The equilibrium price of bubble assets

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17/10/2025, FiME seminar.
Joint work with J.-M. Lasry and P.-L. Lions

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- Such assets are called bubble assets.
- Hence, a value on money requires a form of consensus, that is of equilibrium.
- Can we propose mathematical models for such equilibria ?

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- Money is an extremely stable example of a bubble : not all national currencies collapse (even they might sometimes, namely because of inflation)
- The housing price in Paris is typically a bubble, especially in neighborhoods like where we are.
- Recently, cryptocurrencies have proven to be also quite stable, despite being purely bubble assets.

Why a model ?

- The surprising (or not) stability of some bubbles hints that maybe a strong phenomenon maintains bubbles.
- This stability is also a good sign for a mathematical model.

Bibliography

- Pioneering work of Samuelson on the value/need of money
- Several works have followed this overlapping generations model : Scheinkman, Tirole,...
- Recent work and survey of A. Toda
- Our modeling is inspired from Biais, Rochet and Villeneuve.

Model and derivation of the main equation

Model setup

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- The two goods have some returns $r_0(x)$ and $r_1(x)$. That is, if I hold a quantity λ_t of the bubble good at time t, that I do not buy or sell, then

$$d\lambda_t = (1 + r_1(X_t))\lambda_t dt.$$

- $r_0, r_1 : [0,1] \to \mathbb{R}$, not necessary non-negative.

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Proposition

The maximization problem of the growth rate of the portfolio of an agent with wealth q, when the economy is in state $x \in (0,1)$, is

$$\max_{\theta \in \mathbb{R}} \left\{ qU'(q)\Big(+\theta(u(x))^{-1}(u'(x)\cdot b(x) + \frac{\sigma^2}{2}u''(x)) + r_0(x)(1-\theta) + r_1(x)\theta + \frac{U''(q)q}{U'(q)}\frac{\theta^2\sigma^2}{2u^2(x)}|u'(x)|^2 \Big) \right\}.$$

Proof of the problem of the agent

- Consider the wealth of an agent who choose θ as its repartition of wealth at time, evaluated at time s

$$Y_{t,s} = q(1-\theta)e^{\int_t^s r_{\mathbf{0}}(X_{t'})dt'} + \frac{\theta q}{p_t} p_s e^{\int_t^s r_{\mathbf{1}}(X_{t'})dt'}.$$

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- By using Ito's Lemma 3 times, we arrive at

$$\mathbb{E}[U(Y_{t,t+h})|X_t] = U(q)+$$
$$\mathbb{E}\Bigg[\int_t^{t+h} U'(Y_{t,s})\, q\Bigg(r_0(X_s)(1-\theta)e^{\int_t^s r_0(X_{t'})dt'} + r_1(X_s)\theta\frac{u(X_s)}{u(X_t)}e^{\int_t^s r_1(X_{t'})dt'} +$$
$$+\frac{\theta}{u(X_t)}e^{\int_t^s r_1(X_{t'})dt'}(u'(X_s)\cdot b(X_s) + \frac{\sigma^2}{2}u''(X_s))$$
$$+\frac{U''(Y_{t,s})q}{U'(Y_{t,s})}\frac{\theta^2\sigma^2}{2u^2(X_t)}e^{2\int_t^s r_1(X_{t'})dt'}|u'(X_s)|^2\Bigg)ds|X_t\Bigg].$$

Demand of the agents and market clearing

- Choose $U(q) = q^{1-\gamma}$. Hence, the demand of the agents is expressed as

$$\theta^* = u(X_t)\frac{u(X_t)(r_1(X_t) - r_0(X_t)) + u'(X_t)\cdot b(X_t) + \frac{\sigma^2}{2}u''(X_t)}{\gamma\sigma^2|u'(X_t)|^2}.$$

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- Denote Q_t the total wealth (in numéraire) and K_t the total supply (in bubble asset). Demand = Supply implies

$$\left(u(X_t)(r_1(X_t) - r_0(X_t)) + u'(X_t)\cdot b(X_t) + \frac{\sigma^2}{2}u''(X_t)\right)Q_t = K_t\gamma\sigma^2|u'(X_t)|^2.$$

