An Efficient SSP-based Methodology for Assessing Climate Risks of a Large Credit Portfolio

based on joint work with

F. Bourgey[1,2] E. Gobet[1] Y. Jiao[3]

[1]École Polytechnique, [2]Bloomberg L.P., [3]Université Lyon 1

https://hal.science/hal-04665712

Séminaire FiME - IHP, Paris - 24 Janvier 2025

Objective

To model and sample the cumulative credit loss

$$\mathcal{L} = \sum_{i=1}^{n} \Lambda^i \times \mathbf{1}_{\{X^i \le d^i\}}$$

where

- $\Lambda^i = \mathrm{EAD}^i \times \mathrm{LGD}^i$ and large n ($\approx 1E6$) of obligors,
- d^i represents the default barrier of the ith obligor,
- X^i is the obligor's default-relevant variable.

⚠How to include climate risks in the picture?

Risk management in climate finance

Climate risks in finance: Carney speech in 2015 on "breaking the tragedy of the horizon". *Physical and transition risks.*

- Physical risks: Economic costs and financial losses resulting from the increasing severity and frequency of extreme climate change-related weather events (heatwaves, landslides, floods, wildfires and storms), longer-term gradual shifts of the climate (extreme weather variability, ocean acidification, rising sea levels and average temperatures), loss of ecosystem and services (desertification, water shortage, degradation of soil quality or marine ecology)...
- Transition risks: risks related to the process of adjustment towards a low-carbon economy, it concerns social and political instability of policies in this period as well as technological changes.
- See BIS document "Climate-related risk drivers and their transmission channels", April 2021.

Today's talk:

⋆ modelling of the transmission channel "transition risk" and "physical risk" to "credit risk"

⋆ aggregate them in a large portfolio of obligors

Risk management in climate finance

Climate risks in finance: Carney speech in 2015 on "breaking the tragedy of the horizon". *Physical and transition risks.*

- Physical risks: Economic costs and financial losses resulting from the increasing severity and frequency of extreme climate change-related weather events (heatwaves, landslides, floods, wildfires and storms), longer-term gradual shifts of the climate (extreme weather variability, ocean acidification, rising sea levels and average temperatures), loss of ecosystem and services (desertification, water shortage, degradation of soil quality or marine ecology)...
- Transition risks: risks related to the process of adjustment towards a low-carbon economy, it concerns social and political instability of policies in this period as well as technological changes.
- See BIS document "Climate-related risk drivers and their transmission channels", April 2021.

Today's talk:

⋆ modelling of the transmission channel "transition risk" and "physical risk" to "credit risk"

⋆ aggregate them in a large portfolio of obligors

Building Forward-looking scenarios

- Since 1988, the Intergovernmental Panel on Climate Change (IPCC) has published Special Reports summarizing different potential macro-scenarios RCPs (Representative Concentration Pathways) of global warming and related risks.

  These scenarios are based on IAMs (Integrated Assessment Models), developed by scientists and economists.

  Examples: possible increases of global temperature by 2100 are 0.3°C-1.7°C (RCP2.6), 1.1°C-2.6°C (RCP4.5), 1.4°C-3.1°C (RCP6.0), and 2.6°C-4.8°C (RCP8.5).
- Many other possible scenarios are being developed in the scientific literature subject to different ecological transition trajectories, sectors, countries, etc.

  Shared Socioeconomic Pathways (SSP)

  See [RvVK+17] for an overview.

  Data available on the SSP Public Database https://tntcat.iiasa.ac.at/SspDb

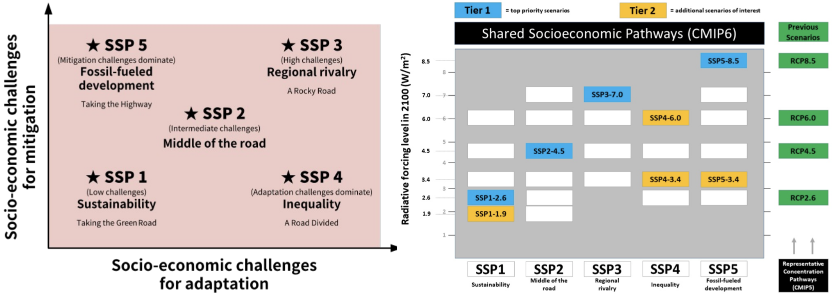


Figure: Left: SSPs narratives overview. Right: RCP/SSP combinations.

Examples of SSP

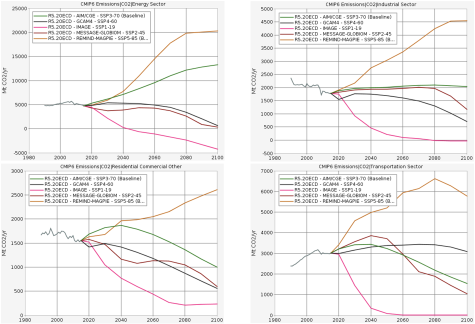


Figure: Historical and scenario-based CO2 emission, from 1980 to 2100, in Mt/yr in the OECD, according to the activity sectors: Energy (top left), Industry (top right), Residential Commercial (bottom left), Transportation (bottom right).

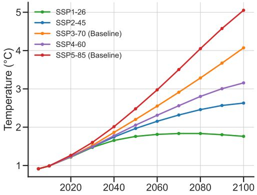
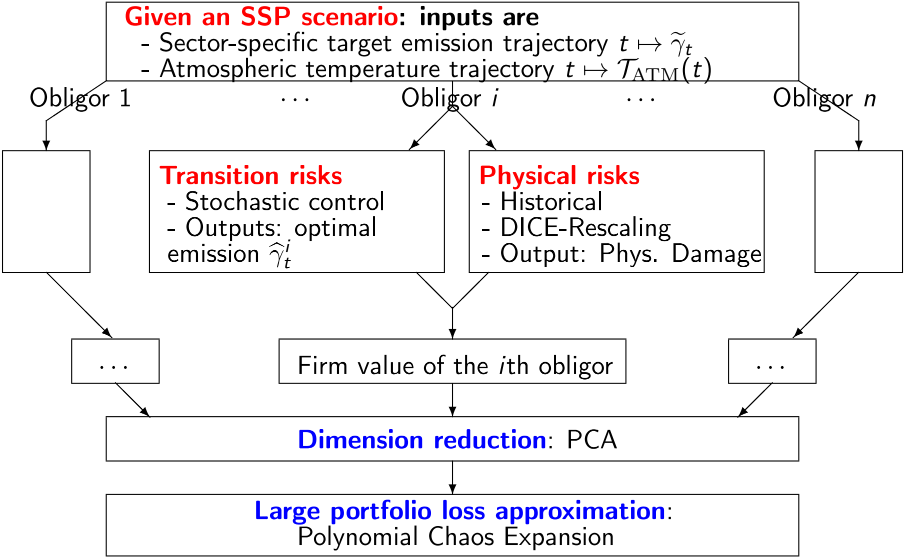


Figure: Global average temperature increase $\mathcal{T}_{\mathrm{ATM}}$ relative to the pre-industrial era (year 1750) for different SSPs.


