A pure dual approach for hedging Bermudan options

Aurélien Alfonsi

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Outline

1. Introduction
2. Main results and algorithm
3. The financial framework
4. Numerical experiments

Computing Bermudan options prices

- A discrete time (discounted) payoff process $(Z_k)_{0\leq k\leq N}$ adapted to $(\mathcal{F}_k)_{0\leq k\leq N}$. $\max_{0\leq k\leq N}|Z_k|\in L^p$, $p\geq 1$.
- The (discounted) value of the Bermudan option is given by

$$U_k = \operatorname{esssup}_{\tau\in\mathcal{T}_k}\mathbb{E}[Z_\tau|\mathcal{F}_k]$$

where $\mathcal{T}_k$ is the set of all $\mathcal{F}-$ stopping times with values in $\{k, k+1, \dots, N\}$.
- From the Snell enveloppe theory, we derive the standard dynamic programming algorithm

$$\begin{cases} U_N = Z_N \\ U_k = \max\left(Z_k, \mathbb{E}[U_{k+1}|\mathcal{F}_k]\right) \end{cases} \tag{1}$$

Classical (primal) valuation methods

- Monte-Carlo Least square regression is used to approximate $\mathbb{E}[U_{k+1}|\mathcal{F}_k]$ on a regression basis.
- [Tsitsiklis and Van Roy(1997)] use then directly (1) to approximate the value.
- [Longstaff and Schwartz(2001)] instead approximate the optimal stopping rule $\tau^\star$ and then approximate the value. This gives a lower bound.

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- However, the option price should be for the option seller the value of the hedging portfolio.
- These prices are pointless if we do not know how to build the corresponding hedging portfolio.

Some existing (primal) methods for hedging

- Several papers such as [Wang and Caflisch(2010)] or [Belomestny, Milstein, and Schoenmakers(2010)] have proposed algorithms to compute the sensitivies (Greeks) for Bermudan options.
- However, when there are many underlyings, it may be not straightforward to find/select a portfolio with given financial instruments that matches these sensitivities.

The dual formulation of the price

- Dual representation ([Rogers(2002), Rogers(2010), Haugh and Kogan(2004)])

$$U_n = \inf_{M \in \mathbb{H}^p} \mathbb{E}\left[\max_{n \le j \le N} \{Z_j - (M_j - M_n)\} \middle| \mathcal{F}_n \right] \quad (2)$$

where $\mathbb{H}^p$ is the set of $\mathcal{F}$-martingales that are L^p integrable.

- From the Doob-Meyer decomposition

$$U_n = U_0 + M_n^\star - A_n^\star, \quad (3)$$

where $M^\star \in \mathbb{H}^p$ vanishes at 0 and $A^\star$ is a predictable, nondecreasing and L^p-integrable process. Then, $M^\star$ solves (2) and

$$U_n = \max_{n \le j \le N} \{Z_j - (M_j^\star - M_n^\star)\}$$

(almost surely optimal martingales).

The dual formulation as an hedging portfolio

- Let $M \in \mathbb{H}^p$ be a martingale such that $M_0 = 0$. Then, $V_0 = \mathbb{E}[\max_{0 \le n \le N}\{Z_n - M_n\}] \ge U_0$.
- $V_0 + M_n$ can be interpreted as the value at time n of a self-financing portfolio.
- We can prove for $p = 2$ that $\mathbb{E}[|Z_{\tau^\star} - (V_0 + M_{\tau^\star})|^2]^{1/2} \le 3\mathbb{E}[|M_N^\star - M_N|^2]^{1/2}$.
- As noticed by Rogers, if $M^\star$ is tradable, it is a perfect hedge.
- The dual problem is convex but may admit many solutions. See [Schoenmakers, Zhang, and Huang(2013)] for the characterization of almost surely optimal martingales.
- How to approximate $M^\star$? $\Rightarrow$ Find a new dual representation.

