Buy it, Store it, Sell it: On the Optimality Gap of the Rolling Intrinsic Strategy

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Introduction: BESS and Intraday Electricity Markets

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View Market Dynamics

Intraday Electricity Markets: Modelling Forward Curves

Stylized Facts

- Intraday pattern of futures price curves.
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Not my topic today.

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In this Talk

- Analysis of the classical heuristic strategy: rolling intrinsic strategy.
- Mathematical framework for optimisation.
- Examples in the case of (correlated) binomial trees.

Market Framework: Physical Delivery & Trading

The Asset and Contracts

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Price Matrix View

$$\begin{pmatrix} X_{0,0} & \cdots & X_{0,N} \\ & \ddots & \vdots \\ & & X_{N,N} \end{pmatrix}$$

- **Rows**: Observation time (n).
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- **Innovations**: $\epsilon_{n,m} := X_{n,m} - X_{n-1,m}$.
- Often a martingale assumption.
- The dependence structure of $(\epsilon_{n,m})_{m \geq n}$ for fixed n is important.

The Rolling Intrinsic Strategy

- **Context**: A benchmark strategy for storage valuation (common in Natural Gas and BESS).
- **The Goal**: Buy one unit of the asset, store it, and sell it later at a profit.
- **Intrinsic Value**: The maximum spread between two futures contracts available *now*:

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- **The "Rolling" Logic**:
  - Myopically locks in the best spread.
  - **Dynamic**: Re-computes the best spread as new info arrives.
  - **Execution**: Swaps existing positions for better ones if the gain exceeds transaction costs.

Step-by-Step: The Sell-Only Strategy

Suppose the agent already holds the asset at $n = 0$:

1. Initial Position ($n = 0$)

Sell the contract with highest price: $m_0 \in \arg\max_{0 \leq k \leq N} X_{0,k}$.

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2. Iterative Update at time n

If $X_{n,m_{n-1}} < \max_{n \le k \le N} X_{n,k} - 2\psi$:

- **Action**: Buy back m_{n-1}, sell new $m_n \in \arg\max_{n \le k \le N} X_{n,k}$.
- **PnL Gain**: $\Delta \text{PnL}_n = X_{n,m_n} - X_{n,m_{n-1}} - 2\psi > 0$.

Otherwise, keep the current position ($m_n = m_{n-1}$).

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- **Termination**: The strategy stops at the first time the locked maturity is current: $\tau = \min\{n \in \mathcal{N} \mid m_n = n\}$.

Decision Variables: Buy and Sell Maturities

At each time n, the strategy is defined by the pair of maturities $(l_n, m_n) \in \overline{\mathcal{N}}^2$:

Definitions

- l_n: Maturity of the **buy** contract (entry leg).
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l_n and m_n are updated with the same logic of myopically increasing the PnL.

Key Stopping Times

The strategy is governed by three critical dates:

Market Entry (τ_i) First time a profitable spread is locked:

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Temporal Hierarchy

On the set $\{\tau_i < +\infty\}$, the sequence is strictly ordered:

$$\tau_i \leq \tau_b < \tau_s \leq N$$

Mathematical Results: PnL Formulation

Proposition

The terminal PnL of the Rolling Intrinsic strategy is:

$$PnL_N = \mathbb{1}_{\tau_i < +\infty} \left(X_{\tau_s,\tau_s} - X_{\tau_b,\tau_b} - \sum_{\tau_i < n \leq \tau_s} \epsilon_{n,m_{n-1}} + \sum_{\tau_i < n \leq \tau_b} \epsilon_{n,l_{n-1}} \right) - \sum_{n \in \mathcal{N}} \Psi(m_{n-1}, m_n) - \sum_{n \in \mathcal{N}} \Psi(l_{n-1}, l_n)$$

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- **Realized Spread**: $X_{\tau_s,\tau_s} - X_{\tau_b,\tau_b}$.
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Corollary

If price processes are **martingales** then, in the absence of transaction costs:

$$\mathbb{E}[\mathrm{PnL}_N] = \mathbb{E}\left[\mathbb{1}_{\tau_i<+\infty}\left(X_{\tau_s,\tau_s} - X_{\tau_b,\tau_b}\right)\right]$$

The Rolling Intrinsic Strategy: Example and Remarks

Open Strategy Illustration (PDF)

Conceptual Advantages

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Research Motivation

- How efficient is this heuristic?
- What about risk aversion?
- What is the impact of **model risk** on more complex, model-dependent strategies?