Given an SSP scenario: inputs are
- Sector-specific target emission trajectory $t \mapsto \widetilde{\gamma}_t$
- Atmospheric temperature trajectory $t \mapsto \mathcal{T}_{\mathrm{ATM}}(t)$
Obligor 1
...
Obligor i
...
Obligor n
Transition risks
- Stochastic control
- Outputs: optimal emission $\widehat{\gamma}^i_t$
Physical risks
- Historical
- DICE-Rescaling
- Output: Phys. Damage
...
Firm value of the ith obligor
...
Dimension reduction: PCA
Large portfolio loss approximation:
Polynomial Chaos Expansion

Modeling transition risks

▷ We model how the obligor production can be impacted by the future policies of CO2 emission. Stochastic model with energy parameters as inputs.

Depending on how and where these energy sources are produced, their climate impact and price are different.

Source of Energy	Emission factors (kgCO_2e per kWh)
Electricity (Coal)	0.820
Electricity (Gas)	0.490
Electricity (Wind)	0.011
Electricity (Nuclear)	0.012
Electricity (Hydro)	0.024
Charcoal	0.403
Crude Oil	0.264
Natural Gas	0.202

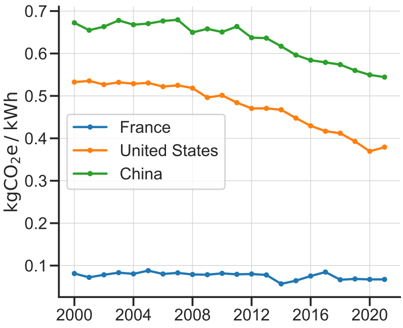
Table: A few examples of emission factors (sometimes called emission intensity or carbon output rate).

Supplier	Coal	Gas	Nuclear	Renewable	Other	Emission factor ($\mathtt{kgCO_2e}$ per $\mathtt{kWh}$)
Utilita	9.0	74.0	7.0	3.0	8.0	0.429
e.on	2.0	16.1	1.5	78.8	1.6	0.093
EDF Energy	1.6	15.1	63.1	19.0	1.2	0.082
Bulb	0	0	0	100.0	0	≈ 0

Table: Fuel Mix (in %) of some of the UK Domestic Electricity Suppliers as of 2022.

NB: Because of The Electricity (Fuel Mix Disclosure) Regulations Act of 2005, electric suppliers are obligated to provide details of the mix of fuels used to produce electricity.

▷ Impact of location and date



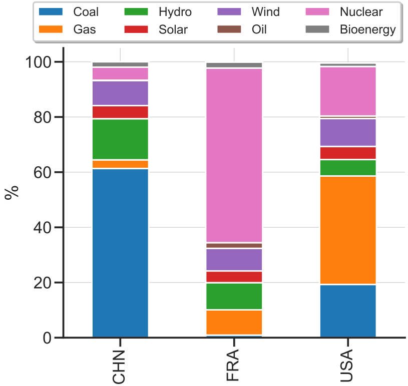


Figure: Left: emission factors of electricity in $\mathtt{kgCO_2e/kWh}$ as a function of time for France, the USA, and China. Right: Bar plots of the electricity mix as of 2022 for the same countries.

▷ Impact of energy price

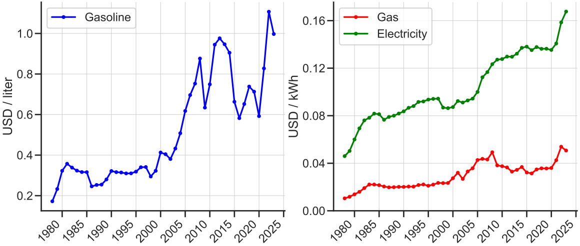


Figure: Left: Yearly US average price of Gasoline in USD/liter. Right: Yearly US average prices of household gas and electricity in USD/kWh.

Stochastic model for the production

- n obligors
- $n+1$ independent standard Brownian motions
- one common systemic factor B, n independent idiosyncratic risk factors B^i
- Log-production $p^i_t = \log(P^i_t)$ solves:

$$\mathrm{d}p^i_t = (a^i - b^i p^i_t + c^{i,\theta}_\bullet \cdot \gamma^i_{\bullet,t},)\,\mathrm{d}t + \sigma^i \left(\rho^i\,\mathrm{d}B_t + \sqrt{1-(\rho^i)^2}\,\mathrm{d}B^i_t\right),$$

$$c^{i,\theta}_\bullet := (c^i_{\mathrm{e}} \times \theta^i_{\mathrm{e},t})_{\mathrm{e}\in\mathscr{E}^i},$$

where $\theta^i_{\mathrm{e},t}$ are inverse emission factors.

Mathematically, we solve a stochastic control problem

$$\widehat{\gamma}^i_\bullet := \arg\sup_{\gamma^i_\bullet \in \mathcal{A}^i} \mathcal{J}^i\left(\gamma^i_\bullet\right),$$

$$\mathcal{J}^i\left(\gamma^i_\bullet\right) = \mathbb{E}\Big[\int_0^\infty e^{-rt}\big(\mathrm{AP}^i p^i_t - \alpha^{i,\theta}_{\bullet,t}\cdot\gamma^i_{\bullet,t} - \beta^{i,\theta}_{\bullet,t}\cdot(\gamma^2)^i_{\bullet,t} - \ell^i_1(\mathbf{1}\cdot\gamma^i_{\bullet,t} - \widetilde{\gamma}^i_t) + \ell^i_2(\widetilde{\gamma}^i_t - \mathbf{1}\cdot\gamma^i_{\bullet,t})\big)\mathrm{d}t$$

Stochastic model for the production

- n obligors
- $n+1$ independent standard Brownian motions
- one common systemic factor B, n independent idiosyncratic risk factors B^i
- Log-production $p_t^i = \log(P_t^i)$ solves:

$$\mathrm{d}p_t^i = (a^i - b^i p_t^i + c_\bullet^{i,\theta} \cdot \gamma_{\bullet,t}^i,)\,\mathrm{d}t + \sigma^i \left(\rho^i \,\mathrm{d}B_t + \sqrt{1-(\rho^i)^2}\,\mathrm{d}B_t^i\right),$$

$$c_\bullet^{i,\theta} := (c_{\mathrm{e}}^i \times \theta_{\mathrm{e},t}^i)_{\mathrm{e}\in\mathscr{E}^i},$$

where $\theta_{\mathrm{e},t}^i$ are inverse emission factors.

