# Asset pricing under transition scenario uncertainty

Maria Flora[1] Théo Le Guénédal[2] Peter Tankov[3]

[1]CREST, CNRS, IP Paris & CFM

[2]CREST, ENSAE, IP Paris & Amundi

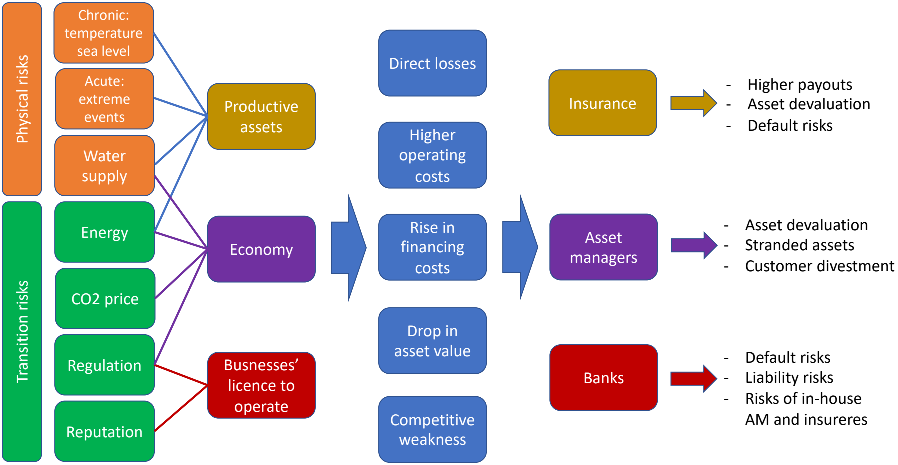
[3]CREST, ENSAE, IP Paris

FIME days, September 14, 2023

# Outline

1. Introduction: climate financial risks and the scenario approach
2. Pricing energy assets under transition scenario uncertainty
3. Pricing defaultable bonds under transition scenario uncertainty

# Climate financial risks: a tragedy of the horizons



# The scenario approach

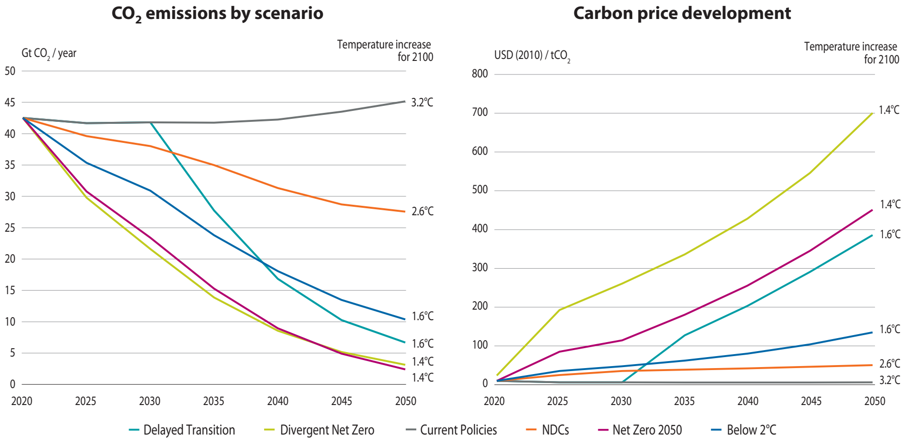
- Modern approaches to transition risk management are based on scenarios (Net Zero 2050, Delayed Transition, Disorderly Transition etc.);
- The risk is evaluated e.g., by comparing a bank's portfolio under orderly and disorderly transition scenarios;
- Scenario: representation of future evolution of a range of variables, such as CO2 emissions, technology penetration, energy demand, etc., resulting from an integrated assessment model;
- Produced and maintained by international organisms: IEA, IPCC, IIASA;
- A scenario is by construction exogeneous to the exercise of risk evaluation and may not be adapted to the problem at hand;
- A scenario has a reference value: different assessment exercises using the same scenaio may be compared between each other.

# NGFS scenario database

The Network for Greening the Financial System maintains a database of scenarios with annual updates, for use by central banks.