Optimal Control: States and Admissibility

- **Objective**: Maximize expected utility of terminal PnL: $\mathbb{E}[-\exp(-\gamma \mathrm{PnL}_N)]$.
- **State Variables**: PnL_n, $x_{n,m}$ for $m \geq n$, buy/sell maturities $(l_n, m_n) \in \overline{\mathcal{N}}^2$ (+ constraints).

Admissibility Constraints

To be physically consistent, actions must satisfy:

Post-Cycle If $m_{n-1} < n$: Strategy is over (positions are fixed).

Storage If $l_{n-1} < n \leq m_{n-1}$: Asset is held. Only m_n can be updated (must sell by N).

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Pre-Buy If asset not yet bought:

- $n \leq l_n < N$: Must buy before the last period.
- $max(n, l_n) < m_n \leq N$: Cannot deliver without purchasing first.

Markovian Framework and Ansätze

We assume price innovations ϵ_n are mutually independent.

Value Function $U_n(p, x_n, l, m)$

Represents the maximal expected utility given current PnL p, prices $x_n = (x_{n,n}, \ldots, x_{n,N})$, and positions (l, m).

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The Four Specific Ansätze

To solve the problem, we decompose U_n based on the agent's position:

$$U_n = -\exp\left(-\gamma(p + \text{Value Component})\right)$$

- $u_n(x_n)$: No positions held ($l = m = +\infty$).
- $v_n^l(x_n)$: Long position only (l finite, $m = +\infty$).
- $w_n^m(x_n)$: Short position only ($l = +\infty$, m finite).
- $z_n^{l,m}(x_n)$: Both positions held (Full buy-sell cycle).

Dynamic Programming

To simplify the Bellman equations, we define $\mathcal{T}_\gamma$:

$$\mathcal{T}_\gamma[f](x_n) := -\frac{1}{\gamma}\log\left(\mathbb{E}\left[\exp\left(-\gamma f(\widehat{x}_{n+1}(x_n,\epsilon_{n+1}))\right)\right]\right)$$

- This operator computes the **Certainty Equivalent** of the future value.
- If $\gamma \to 0$, $\mathcal{T}_\gamma[f] \to \mathbb{E}[f]$ (Risk-neutral case).

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State Space Transitions

At each step n, the agent chooses an action to transition between:

- Empty position.
- Long position (maturity l).
- Short position (maturity m).
- Full buy-store-sell cycle (maturities l, m).

Bellman Equations

- **No position (u_n)**: $u_n = \max\left\{\mathcal{T}_\gamma[u_{n+1}], \max_{l\geq n}\{-x_{n,l} - \psi + \mathcal{T}_\gamma[v^l_{n+1}]\}, \max_{m>n}\{x_{n,m} - \psi + \mathcal{T}_\gamma[w^m_{n+1}]\}, \ldots\right\}$
- **Long position (v^l_n)**: $v^l_n = \begin{cases} \max\left\{\mathcal{T}_\gamma[v^l_{n+1}], \max_{m\geq n}\{x_{n,m} - \psi + \mathcal{T}_\gamma[z^{l,m}_{n+1}]\}\right\} & l < n \\ \max\left\{\mathcal{T}_\gamma[v^l_{n+1}], \max_{m>l}\{x_{n,m} - \psi + \mathcal{T}_\gamma[z^{l,m}_{n+1}]\}, x_{n,l} - \psi + u_n\right\} & l \geq n \end{cases}$
- **Short position (w^m_n)**:
  $$w^m_n = \begin{cases} -x_{n,n} - \psi + u_n & m = n \\ \max\left\{\mathcal{T}_\gamma[w^m_{n+1}], \max_{m>l\geq n}\{-x_{n,l} - \psi + \mathcal{T}_\gamma[z^{l,m}_{n+1}]\}, -x_{n,m} - \psi + u_n\right\} & m > n \end{cases}$$
- **Full cycle ($z^{l,m}_n$)**: $z^{l,m}_n = \begin{cases} 0 & l < m < n \\ \max\left\{\mathcal{T}_\gamma[z^{l,m}_{n+1}], -x_{n,m} - \psi + v^l_n\right\} & l < n \leq m \\ \max\left\{\mathcal{T}_\gamma[z^{l,m}_{n+1}], x_{n,l} - \psi + w^m_n, -x_{n,m} - \psi + v^l_n\right\} & n \leq l < m \end{cases}$