Mathematically, we solve a stochastic control problem

$$\widehat{\gamma}_\bullet^i := \underset{\gamma_\bullet^i \in \mathcal{A}^i}{\arg\sup}\, \mathcal{J}^i\left(\gamma_\bullet^i\right),$$

$$\mathcal{J}^i\left(\gamma_\bullet^i\right) = \mathbb{E}\Big[\int_0^\infty e^{-rt}(\mathrm{AP}^i p_t^i - \alpha_{\bullet,t}^{i,\theta}\cdot\gamma_{\bullet,t}^i - \beta_{\bullet,t}^{i,\theta}\cdot(\gamma^2)_{\bullet,t}^i - \ell_1^i(\mathbf{1}\cdot\gamma_{\bullet,t}^i - \widetilde{\gamma}_t^i) + \ell_2^i(\widetilde{\gamma}_t^i - \mathbf{1}\cdot\gamma_{\bullet,t}^i))\mathrm{d}t$$

Solutions

- Under mild conditions, we have existence/uniqueness results (see the paper for details).
- Optimal emission paths may be deterministic.

Theorem 1

Assume $\ell_1^i(x) = \omega_1^i\ x_+^2$ and $\ell_2^i(x) = \omega_2^i\ x_+^2$. For any $t \geq 0$, define

$$\Gamma_t^i := \frac{\mathbf{1}}{2\beta_{\bullet,t}^{i,\theta}} \cdot \left(\frac{\mathrm{AP}^i\ c_\bullet^{i,\theta}}{r + b^i} - \alpha_{\bullet,t}^{i,\theta}\right), \quad \xi_{1,t}^i := \omega_1^i \left(\mathbf{1} \cdot \frac{1}{\beta_{\bullet,t}^{i,\theta}}\right), \quad \xi_{2,t}^i := \omega_2^i \left(\mathbf{1} \cdot \frac{1}{\beta_{\bullet,t}^{i,\theta}}\right).$$

Under a small parameter condition (...), the optimal emission strategy for the energy source e *has the explicit form*

$$\widehat{\gamma}_{\mathrm{e},t}^i = \frac{1}{2\beta_{\mathrm{e},t}^{i,\theta}} \left(\frac{\mathrm{AP}^i\ c_\mathrm{e}^{i,\theta}}{r + b^i} - \alpha_{\mathrm{e},t}^{i,\theta} - \frac{2\omega_1^i}{1 + \xi_{1,t}^i} \left(\Gamma_t^i - \widetilde{\gamma}_t^i\right)^+ - \frac{2\omega_2^i}{1 - \xi_{2,t}^i} \left(\widetilde{\gamma}_t^i - \Gamma_t^i\right)^+\right).$$

Optimal emissions for three types of energy (natural gas, electricity from nuclear and from coal)

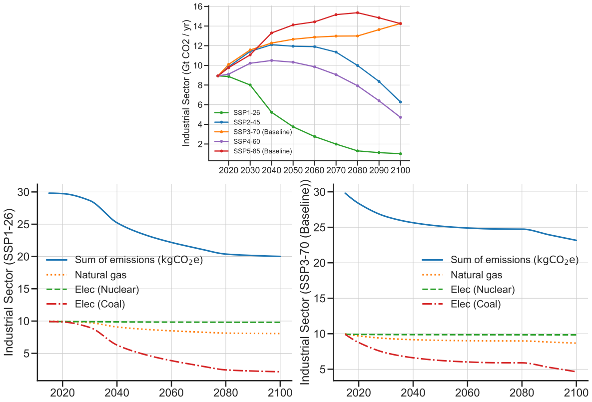


Figure: Parameters: $r = 0.05$, $a = b = 0$, $c = (1, 1, 1)$, $\omega_1 = 10^{-4}$, $\omega_2 = 0$, $\alpha = (0.05, 0.16, 0.16)$, $\beta = (1, 1, 1)$, $\theta = (1/0.202, 1/0.012, 1/0.820)/3600$.

Projecting physical risks

▷ Types of climate extremes and weather extreme events:

$$\mathcal{CW} := \{\text{storm/typhoon/hurricane/tornado/tropical cyclone, hailstorm, flood, winter storm, level sea rise, and storm surge, heat wave, drought, wildfire, extreme cold spells, snowstorms,} \dots\}.$$

Occurence of climate and weather event $\texttt{cw}$ at geographical zone l is modeled with Poisson process $N^{\texttt{cw},l}$.

Cumulative loss over $[t_0, t_1]$: $\int_{t_0}^{t_1} \sum_{\texttt{cw},l} Z_u^{i,\texttt{cw},l} \, \mathrm{d}N_u^{\texttt{cw},l}$.

▷ Expected Physical Damage (EPL) for obligor i:

$$\mathrm{EPL}^i(t) := \mathbb{E}\Big[\sum_{\texttt{cw},l} \int_t^\infty e^{-r(u-t)} Z_u^{i,\texttt{cw},l} \mathrm{d}N_u^{\texttt{cw},l} \Big| \mathcal{F}_t\Big],$$

$$= \int_t^\infty e^{-r(u-t)} \Big(\sum_{\texttt{cw},l} \mathbb{E}\big[Z_u^{i,\texttt{cw},l}\big] \lambda_u^{\texttt{cw},l}\Big) \mathrm{d}u.$$

Projecting physical risks

▷ Types of climate extremes and weather extreme events:

$$\mathcal{CW} := \{\text{storm/typhoon/hurricane/tornado/tropical cyclone, hailstorm, flood, winter storm, level sea rise, and storm surge, heat wave, drought, wildfire, extreme cold spells, snowstorms,} \dots\}.$$

Occurence of climate and weather event $\texttt{cw}$ at geographical zone l is modeled with Poisson process $N^{\texttt{cw},l}$.

Cumulative loss over $[t_0, t_1]$: $\int_{t_0}^{t_1} \sum_{\texttt{cw},l} Z_u^{i,\texttt{cw},l} \, \mathrm{d}N_u^{\texttt{cw},l}$.