1. **Current Policies :** existing climate policies remain in place
2. **Nationally Determined Contributions (NDCs) :** currently pledged unconditional NDCs are implemented fully, and respective targets on energy and emissions in 2025 and 2030 are reached in all countries;
3. **Delayed Transition (Disorderly) :** there is a "fossil recovery" from 2020 to 2030; Only thereafter countries with a clear commitment to a specific net-zero policy target at the end of 2020 are assumed to meet the target
4. **Below 2°C :** the 67-percentile of warming is kept below 2°C throughout the 21st century
5. **Divergent Net Zero (Disorderly) :** median temperature below 1.5°C in 2100, after a limited temporary overshoot
6. **Net Zero 2050 :** global $CO_2$ emissions are at net-zero in 2050

# NGFS scenarios



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The aim is to enable dynamic pricing and hedging of assets sensitive to transition risks and transition scenario uncertainty.

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# The model

- A risk-neutral economic agent (owner of a power-generating asset), faces revenue risk and scenario uncertainty.
- Option to sell/decommission the asset at a future date $\tau$.
- Revenues of the asset depend on future values of risk factors (fuel prices, electricity price, carbon price) whose distribution depends on the realized scenario.
- There are $N$ scenarios corresponding to different climate, economic and policy assumptions.
- True scenario is not known but the agent observes a noisy signal (e.g., global CO2 emissions) and updates posterior probabilities of scenarios.

# Modeling the risk factors

- The risk factors $P_k$ (e.g. electricity price, fuel price, carbon emission allowances price) follow an autoregressive dynamics with mean-reversion rate $\phi_k$, volatility $\sigma_k$, and scenario-dependent mean $\mu^i_{k,t}$:

$$P_{k,t} = \mu^i_{k,t} + \widetilde{P}^k_t,$$

where $\widetilde{P}^k$ is an autoregressive component such that

$$\widetilde{P}^k_t = \phi_k \widetilde{P}^k_{t-1} + \sigma_k \varepsilon^k_t,$$

and $(\varepsilon^k_t)$ are i.i.d. standard Gaussian.

- The mean $\mu^i$ is taken from an integrated assessment model scenario.
- The autoregressive part models the local and high-frequency noises.

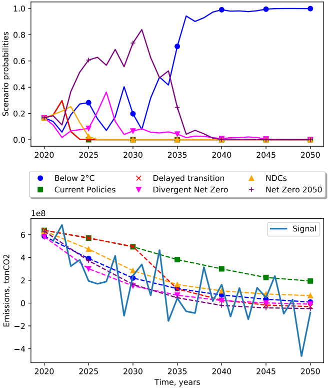
# Bayesian learning approach

- The signal is normally distributed with mean $\mu^i_{y,t}$ and volatility $\sigma_y$, that is

$$y_t = \mu^i_{y,t} + \sigma_y \eta_t\,, \qquad \eta_t \sim N(0,1) \text{ i.i.d.}$$

- The information about the scenario is encoded in a vector $\pi_t$ of subjective scenario probabilities:

$$\pi^i_t = \mathbb{P}[I = i|\mathcal{F}_t], \quad \mathcal{F}_t = \sigma(y_s, s \le t)\,.$$




- Bayesian update at each step:

$$\pi^i_t = \mathbb{P}[I = i|y_t, \mathcal{F}_{t-1}] = \frac{\mathbb{P}[I = i, y_t \in dy|\mathcal{F}_{t-1}]}{\mathbb{P}[y_t \in dy|\mathcal{F}_{t-1}]} = \pi^i_{t-1}\frac{\mathbb{P}[y_t \in dy|I = i, \mathcal{F}_{t-1}]}{\mathbb{P}[y_t \in dy|\mathcal{F}_{t-1}]}.$$

# Pricing the real option: exit from a brown plant

- Agent owns a brown plant and optimizes the time to decommission / sell the plant with P&L function $h^b(\mathrm{P}_t)$ in year $t$
- The optimization problem of the agent is of the form

$$\sup_{\tau \in \mathcal{T}} \mathbb{E}\left[\sum_{s=1}^{\tau} \beta^s h^b(\mathrm{P}_s) - \beta^\tau K(\tau)\right]$$

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$$\sup_{\tau \in \mathcal{T}} \mathbb{E}\left[\sum_{s=1}^{\tau} \beta^s h^b(\mathsf{P}_s) - \beta^\tau K(\tau)\right]$$