Example of Optimal Strategies

Binomial Model (with correlation)

Risk-Neutral ($\gamma = 0$)

Risk-Averse ($\gamma = 1$)

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Risk-Neutral ($\gamma = 0$)

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Key Observation

Very different strategies from the Rolling Intrinsic one:

- Spot-only strategies when $\gamma = 0$.
- Open positions in both cases.

PnL Distributions

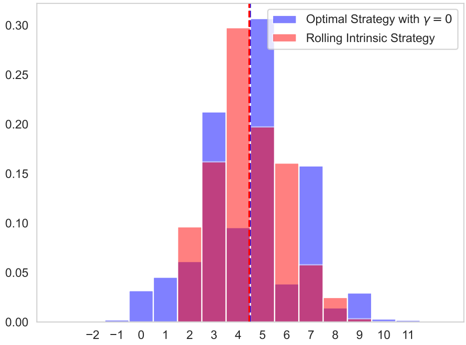

Figure: $\gamma = 0$

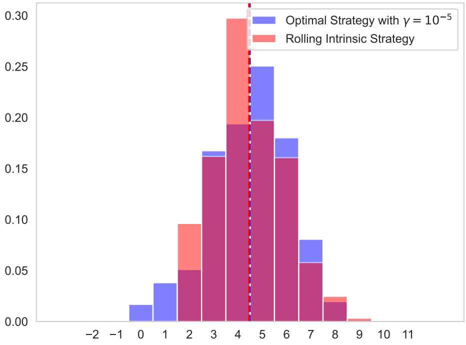

Figure: $\gamma = 10^{-5}$

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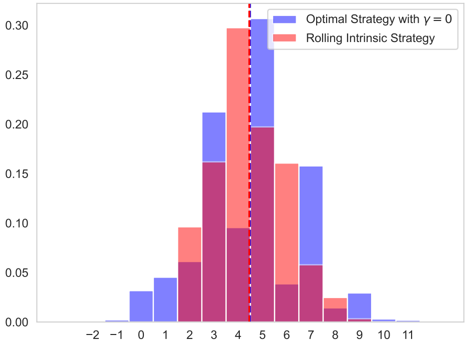

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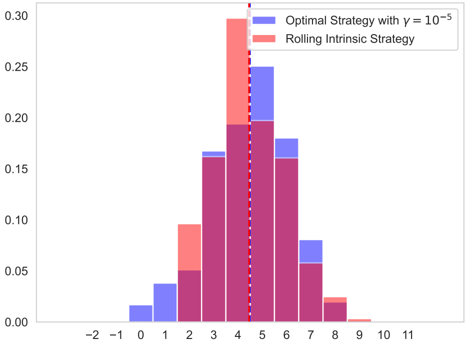