▷ Expected Physical Damage (EPL) for obligor i:

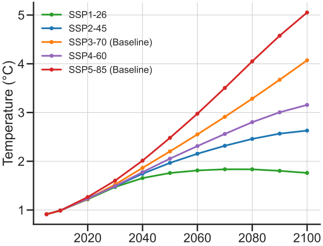
$$\mathrm{EPL}^i(t) := \mathbb{E}\Big[\sum_{\texttt{cw},l} \int_t^\infty e^{-r(u-t)} Z_u^{i,\texttt{cw},l} \mathrm{d}N_u^{\texttt{cw},l} \Big| \mathcal{F}_t\Big],$$

$$= \int_t^\infty e^{-r(u-t)} \Big(\sum_{\texttt{cw},l} \mathbb{E}\big[Z_u^{i,\texttt{cw},l}\big] \, \lambda_u^{\texttt{cw},l}\Big) \mathrm{d}u.$$

From the DICE model (Dynamic Integrated model of Climate and the Economy, Nordhaus 1992), the global damage increases with the atmospheric temperature $\mathcal{T}_{\mathrm{ATM}}$:

$$\mathcal{D}(\mathcal{T}_{\mathrm{ATM}}) = a_1 \mathcal{T}_{\mathrm{ATM}} + a_2 \mathcal{T}_{\mathrm{ATM}}^2$$

with $a_1 = 0$, $a_2 = 0.0028388$ (2008 version of DICE).



Global average temperature increase relative to the pre-industrial era (year 1750) for different SSPs.

Projection in the future (using temperature SSP scenarios):

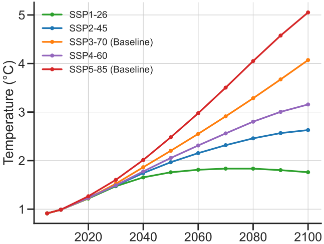
$$\mathrm{EPL}^i(t) := \Big(\sum_{\mathrm{cw},l} \mathbb{E}\big[Z_{t_{\mathrm{ref}}}^{i,\mathrm{cw},l}\big] \lambda_{t_{\mathrm{ref}}}^{\mathrm{cw},l} \Big) \int_t^{\infty} e^{-r(u-t)} \frac{\mathcal{D}(\mathcal{T}_{\mathrm{ATM}}(u))}{\mathcal{D}(\mathcal{T}_{\mathrm{ATM}}(t_{\mathrm{ref}}))} \mathrm{d}u,$$

where t_{ref} is the time at which the model is calibrated.

From the DICE model (Dynamic Integrated model of Climate and the Economy, Nordhaus 1992), the global damage increases with the atmospheric temperature $\mathcal{T}_{\mathrm{ATM}}$:

$$\mathcal{D}(\mathcal{T}_{\mathrm{ATM}}) = a_1 \mathcal{T}_{\mathrm{ATM}} + a_2 \mathcal{T}_{\mathrm{ATM}}^2$$

with $a_1 = 0$, $a_2 = 0.0028388$ (2008 version of DICE).



Global average temperature increase relative to the pre-industrial era (year 1750) for different SSPs.

Projection in the future (using temperature SSP scenarios):

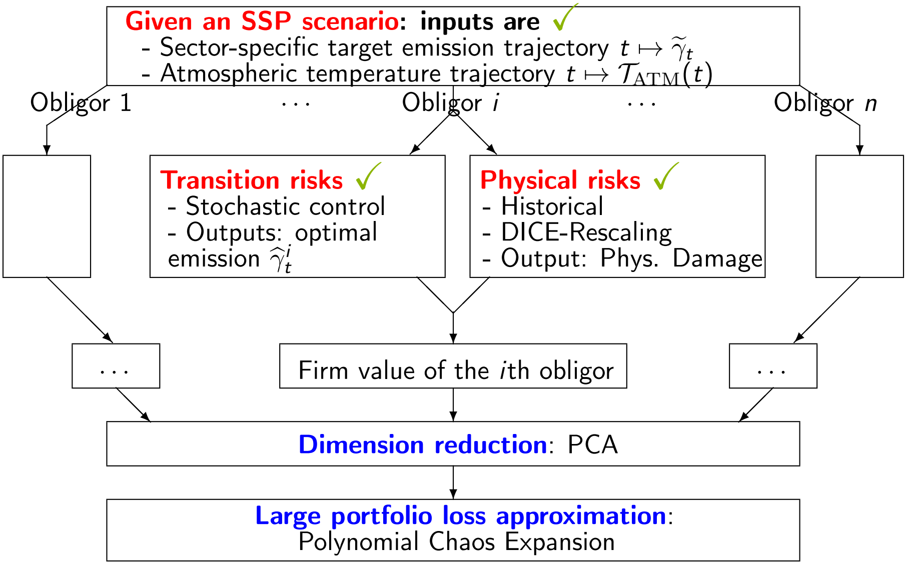
$$\mathrm{EPL}^i(t) := \left(\sum_{\mathtt{cw},l} \mathbb{E}\left[Z^{i,\mathtt{cw},l}_{t_{\mathrm{ref}}} \right] \lambda^{\mathtt{cw},l}_{t_{\mathrm{ref}}} \right) \int_t^{\infty} e^{-r(u-t)} \frac{\mathcal{D}(\mathcal{T}_{\mathrm{ATM}}(u))}{\mathcal{D}(\mathcal{T}_{\mathrm{ATM}}(t_{\mathrm{ref}}))} \mathrm{d}u,$$

where t_{ref} is the time at which the model is calibrated.

Alternative to EPL

- Purpose: to better capture the full distribution of physical losses
- Using Value-at-Risk-based measure at risk level α
- Define $\mathrm{PL}^i_u := \sum_{\mathtt{cw},l} \int_{u-1}^{u} Z_s^{i,\mathtt{cw},l}\,\mathrm{d}N_s^{\mathtt{cw},l}\,\mathrm{d}s$ for the physical loss incurred by the ith obligor over a one-year period ending at time u.
- Replace EPL^i by

$$\mathrm{VaR}_\alpha\text{-}\mathrm{PL}^i(t) = \int_t^\infty e^{-r(u-t)}\,\mathrm{VaR}_\alpha\Big[\sum_{\mathtt{cw},l}\int_{u-1}^{u} Z_s^{i,\mathtt{cw},l}\,\mathrm{d}N_s^{\mathtt{cw},l}\,\mathrm{d}s\Big]\,\mathrm{d}u$$
$$\approx \mathrm{VaR}_\alpha\big[\mathrm{PL}^i_{t_{\mathrm{ref}}}\big]\int_t^\infty e^{-r(u-t)}\,\frac{\mathcal{D}(\mathcal{T}_{\mathrm{ATM}}(u))}{\mathcal{D}(\mathcal{T}_{\mathrm{ATM}}(t_{\mathrm{ref}}))}\,\mathrm{d}u.$$


Given an SSP scenario: inputs are ✓
- Sector-specific target emission trajectory $t \mapsto \widetilde{\gamma}_t$
- Atmospheric temperature trajectory $t \mapsto \mathcal{T}_{\mathrm{ATM}}(t)$
Obligor 1
· · ·
Obligor i
· · ·
Obligor n
Transition risks ✓
- Stochastic control
- Outputs: optimal emission $\widehat{\gamma}^i_t$
Physical risks ✓
- Historical
- DICE-Rescaling
- Output: Phys. Damage
· · ·
Firm value of the ith obligor
· · ·
Dimension reduction: PCA
Large portfolio loss approximation:
Polynomial Chaos Expansion