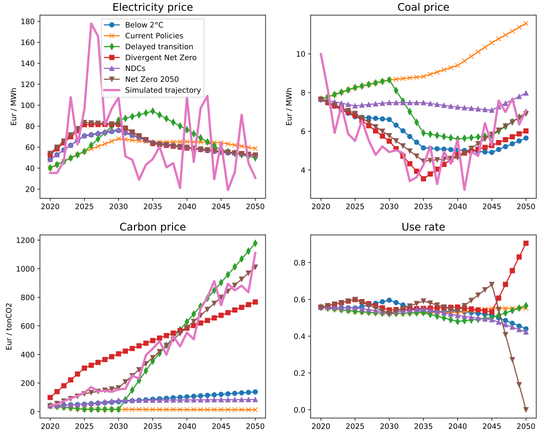
- We define the associated value function

$$V(\widetilde{\mathsf{P}}, \hat{\boldsymbol{\pi}}) = \sup_{\tau \in \mathcal{T}_t} \mathbb{E}\left[\sum_{s=t+1}^{\tau} \beta^{(s-t)} h^b(\mathsf{P}_s) - \beta^{\tau-t} K(\tau) \,\middle|\, (\widetilde{\mathsf{P}}_t, \hat{\boldsymbol{\pi}}_t) = (\widetilde{\mathsf{P}}, \hat{\boldsymbol{\pi}})\right]$$

and solve the optimal stopping problem using least squares Monte Carlo.

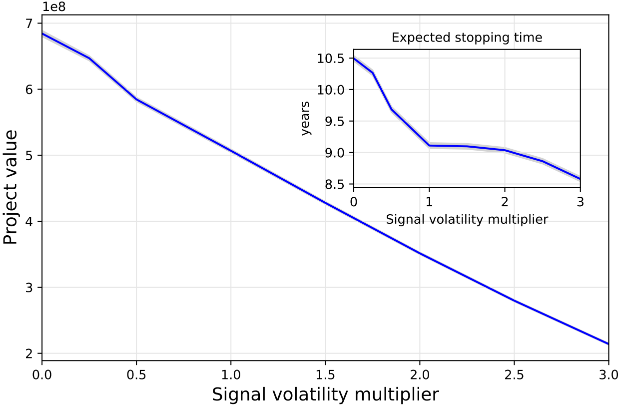
# Optimal exit problem

- Integrated coal gasification plant without CCS, located in Germany
- 3 risk factors: price of electricity, price of coal and price of carbon
- Utilization rate is deterministic in each scenario



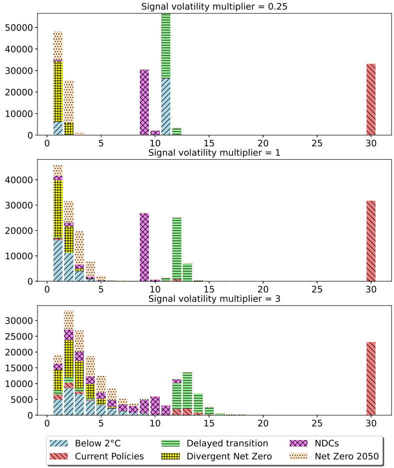
# Optimal exit problem: Results

- Sensitivity of the RO value to the volatility of the signal $\sigma_y$ (signal: total GHG emissions)



# Distribution of optimal exit times

- Distribution of optimal exit times for different uncertainty levels in different scenarios
- As uncertainty level increases, the asset becomes stranded earlier and the exit decision is taken at suboptimal time



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# A structural default model under transition scenario uncertainty

- We assume that the carbon price dynamics is given by $\Delta C \cdot N_t$, where $N$ is a Poisson process, whose intensity $\Lambda$ is an unobservable random variable taking values $\lambda_1, \dots, \lambda_n$ in $n$ different transition scenarios.
- Simplified framework with constant intensities to reduce problem dimension

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- Simplified framework with constant intensities to reduce problem dimension
- The cash flow dynamics of the company with carbon intensity $I$ is affected by the carbon price and given by

$$\frac{dV_t}{V_t} = \mu dt + \sigma dW_t - \alpha dN_t, \quad \alpha = I \Delta C.$$

- We are interested in computing the value of the company, its optimal default threshold, and the price of a bond emitted by the company.