Figure: $\gamma = 10^{-5}$

PnL Distributions

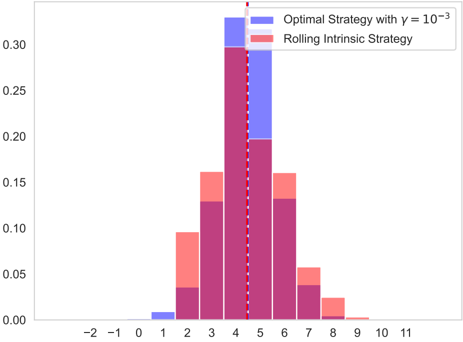

Figure: $\gamma = 10^{-3}$

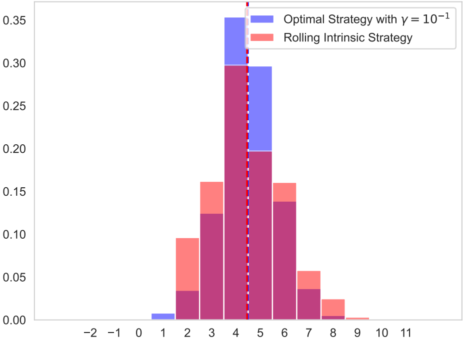

Figure: $\gamma = 10^{-1}$

PnL Distributions

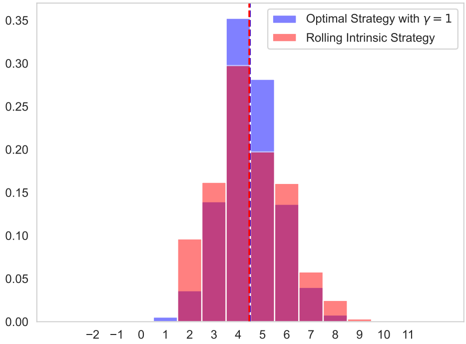

Figure: $\gamma = 1$

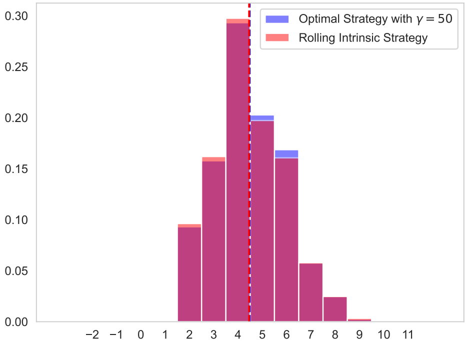

Figure: $\gamma = 50$

Analysis of the Optimality Gap

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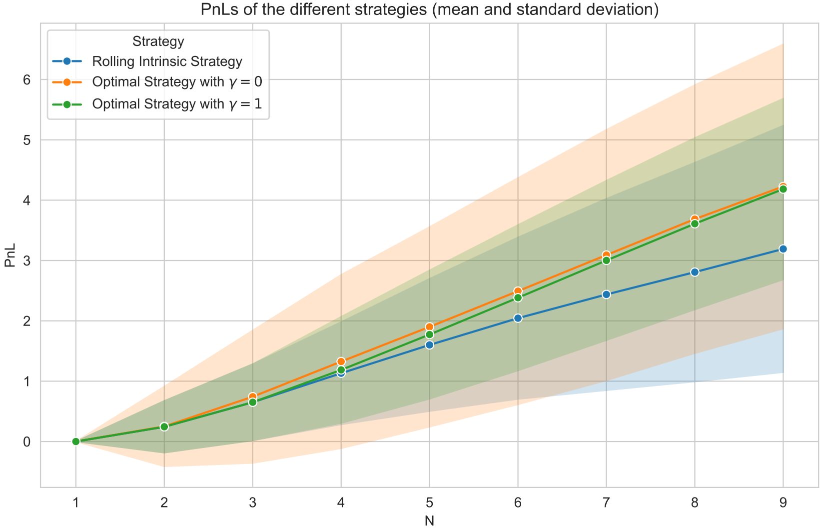
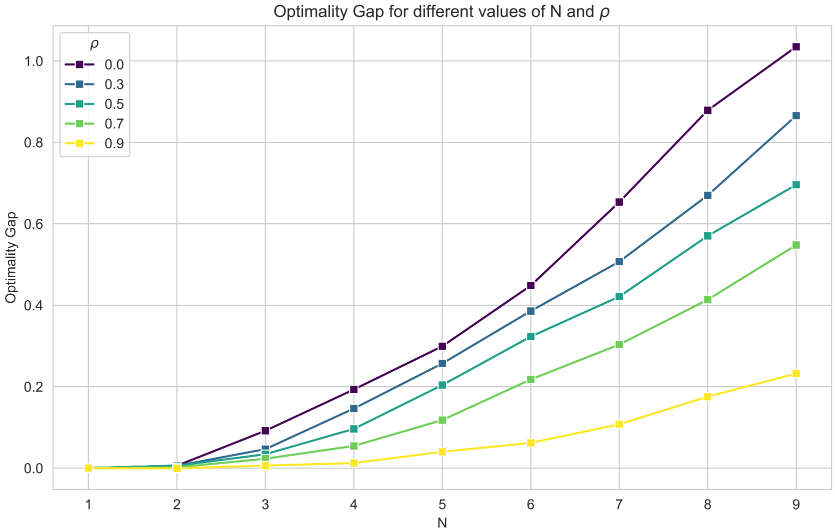
Let us show some empirical results with a flat initial price curve.

Dependence on N



Dependence on N and ρ



Misspecification of ρ