Portfolio loss

Recall

$$\mathcal{L}_t = \sum_{i=1}^{n} \Lambda^i \times \mathbf{1}_{\{\widehat{V}_t^i \le L^i(t)\}}$$

where $\Lambda^i = \mathrm{EAD}^i \times \mathrm{LGD}^i$ and large n ($\approx 1E6$).
Obligor firm value:

$$\widehat{V}_t^i := \mathbb{E}\Bigg[\int_t^{\infty} e^{-r(u-t)} \Big(\mathrm{AP}^i \times \widehat{P}_u^i - \alpha_{\bullet,u}^{i,\theta} \cdot \widehat{\gamma}_{\bullet,u}^i - \beta_{\bullet,u}^{i,\theta} \cdot (\widehat{\gamma}^2)_{\bullet,u}^i$$

$$- \omega_1^i \left(\mathbf{1} \cdot \widehat{\gamma}_{\bullet,u}^i - \widetilde{\gamma}_u^i\right)_+^2 + \omega_2^i \left(\widetilde{\gamma}_u^i - \mathbf{1} \cdot \widehat{\gamma}_{\bullet,u}^i\right)_+^2 \Big) \mathrm{d}u \Bigg| \mathcal{F}_t \Bigg] - \mathrm{EPL}^i(t),$$

After some computations: $\mathcal{L}_t = \sum_{i=1}^{n} \Lambda^i \, \mathbf{1}_{\{A_t^i \le X_t^i\}}$ with

$$A_t^i := \sqrt{1 - (\rho^i)^2} \int_0^t e^{-b^i(t-s)} \, \mathrm{d}B_s^i + \text{ deterministic function of time } t,$$

$$X_t^i := -\rho^i \int_0^t e^{-b^i(t-s)} \, \mathrm{d}B_s.$$

⚠Time consuming to sample.

PCA approximation

Consider $X_t := (X_t^i)_{1 \le i \le n}$ with covariance matrix

$$(K_{X_t})^{i,j} := \mathrm{Cov}(X_t^i, X_t^j) = \rho^i \rho^j \int_0^t e^{-b^i(t-s)} e^{-b^j(t-s)} \mathrm{d}s = \rho^i \rho^j \frac{1 - e^{-(b^i+b^j)t}}{b^i + b^j}.$$

Spectral decomposition: $K_{X_t} = \sum_{k=1}^n \nu^k u^k (u^k)^\top$.

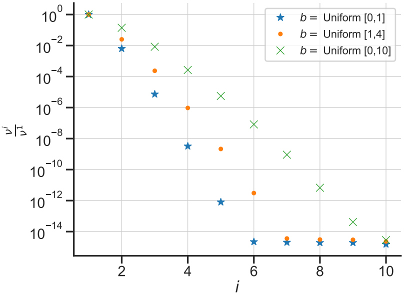


Figure: Ratio $\frac{\nu^k}{\nu^1}$ for k from 1 to 10 with $n = 1000$, $\rho \sim \mathcal{U}[-1,1]$ and $b \sim \mathcal{U}[0,1]$, $\mathcal{U}[1,4]$, and $\mathcal{U}[0,10]$.

Mostly, only two eigenvalues matter !! So far, lack of quantitative estimates.

PCA error bound

Approximate PCA model:

$$\mathcal{L}_t \approx \mathcal{L}_t^{\mathrm{PCA}} := \sum_{i=1}^{n} \Lambda^i \, \mathbf{1}_{\left\{A_t^i \leq \sqrt{\nu^1} G^1 u^{1,i} + \sqrt{\nu^2} G^2 u^{2,i}\right\}}.$$

Theorem 2

The L_1 error between the original and approximated loss is bounded as follows:

$$\mathbb{E}\left[\left|\mathcal{L}_t - \mathcal{L}_t^{\mathrm{PCA}}\right|\right] \leq \sum_{i=1}^{n} \frac{\Lambda^i}{\pi} \frac{|\rho^i|}{\sqrt{1-(\rho^i)^2}} \sqrt{\frac{\sum_{k=3}^{n} \nu^k (u^{k,i})^2}{\sum_{k=1}^{n} \nu^k (u^{k,i})^2}}.$$

Polynomial chaos expansion

$$\mathcal{L}_t^{\mathrm{PCA}} = \sum_{i=1}^{n} \Lambda^i \mathbf{1}_{\left\{\frac{A_t^i}{\sqrt{\nu^1(u^{1,i})^2+\nu^2(u^{2,i})^2}} \le \frac{\sqrt{\nu^1}u^{1,i}G^1+\sqrt{\nu^2}u^{2,i}G^2}{\sqrt{\nu^1(u^{1,i})^2+\nu^2(u^{2,i})^2}}\right\}} \text{ with } (L^{1,i})^2+(L^{2,i})^2=1$$

$$=: \sum_{i=1}^{n} \Lambda^i \mathbf{1}_{\{\tilde{A}_t^i \le L^{1,i}G^1+L^{2,i}G^2\}}$$

$$\approx \sum_{i=1}^{n} \Lambda^i \sum_{m=0}^{M} \tau_m(\tilde{A}_t^i)\,\mathrm{He}_m(L^{1,i}G^1+L^{2,i}G^2) \quad \text{(Hermite polynomial chaos expansion)}$$

$$= \sum_{i=1}^{n} \Lambda^i \sum_{m=0}^{M} \tau_m(\tilde{A}_t^i) \sum_{\substack{m_1,m_2\ge 0\\ m_1+m_2=m}} \frac{m!}{m_1!m_2!}(L^{1,i})^{m_1}(L^{2,i})^{m_2}\mathrm{He}_{m_1}(G^1)\mathrm{He}_{m_2}(G^2)$$

$$= \sum_{m=0}^{M} \sum_{\substack{m_1,m_2\ge 0\\ m_1+m_2=m}} \frac{m!}{m_1!m_2!}\mathrm{He}_{m_1}(G^1)\mathrm{He}_{m_2}(G^2) \underbrace{\sum_{i=1}^{n} \Lambda^i \tau_m(\tilde{A}_t^i)(L^{1,i})^{m_1}(L^{2,i})^{m_2}}_{\sim\text{Gaussian (CLT, } n\to+\infty)} =: \mathcal{L}_t^{\mathrm{PCA,PCE}}.$$

L_2 Error control of the PCE: $(\sum_{i=1}^n \Lambda^i)M^{-\frac{1}{4}}$, see [BGR20] and [BGR22, Theorem 2.7]

In practice, taking M small is enough.