Existing algorithms based on the dual formulation

- The dual formula has mostly been used to give a price upper bound. [Andersen and Broadie(2004)] proposes indeed to approximate $M^\star$ with the LS algorithm and then to use the dual formula (2).
- [Rogers(2002)] presents an algorithm minimizing for a given martingale M^0:

$$\min_{\lambda \in \mathbb{R}} \mathbb{E}\left[\max_{0 \leq j \leq N} \{Z_j - \lambda M_j^0\}\right].$$

  Works well if M^0 is "close to $M^\star$".
- [Rogers(2010)] advocates for a "pure dual" algorithm (i.e. only based on the dual formula) and proposes a formal algorithm.
- [Desai, Farias, and Moallemi(2012)] propose to solve (2) in a Markovian setting ($Z_j = \Psi(X_j)$) with linear programming:

$$\text{minimize} \frac{1}{Q}\sum_{q=1}^{Q} u_q \text{ s.t. } u_q \geq \Psi(X_j) - \sum_{r=1}^{R} \alpha_r \mathbb{E}[\Phi_r(X_N)|X_j = x_j^q],\ 0 \leq j \leq N, 1 \leq q \leq Q.$$

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The excess reward representation I

With $\Delta M_n = M_n - M_{n-1}$,

$$\max_{0\le j\le N}\{Z_j-(M_j-M_0)\}$$

$$= Z_N-(M_N-M_0)+\sum_{n=0}^{N-1}\max_{n\le j\le N}\{Z_j-M_j\}-\max_{n+1\le j\le N}\{Z_j-M_j\}$$

$$= Z_N-(M_N-M_0)+\sum_{n=0}^{N-1}\max_{n\le j\le N}\{Z_j-(M_j-M_n)\}-\max_{n+1\le j\le N}\{Z_j-(M_j-M_n)\}$$

$$= Z_N-(M_N-M_0)+\sum_{n=0}^{N-1}\left(Z_n+\Delta M_{n+1}-\max_{n+1\le j\le N}\left\{Z_j-\sum_{i=n+2}^{j}\Delta M_i\right\}\right)_+ .$$

The excess reward representation II

By taking expectation,

$$\mathbb{E}\left[\max_{0\le j\le N}\{Z_j-(M_j-M_0)\}\right]$$
$$=\mathbb{E}[Z_N]+\sum_{n=0}^{N-1}\mathbb{E}\left[\left(Z_n+\Delta M_{n+1}-\max_{n+1\le j\le N}\left\{Z_j-\sum_{i=n+2}^{j}\Delta M_i\right\}\right)_+\right].$$

For $M=M^\star$, the red terms are equal to

$$\mathbb{E}[(Z_n-\mathbb{E}[U_{n+1}|\mathcal{F}_n])_+]$$

and represents the values of having the right to exercise the option at time $n\in\{0,\dots,N-1\}$.

A sequence of optimization problems I

Introduce the space of mg increments between $n-1$ and n:

$$\mathcal{H}_n^p = \{Y \in \mathbb{L}^p(\Omega) : Y \text{ is real valued}, \mathcal{F}_n-\text{mesurable and } \mathbb{E}[Y|\mathcal{F}_{n-1}] = 0\}.$$

It is tempting to solve backward from $n = N-1$ to $n = 0$

$$\inf_{\Delta M_{n+1} \in \mathcal{H}_{n+1}^p} \mathbb{E}\left[\left(Z_n + \Delta M_{n+1} - \max_{n+1 \leq j \leq N}\left\{Z_j - \sum_{i=n+2}^{j} \Delta M_i\right\}\right)_+\right].$$

However, the non strict convexity of the positive part raises some issues in the back propagation of the minimisation problems.

A sequence of optimization problems II

Theorem

Let $\varphi : \mathbb{R} \to \mathbb{R}$ be a convex function such that $|\varphi(x)| \leq C(1 + |x|^p)$. Then,

$$\mathbb{E}[Z_N] + \sum_{n=0}^{N-1} \mathbb{E}\left[\varphi\left(Z_n + \Delta M_{n+1} - \max_{n+1 \leq j \leq N}\left\{Z_j - \sum_{i=n+2}^{j} \Delta M_i\right\}\right)\right]$$

$$\geq \mathbb{E}[Z_N] + \sum_{n=0}^{N-1} \mathbb{E}\left[\varphi\left(Z_n + \Delta M^\star_{n+1} - \max_{n+1 \leq j \leq N}\left\{Z_j - \sum_{i=n+2}^{j} \Delta M^\star_i\right\}\right)\right],$$

and $M^\star$ is a solution of the following problems for $n = N-1, \dots, 0$

$$\inf_{\Delta M_{n+1} \in \mathcal{H}^p_{n+1}} \mathbb{E}\left[\varphi\left(Z_n + \Delta M_{n+1} - \max_{n+1 \leq j \leq N}\left\{Z_j - \sum_{i=n+2}^{j} \Delta M_i\right\}\right)\right]. \quad (4)$$