# Filtering scenario probabilities

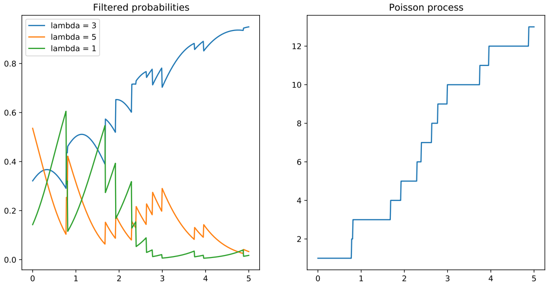
Denote by $(\mathcal{F}_t)$ the observation filtration and let $\hat{p}_t^i = \mathbb{E}[\Lambda = \lambda_i | \mathcal{F}_t]$.

In $(\mathcal{F}_t)$, the intensity of $N$ is $\hat{\lambda}_t = \sum_{i=1}^n \lambda_i \hat{p}_t^i$, and the filtered probabilities are

$$\hat{p}_t^i = \frac{e^{-\lambda_i t} \lambda_i^{N_t} \hat{p}_0^i}{\sum_j e^{-\lambda_j t} \lambda_j^{N_t} \hat{p}_0^j}$$

so that

$$\hat{\lambda}_t = \frac{\sum_i e^{-\lambda_i t} \lambda_i^{N_t+1} \hat{p}_0^i}{\sum_j e^{-\lambda_j t} \lambda_j^{N_t} \hat{p}_0^j},$$




$\Rightarrow$ in $(\mathcal{F}_t)$, the couple $(V_t, N_t)$ is a (time-inhomogeneous) Markov process,

# Computing firm value

- A model with endogenous bankruptcy and optimal default threshold, inspired by Leland and Toft '96, and subsequent literature.
- The company pays a continuous stream of coupons $b(t)$ until maturity date $T$ or until default date $\tau$, whichever occurs first.
- At maturity, the creditors are entitled to receive the notional amount, denoted by $K$.
- The default date is determined by the company's owners to maximize equity value.

# Computing firm value

In the absence of arbitrage, the value of the future cash flows at time $t$ is

$$\widehat{V}_t = \mathbb{E}\left[\int_t^\infty e^{-r(s-t)} V_s \middle| \mathcal{F}_t\right] = \sum_{i=1}^n \mathbb{P}[I = i|] \, \mathbb{E}\left[\int_t^\infty e^{-r(s-t)} V_s \middle| \mathcal{F}_t, I = i\right]$$

$$= V_t \sum_{i=1}^n \frac{\hat{p}_t^i}{r + e\lambda_i - \mu} = V_t \, \alpha_t^{N_t},$$

where we denote

$$\alpha_t^N := \sum_{i=1}^n \frac{1}{r + \alpha\lambda_i - \mu} \frac{e^{-\lambda_i t} \lambda_i^N \hat{p}_0^i}{\sum_j e^{-\lambda_j t} \lambda_j^N \hat{p}_0^j}.$$

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The equity value at time $t$ is then given by

$$\sup_{\tau \in \mathcal{T}([t,T])} \mathbb{E}\left[\int_t^{\tau\wedge T} e^{-r(s-t)} (V_s^{t,V,N} - b(s))ds + e^{-r(T\wedge\tau - t)} (\widehat{V}_{T\wedge\tau}^{t,V,N} - K)^+\right],$$

where $\mathcal{T}([t,T])$ is the set of $(\mathcal{F}_t)$-stopping times with values in $[t,T]$.

# Computing the bond price

The bond price is given by

$$B^N(t,V) = V\alpha_t^N - \sup_{\tau\in\mathcal{T}([t,T])} \mathbb{E}\Big[\int_t^{\tau\wedge T} e^{-r(s-t)}(V_s^{t,V,N} - b(s))ds \\ + e^{-r(T\wedge\tau - t)}(\widehat{V}_{T\wedge\tau}^{t,V,N} - K)^+\Big] \\ = \inf_{\tau\in\mathcal{T}([t,T])} \mathbb{E}\Big[\int_t^{\tau\wedge T} e^{-r(s-t)} b(s)\, ds + e^{-r(T\wedge\tau - t)}\widehat{V}_{T\wedge\tau}^{t,V,N} \wedge K\Big].$$

We define contunuation and default/restructuring regions:

$$\mathcal{C}^N = \{(t,V) : B^N(t,V) < \alpha_t^N V \wedge K\}$$

and exercise (default or restructuring) region

$$\mathcal{E}^N = \{(t,V) : B^N(t,V) = \alpha_t^N V \wedge K\}$$

# Default and restructuring thresholds

i. Assume that $rK \geq b$. Then $\mathcal{C}^N = \{(t, V) : t \in [0, T], V > V_D^N(t)\}$, where the default threshold satisfies $V_D^N(t) \leq b$ for all $t \in [0, T]$.