Polynomial chaos expansion

$$\mathcal{L}_t^{\mathrm{PCA}} = \sum_{i=1}^{n} \Lambda^i \mathbf{1}_{\left\{ \frac{A_t^i}{\sqrt{\nu^1 (u^{1,i})^2 + \nu^2 (u^{2,i})^2}} \leq \frac{\sqrt{\nu^1} u^{1,i} G^1 + \sqrt{\nu^2} u^{2,i} G^2}{\sqrt{\nu^1 (u^{1,i})^2 + \nu^2 (u^{2,i})^2}} \right\}} \text{ with } (L^{1,i})^2 + (L^{2,i})^2 = 1$$

$$=: \sum_{i=1}^{n} \Lambda^i \mathbf{1}_{\left\{ \tilde{A}_t^i \leq L^{1,i} G^1 + L^{2,i} G^2 \right\}}$$

$$\approx \sum_{i=1}^{n} \Lambda^i \sum_{m=0}^{M} \tau_m(\tilde{A}_t^i) \, \mathrm{He}_m(L^{1,i} G^1 + L^{2,i} G^2) \quad \text{(Hermite polynomial chaos expansion)}$$

$$= \sum_{i=1}^{n} \Lambda^i \sum_{m=0}^{M} \tau_m(\tilde{A}_t^i) \sum_{\substack{m_1, m_2 \geq 0 \\ m_1 + m_2 = m}} \frac{m!}{m_1! m_2!} (L^{1,i})^{m_1} (L^{2,i})^{m_2} \mathrm{He}_{m_1}(G^1) \mathrm{He}_{m_2}(G^2)$$

$$= \sum_{m=0}^{M} \sum_{\substack{m_1, m_2 \geq 0 \\ m_1 + m_2 = m}} \frac{m!}{m_1! m_2!} \mathrm{He}_{m_1}(G^1) \mathrm{He}_{m_2}(G^2) \underbrace{\sum_{i=1}^{n} \Lambda^i \tau_m(\tilde{A}_t^i) (L^{1,i})^{m_1} (L^{2,i})^{m_2}}_{\sim \text{Gaussian (CLT, } n \to +\infty)} =: \mathcal{L}_t^{\mathrm{PCA,PCE}}.$$

L_2 Error control of the PCE: $(\sum_{i=1}^{n} \Lambda^i) M^{-\frac{1}{4}}$, see [BGR20] and [BGR22, Theorem 2.7]

In practice, taking M small is enough.

Polynomial chaos expansion

$$\mathcal{L}_t^{\mathrm{PCA}} = \sum_{i=1}^{n} \Lambda^i \mathbf{1}_{\left\{\frac{A_t^i}{\sqrt{\nu^1(u^{1,i})^2+\nu^2(u^{2,i})^2}} \leq \frac{\sqrt{\nu^1}u^{1,i}G^1+\sqrt{\nu^2}u^{2,i}G^2}{\sqrt{\nu^1(u^{1,i})^2+\nu^2(u^{2,i})^2}}\right\}} \text{ with } (L^{1,i})^2+(L^{2,i})^2=1$$

$$=: \sum_{i=1}^{n} \Lambda^i \mathbf{1}_{\{\tilde{A}_t^i \leq L^{1,i}G^1+L^{2,i}G^2\}}$$

$$\approx \sum_{i=1}^{n} \Lambda^i \sum_{m=0}^{M} \tau_m(\tilde{A}_t^i)\,\mathrm{He}_m(L^{1,i}G^1+L^{2,i}G^2) \quad \text{(Hermite polynomial chaos expansion)}$$

$$= \sum_{i=1}^{n} \Lambda^i \sum_{m=0}^{M} \tau_m(\tilde{A}_t^i) \sum_{\substack{m_1,m_2\geq 0\\ m_1+m_2=m}} \frac{m!}{m_1!m_2!}(L^{1,i})^{m_1}(L^{2,i})^{m_2}\mathrm{He}_{m_1}(G^1)\mathrm{He}_{m_2}(G^2)$$

$$= \sum_{m=0}^{M} \sum_{\substack{m_1,m_2\geq 0\\ m_1+m_2=m}} \frac{m!}{m_1!m_2!}\mathrm{He}_{m_1}(G^1)\mathrm{He}_{m_2}(G^2) \underbrace{\sum_{i=1}^{n} \Lambda^i \tau_m(\tilde{A}_t^i)(L^{1,i})^{m_1}(L^{2,i})^{m_2}}_{\sim\text{Gaussian (CLT, } n\to+\infty)} =: \mathcal{L}_t^{\mathrm{PCA,PCE}}.$$

L_2 Error control of the PCE: $(\sum_{i=1}^n \Lambda^i)M^{-\frac{1}{4}}$, see [BGR20] and [BGR22, Theorem 2.7]

In practice, taking M small is enough.

Computational gain

$n = 10,000$ obligors, $n_{\mathrm{mc}} = 10^5$ Monte Carlo samples

Algorithm 1: Crude Monte Carlo: Sampling of the portfolio loss $\mathcal{L}_t$

Output: n_{mc} i.i.d. samples of $\mathcal{L}_t$;
for $j \leftarrow 1$ **to** n_{mc} **do**
- Compute vector of optimal emissions $(\widehat{\gamma}_t^1, \dots, \widehat{\gamma}_t^n)$;
- Sample Gaussian vector $(\widehat{p}_t^1, \dots, \widehat{p}_t^n)$;
- Compute vector of optimal values $(\widehat{V}_t^1, \dots, \widehat{V}_t^n)$;
- Compute the portfolio loss $\mathcal{L}_t = \sum_{i=1}^n \Lambda^i \times \mathbf{1}_{\{\widehat{V}_t^i \leq L^i(t)\}}$.

- Algorithm 1 $\approx$ 75 seconds
- Algorithm 2 $\approx$ 2 seconds.

Algorithm 2: Monte Carlo: Sampling of the PCA-PCE portfolio loss with Gaussian approximation

Output: n_{mc} i.i.d. samples of $\mathcal{L}_t^{\mathrm{PCA,PCE,G}}$;
Offline computation:
Find PCA decomposition of K_{X_t};
Compute parameters for the PCE;
for $j \leftarrow 1$ **to** n_{mc} **do**
- Sample $\left(\varepsilon^{\mathrm{G}}_{n,m_1,m_2}\right)_{m_1+m_2=m,\, 0\leq m\leq M} \sim \mathcal{N}(\mathcal{M}, \mathcal{K})$;
- Sample two independent G^1, $G^2 \sim \mathcal{N}(0,1)$;
- Compute PCA-PCE portfolio loss $\mathcal{L}_t^{\mathrm{PCA,PCE,G}} = \sum_{m=0}^M \sum_{\substack{m_1,m_2\geq 0 \\ m_1+m_2=m}} \varepsilon^{\mathrm{G}}_{n,m_1,m_2} \mathrm{He}_{m_1}(G^1)\mathrm{He}_{m_2}(G^2)$;

Numerical experiments

We focus on three distinct SSP scenarios: SSP1-26, SSP3-70 (Baseline), and SSP5-85 (Baseline) for the transportation sector.