When φ is strictly convex, $M^\star$ is the unique solution of (4).

A sequence of optimization problems III

Proof by backward induction. By Jensen's inequality, we have

$$\mathbb{E}\left[\varphi\left(Z_n + \Delta M_{n+1} - \max_{n+1\le j\le N}\left\{Z_j - \sum_{i=n+2}^{j}\Delta M_i\right\}\right)\middle|\mathcal{F}_n\right]$$
$$\ge \varphi\left(\mathbb{E}\left[Z_n + \Delta M_{n+1} - \max_{n+1\le j\le N}\left\{Z_j - \sum_{i=n+2}^{j}\Delta M_i\right\}\middle|\mathcal{F}_n\right]\right),$$

with equality if

$$Z_n + \Delta M_{n+1} - \max_{n+1\le j\le N}\left\{Z_j - \sum_{i=n+2}^{j}\Delta M_i\right\} \in \mathcal{F}_n,$$

which is satisfied by $M^\star_n$ (this quantity then equals $Z_n - \mathbb{E}[U_{n+1}|\mathcal{F}_n]$).

Our theoretical algorithm

1. Take $p = 2$, $\phi(x) = x^2$
2. For each $n \in \{1, \dots, N\}$, choose a finite dimensional linear subspace $\mathcal{H}_n^{pr}$ of $\mathcal{H}_n^2$.
3. For $n = N - 1$ to $n = 0$, use an optimisation algorithm to minimise

$$\inf_{\Delta M_{n+1} \in \mathcal{H}_{n+1}^{pr}} \mathbb{E}\left[\left(Z_n + \Delta M_{n+1} - \max_{n+1 \le j \le N}\left\{Z_j - \sum_{i=n+2}^{j} \Delta M_i\right\}\right)^2\right].$$

ΔM_{n+1} solves a classical least square problem.

Two approximations are needed:

1. Use a finite dimensional subspace of $\mathcal{H}_n^{pr}$
2. Approximate $\mathbb{E}$ by Monte-Carlo.

Finite dimensional subspace approximation

We assume that the subspaces $\mathcal{H}_n$, $1 \leq n \leq N$, are spanned by $L \in \mathbb{N}^*$ martingale increments $\Delta X_{n,\ell} \in \mathcal{H}_n^2$, $1 \leq \ell \leq L$:

$$\mathcal{H}_n^{pr} = \left\{\alpha \cdot \Delta X_n \ : \ \alpha \in \mathbb{R}^L\right\}.$$

The minimisation problem becomes

$$\inf_{\alpha \in \mathbb{R}^L} \mathbb{E}\left[\left(Z_n + \alpha \cdot \Delta X_{n+1} - \max_{n+1 \leq j \leq N}\left\{Z_j - \sum_{i=n+2}^{j} \Delta M_i\right\}\right)^2\right].$$

If $\mathbb{E}[\Delta X_{n+1} \Delta X_{n+1}^T]$ is invertible, the minimum is given by

$$\alpha_{n+1} = \left(\mathbb{E}[\Delta X_{n+1} \Delta X_{n+1}^T]\right)^{-1} \mathbb{E}\left[\left(\max_{n+1 \leq j \leq N}\left\{Z_j - \sum_{i=n+2}^{j} \Delta M_i\right\}\right) \Delta X_{n+1}\right].$$