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- If we assume that Q_t and K_t are constant, we find

$$u(x)(r_1(x) - r_0(x)) + u'(x)\cdot b(x) + \frac{\sigma^2}{2}u''(x) = \frac{\gamma K\sigma^2}{Q}|u'(x)|^2 \text{ in } (0,1).$$

Summary of the modelling

- We end with an equation to characterize the price $p_t = u(X_t)$ of the bubble good (expressed in numéraire).

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- The bubble yields nothing, $u = 0$ is solution, thank you for coming

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- We end with an equation to characterize the price $p_t = u(X_t)$ of the bubble good (expressed in numéraire).
- If $U(q) = 1 - e^{-cq}$

$$-u'(x) \cdot b(x) - \frac{\sigma^2}{2} u''(x) + \frac{cK\sigma^2}{N} |u'(x)|^2 + u(x)(r_0(x) - r_1(x)) = 0 \quad \text{in } (0,1).$$

- Same sort of computations

Mathematical analysis of the equation

Statement of the problem

- Understand existence and uniqueness of the equation

$$
\begin{aligned}
-\nu\Delta u + \frac{1}{2}|\nabla_x u|^2 &= a(x)u \text{ in } \Omega,\\
u(x) &> 0 \text{ in } \Omega,\\
\partial_n u &= 0 \text{ on } \partial\Omega.
\end{aligned}
$$

- Ω is a smooth domain, ν a constant.
- The assumptions are on a.

Main result

- The operator $\phi \to -\nu\Delta\phi - a(x)\phi$ is a well defined self-adjoint operator with compact inverse.

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Theorem

Assume that

- $\lambda_1(-\nu\Delta - a) < 0,$
- $\min_\Omega a < 0.$

Then, there exists a unique smooth solution.

Existence

Lemma

Assume that the function a takes both positive and negative values. Then, there exists a constant $C > 0$ depending only on the function a and Ω such that, for any smooth non-negative function u satisfying

$$-\nu\Delta u + \frac{1}{2}|\nabla_x u|^2 \leq a(x)u \text{ in } \Omega,$$
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we have $u \leq C$.

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Proof is simply that we have

$$\int_\Omega |\nabla u|^2 \leq 2 \max_\Omega a \int_\Omega u \quad + \text{ standard stuff.}$$

Uniqueness

Lemma

Assume that the function a takes both positive and negative values. Consider $u >> 0$ and $v >> 0$ two smooth functions on Ω. Assume that

$$-\nu\Delta u + \frac{1}{2}|\nabla_x u|^2 \leq a(x)u \text{ in } \Omega,$$

$$-\nu\Delta v + \frac{1}{2}|\nabla_x v|^2 \geq a(x)v \text{ in } \Omega.$$

Then, $u \leq v$ and either $u = v$ or $u < v$.

Proof of comparison

- The argument follows from Laetsch (1975).

$$\theta = \max\{\tilde{\theta} \in [0,1], \forall x \in \Omega, \tilde{\theta} u(x) \leq v(x)\}.$$

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- $u_\theta := \theta u$ is a solution of

$$-\nu \Delta u_\theta + \frac{1}{2}|\nabla_x u_\theta|^2 + \left(\frac{1}{2\theta} - \frac{1}{2}\right)|\nabla_x u_\theta|^2 \leq a(x) u_\theta \text{ in } \Omega.$$

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- $w = v - u_\theta$ is a solution of

$$-\nu \Delta w + \frac{1}{2}\left(|\nabla_x v|^2 - |\nabla_x u_\theta|^2\right) \geq a(x) w \text{ in } \Omega.$$

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- Both $w \geq 0$ and for some B

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- This implies that either $w > 0$ or $w = 0$.

