,dr \;\middle|\; \begin{aligned} &\mu = \epsilon(R) \\ &V(R, \mu) \geq V^{\text{pi}} \end{aligned} \right\} \qquad (P^{\text{ret}})$$

Optimal reward – Homogeneous population ($K = 1$)

Principal's problem:

Idea:

$$\max_{\beta \in \mathcal{B}} \left\{ (p - c_p) m_\mu - s(m_\mu) - x^{\text{nom}} \int_0^1 \beta(r)\, dr \;\middle|\; \begin{array}{l} \mu = \epsilon(R) \\ V(R, \mu) \geq V^{\text{pi}} \end{array} \right\} \qquad (P^{\text{ret}})$$

μ distrib.

$R = \epsilon^{-1}(\mu)$

$+\beta$ bounded and decreasing

Optimal reward – Homogeneous population ($K = 1$)

Principal's problem:

Idea: μ distrib. $\quad R = \epsilon^{-1}(\mu)$

$$\max_{\beta \in \mathcal{B}} \left\{ (p - c_p) m_\mu - s(m_\mu) - x^{\text{nom}} \int_0^1 \beta(r)\, dr \;\middle|\; \begin{array}{l} \mu = \epsilon(R) \\ V(R, \mu) \geq V^{\text{pi}} \end{array} \right\} \qquad (P^{\text{ret}})$$

$+\beta$ bounded and decreasing

Using the characterization of the equilibrium,

$$R_\mu(r) = V^{\text{pi}} + 2c\sigma^2 \ln(\zeta_\mu(q_\mu(r))) \qquad \left(= \epsilon^{-1}(\mu)\right) \;,$$

with $\zeta_\mu := f_\mu / f^{\text{nom}}$.

Reformulation in the distribution space:

$$(P^{\text{ret}}) \begin{cases} \min\limits_{\mu} \quad \kappa\left(\int_{-\infty}^{+\infty} y f_\mu(y)\, dy\right) + 2c\sigma^2 \int_{-\infty}^{+\infty} \ln\left(\frac{f_\mu(y)}{f^{\text{nom}}(y)}\right) f_\mu(y)\, dy \\ \text{s. t.} \quad \int_{-\infty}^{+\infty} f_\mu(y)\, dy = 1, \quad f_\mu(y) \geq 0 \\ \qquad y \mapsto \ln\left(\frac{f_\mu(y)}{f^{\text{nom}}(y)}\right) + \frac{p}{2c\sigma^2} y \text{ bounded and decreasing} \end{cases}$$

Optimal reward – Homogeneous population ($K = 1$)

Principal's problem:

$R = \epsilon^{-1}(\mu)$

Idea:
$$\max_{\beta \in \mathcal{B}} \left\{ (p - c_p) m_\mu - s(m_\mu) - x^{\text{nom}} \int_0^1 \beta(r)\, dr \;\middle|\; \begin{array}{l} \mu = \epsilon(R) \\ V(R, \mu) \geq V^{\text{pi}} \end{array} \right\} \qquad (P^{\text{ret}})$$

μ distrib.

$+\beta$ bounded and decreasing

Using the characterization of the equilibrium,

$$R_\mu(r) = V^{\text{pi}} + 2c\sigma^2 \ln\left(\zeta_\mu(q_\mu(r))\right) \qquad \left(= \epsilon^{-1}(\mu)\right) \ ,$$

with $\zeta_\mu := f_\mu / f^{\text{nom}}$.

Reformulation in the distribution space:

Relaxation

(P^{ret}) $(\widetilde{P}^{\text{ret}})$
$$\begin{cases} \min_\mu \quad \kappa \left(\int_{-\infty}^{+\infty} y f_\mu(y)\, dy \right) + 2c\sigma^2 \int_{-\infty}^{+\infty} \ln\left(\frac{f_\mu(y)}{f^{\text{nom}}(y)} \right) f_\mu(y)\, dy \\ \text{s.t.} \quad \int_{-\infty}^{+\infty} f_\mu(y)\, dy = 1, \quad f_\mu(y) \geq 0 \\ \quad y \mapsto \ln\left(\frac{f_\mu(y)}{f^{\text{nom}}(y)} \right) + \frac{p}{2c\sigma^2} y \text{ bounded and decreasing} \end{cases}$$

Optimal reward – Homogeneous population ($K = 1$)

Assumption: The function $s : \mathbb{R} \to \mathbb{R}$ is supposed to be increasing, convex and differentiable. Moreover, $\kappa'(0) \leq p \leq \kappa'(x^{\text{pi}})$.

Lemma

The optimal distribution μ^* for $(\widetilde{P}^{\text{ret}})$ satisfies the following equation:

$$f_\mu(y) = f^{\text{nom}}(y) \exp\left(-y\frac{\kappa'(m_\mu)}{2c\sigma^2}\right) \tag{11}$$

Theorem – Analytic formula of the optimal reward

Let $\delta(m) = p - \kappa'(m)$. The distribution $\mu^* = \mathcal{N}(m^*, \sigma\sqrt{T})$, where m^* satisfies

$$m^* = x^{\text{pi}} + \frac{T}{2c}\delta(m^*) \ , \tag{12}$$

is optimal for $(\widetilde{P}^{\text{ret}})$. Moreover, the associated reward B^* is

$$B_{\mu^*}(r) = \frac{c}{T}\left[(x^{\text{pi}})^2 - (m^*)^2\right] + q_{\mu^*}(r)\delta(m^*) \ . \tag{13}$$

Remark: The function $\delta(\cdot)$ is viewed as the *reduction desire* of the provider.