The portfolio loss maturity is set at $t = 5$ years, where $t = 0$ corresponds to 2015. We take $n_{\mathrm{mc}} = 10^5$ Monte Carlo samples for our analysis.

We study the fictitious portfolio with parameters:

- $t = 5$ years, $n = 1,000$, $r = 2\%$,
- 3 different energy sources for e, $c^i_{\mathrm{e}} = (0.01, 0.01, 0.01)$, $\alpha^i_{\bullet,t} = (0, 0, 0)$, $\beta^i_{\bullet,t} = (0.1, 0.5, 0.8)$, $\theta^i_{\bullet,t} = (1, 1, 1)$,
- $\omega_1 = 0.05$, $\omega_2 = 0.02$,
- $P_0 = 1$, $\sigma^i \overset{\text{i.i.d.}}{\sim} \mathcal{U}[0, \frac{1}{2}]$, $a^i \overset{\text{i.i.d.}}{\sim} \mathcal{U}[0, \frac{1}{2}]$, $b^i \overset{\text{i.i.d.}}{\sim} \mathcal{U}[1, 4]$, $\rho^i \overset{\text{i.i.d.}}{\sim} \mathcal{U}[-1, 1]$, $\Lambda^i = \frac{1}{\sqrt{i}}$, $\mathrm{AP}^i = 1$, $\lambda_{\mathrm{ref}} = 3\%$.
- Initial physical damage: $0.001\% \times \hat{V}^i_0$.

Impact of physical risks

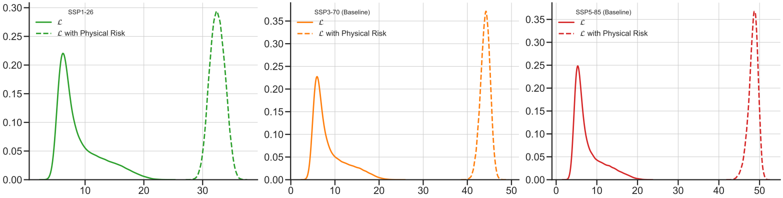


Figure: Portfolio losses with and without physical risk for the scenarios SSP1-26, SSP3-70, and SSP5-85.

⚠ As expected, incorporating a physical risk term significantly shifts the portfolio loss distribution upward.

Impact of Principal Component Analysis

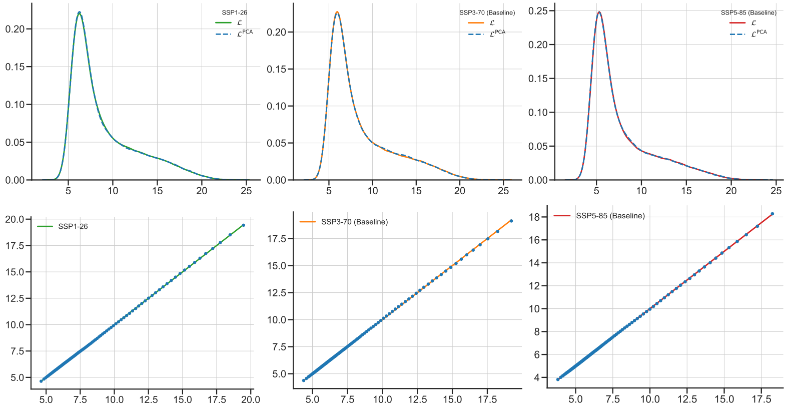


Figure: Portfolio loss $\mathcal{L}$ and two-factor-PCA-approximated loss $\mathcal{L}^{\mathrm{PCA}}$ for the scenarios SSP1-26, SSP3-70, and SSP5-85 (top figures) and associated Q-Q plots (bottom figures).

Impact of Polynomial Chaos Expansion

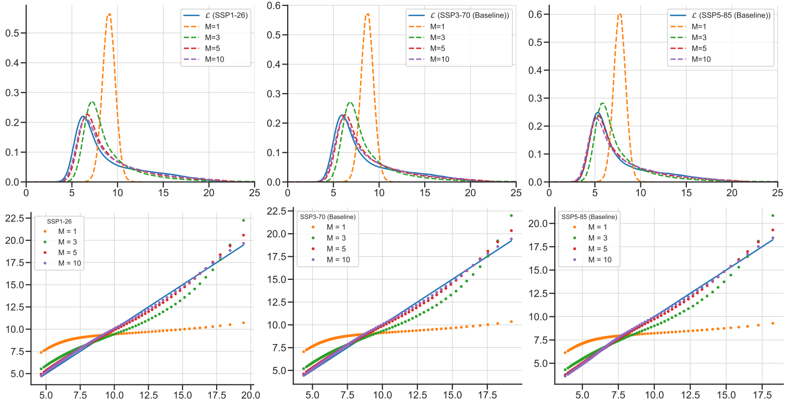


Figure: Comparison of $\mathcal{L}$ and $\mathcal{L}^{\mathrm{PCA,PCE,G}}$ (left figures) and associated Q-Q plots (right figures) with $M = 1, 3, 5, 10$ for the scenarios SSP1-26, SSP3-70, and SSP5-85.

Conclusion

- End-to-end methodology to assess the credit risk of a large portfolio of obligors
- Impact of transition and physical climate risks, given a SSP scenario
- Efficient computations using two steps-reduction techniques (Principal Component Analysis and Polynomial Chaos Expansion)
- Work-in-progress: dealing with real data

References I

Maximilian Auffhammer.
Quantifying economic damages from climate change.
Journal of Economic Perspectives, 32(4):33–52, 2018.

Basel Committee on Banking Supervision.
Climate-related risk drivers and their transmission channels.
Bank for International Settlements, d517, 2021.
https://www.bis.org/bcbs/publ/d517.htm.

Florian Bourgey, Emmanuel Gobet, and Ying Jiao.
Bridging socioeconomic pathways of CO2 emission and credit risk.
Annals of Operations Research, 336:1197–1218, 2024.

Florian Bourgey, Emmanuel Gobet, and Clément Rey.
Metamodel of a Large Credit Risk Portfolio in the Gaussian Copula Model.
SIAM Journal on Financial Mathematics, 11(4):1098–1136, 2020.