The case of a source term

$$\begin{aligned} -\nu\Delta u + \frac{1}{2}|\nabla_x u|^2 = a(x)u + f \text{ in } \Omega, \\ u >> 0, \\ \partial_n u = 0 \text{ on } \partial\Omega, \end{aligned} \quad (1)$$

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Theorem

Under the assumptions of the first theorem, there exists a unique smooth solution of (1)*.*

Corollary

If we denote by $u(f)$ *the solution of* (1) *for a smooth* $f \geq 0$*, then* u *is increasing with respect to* f*.*

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$$v_\epsilon(x) := \inf_{(\alpha_t)_{t\geq 0}} \mathbb{E}\left[\int_0^{+\infty} e^{\int_0^t a(X_s^{x,\alpha})ds}\left(\frac{1}{2}|\alpha_t|^2 + \epsilon\right) dt\right],$$

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Proposition

$$\lim_{\epsilon\to 0^+} \|v_\epsilon - u\|_\infty = 0.$$

Other properties of the solution

Proposition (Stability)

Let (a_n) be a sequence converging to a such that the assumptions of the main Theorem are uniformly satisfied. Then $\|u_n - u\| \to 0$ as $n \to \infty$.

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Proposition

Let u_ϵ be the solution of

$$\begin{aligned} -\nu\Delta u_\epsilon + \epsilon|\nabla_x u_\epsilon|^2 = a(x)u_\epsilon \text{ in } \Omega, \\ u_\epsilon(x) > 0 \text{ in } \Omega, \\ \partial_n u_\epsilon = 0 \text{ on } \partial\Omega. \end{aligned}$$

Then $\epsilon \to u_\epsilon$ is decreasing.

Economic interpretation of our results

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- Agents have CARA utility with parameter c, K the total mass of crypto, N the number of agents, $\nu = \sigma^2/2$. Note $\epsilon = c2K\nu/N$.

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\begin{aligned}
&-\nu u'' + \epsilon(u')^2 = (r_1 - r_0)(x)u \quad \text{in } (0,1),\\
&u' = 0 \text{ at } 0 \text{ and } 1,\\
&u \geq 0.
\end{aligned}
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Consequences

Stylized fact (number 1)

If the government can be bad enough, then a C-value u exists.

Indeed, we have $r_0(0) \geq 0$. For ant $\delta > 0$, there exists $\kappa > 0$ such that the assumptions are satisfied if $r_0(x) \leq -\kappa$ for $x > 1 - \delta$.

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Stylized fact (number 2)

If it exists, the C-value u increases with x.

We assumed that r_0 is non-increasing, so $r_1 - r_0$ is non-decreasing, and thus so is u.

Consequences 2

Stylized fact (number 3)

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This is the monotonicity of u with respect to the parameter ϵ.

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Stylized fact (number 4)

The C-value u can collapse with change of r_0 or r_1 but not of N.

The only threshold in the assumptions are in terms of ν or r_0 and r_1. So the value of N does not change the fact that they are satisfied. This means that even with very few actors, it is reasonable to form a consensus on the bubble.

Real estate model

- $x \in (0, 1)$ is the proportion of the government to generate inflation.
- The return of real estate is supposed negative, some works have to be made.
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Consequence

Stylized fact

Even with a strong limit on the rent, the price of real estate can still be rationally arbitrary large in certain prestigious locations, i.e. it can contain a C-value component which can be arbitrary large compared to f.

See picture.

Link with no-arbitrage

- What happens when arbitrageurs are present ?
- They will be similar agents, but with no risk aversion, or a smaller one.
- Hence, they lower the value of ϵ in

$$
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u' &= 0 \text{ at } 0 \text{ and } 1,\\
u &\geq 0.
\end{aligned}
$$

- In the limit $\epsilon \to 0$, u explodes, but in the limit case $\epsilon = 0$, generically only 0 is the solution.

Final comment

- Have bubbles disappeared ?
- No !
- We were only interested in prices of the form $p_t = u(X_t)$.
- Explosive bubbles are typically not of this form...

Thank you for your coming

Thank you for your attention