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ii. Assume that $rK < b$. Then $\mathcal{C}^N = \{(t, V) : t \in [0, T], V_U^N(t) > V > V_D^N(t)\}$, where the default threshold satisfies $V_D^N(t) \leq b$ and the restructuring threshold satisfies $\alpha^N(t)V_R^N(t) \geq K$ for all $t \in [0, T]$.

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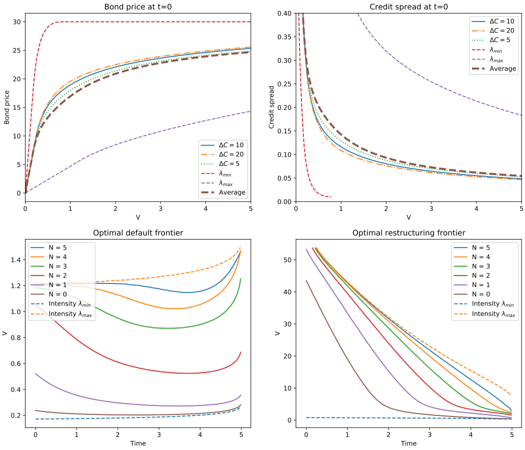
   $\Rightarrow$ Default may only occur when the cash flow of the company is smaller than the coupon, and the restructuring may occur only when the value of the future cash flows is greater than the notional of the debt.

# Numerical illustration




Top: bond price and credit spread as function of revenues $V$;

Bottom: default and restructuring thresholds as function of time.

Carbon price sensitivity: $\alpha_{10} = 10\%$.

Faster discovery of scenario information leads to higher credit spreads since better information allows the shareholders to optimize the timing of default.

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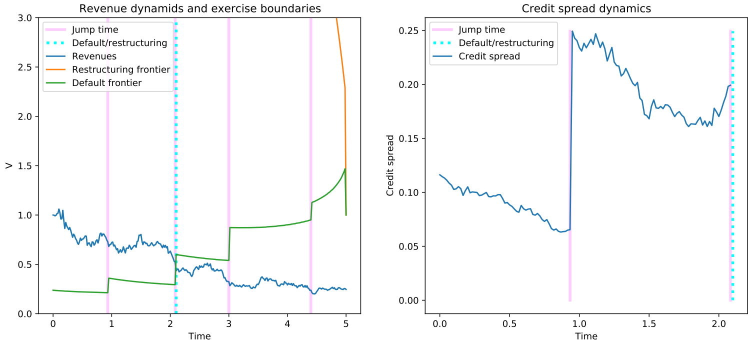


Sample trajectories of the firm's annualized revenues (left) and credit spread (right).

Under transition scenario uncertainty, carbon price adjustments are more likely to trigger a default because after each adjustment the more stringent scenario becomes more likely.

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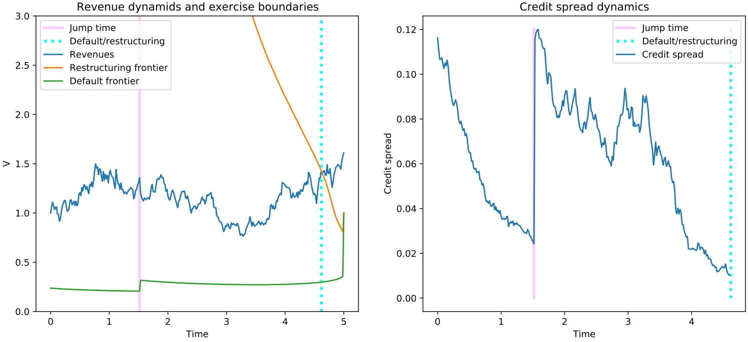


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# Extensions / ongoing work

- Model calibration
- Technological change
- Beyond scenario uncertainty: model ambiguity