References II

Florian Bourgey, Emmanuel Gobet, and Clément Rey.
A Comparative Study of Polynomial-Type Chaos Expansions for Indicator Functions.
SIAM/ASA Journal on Uncertainty Quantification, 10(4):1350–1383, 2022.

Vincent Bouchet and Theo Le Guenedal.
Credit risk sensitivity to carbon price.
Available at SSRN 3574486, 2020.

Stefano Battiston, Antoine Mandel, Irene Monasterolo, and Alan Roncoroni.
Climate Credit Risk and Corporate Valuation.
Available at SSRN 4124002, 2023.
https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4124002.

M. Carney.
Speech: "breaking the tragedy of the horizon".
Bank of England, https://www.bankofengland.co.uk/speech/2015/breaking-the-tragedy-of-the-horizon-climate-change-and-financial-stabilit
September, 29th 2015.

References III

Josselin Garnier, Jean-Baptiste Gaudemet, and Anne Gruz.
The Climate Extended Risk Model (CERM).
Preprint arXiv:2103.03275, 2021.

Michael B. Gordy.
A risk-factor model foundation for ratings-based bank capital rules.
Journal of Financial Intermediation, 12(3):199–232, 2003.

M. J. Gidden, K. Riahi, S. J. Smith, S. Fujimori, G. Luderer, E. Kriegler, D. P. van Vuuren, M. van den Berg, L. Feng, D. Klein, K. Calvin, J. C. Doelman, S. Frank, O. Fricko, M. Harmsen, T. Hasegawa, P. Havlik, J. Hilaire, R. Hoesly, J. Horing, A. Popp, E. Stehfest, and K. Takahashi.
Global emissions pathways under different socioeconomic scenarios for use in cmip6: a dataset of harmonized emissions trajectories through the end of the century.
Geoscientific Model Development, 12(4):1443–1475, 2019.

References IV

Darío R Gómez, J Wattersson, BB Americano, Chia Ha, Gregg Marland, Emmanuel Matsika, L Namayanga, B Osman, J Saka, and Karen Treanton.
Stationary combustion.
Energy, 2006 IPCC Guidelines for National Greenhouse Gas Emissions Inventories, Intergovernmental Panel on Climate Change, Geneva, 2006.
https://www.ipcc-nggip.iges.or.jp/public/2006gl/pdf/2_Volume2/V2_2_Ch2_Stationary_Combustion.pdf.

Peter H Howard and Thomas Sterner.
Few and Not So Far Between: A Meta-analysis of Climate Damage Estimates.
Environmental and Resource Economics, 68(1):197–225, 2017.

International Energy Agency.
Emissions factors 2022.
IEA website, 2023.
https://www.iea.org/data-and-statistics/data-product/emissions-factors-2022.

References V

Pantelis Kalaitzidakis, Theofanis P. Mamuneas, and Thanasis Stengos.
Greenhouse emissions and productivity growth.
Journal of Risk and Financial Management, 11(3):38, Septembre 2018.

Théo Le Guenedal and Peter Tankov.
Corporate debt value under transition scenario uncertainty.
Mathematical Finance, july 2024.

Alexandra Lefevre and Agnès Tourin.
Incorporating Climate Risk into Credit Risk Modeling: An Application in Housing Finance.
FinTech, 2(3):614–640, 2023.

References VI

Meinshausen Malte, Zebedee R. J. Nicholls, Jared Lewis, Matthew J. Gidden, Elisabeth Vogel, Mandy Freund, Urs Beyerle, Claudia Gessner, Alexander Nauels, Nico Bauer, Josep G. Canadell, John S. Daniel, Andrew John, Paul B. Krummel, Gunnar Luderer, Nicolai Meinshausen, Stephen A. Montzka, Peter J. Rayner, Stefan Reimann, Steven J. Smith, Marten van den Berg, Guus J. M. Velders, Martin K. Vollmer, and Ray H. J. Wang.
The shared socio-economic pathway (SSP) greenhouse gas concentrations and their extensions to 2500.
Geoscientific Model Development, 13:3571–3605, 2020.

Pierre Monnin.
Integrating climate risks into credit risk assessment-current methodologies and the case of central banks corporate bond purchases.
Council on Economic Policies, Discussion Note, 4, 2018.

William D Nordhaus.
An optimal transition path for controlling greenhouse gases.
Science, 258(5086):1315–1319, 1992.

References VII

William Nordhaus and Paul Sztorc.
DICE 2013R Introduction and user's manual.
Yale University and the National Bureau of Economic Research, USA, 2013.
http://acdc2007.free.fr/dicemanual2013.pdf.

Thierry Roncalli.
Handbook of Financial Risk Management.
CRC Press, 2020.

Joeri Rogelj, Alexander Popp, Katherine V. Calvin, Gunnar Luderer, Johannes Emmerling, David Gernaat, Shinichiro Fujimori, Jessica Strefler, Tomoko Hasegawa, Giacomo Marangoni, Volker Krey, Elmar Kriegler, Keywan Riahi, Detlef P. van Vuuren, Jonathan Doelman, Laurent Drouet, Jae Edmonds, Oliver Fricko, Mathijs Harmsen, Petr Havlík, Florian Humpenöder, Elke Stehfest, and Massimo Tavoni.
Scenarios towards limiting global mean temperature increase below 1.5 ^{o}c.
Nature Climate Change, 8(4):325–332, mar 2018.

References VIII

Keywan Riahi, Detlef P. van Vuuren, Elmar Kriegler, Jae Edmonds, Brian C. O'Neill, Shinichiro Fujimori, Nico Bauer, Katherine Calvin, Rob Dellink, Oliver Fricko, Wolfgang Lutz, Alexander Popp, Jesus Crespo Cuaresma, Samir KC, Marian Leimbach, Leiwen Jiang, Tom Kram, Shilpa Rao, Johannes Emmerling, Kristie Ebi, Tomoko Hasegawa, Petr Havlik, Florian Humpenöder, Lara Aleluia Da Silva, Steve Smith, Elke Stehfest, Valentina Bosetti, Jiyong Eom, David Gernaat, Toshihiko Masui, Joeri Rogelj, Jessica Strefler, Laurent Drouet, Volker Krey, Gunnar Luderer, Mathijs Harmsen, Kiyoshi Takahashi, Lavinia Baumstark, Jonathan C. Doelman, Mikiko Kainuma, Zbigniew Klimont, Giacomo Marangoni, Hermann Lotze-Campen, Michael Obersteiner, Andrzej Tabeau, and Massimo Tavoni.
The Shared Socioeconomic Pathways and their energy, land use, and greenhouse gas emissions implications: An overview.
Global Environmental Change, 42:153–168, jan 2017.

Martin L Weitzman.
GHG Targets as Insurance Against Catastrophic Climate Damages.
Journal of Public Economic Theory, 14(2):221–244, 2012.