Monte Carlo approximation

Let $Q > 0$. For $1 \leq q \leq Q$, $(Z_n^q)_{1 \leq n \leq N}$ and $(\Delta X_n^q)_{1 \leq n \leq N}$ be independent sample paths of the underlying process Z and martingale increments ΔX. Solve backward in time, the sequence of optimisation problems

$$\inf_{\alpha \in \mathbb{R}^L} \frac{1}{Q} \sum_{q=1}^{Q} \left(Z_n^q + \alpha \cdot \Delta X_{n+1}^q - \max_{n+1 \leq j \leq N} \left\{ Z_j^q - \sum_{i=n+2}^{j} \alpha_i^Q \cdot \Delta X_i^q \right\} \right)^2 .$$

If $\sum_{q=1}^{Q} \Delta X_{n+1}^q (\Delta X_{n+1}^q)^T$ is positive definite, it has a unique solution α_{n+1}^Q:

$$\left(\sum_{q=1}^{Q} \Delta X_{n+1}^q (\Delta X_{n+1}^q)^T \right) \alpha_{n+1}^Q = \sum_{q=1}^{Q} \max_{n+1 \leq j \leq N} \left\{ Z_j^q - \sum_{i=n+2}^{j} \alpha_i^Q \cdot \Delta X_i^q \right\} \Delta X_{n+1}^q .$$

Convergence results

Proposition

Assume that for $1 \le n \le N$, the matrix $\mathbb{E}[\Delta X_n \Delta X_n^T]$ is invertible. Then,

- *For all $n \in \{1, \dots, N\}$, $\alpha_n^Q \to \alpha_n$ when $Q \to \infty$ a.s.*
- $U_0^Q = \frac{1}{Q} \sum_{q=1}^{Q} \max_{0 \le j \le N} \left\{ Z_j^q - \sum_{i=1}^{j} \alpha_i^Q \cdot \Delta X_i^q \right\} \to \mathbb{E}\left[\max_{0 \le j \le N} \left\{ Z_j - \sum_{i=1}^{j} \alpha_i \cdot \Delta X_i \right\} \right]$ *a.s.*

If we assume moreover that ΔX_i and Z_i have finite moments of order 4, then $\left(\sqrt{Q}(\alpha_n^Q - \alpha_n)\right)_{Q \ge 1}$ and $\left(\sqrt{Q} \left(U_0^Q - \mathbb{E}\left[\max_{0 \le j \le N} \left\{ Z_j - \sum_{i=1}^{j} \alpha_i \cdot \Delta X_i \right\} \right] \right) \right)_{Q \ge 1}$ are tight.

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The financial framework I

- A market with d assets $(S_t^k, t \geq 0)$, $k \in \{1, \dots, d\}$ and $(\mathcal{G}_t, t \geq 0)$ their usual filtration.
- For simplicity the interest rate r is deterministic
- Assume that the discounted assets $(\tilde{S}_t^k, t \geq 0)$ with $\tilde{S}_t^k = e^{-rt} S_t^k$ are square integrable $\mathcal{G}_t$-martingales.
- Consider a time horizon $T > 0$ and a Bermudan option with regular exercising dates

$$T_i = \frac{iT}{N},\ i = 0, \dots, N.$$

The financial framework II

Since perfect hedging holds (or may hold) with a continuous time martingale representation theorem, we further split each interval $[T_i, T_{i+1}]$ for $0 \leq i \leq N-1$ into $\bar{N}$ regular sub-intervals, and we set

$$t_{i,j} = T_i + \frac{j}{\bar{N}} \frac{T}{N}, \text{ for } 0 \leq j \leq \bar{N}. \tag{5}$$

Consider a family of functions $u_p : \mathbb{R}^d \to \mathbb{R}$ for $p \in \{1, \dots, \bar{P}\}$ and a family of discounted assets $(\mathcal{A}^k)_{1 \leq k \leq \bar{d}}$. Then, we define the following elementary martingale increments:

$$X^{p,k}_{t_{i,j}} - X^{p,k}_{t_{i,j-1}} = u^p_{i,j-1}(S_{t_{i,j-1}})(\mathcal{A}^k_{t_{i,j}} - \mathcal{A}^k_{t_{i,j-1}}), \tag{6}$$

for $1 \leq p \leq \bar{P}$ and $1 \leq k \leq \bar{d}$. Thus, $L = \bar{N} \times \bar{P} \times \bar{d}$ is the number of martingale increments between two exercising dates that span $\mathcal{H}^{pr}_i$.