Optimal reward – Heterogeneous population ($K > 1$)

Theorem (Explicit characterization for a sub-class of heterogeneous population)

Let suppose that the following statement holds:

$$\forall k \in [K], \quad \frac{x_k^{\text{nom}}}{x_1^{\text{nom}}} = \frac{\sigma_k}{\sigma_1} = \frac{c_1}{c_k} \quad (:= \theta_k) \ . \tag{14}$$

Then, any $\mu_1, \ldots, \mu_K$ equilibrium distributions associated to a common unitary reward β solution of (P^{ret}) satisfies $f_{\mu_k}(y) = \frac{1}{\theta_k} f_{\mu_1}\left(\frac{y}{\theta_k}\right)$ for all $k \in [K]$. Moreover, the retailer's profit problem simplifies to

$$\pi^* := \bar{\theta} \max_{\beta \in \mathcal{B}} \left\{ p m_{\mu_1} - \tilde{\kappa}(m_{\mu_1}) - x_1^{\text{nom}} \int_0^1 \beta(r)\, dr \ \middle|\ \begin{array}{l} R_1(x,r) = x_1^{\text{nom}} \beta(r) - px \\ \mu_1 = \epsilon_1(R_1) \\ V_1(R_1, \mu_1) \geq V_1^{\text{pi}} \end{array} \right\} , \tag{15}$$

with $\tilde{\kappa}(m) = \bar{\theta}^{-1} \kappa(\bar{\theta} m)$ and $\bar{\theta} = \sum_{k \in [K]} \rho_k \theta_k$.

Section 8

Numerical results

Instance

Based on real data from the French retail market:

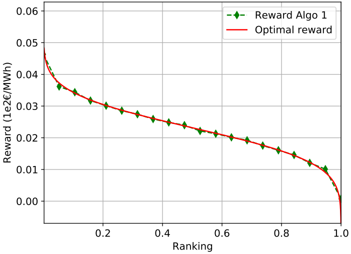
	Distribution	Housing	Heating	Nb occupants	Consumption (mean/year)
Sub-pop. 1	26%	House 70 m^2	Electric	3	9.9 MWh
Sub-pop. 2	49%	House 70 m^2	Non-electric	3	1.5 MWh
Sub-pop. 3	9%	House 150 m^2	Electric	4	20 MWh
Sub-pop. 4	16%	House 150 m^2	Non-electric	4	2.2 MWh

Table: Annual electricity consumption by type of usage.

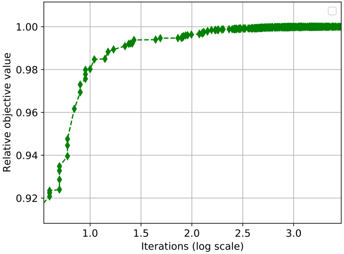
Power plant	Marginal cost (€/MWh)	Production (TWh)
Hydro/Wind/Solar	0 to 15	115
Nuclear	30	380
Gas	70	30
Coal	86	7
Fuel	162	5

Table: Marginal price and annual production.

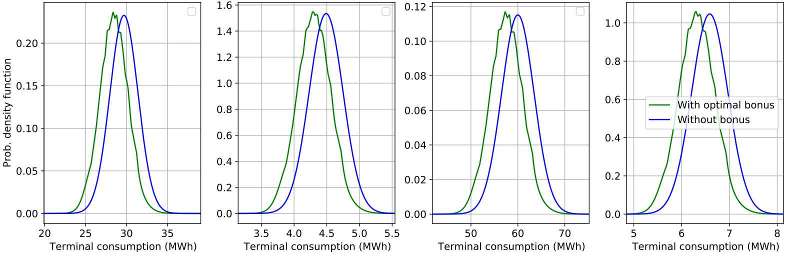
Results – Uniform elasticity



(a) Optimal reward.

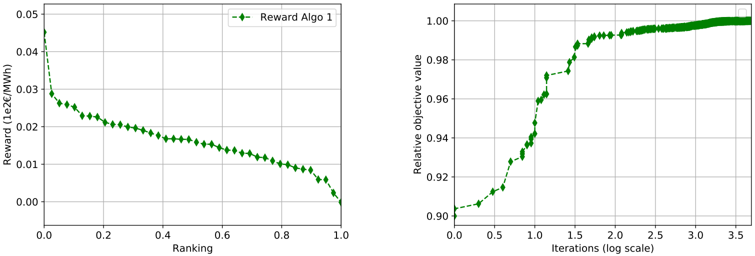


(b) Evolution of the relative objective value.



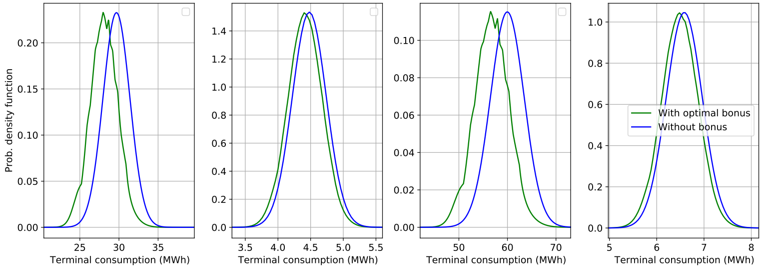
(c) Terminal consumption distribution for the four sub-populations

Results – Non-uniform elasticity



(a) Optimal reward.

(b) Evolution of the relative objective value.



(c) Terminal consumption distribution for the four sub-populations

Thank you!