The financial framework III

Decompose the martingale increments ΔM_{i+1}, $0 \leq i \leq N-1$ as follows

$$\Delta M_{i+1} = \sum_{j=1}^{\bar{N}} \sum_{p,k} \alpha_{i,j}^{p,k} (X_{t_{i,j}}^{p,k} - X_{t_{i,j-1}}^{p,k}). \tag{7}$$

There are $L = \bar{N} \times \bar{P} \times \bar{d}$ coefficients to estimate
Between two exercising dates, the option is European and using the martingale property we can easily show that the coefficients on every sub-intervals can be computed independently.

The use of subticks induces a linear computational cost: instead of solving a linear system of size $L = \bar{N} \times \bar{P} \times \bar{d}$, we solve $\bar{N}$ linear systems of size $\bar{P} \times \bar{d}$.

Typical choice for u^p: local functions ($u^p = \mathbf{1}_{A_p}$ for disjoint sets A_p) or polynomial functions.

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Numerical experiments

Numerical tests presented in the Black-Scholes model. Let

$$U_0^Q = \frac{1}{Q}\sum_{q=1}^{Q} \max_{0\le j\le N}\left\{ Z_j^q - \sum_{i=1}^{j} \alpha_i^Q \cdot \Delta X_i^q \right\}.$$

Because of overfitting, U_0^Q can significantly underestimate $\mathbb{E}\left[\max_{0\le j\le N}\left\{ Z_j - \sum_{i=1}^{j} \alpha_i \cdot \Delta X_i \right\}\right]$ when Q is not sufficiently large, compared to the number of parameters to estimate.

$$\hat{U}_0^Q = \frac{1}{Q}\sum_{q=1}^{Q} \max_{0\le j\le N}\left\{ \hat{Z}_j^q - \sum_{i=1}^{j} \alpha_i^Q \cdot \Delta \hat{X}_i^q \right\},$$

where $(\hat{Z}^q, \Delta\hat{X}^q)_{1\le q\le Q}$ is independent from the sample $(Z^q, \Delta X^q)_{1\le q\le Q}$ used to compute α^Q.

$\hat{U}_0^Q$ has a nonnegative biais. The difference $\hat{U}_0^Q - U_0^Q$ is a measure of the accuracy.

Comparison with Rogers' approach

For the Bermudan Put option, [Rogers(2002)] directly solves

$$U_0 = \inf_{\lambda \in \mathbb{R}} \mathbb{E}\left[\max_{0 \leq j \leq N} \{Z_j - \lambda(M_j - M_0)\}\right]$$

with $M_j = \tilde{P}(t_j, S_{t_j})$, where $\tilde{P}$ is the discounted European Put option price.

Rogers uses the continuous time European hedge.

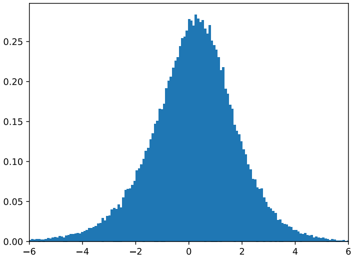
The 1-dimensional put option

Consider a 1-dimensional put options in the Black Scholes models

Q	$\bar{N}$	P	Vanilla	U_0^Q	$\hat{U}_0^Q$
50000	1	1	True	9.91	9.91
100000	1	50	True	9.89	9.91
100000	1	50	False	10.32	10.33
100000	5	50	False	9.99	10.08
100000	10	100	False	9.82	10.19
500000	10	100	False	9.95	10.02
2000000	10	50	False	9.98	9.98
2000000	20	50	False	9.94	9.96

Table: Prices for a put option using a basis of P local functions with $K = S_0 = 100$, $T = 0.5$, $r = 0.06$, $\sigma = 0.4$ and $N = 10$ exercising dates. LS price with a polynomial approximation of order 6: 9.90. [Rogers(2002)] price: 9.94.

The 1d put option: P&L



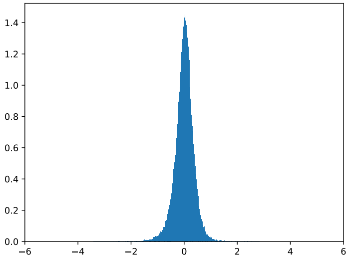


Figure: P&L histograms of the hedging strategy for the Bermudan Put option for the stock only strategy (left, $\bar{N} = 5$, $P = 50$, $Q = 10^5$) and the strategy using European options (right, $\bar{N} = 1$, $P = 50$, $Q = 10^5$).

The 1d put option: P&L (Delta Hedging)

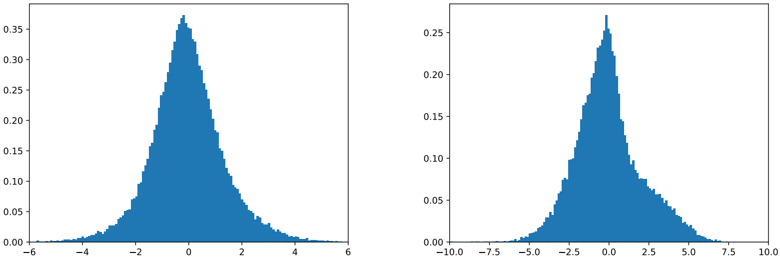


Figure: P&L histograms of the delta hedging strategy for the Bermudan Put option calculated with the CRR approximation (left) and Wang and Caflisch method (right). Parameters: $\bar{N} = 5$, $P = 50$, $Q = 10^5$.

A Bermudan butterfly option

$$\Psi(S) = 2\left(\frac{K_1+K_2}{2} - S\right)_+ - (K_1 - S)_+ - (K_2 - S)_+ .$$

Using the European butterfly to hedge the Bermudan options gives a price way too high with [Rogers(2002)]: 6.49 vs 5.65 (Longstaff Schwartz price)

Q	$\bar{N}$	P	Vanilla strike $\frac{K_1+K_2}{2}$	U_0^Q	$\hat{U}_0^Q$
50000	1	50	False	6.54	6.54
50000	1	50	True	6.25	6.28
100000	10	50	False	5.97	6.00
100000	10	50	True	5.79	5.87
500000	20	50	False	5.86	5.87
500000	20	50	True	5.71	5.74

Table: Prices for a butterfly option with parameters using a basis of P local functions. Parameters: $Q = 10^6$, $S_0 = 95$, $K_1 = 90$, $K_2 = 110$, $T = 0.5$, $r = 0.06$, $\sigma = 0.4$ and $N = 10$, The Longstaff-Schwartz algorithm with order 5 polynomials gives a price of 5.65.

The butterfly option

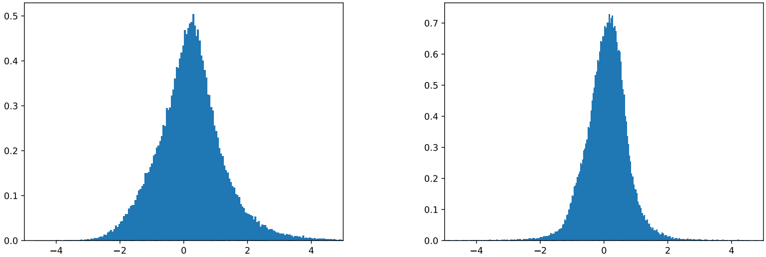


Figure: P&L histograms of the hedging strategy for the Bermudan Butterfly option obtained with $\bar{N} = 20$, $P = 50$, $Q = 5 \times 10^5$ for the stock only strategy (left) and the strategy using extra European options (right).

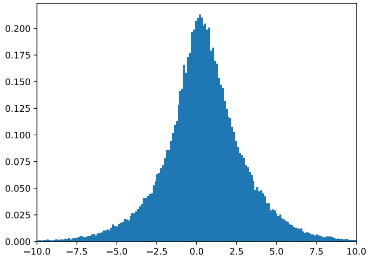
A max-call option on 2 assets

Payoff: $\left(\max(S^1, S^2) - K\right)_+$.
Hedging instruments: 2 assets, 2 ATM Vanilla options.

Q	$\bar{N}$	P	Vanilla	U_0^Q	$\hat{U}_0^Q$
1000000	1	10	False	8.98	8.99
1000000	1	10	True	8.33	8.36
2000000	5	10	False	8.53	8.55
2000000	5	10	True	8.19	8.21
4000000	10	10	False	8.46	8.47
4000000	10	10	True	8.16	8.18

Table: Prices for a call option on the maximum of 2 assets using a basis of $P \times P$ local functions and parameters $S_0 = (90, 90)$, $\sigma = (0.2, 0.2)$, $\rho = 0$, $\delta = (0.1, 0.1)$, $T = 3$, $r = 0.05$, $K = 100$, $N = 9$. The Longstaff-Schwartz algorithm give a price of 8.1.

A max-call option on 2 assets: P&L



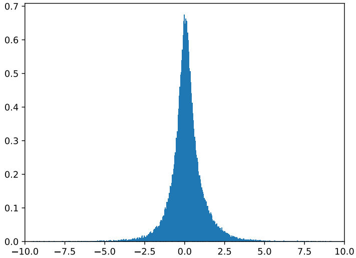


Figure: P&L histograms of the hedging strategy for the Bermudan max-call option of Table 3 obtained with $\bar{N} = 5$, $P = 10$, $Q = 2 \times 10^6$ for the stock only strategy (left) and the strategy using extra European options (center).

A max-call option on 2 assets: P&L (Delta hedging)

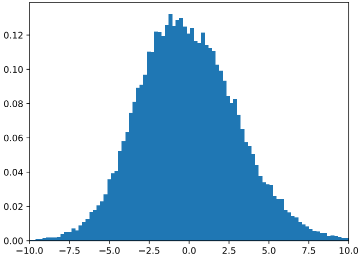


Figure: P&L histograms of the hedging strategy for the Bermudan max-call option of Table 3 with the delta hedging strategy calculated with [Wang and Caflisch(2010)].

A basket option on 3 assets

Payoff: $\left(K - \frac{1}{3}\sum_{\ell=1}^{3} S^\ell\right)_+$.

Hedging instruments: 3 assets, 3 ATM Vanilla options.

Regression on the signed payoff (we replace in (6) $u^p_{i,j-1}(S_{t_{i,j-1}})$ by $\tilde{u}^p_{i,j-1}(K - \frac{1}{3}\sum_{\ell=1}^{3} S^\ell_{t_{i,j-1}}))$

Q	$\bar{N}$	P	Vanilla	U_0^Q	$\hat{U}_0^Q$
100000	1	50	False	4.32	4.34
100000	1	50	True	4.29	4.32
250000	5	50	False	4.11	4.15
250000	5	50	True	4.09	4.16
500000	10	50	False	4.07	4.11
500000	10	50	True	4.05	4.13
1000000	10	50	False	4.08	4.11
1000000	10	50	True	4.07	4.12

Table: Prices for a put option on 3-dimensional basket using a basis of local functions of the signed payoff with $K = S_0 = 100$, $T = 1$, $r = 0.05$, $\sigma^i = 0.2$, $\rho = 0.3$ and 10 exercising dates. LS price with a polynomial approximation of order 3: 4.03.

Conclusion

The key ingredients are

- The reward excess representation: a new dual formula.
- Strictly convexifying the optimisation problem.

This gives a pure dual algorithm that boils down to a sequence of least-square minimisation problems.

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This gives a pure dual algorithm that boils down to a sequence of least-square minimisation problems.

The main advantages of the algorithm are:

- It gives directly the hedging portfolio and the corresponding price.
- It allows to quantify the value of monitoring frequently the hedging portfolio.
- It allows to quantify the value of adding/removing a financial instrument in the hedging portfolio.

Conclusion

The key ingredients are

- The reward excess representation: a new dual formula.
- Strictly convexifying the optimisation problem.

This gives a pure dual algorithm that boils down to a sequence of least-square minimisation problems.

The main advantages of the algorithm are:

- It gives directly the hedging portfolio and the corresponding price.
- It allows to quantify the value of monitoring frequently the hedging portfolio.
- It allows to quantify the value of adding/removing a financial instrument in the hedging portfolio.

Thank you for your attention!

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